Challenges of Sensitivity Analysis with Time Domain EM Solvers

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Outline

- Background
- Reviewed existing results
- Our current results
- Challenges
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Background

- Design Sensitivity Analysis (DSA)
  - DSA concerns the relationship between the objective function and design variables that describe the shape of geometry or the material properties to be optimized.

Objective function:

$$F(\boldsymbol{e},\mathbf{p}) = \int_z \int_y \int_x \int_0^T G(\boldsymbol{e},\mathbf{p})\,dtdxdydz$$

T: fixed final time

G: arbitrary differentiable function

$\boldsymbol{e}$: field variable vector, $\boldsymbol{p}$: design variable vector

- Purpose of DSA: to evaluate the derivative of the system's response with respect to variations in the design parameters

Design sensitivity:

$$\frac{\partial F}{\partial p_i} = \int_z \int_y \int_x \int_0^T \left[\frac{\partial G}{\partial \boldsymbol{e}} \frac{\partial \boldsymbol{e}}{\partial p_i} + \frac{\partial^e G}{\partial p_i} \right] dtdxdydz \qquad i = 1,\ 2,\ \ldots,\ n_p$$

n_p: the total number of the design variable p_i

$\partial^e G / \partial p_i$: the explicit dependence of G on p_i

- Adjoint Variable Method (AVM)
  - AVM is an efficient design approach to complex linear and nonlinear problems and it has been proposed in many areas, such as structural design, circuit theory and control theory, etc.
  - Comparing with direct difference method (DDM), AVM offers significant reduction in CPU time by producing the response and its gradient through a single full-wave analysis.
    - DDM: requires solving the system equation for each design variable.
    - AVM: only need to solve the adjoint variable equation in the introduced adjoint system and it requires only a moderate amount of computation.

- Implementation of AVM: by introducing adjoint system and solving the adjoint variable equation as well as the original system.

Vector wave equation:

$$\nabla \times \frac{1}{m_r} \nabla \times \mathbf{E} + \frac{e_r}{c^2} \frac{\partial^2}{\partial t^2} \mathbf{E} = -m_0 \frac{\partial}{\partial t} \mathbf{J}$$

Original system:

$$\mathbf{M} \frac{\partial^2 \boldsymbol{e}}{\partial t^2} + \mathbf{K} \boldsymbol{e} = \mathbf{Q}, \qquad \mathrm{M} = e_r / c^2$$

$$\boldsymbol{e}(0) = 0, \qquad \mathrm{K} = \nabla \times m_r^{-1} \nabla \times$$

$$\frac{\partial \boldsymbol{e}(0)}{\partial t} = 0 \qquad \mathrm{Q} = -m_0 \partial \mathbf{J} / \partial t$$

Adjoint system:

$$\mathbf{M} \frac{\partial^2 l}{\partial t^2} + \mathbf{K} l = \left\{ \frac{\partial G}{\partial \boldsymbol{e}} \right\}^T \qquad \mathbf{M} \frac{\partial^2 \bar{l}}{\partial t^2} + \mathbf{K} \bar{l} = \left\{ \frac{\partial G}{\partial \boldsymbol{e}} \right\}^T$$

$$l(T) = 0 \qquad \bar{l}(0) = 0$$

$$\frac{\partial l}{\partial t}(T) = 0 \qquad \frac{\partial \bar{l}}{\partial t}(0) = 0$$

$t = \mathrm{T} - t$

Backward time scheme

Terminal condition for t Initial condition for $\boldsymbol{\tau}$

Design sensitivity evaluated by AVM:

$$\frac{\partial F}{\partial p_i} = \int_z \int_y \int_x \int_0^T \left(l^T \frac{\partial \mathbf{R}}{\partial p_i} + \frac{\partial^e G}{\partial p_i} \right) dtdxdydz$$

$$\mathbf{R} = \mathbf{Q} - \mathbf{M} \frac{\partial^2 \tilde{\boldsymbol{e}}}{\partial t^2} - \mathbf{K} \tilde{\boldsymbol{e}}$$

$\tilde{\boldsymbol{e}}$ is independent of the design variable p_i

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Published achievements

- Results by FETD

  (Y.S. Chung et al. 2001)
  - Design microwave passive devices with small return loss by FETD technique
  - AVM is used to evaluate design sensitivity
  - Unstructured triangular elements used

- Results by FDTD

  (Y.S. Chung et al. 2000, 2001)
  - Using uniqueness theorem, adjoint variable equations from FETD are derived as coupled Maxwell's curl equations and can be solved by the FDTD method.
  - AVM is used to evaluate design sensitivity.
  - Unstructured quadrilateral grids and DSI technique are used to model the design space, and PML is employed as ABC.

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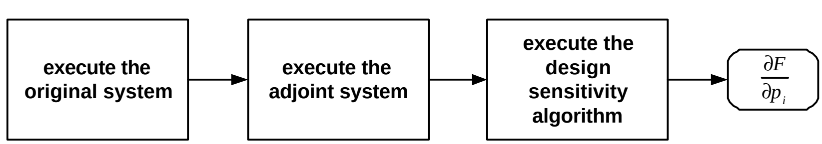
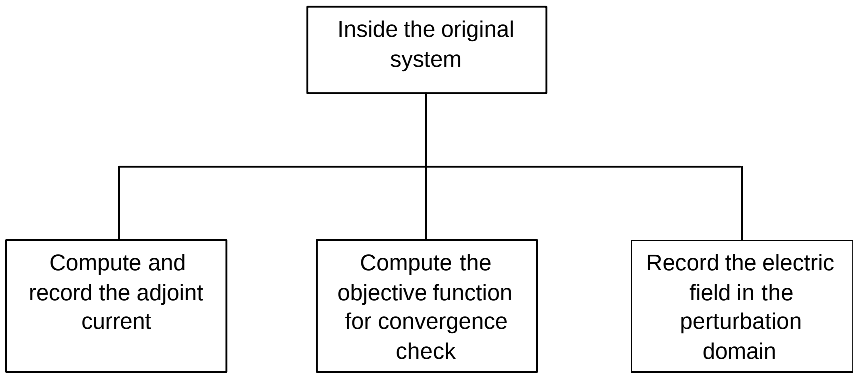
Our current results

- Methodology

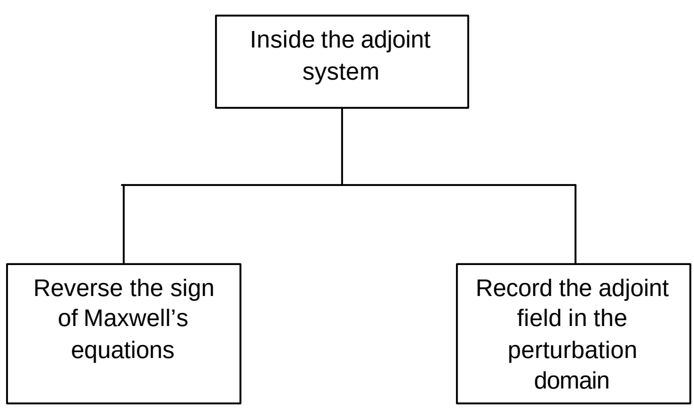
  To evaluate the design sensitivity of the design variables by employing AVM in the FDTD method.



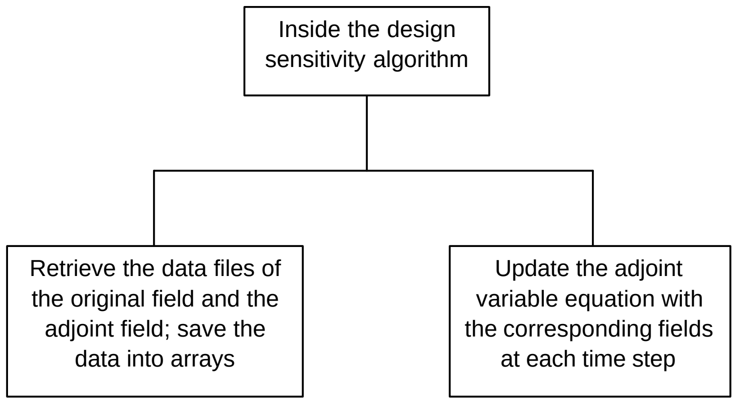
Original system



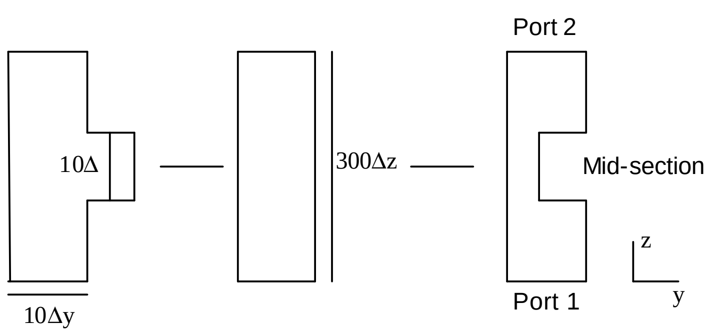
Adjoint system



- Design sensitivity algorithm



- Numerical results and comparison
  - Step waveguide junction structure



The flow of the width variation in the step waveguide junction structure (symmetric about *y*-axis)

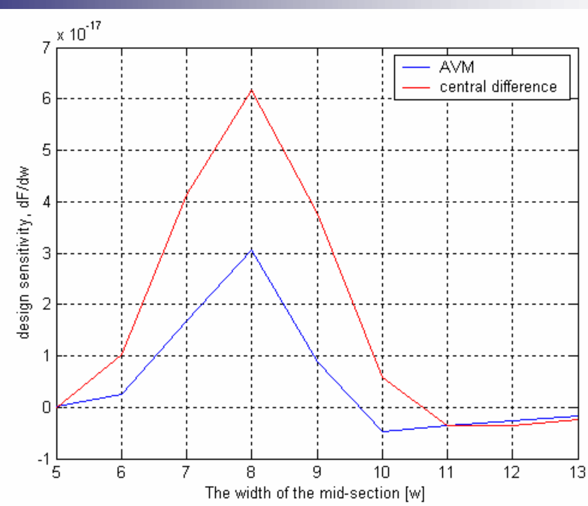
Objective function:

$$F = \int_0^T \int_x \int_y \frac{\boldsymbol{e}_x^2}{2} \Delta z dx dy dt$$

$$F = \int_0^T \int_{y=1}^{ymax} \frac{\boldsymbol{e}_x^2}{2} \Delta z \Delta x dy dt$$

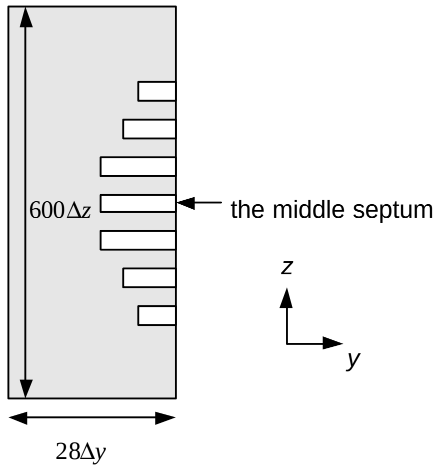
Design variable: the width of the mid section y.

	Height	Width	Length
Port 1	$1\Delta x$	$10\Delta y$	$147\Delta z$
Mid-section	$1\Delta x$	$5\Delta y - 13\Delta y$	$10\Delta z$
Port 2	$1\Delta x$	$10\Delta y$	$147\Delta z$



Comparison of the design sensitivity between the adjoint variable method and the central finite difference

□ Waveguide H-plane filter



The layout of the waveguide H-plane filter
(symmetric about *y*-axis)

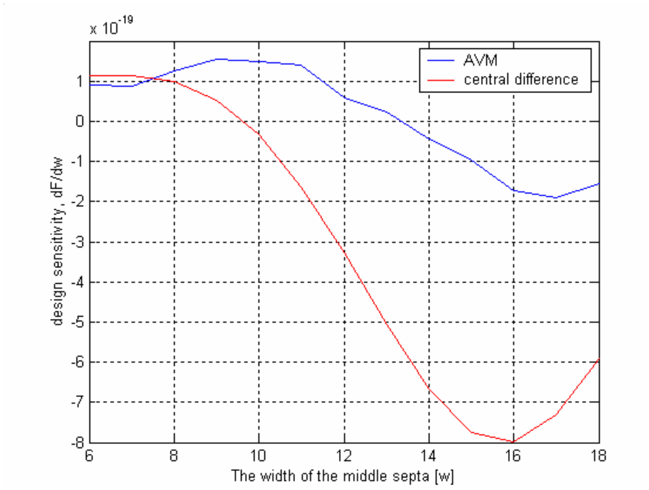
Objective function:

$$F = \int_0^T \int_x \int_y \frac{e_x^2}{2} \Delta z dx dy dt$$

$$F = \int_0^T \int_{y=1}^{ymax} \frac{e_x^2}{2} \Delta z \Delta x dy dt$$

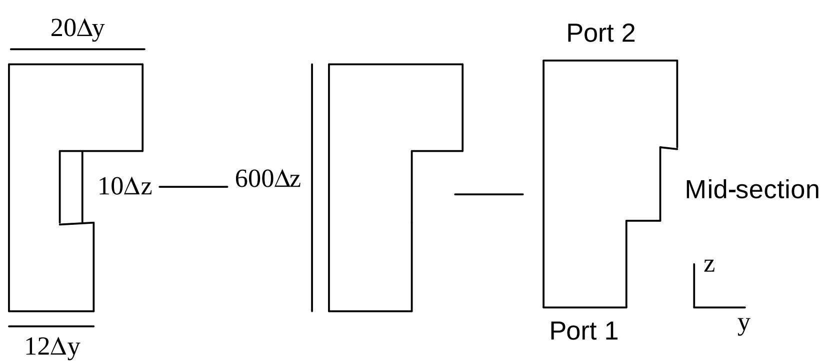
Design variable: the width of the mid section y.

Septa	Height	Width	Length
1st and 7th	$1\Delta x$	$7\Delta y$	$1\Delta z$
2nd and 6th	$1\Delta x$	$9\Delta y$	$1\Delta z$
3rd and 5th	$1\Delta x$	$10\Delta y$	$1\Delta z$
4th	$1\Delta x$	$6\Delta y - 15\Delta y$	$1\Delta z$

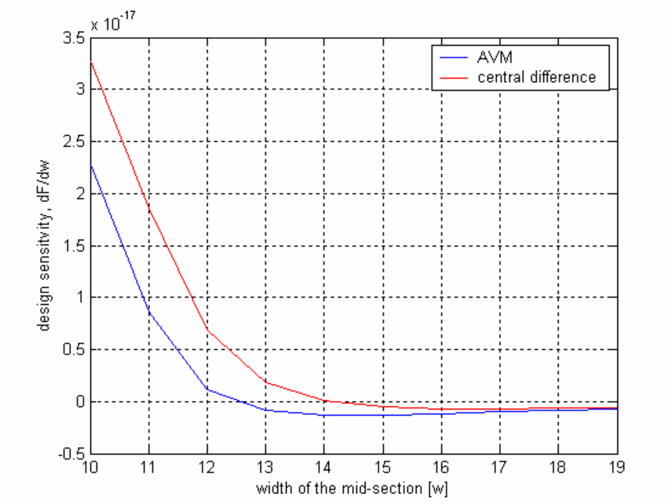


The design sensitivity of the septum waveguide

- Single-section transformer



The layout of the single-section transformer
(symmetric about *y*-axis)



The design sensitivity of the single-section transformer

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Challenges

- Advantages of AVM
  - Computationally efficient as it only requires two simulations regardless of the number of design variables.
  - AVM with FDTD can provide broad-band sensitivity estimation.

- Limitations of current algorithm
  - Excitation cannot have a DC component.
  - The design variables can not be freely chosen.
  - The two ports are expected to have same width in order to reduce the reflection.
  - For those with few field points, the results will be less accurate.
  - Absorbing boundary conditions affect the accuracy.

- Ways to improve the algorithm
  - Increase the number of observation field points, i.e., increase the observation domain.
  - Use more efficient absorbing boundary conditions, such as PML (Perfectly Matched Layer) as ABC. The accuracy of the excitation of the adjoint system depends much on the efficiency of ABC.

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Conclusions

- Overview of DSA and AVM techniques.
- Implementation with FETD and FDTD methods.
- Advantages and limitations.
- Challenges and improvements.

References

- Y.S. Chung, J. Ryu, C. Cheon, I.H. Park and S.Y. Hahn, “Optimal design method for microwave device using time domain method and design sensitivity analysis – Part I: FETD case,” pp. 3289-3293, *IEEE Trans. Magnetics*, vol. 37, No. 5, Sep. 2001.
- Y.S. Chung, C. Cheon, I.H. Park and S.Y. Hahn, “Optimal design method for microwave device using time domain method and design sensitivity analysis – Part I: FDTD case,” pp. 3255-3259, *IEEE Trans. Magnetics*, vol. 37, No. 5, Sep. 2001.
- Y.S. Chung, C. Cheon, I.H. Park and S.Y. Hahn, “Optimal shape design of microwave device using FDTD and design sensitivity analysis,” pp. 2289-2296, *IEEE Trans. Microwave Theory and Techniques*, vol. 48, No. 12, Dec. 2000.