Rosenbrock-like Problems: SMF Versus Other SBO Implementations

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Outline

space mapping surrogate

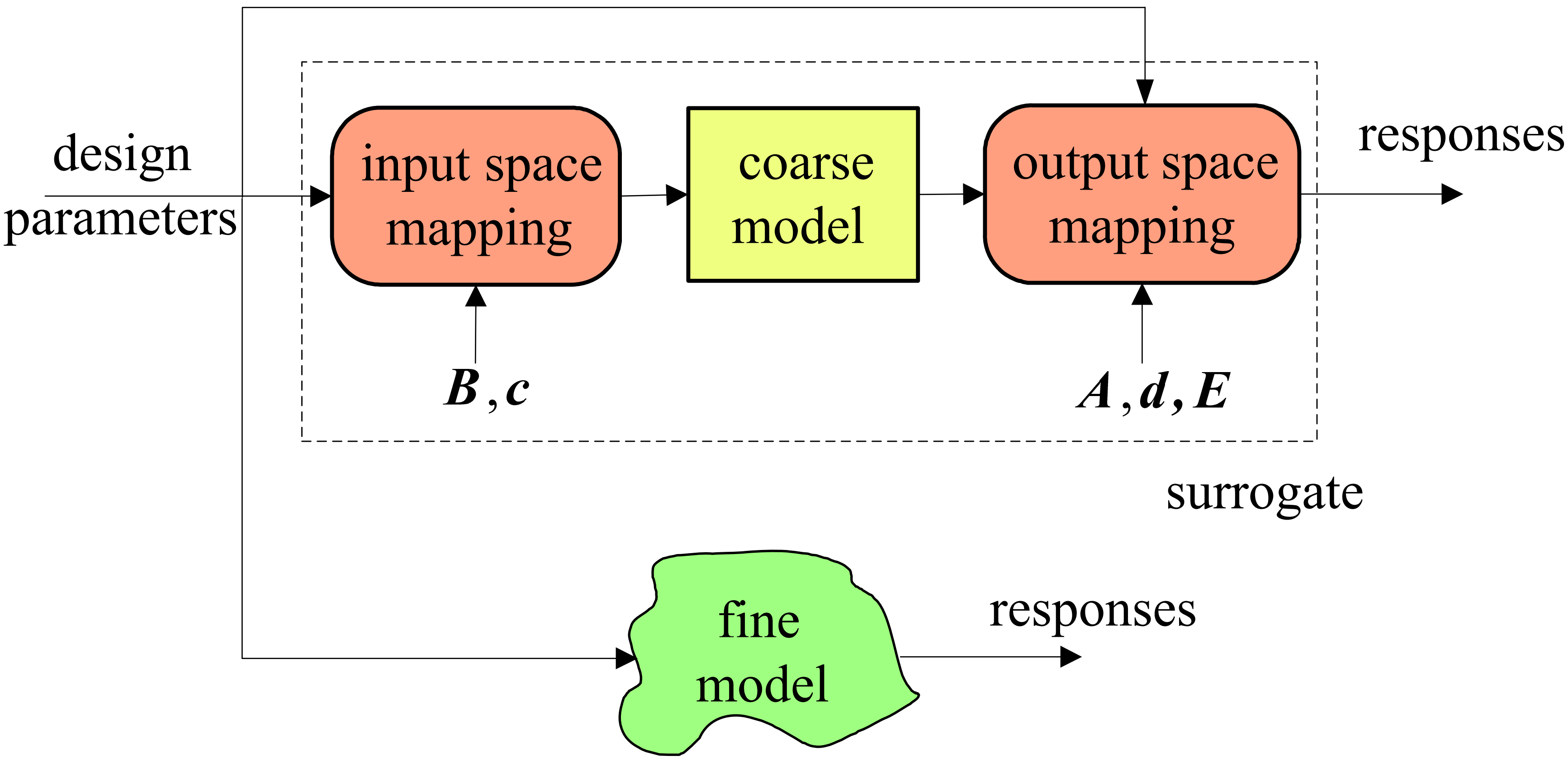
Rosenbrock function: the benchmark

our Rosenbrock test examples

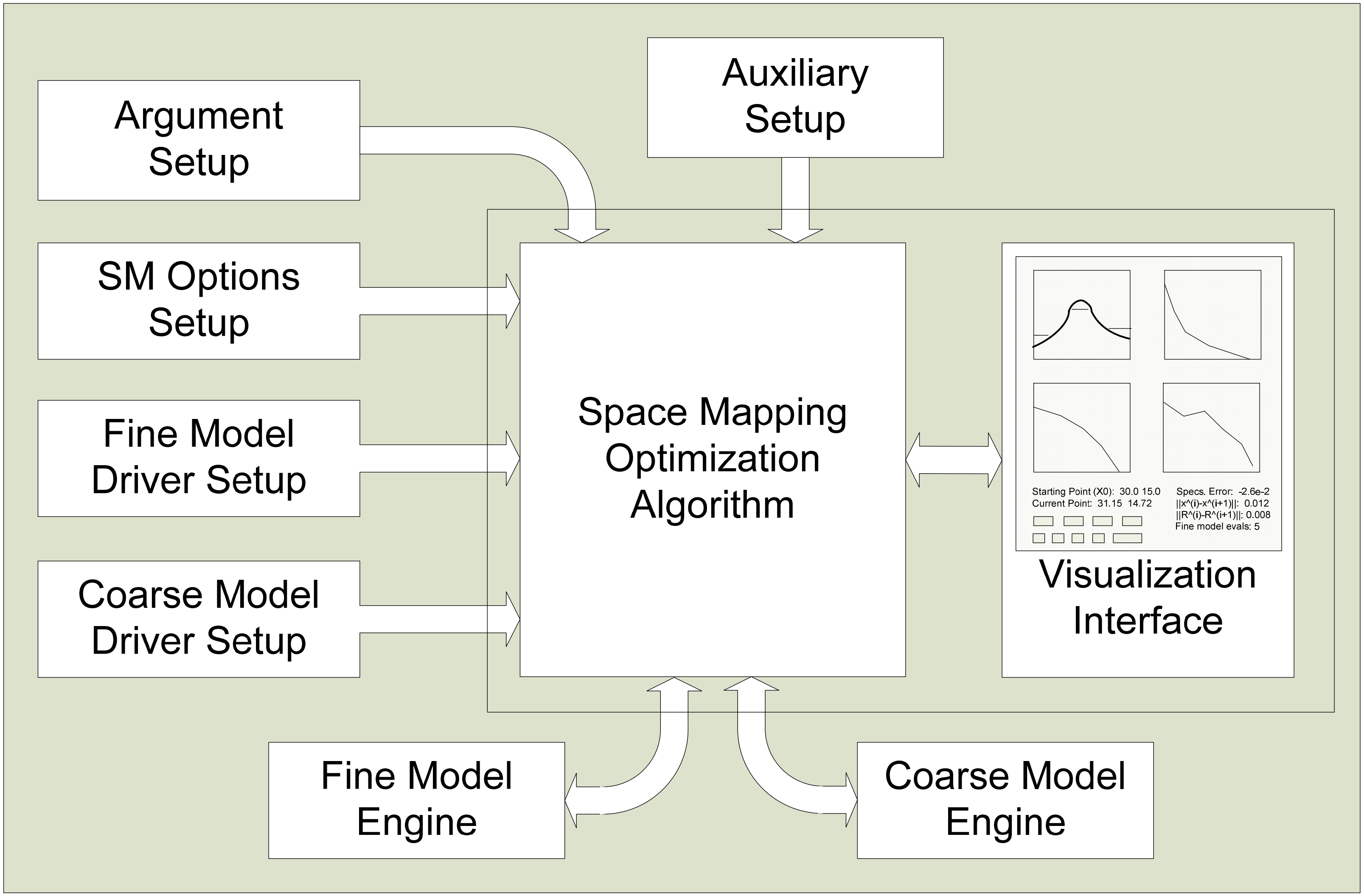
SMF and other SBO implementations comparison

conclusions

A Space-Mapping-based Surrogate



SMF: Optimization Flowchart



Generalized Space Mapping (GSM) Framework

(*Koziel, Bandler, and Madsen, 2006*)

at iteration i, a surrogate model $\boldsymbol{R}_s^{(i)} : X \to R^m$ used by the GSM optimization algorithm is defined as

$$\boldsymbol{R}_s^{(i)}(\boldsymbol{x}) = \boldsymbol{A}^{(i)} \cdot \boldsymbol{R}_c(\boldsymbol{B}^{(i)} \cdot \boldsymbol{x} + \boldsymbol{c}^{(i)}) + \boldsymbol{d}^{(i)} + \boldsymbol{E}^{(i)} \cdot (\boldsymbol{x} - \boldsymbol{x}^{(i)})$$

where

$$(\boldsymbol{A}^{(i)}, \boldsymbol{B}^{(i)}, \boldsymbol{c}^{(i)}) = \arg \min_{(\boldsymbol{A},\boldsymbol{B},\boldsymbol{c})} \Big\{ \sum_{k=0}^{i} w_k \| \boldsymbol{R}_f(\boldsymbol{x}^{(k)}) - \boldsymbol{A} \cdot \boldsymbol{R}_c(\boldsymbol{B} \cdot \boldsymbol{x}^{(k)} + \boldsymbol{c}) \| + $$
$$+ \sum_{k=0}^{i} v_k \| \boldsymbol{J}_{\boldsymbol{R}_f}(\boldsymbol{x}^{(k)}) - \boldsymbol{A} \cdot \boldsymbol{J}_{\boldsymbol{R}_c}(\boldsymbol{B} \cdot \boldsymbol{x}^{(k)} + \boldsymbol{c}) \cdot \boldsymbol{B} \| \Big\}$$

$$\boldsymbol{E}^{(i)} = \boldsymbol{J}_{\boldsymbol{R}_f}(\boldsymbol{x}^{(i)}) - \boldsymbol{A}^{(i)} \cdot \boldsymbol{J}_{\boldsymbol{R}_c}(\boldsymbol{B}^{(i)} \cdot \boldsymbol{x}^{(i)} + \boldsymbol{c}^{(i)}) \cdot \boldsymbol{B}^{(i)}$$

$$\boldsymbol{d}^{(i)} = \boldsymbol{R}_f(\boldsymbol{x}^{(i)}) - \boldsymbol{A}^{(i)} \cdot \boldsymbol{R}_c(\boldsymbol{B}^{(i)} \cdot \boldsymbol{x}^{(i)} + \boldsymbol{c}^{(i)})$$

Rosenbrock Banana Function

Rosenbrock, 1960

Fletcher, *Practical Methods of Optimization*, 1987

Bakr, Bandler, Georgieva, and K. Madsen, 1999

Bandler, Mohamed, Bakr, Madsen, and Søndergaard, 2002

Søndergaard, 2003

Bandler, Cheng, Dakroury, Mohamed, Bakr, Madsen, and Søndergaard, 2004

Giunta and Eldred, 2000; Eldred, Giunta, and Collis, 2004

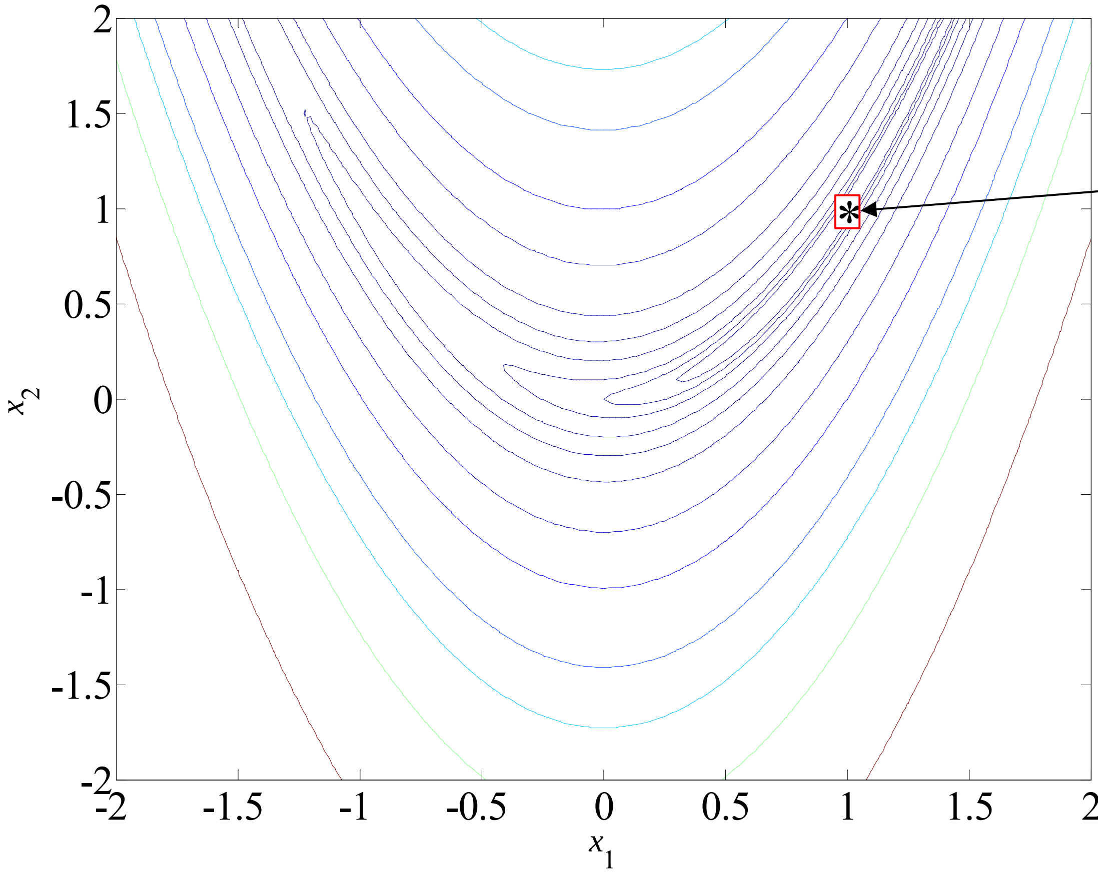
Robinson, Eldred, Willcox, and Haimes, 2006

Original Rosenbrock Function (Coarse Model)

(*Bandler et al., 1999, 2002*)

$$R_c(\boldsymbol{x}_c) = 100(x_2 - x_1^2)^2 + (1 - x_1)^2$$

$$\text{where } \boldsymbol{x}_c = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \text{ and } \boldsymbol{x}_c^* = \begin{bmatrix} 1.0 \\ 1.0 \end{bmatrix}$$



$$\boldsymbol{x}_c^* = \begin{bmatrix} 1.0 \\ 1.0 \end{bmatrix}$$

$$R_c(\boldsymbol{x}_c^*) = 0$$

Transformed Rosenbrock Function (Fine Model)
(*Bandler et al., 2002*)

parameter transformation of the original Rosenbrock function

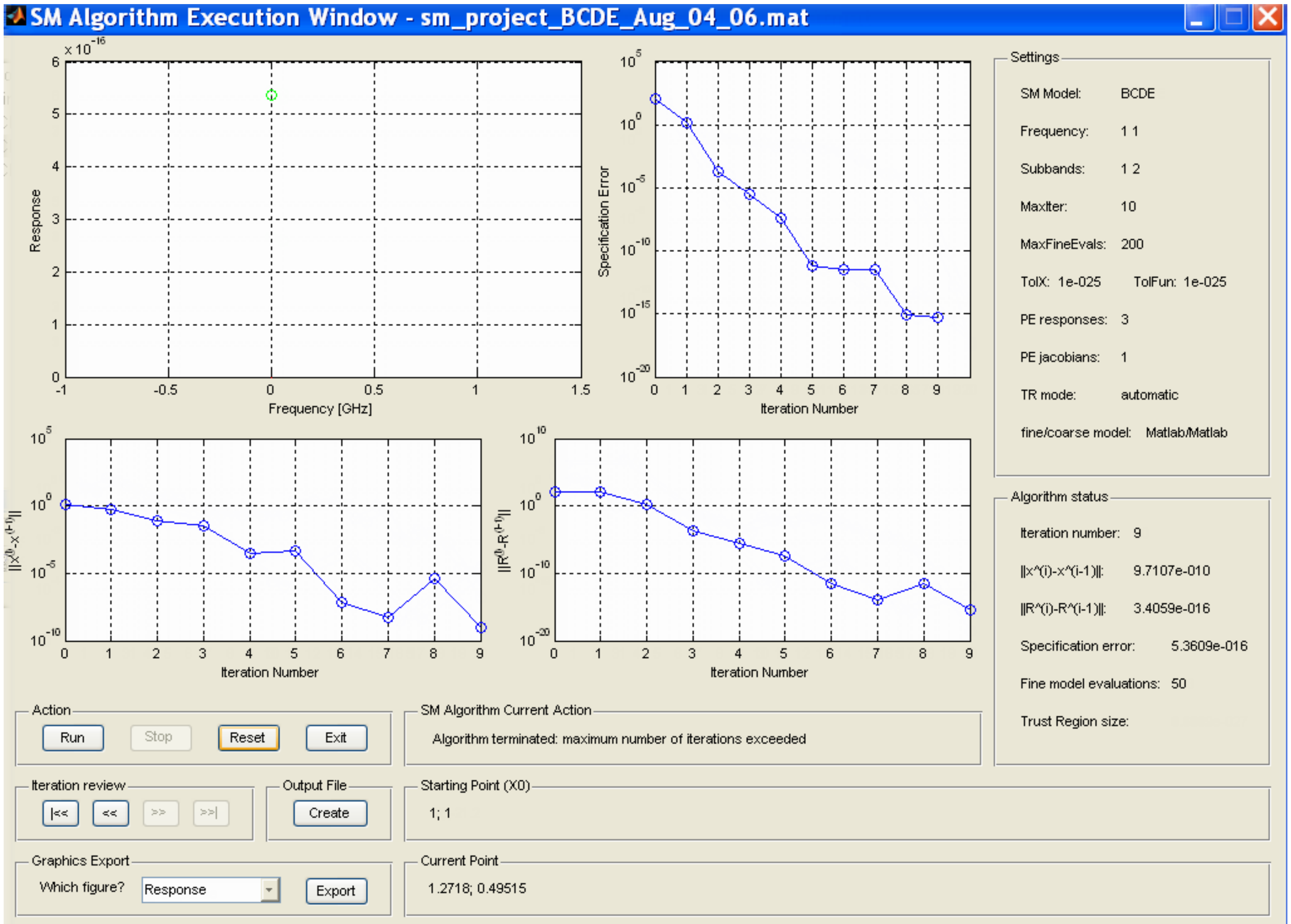
$$R_f(\boldsymbol{x}_f) = 100(u_2 - u_1^2)^2 + (1 - u_1)^2$$

$$\text{where } \boldsymbol{u} = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} 1.1 & -0.2 \\ 0.2 & 0.9 \end{bmatrix} \boldsymbol{x}_f + \begin{bmatrix} -0.3 \\ 0.3 \end{bmatrix}$$

$$\boldsymbol{x}_f^* = \begin{bmatrix} 1.2718447 \\ 0.4951456 \end{bmatrix}$$

Transformed Rosenbrock Function

(*Mohamed et al., 2006*)



Transformed Rosenbrock Function

(*Mohamed et al., 2006*)

$$\boldsymbol{B}^{(9)} = \begin{bmatrix} 1.1083 & -0.2035 \\ 0.2177 & 0.8928 \end{bmatrix} \qquad \boldsymbol{B}^{(true)} = \begin{bmatrix} 1.1 & -0.2 \\ 0.2 & 0.9 \end{bmatrix}$$

$$\boldsymbol{c}^{(9)} = \begin{bmatrix} -0.3088 \\ 0.2810 \end{bmatrix} \qquad \boldsymbol{c}^{(true)} = \begin{bmatrix} -0.3 \\ 0.3 \end{bmatrix}$$

$$\boldsymbol{x}_f^{(9)} = \begin{bmatrix} 1.2718446 \\ 0.4951456 \end{bmatrix} \qquad \boldsymbol{x}_f^{*} = \begin{bmatrix} 1.2718447 \\ 0.4951456 \end{bmatrix}$$

$$R_f^{(9)} = 5.4\mathrm{e}-16 \qquad R_f^{*} = 0$$

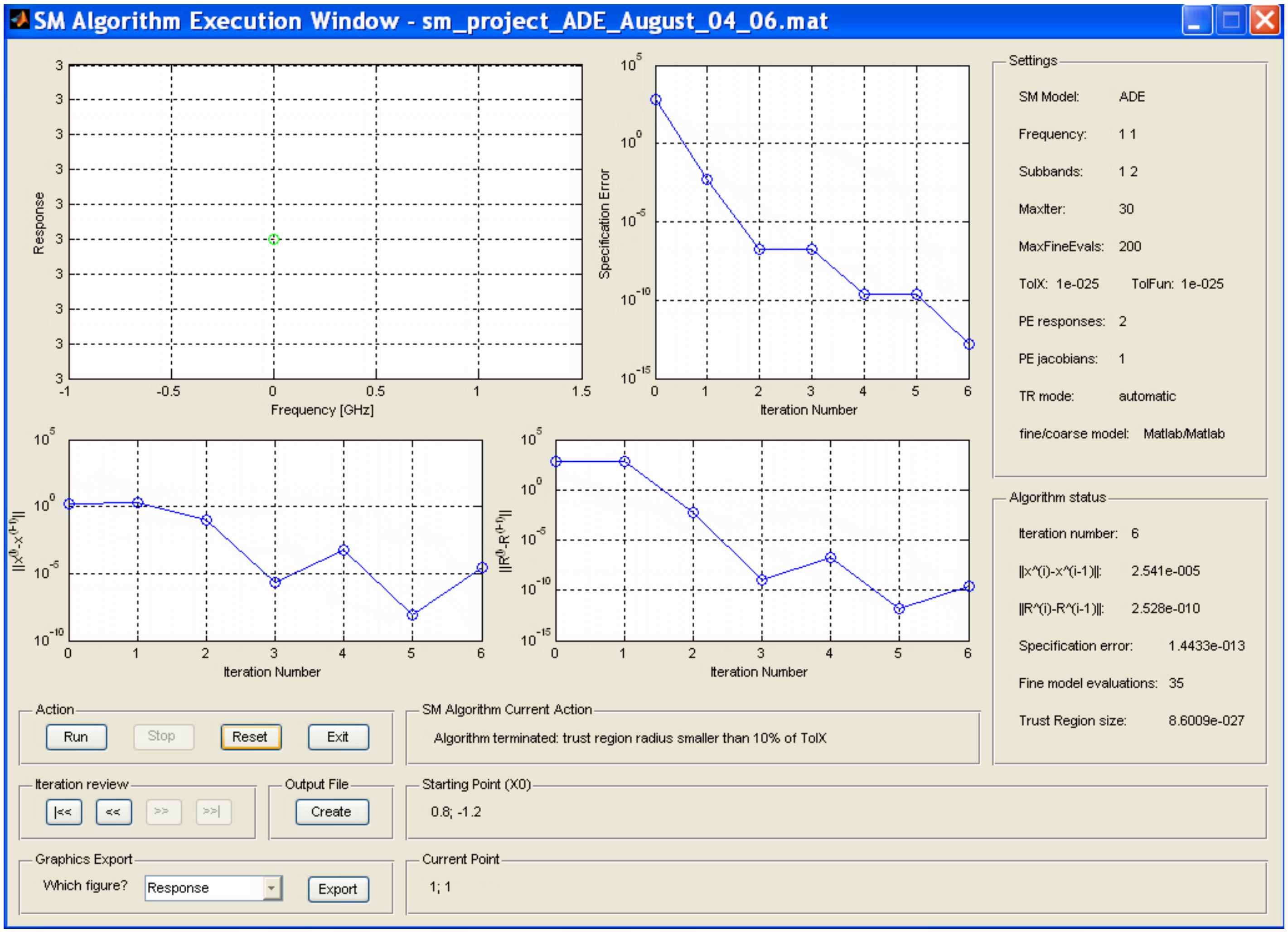
Response-Transformed Rosenbrock Function (Fine Model)
(*Mohamed et al., 2006*)

a response linear transformation of the original Rosenbrock function

$$R_f(\boldsymbol{x}_f) = 2\left[100(x_2 - x_1^2)^2 + (1 - x_1)^2\right] + 3$$

where $\boldsymbol{x}_f = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ and $\boldsymbol{x}_f^* = \begin{bmatrix} 1.0 \\ 1.0 \end{bmatrix}$

Response-Transformed Rosenbrock Function
(*Mohamed et al., 2006*)



Response-Transformed Rosenbrock Function

(*Mohamed et al., 2006*)

$$A^{(6)} = 2.0007 \qquad A^{(true)} = 2.0$$

$$D^{(6)} = 3.0 \qquad D^{(true)} = 3.0$$

$$\boldsymbol{x}_f^{(6)} = \begin{bmatrix} 1.0000003 \\ 1.0000005 \end{bmatrix} \qquad \boldsymbol{x}_f^{*} = \begin{bmatrix} 1.0 \\ 1.0 \end{bmatrix}$$

$$R_f^{(6)} = 1.4\text{e}-13 \qquad R_f^{*} = 0$$

Response and Parameter-Transformed Rosenbrock Function (Fine Model) (*Mohamed et al., 2006*)

a response (scale + shift) and parameter (rotation + shift) transformation of the original Rosenbrock function

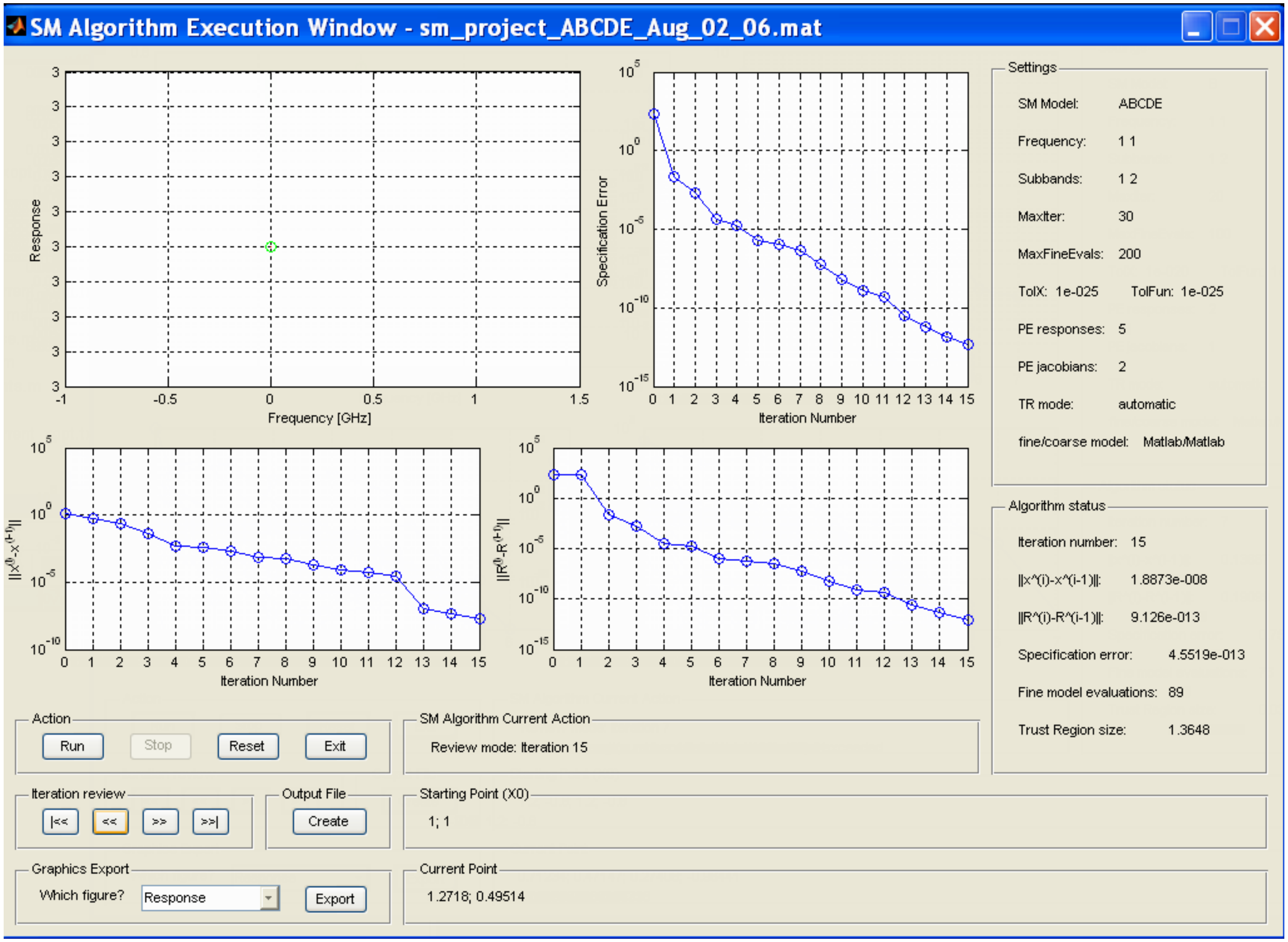
$$R_f(\boldsymbol{x}_f) = 2\left[100(u_2 - u_1^2)^2 + (1 - u_1)^2\right] + 3$$

$$\text{where } \boldsymbol{u} = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} 1.1 & -0.2 \\ 0.2 & 0.9 \end{bmatrix} \boldsymbol{x}_f + \begin{bmatrix} -0.3 \\ 0.3 \end{bmatrix}$$

$$\boldsymbol{x}_f^* = \begin{bmatrix} 1.2718447 \\ 0.4951456 \end{bmatrix}$$

Response and Parameter-Transformed Rosenbrock Function

(*Mohamed et al., 2006*)



Response and Parameter-Transformed Rosenbrock Function
(*Mohamed et al., 2006*)

$$A^{(15)} = 4.8715 \qquad A^{(true)} = 2.0$$

$$\boldsymbol{B}^{(15)} = \begin{bmatrix} 0.9862 & -0.6372 \\ 1.8238 & -1.1784 \end{bmatrix} \qquad \boldsymbol{B}^{(true)} = \begin{bmatrix} 1.1 & -0.2 \\ 0.2 & 0.9 \end{bmatrix}$$

$$\boldsymbol{c}^{(15)} = \begin{bmatrix} 0.8446 \\ 1.449 \end{bmatrix} \qquad \boldsymbol{c}^{(true)} = \begin{bmatrix} -0.3 \\ 0.3 \end{bmatrix}$$

$$d^{(15)} = 0.0 \qquad d^{(true)} = 3.0$$

$$\boldsymbol{x}_f^{(15)} = \begin{bmatrix} 1.2718442 \\ 0.4951449 \end{bmatrix} \qquad \boldsymbol{x}_f^{*} = \begin{bmatrix} 1.2718447 \\ 0.4951456 \end{bmatrix}$$

$$R_f^{(15)} = 3 - (4.6\mathrm{e}-13) \qquad R_f^{*} = 3$$

Rosenbrock Function (Low Fidelity Model with Offsets)
(*Eldred, Giunta, and Collis, AIAA, 2004*)

low fidelity model

$$R_c(\boldsymbol{x}_c) = 100(x_2 - x_1^2 + 0.2)^2 + (0.8 - x_1)^2$$

$$\text{where } \boldsymbol{x}_c = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \text{ and } \boldsymbol{x}_c^* = \begin{bmatrix} 0.8 \\ 0.44 \end{bmatrix}$$

high fidelity model

$$R_f(\boldsymbol{x}_f) = 100(x_2 - x_1^2)^2 + (1 - x_1)^2$$

$$\text{where } \boldsymbol{x}_f = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \text{ and } \boldsymbol{x}_f^* = \begin{bmatrix} 1.0 \\ 1.0 \end{bmatrix}$$

Rosenbrock Function (Low Fidelity Model with Offsets)
(*Mohamed et al., 2006*)

method	#of iters	FM Evals	R_f
Full 2nd add[1]	5	11	1.24e–15
Full 2nd mult[1]	31	59	8.96e–15
SR1 2nd comb[1]	23	42	4.73e–15
FD 2nd add[1]	5	23	1.53e–10
SMF	6	35	2.79e–14

[1]*Eldred, Giunta, and Collis, AIAA, 2004*

Rosenbrock Function (Low Fidelity Model with Scalings)
(*Eldred, Giunta, and Collis, AIAA, 2004*)

low fidelity model

$$R_c(\boldsymbol{x}_c) = 100(1.25x_2 - x_1^2)^2 + (1 - 1.25x_1)^2$$

$$\text{where } \boldsymbol{x}_c = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \text{ and } \boldsymbol{x}_c^* = \begin{bmatrix} 0.8 \\ 0.512 \end{bmatrix}$$

high fidelity model

$$R_f(\boldsymbol{x}_f) = 100(x_2 - x_1^2)^2 + (1 - x_1)^2$$

$$\text{where } \boldsymbol{x}_f = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \text{ and } \boldsymbol{x}_f^* = \begin{bmatrix} 1.0 \\ 1.0 \end{bmatrix}$$

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Rosenbrock Function (Low Fidelity Model with Scalings)

(*Mohamed et al., 2006*)

method	#of iters	FM Evals	R_f
BFGS 2nd comb[1]	292	514	1.68e–14
BFGS 2nd mult[1]	87	154	1.38e–13
Full 2nd mult[1]	42	76	2.59e–12
FD 2nd add[1]	17	68	4.58e–9
SMF	14	77	9.39e–15

[1]*Eldred, Giunta, and Collis, AIAA, 2004*

Multi-Fidelity Optimization (MFO) Algorithm
(*Castro, Gray, Giunta, and Hough, 2006*)

the MFO algorithm incorporates a derivative free optimization approach based on two techniques:

1. Asynchronous Parallel Pattern Search (APPS)
2. Space Mapping (SM)

Multi-Variable Rosenbrock Function (Case 1)
(*Castro, Gray, Giunta, and Hough, 2006*)

high fidelity model

$$R_f(\boldsymbol{x}_f) = 100(x_2 - x_1^2)^2 + (1 - x_1)^2 + 100(x_3 - x_2^2)^2 + (1 - x_2)^2$$

where $\boldsymbol{x}_f = \begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix}^T$ and $\boldsymbol{x}_f^* = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix}^T$

low fidelity model

$$R_c(\boldsymbol{x}_c) = 100(x_2 - x_1^2)^2 + (1 - x_1)^2$$

where $\boldsymbol{x}_c = \begin{bmatrix} x_1 & x_2 \end{bmatrix}^T$ and $\boldsymbol{x}_c^* = \begin{bmatrix} 1 & 1 \end{bmatrix}^T$

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Multi-Variable Rosenbrock Function (Case 1, using B)
(*Mohamed et al., 2006*)

six SM parameters

$$\boldsymbol{B}^{(6)} = \begin{bmatrix} 0.05 & -0.15 & -0.45 \\ 0.19 & 1.03 & -1.02 \\ 0.0 & 0.0 & 1.0 \end{bmatrix}$$

method	$\boldsymbol{x}_f^*$	R_f	# of function evaluations
MFO[1]	$[0.3 \quad 0.68 \quad 0.46]^T$	1.35	87
SMF	$[1.05 \quad 1.09 \quad 1.14]^T$	0.38	30

[1]*Castro, Gray, Giunta, and Hough, 2006*

Multi-Variable Rosenbrock Function (Case 2)
(*Castro, Gray, Giunta, and Hough, 2006*)

high fidelity model

$$R_f(\boldsymbol{x}_f) = 100(x_2 - x_1^2)^2 + (1 - x_1)^2 + 100(x_3 - x_2^2)^2 + (1 - x_2)^2 + 100(x_4 - x_3^2)^2 + (1 - x_3)^2$$

where $\boldsymbol{x}_f = \begin{bmatrix} x_1 & x_2 & x_3 & x_4 \end{bmatrix}^T$ and $\boldsymbol{x}_f^* = \begin{bmatrix} 1 & 1 & 1 & 1 \end{bmatrix}^T$

low fidelity model

$$R_c(\boldsymbol{x}_c) = 100(x_2 - x_1^2)^2 + (1 - x_1)^2$$

where $\boldsymbol{x}_c = \begin{bmatrix} x_1 & x_2 \end{bmatrix}^T$ and $\boldsymbol{x}_c^* = \begin{bmatrix} 1 & 1 \end{bmatrix}^T$

Multi-Variable Rosenbrock Function (Case 2, using B)

(*Mohamed et al., 2006*)

eight SM parameters

$$\boldsymbol{B}^{(9)} = \begin{bmatrix} 5.49 & -1.92 & -2.81 & 0.31 \\ 3.56 & 2.56 & -5.07 & 0.68 \\ 0.0 & 0.0 & 1.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 1.0 \end{bmatrix}$$

method	$\boldsymbol{x}_f^*$	R_f	# of function evaluations
MFO[1]	$\begin{bmatrix}0.55 & 0.29 & 0.087 & -0.003\end{bmatrix}^T$	1.58	154
SMF	$\begin{bmatrix}0.99 & 0.93 & 0.89 & -0.64\end{bmatrix}^T$	0.451	103

[1] *Castro, Gray, Giunta, and Hough, 2006*

Multi-Variable Rosenbrock Function (Case 2, using B and E)

(*Mohamed et al., 2006*)

eight SM parameters

$$\boldsymbol{B}^{(11)} = \begin{bmatrix} 0.56 & 0.08 & 1.66 & 0.25 \\ 0.93 & 1.19 & 0.94 & -2.70 \\ 0.0 & 0.0 & 1.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 1.0 \end{bmatrix}$$

method	$\boldsymbol{x}_f^*$	R_f	# of function evaluations
MFO[1]	$[0.55 \quad 0.29 \quad 0.087 \quad -0.003]^T$	1.58	154
SMF	$[0.99 \quad 1.01 \quad 1.02 \quad -0.60]^T$	0.056	110

[1] *Castro, Gray, Giunta, and Hough, 2006*

Multi-Variable Rosenbrock Function (Case 2, using B)

(*Mohamed et al., 2006*)

four SM parameters

$$\boldsymbol{B}^{(7)} = \begin{bmatrix} 0.96 & 0.0 & 0.0 & 0.0 \\ 0.0 & 1.85 & -0.98 & 0.21 \\ 0.0 & 0.0 & 1.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 1.0 \end{bmatrix}$$

method	$\boldsymbol{x}_f^*$	R_f	# of function evaluations
MFO[1]	$[0.49 \quad 0.24 \quad 0.081 \quad 0.009]^T$	1.73	80
SMF	$[0.71 \quad 0.47 \quad 0.27 \quad -0.96]^T$	0.76	76

[1] *Castro, Gray, Giunta, and Hough, 2006*

Multi-Variable Rosenbrock Function (Case 2, using B and E)
(*Mohamed et al., 2006*)

four SM parameters

$$\boldsymbol{B}^{(7)} = \begin{bmatrix} 0.74 & 0.0 & 0.0 & 0.0 \\ 0.0 & 7.35 & -3.82 & 0.67 \\ 0.0 & 0.0 & 1.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 1.0 \end{bmatrix}$$

method	$\boldsymbol{x}_f^*$	R_f	# of function evaluations
MFO[1]	$[0.49 \quad 0.24 \quad 0.081 \quad 0.009]^T$	1.73	80
SMF	$[1.20 \quad 1.43 \quad 2.06 \quad -2.68]^T$	0.36	76

[1] *Castro, Gray, Giunta, and Hough, 2006*

Multi-Variable Rosenbrock Function (Case 3)
(*Castro, Gray, Giunta, and Hough, 2006*)

high fidelity model

$$R_f(\boldsymbol{x}_f) = 100(x_2 - x_1^2)^2 + (1 - x_1)^2 + 100(x_3 - x_2^2)^2 + (1 - x_2)^2$$

where $\boldsymbol{x}_f = \begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix}^T$ and $\boldsymbol{x}_f^* = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix}^T$

low fidelity model

$$R_c(\boldsymbol{x}_c) = 100(x_2 - x_1^2)^2 + (1 - x_1)^2$$

where $\boldsymbol{x}_c = \begin{bmatrix} x_1 & x_2 \end{bmatrix}^T$ and $\boldsymbol{x}_c^* = \begin{bmatrix} 1 & 1 \end{bmatrix}^T$

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Multi-Variable Rosenbrock Function (Case 3, using B and c)

(*Mohamed et al., 2006*)

five SM parameters

$$\boldsymbol{B}^{(4)} = \begin{bmatrix} 0.90 & 0.0 & 0.0 \\ 0.0 & 1.23 & -0.05 \\ 0.0 & 0.0 & 1.0 \end{bmatrix}, \boldsymbol{c}^{(4)} = \begin{bmatrix} -1.14 \\ -1.28 \\ 0.0 \end{bmatrix}$$

method	$\boldsymbol{x}_f^*$	R_f	# of function evaluations
MFO[1]	$\begin{bmatrix} 0.55 & 0.32 & 0.12 \end{bmatrix}^T$	0.728	50
SMF	$\begin{bmatrix} 1.06 & 1.13 & 1.25 \end{bmatrix}^T$	0.062	42

[1]*Castro, Gray, Giunta, and Hough, 2006*

Multi-Variable Rosenbrock Function (Case 3, using B and c)
(*Mohamed et al., 2006*)

six SM parameters

$$\boldsymbol{B}^{(5)} = \begin{bmatrix} 0.57 & 0.0 & -0.12 \\ 0.0 & 1.58 & -0.71 \\ 0.0 & 0.0 & 1.0 \end{bmatrix}, \boldsymbol{c}^{(5)} = \begin{bmatrix} 0.38 \\ -0.18 \\ 0.0 \end{bmatrix}$$

method	$\boldsymbol{x}_f^*$	R_f	# of function evaluations
MFO[1]	$\begin{bmatrix} 0.35 & 0.12 & 0.007 \end{bmatrix}^T$	1.2	62
SMF	$\begin{bmatrix} 1.04 & 1.07 & 1.15 \end{bmatrix}^T$	0.015	56

[1] *Castro, Gray, Giunta, and Hough, 2006*

Multi-Variable Rosenbrock Function (Case 3, using B and c)

(*Mohamed et al., 2006*)

eight SM parameters

$$\boldsymbol{B}^{(4)} = \begin{bmatrix} 0.92 & -0.05 & 0.67 \\ 0.38 & 0.78 & 0.01 \\ 0.0 & 0.0 & 1.0 \end{bmatrix}, \boldsymbol{c}^{(4)} = \begin{bmatrix} -0.23 \\ 0.71 \\ 0.0 \end{bmatrix}$$

method	$\boldsymbol{x}_f^*$	R_f	# of function evaluations
MFO[1]	$[0.95 \quad 0.91 \quad 0.84]^T$	0.032	91
SMF	$[1.02 \quad 1.04 \quad 1.09]^T$	0.011	38

[1]*Castro, Gray, Giunta, and Hough, 2006*

MIT Rosenbrock Function
(*Robinson, Eldred, Willcox, and Haimes, 2006*)

high fidelity model

$$R_f(\boldsymbol{x}_f) = 4(x_2 - x_1^2)^2 + (1 - x_1)^2$$

$$\text{where } \boldsymbol{x}_f = \begin{bmatrix} x_1 & x_2 \end{bmatrix}^T \text{ and } \boldsymbol{x}_f^* = \begin{bmatrix} 1.0 & 1.0 \end{bmatrix}^T$$

low fidelity model

$$R_c(\boldsymbol{x}_c) = x_1^2 + x_2^2$$

$$\text{where } \boldsymbol{x}_c = \begin{bmatrix} x_1 & x_2 \end{bmatrix}^T \text{ and } \boldsymbol{x}_c^* = \begin{bmatrix} 0.0 & 0.0 \end{bmatrix}^T$$

MIT Rosenbrock Function
(*Mohamed et al., 2006*)

POD: Proper Orthogonal Decomposition

method	FM Evals	R_f
Multi-fidelity with corrected SM[1]	20	1.0e–14
Multi-fidelity with corrected POD[1]	20	1.0e–15
SMF	24	8.2e–14

[1]*Robinson, Eldred, Willcox, and Haimes, 2006*

Test Problem	R_f		# fine model evaluations	
	SMF	Other SBO	SMF	Other SBO
Eldred *et al.*, 2004 (Case 1)	2.79e–14	1.25e–15 8.96e–15 4.73e–10 1.53e–10	35	11 59 42 23
Eldred *et al.*, 2004 (Case 2)	9.39e–15	1.68e–14 1.38e–13 2.59e–12 4.58e–9	77	514 154 76 68
Castro *et al.*, 2006 (Case 1)	0.38	1.35	30	87
Castro *et al.*, 2006 (Case 2a)	0.451 0.056	1.58	103 110	154
Castro *et al.*, 2006 (Case 2b)	0.76 0.36	1.73	76 76	80
Castro *et al.*, 2006 (Case 3a)	0.015	1.2	56	62
Castro *et al.*, 2006 (Case 3b)	0.062	0.728	42	50
Castro *et al.*, 2006 (Case 3c)	0.011	0.032	38	91
Robinson *et al.*, 2006 (Case 1)	8.2e–14	1.0e–14 1.0e–15	24	20 20

Conclusion

we utilize SMF to solve several Rosenbrock-like test problems

we compare SMF with other SBO implementations

within its current configuration, SMF manages to behave as well as or better than the other SBO implementations

Bibliography 1

J.W. Bandler, R.M. Biernacki, S.H. Chen, P.A. Grobelny, and R.H. Hemmers, "Space mapping technique for electromagnetic optimization," *IEEE Trans. Microwave Theory Tech.*, vol. 42, no. 12, pp. 2536–2544, Dec. 1994.

J.W. Bandler, R.M. Biernacki, S.H. Chen, R.H. Hemmers, and K. Madsen, "Electromagnetic optimization exploiting aggressive space mapping," *IEEE Trans. Microwave Theory Tech*., vol. 43, no. 12, pp. 2874–2882, Dec. 1995.

N. Alexandrov, J. E. Dennis, R. M. Lewis, and V. Torczon, "A trust region framework for managing the use of approximation models in optimization," *Structural Optimization*, vol. 15, pp. 16–23, 1998.

A.J. Booker, J.E. Dennis, Jr., P.D. Frank, D. B. Serafini, V. Torczon, and M.W. Trosset, "A rigorous framework for optimization of expensive functions by surrogates," *Structural Optimization*, vol. 17, pp. 1–13 , 1999.

J.W. Bandler, S. Koziel, and K. Madsen, "Space mapping for engineering optimization," *SIAG/Optimization Views-and-News Special Issue on Surrogate/Derivative-free Optimization*, vol. 17, Mar. 2006.

S. Koziel, J.W. Bandler, and K. Madsen, "Space mapping optimization algorithms for engineering design," *IEEE MTT-S Int. Microwave Symp. Digest* (San Francisco, CA, June 2006).

Bibliography 2

H.H. Rosenbrock, "An automatic method for finding the greatest or least value of a function," *Computer Journal*, vol. 3, no. 3, pp. 175–184, 1960.

R. Fletcher, *Practical Methods of Optimization*, 2nd ed. New York: Wiley, 1987.

M.H. Bakr, J.W. Bandler, N.K. Georgieva, and K. Madsen, "A hybrid aggressive space-mapping algorithm for EM optimization," *IEEE Trans. Microwave Theory Tech*., vol. 47, no. 12, pp. 2440–2449, Dec. 1999.

J.W. Bandler, A.S. Mohamed, M.H. Bakr, K. Madsen, and J. Søndergaard, "EM-based optimization exploiting partial space mapping and exact sensitivities," *IEEE Trans. Microwave Theory Tech.*, vol. 50, no. 12, pp. 2741–2750, Dec. 2002.

J. Søndergaard, *Optimization using surrogate models—by the space mapping technique*, Ph.D. Thesis, Informatics and Mathematical Modelling (IMM), Technical University of Denmark (DTU), Lyngby, Denmark, 2003.

J.W. Bandler, Q.S. Cheng, S.A. Dakroury, A.S. Mohamed, M.H. Bakr, K. Madsen, and J. Søndergaard, "Space mapping: the state of the art," *IEEE Trans. Microwave Theory Tech.*, vol. 52, no. 1, pp. 337–361, Jan. 2004.

SMF, Bandler Corporation, P.O. Box 8083, Dundas, ON, Canada L9H 5E7, 2006.

Bibliography 3

M.S. Eldred, A.A. Giunta, and S.S. Collis, "Second-order corrections for surrogate-based optimization with model hierarchies," paper AIAA-2004-4457 in *Proceedings of the 10th AIAA/ISSMO Multidisciplinary Analysis and Optimization Conference*, Albany, NY, Aug. 30–Sept. 1, 2004.

J.P. Castro, G.A. Gray, A.A. Giunta, and P.D. Hough, "Developing a computationally efficient dynamic multilevel hybrid optimization scheme using multifidelity model interactions," *Technical Report SAND2005-7498*, Sandia National Laboratories, Livermore, CA, Jan 2006.

T.D. Robinson, M.S. Eldred, K.E. Willcox, and R. Haimes, "Strategies for multifidelity optimization with variable dimensional hierarchical models," *Proceedings of the 47th AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics, and Materials Conference* (2nd AIAA Multidisciplinary Design Optimization Specialist Conference), Newport, Rhode Island, May 1–4, 2006.

A.A. Giunta and M.S. Eldred, "Implementation of a trust region model management strategy in the DAKOTA optimization toolkit," paper AIAA-2000-4935 in *Proceedings of the 8th AIAA/USAF/NASA/ISSMO Symposium on Multidisciplinary Analysis and Optimization*, Long Beach, CA, September 6–8, 2000.

D. Echeverria and P.W. Hemker, "Space mapping and defect correction," *CMAM Int. Mathematical J. Computational Methods in Applied Mathematics*, vol. 5, pp. 107–136, 2005.

D. Echeverria, D. Lahaye, L. Encica, and P.W. Hemker, "Optimisation in electromagnetics with the space-mapping technique," *COMPEL Int. J. Comp. and Math. in Electrical and Electronic Engineering*, vol. 24, pp. 952–966, 2005.

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