Space Mapping Optimization Exploiting Exact Sensitivities

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Outline

ASM for microwave circuit design

Gradient Parameter Extraction (GPE)

mapping update

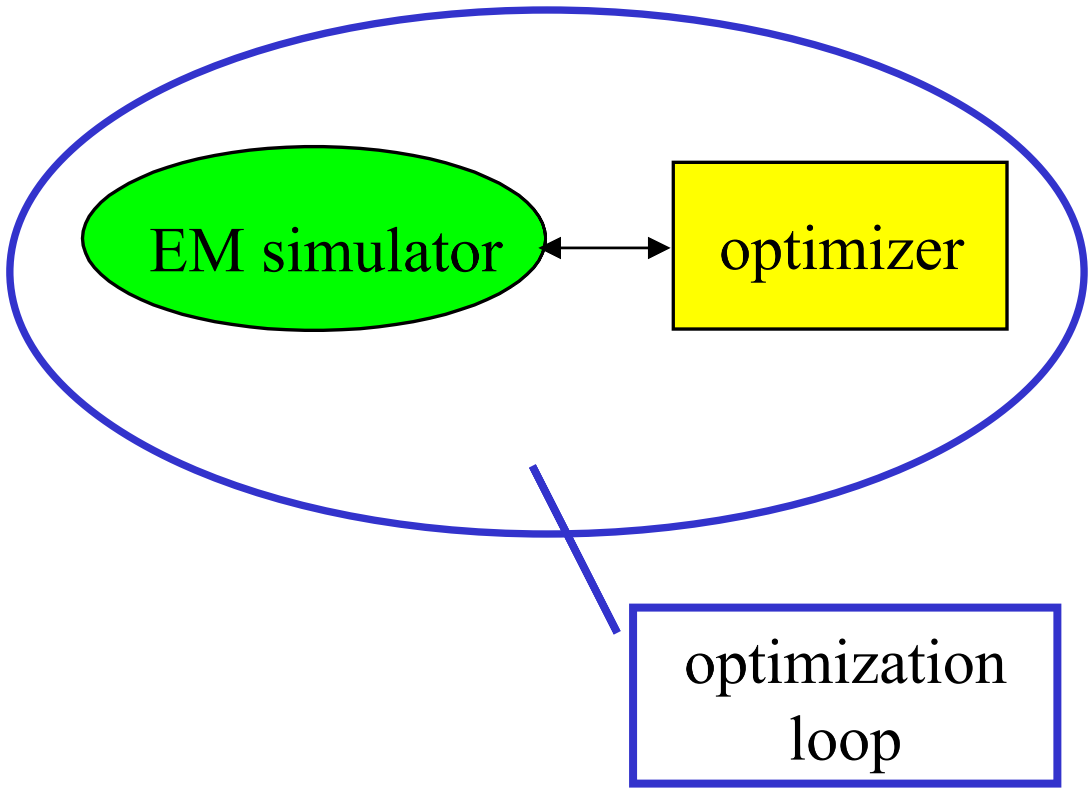
proposed algorithm

examples

conclusions

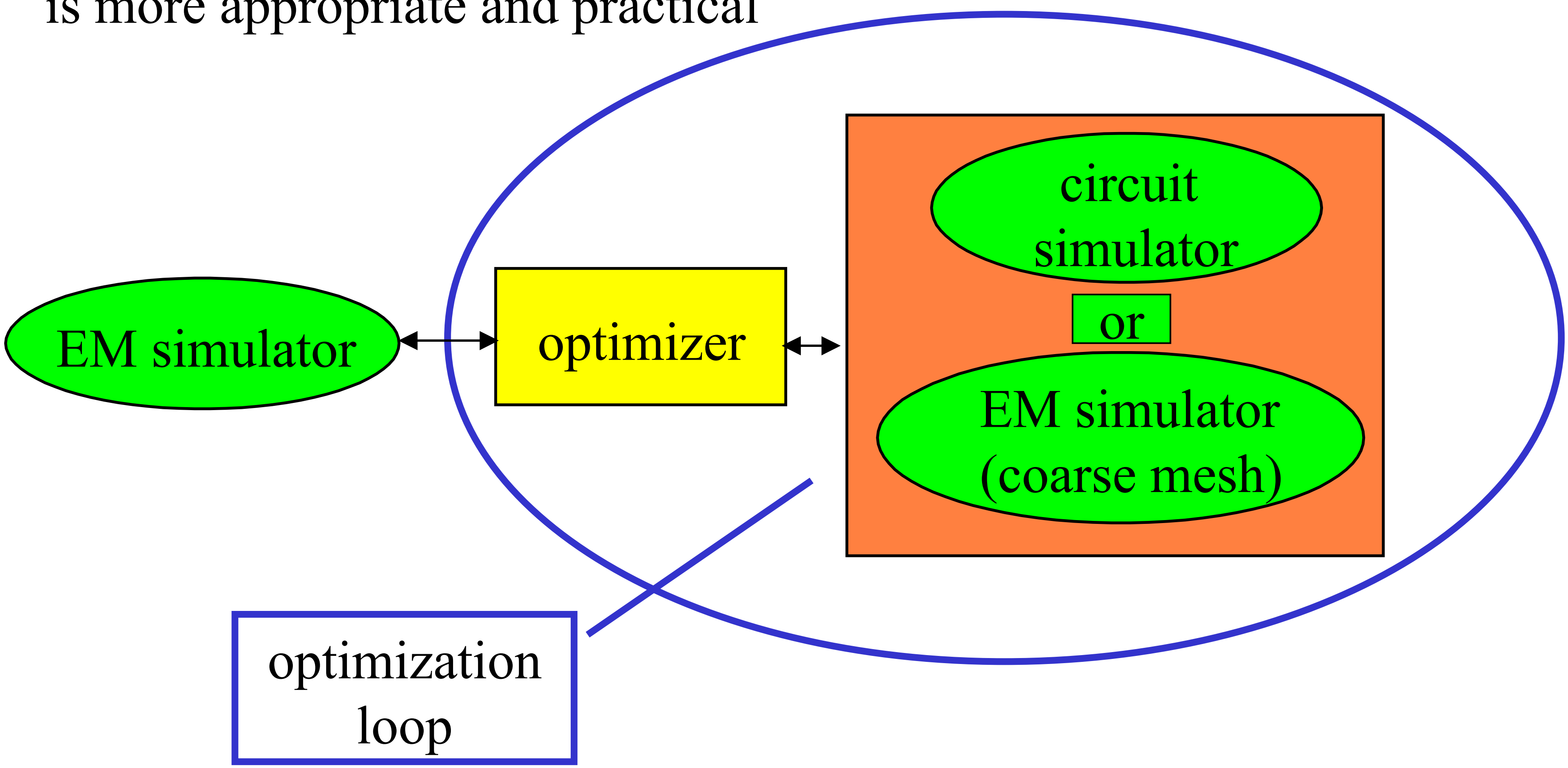
Introduction

using full wave EM simulator (fine model) inside the optimization loop is prohibitive



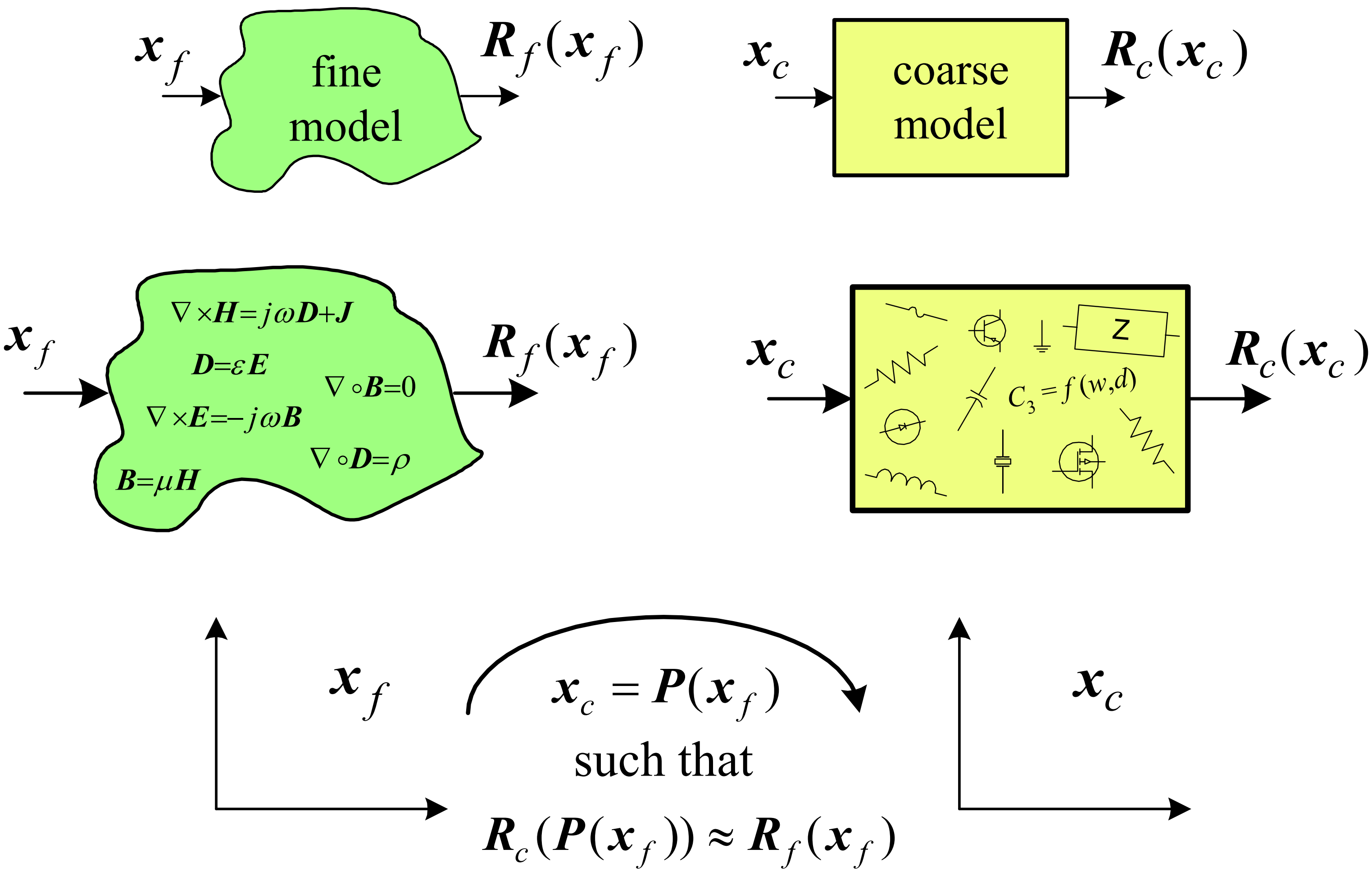
Introduction

using simpler (less accurate) model inside the optimization loop is more appropriate and practical



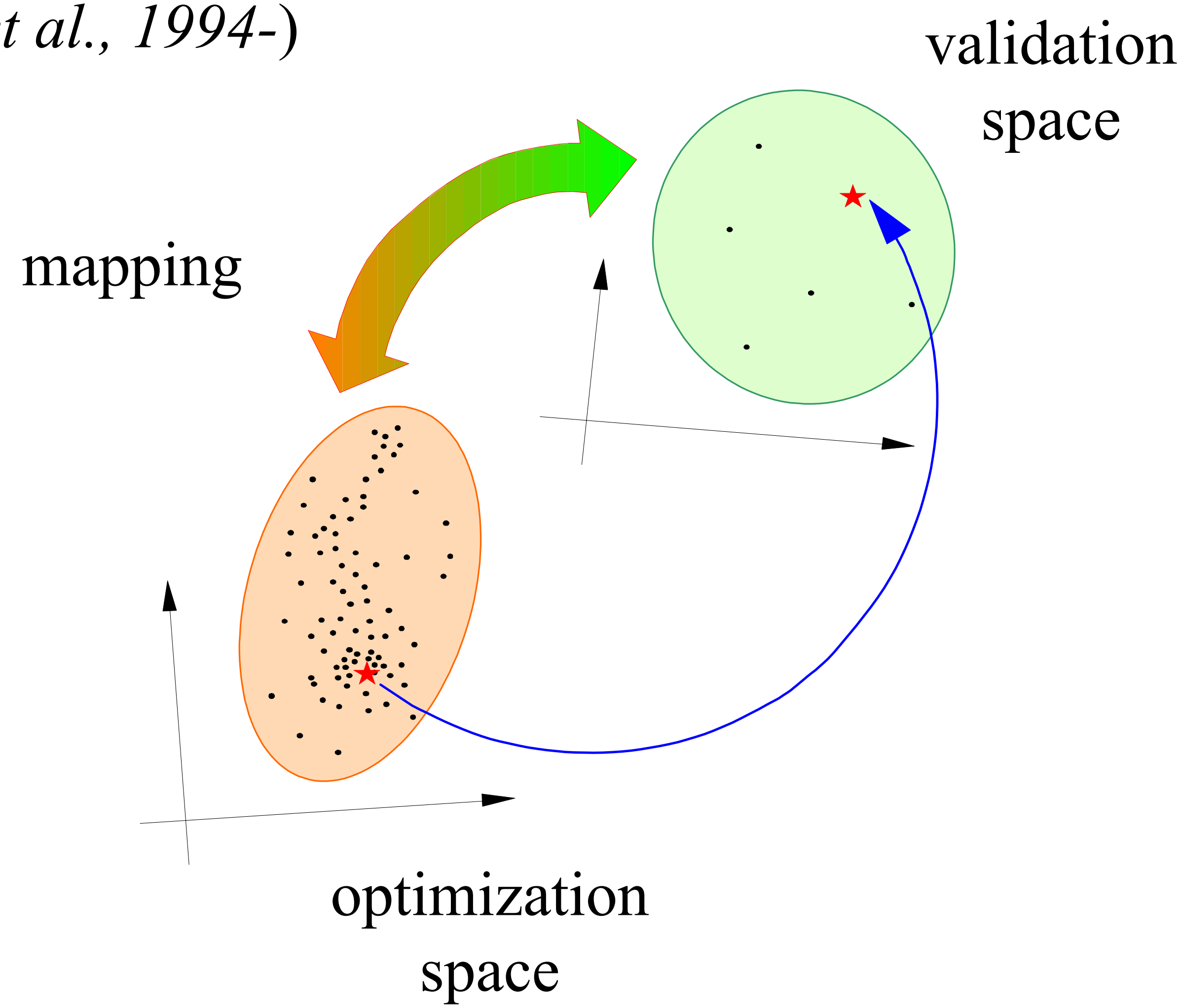
The Space Mapping Concept

(Bandler et al., 1994-)



The Space Mapping Concept

(*Bandler et al., 1994-*)



Jacobian-Space Mapping Relationship

(*Bakr et al., 1999*)

through PE we match the responses

$$\boldsymbol{R}_f(\boldsymbol{x}_f) \approx \boldsymbol{R}_c(\boldsymbol{P}(\boldsymbol{x}_f))$$

by differentiation

$$\left(\frac{\partial \boldsymbol{R}_f^T}{\partial \boldsymbol{x}_f}\right)^T \approx \left(\frac{\partial \boldsymbol{R}_c^T}{\partial \boldsymbol{x}_c}\right)^T \cdot \left(\frac{\partial \boldsymbol{x}_c^T}{\partial \boldsymbol{x}_f}\right)^T$$

Jacobian-Space Mapping Relationship

(*Bakr et al., 1999*)

given coarse model Jacobian $\boldsymbol{J}_c$ and space mapping matrix $\boldsymbol{B}$ we estimate

$$\boldsymbol{J}_f(\boldsymbol{x}_f) \approx \boldsymbol{J}_c(\boldsymbol{x}_c)\boldsymbol{B}$$

given $\boldsymbol{J}_c$ and $\boldsymbol{J}_f$ we estimate (least squares)

$$\boldsymbol{B} \approx (\boldsymbol{J}_c^T \boldsymbol{J}_c)^{-1} \boldsymbol{J}_c^T \boldsymbol{J}_f$$

Gradient Parameter Extraction (GPE)

(*Bandler et al., 2002*)

at the *j*th iteration

$$\boldsymbol{x}_c^{(j)} = \arg\min_{\boldsymbol{x}_c} \left\| [\boldsymbol{e}_0^T \quad \lambda\boldsymbol{e}_1^T \quad \cdots \quad \lambda\boldsymbol{e}_n^T]^T \right\|, \ \lambda \geq 0$$

where λ is a weighting factor and $\boldsymbol{E} = [\boldsymbol{e}_1\ \boldsymbol{e}_2\ \ldots\ \boldsymbol{e}_n]$

$$\boldsymbol{e}_0 = \boldsymbol{R}_f(\boldsymbol{x}_f^{(j)}) - \boldsymbol{R}_c(\boldsymbol{x}_c)$$

$$\boldsymbol{E} = \boldsymbol{J}_f(\boldsymbol{x}_f^{(j)}) - \boldsymbol{J}_c(\boldsymbol{x}_c)\boldsymbol{B}$$

Mapping Update Using Exact Derivatives

$$\boldsymbol{B}^{(j)} = (\boldsymbol{J}_c^{(j)T} \boldsymbol{J}_c^{(j)})^{-1} \boldsymbol{J}_c^{(j)T} \boldsymbol{J}_f^{(j)}$$

Mapping Update Using Hybrid Approach

finite difference initialization used

$$\boldsymbol{B}^{(0)} = (\boldsymbol{J}_c^{(0)T} \boldsymbol{J}_c^{(0)})^{-1} \boldsymbol{J}_c^{(0)T} \boldsymbol{J}_f^{(0)}$$

then update using Broyden formula

Mapping Update By Constraining *B*

(*Bakr et al., 2000*)

$$\boldsymbol{B} = (\boldsymbol{J}_c^T \boldsymbol{J}_c + \eta^2 \boldsymbol{I})^{-1} (\boldsymbol{J}_c^T \boldsymbol{J}_f + \eta^2 \boldsymbol{I})$$

Notation

(*Bandler et al., 1995*)

$$\boldsymbol{f}^{(j)} = \boldsymbol{x}_c^{(j)} - \boldsymbol{x}_c^*$$

$$\boldsymbol{h}^{(j)} = \boldsymbol{x}_f^{(j+1)} - \boldsymbol{x}_f^{(j)}$$

$$\boldsymbol{B}^{(j)}\boldsymbol{h}^{(j)} = -\boldsymbol{f}^{(j)}$$

Proposed Algorithm (*Bandler et al., 2002*)

Step 1 set $j = 1$, $\boldsymbol{B} = \boldsymbol{I}$ for the PE process

Step 2 obtain the optimal coarse model design $\boldsymbol{x}_c^*$

Step 3 set $\boldsymbol{x}_f^{(1)} = \boldsymbol{x}_c^*$

Step 4 if derivatives exist execute GPE
otherwise, execute the traditional PE with $\lambda = 0$

Step 5 initialize the mapping matrix $\boldsymbol{B}$ exploiting derivatives

Step 6 stop if

$$\left\| \boldsymbol{f}^{(j)} \right\| < \varepsilon$$

Proposed Algorithm (continued)

Step 7 evaluate $\boldsymbol{h}^{(j)}$ using quasi-Newton step

$$\boldsymbol{B}^{(j)}\boldsymbol{h}^{(j)} = -\boldsymbol{f}^{(j)}$$

Step 8 find the next $\boldsymbol{x}_f^{(j+1)}$

Step 9 perform GPE or PE as in Step 4

Step 10 if derivatives exist obtain $\boldsymbol{B}^{(j)}$ using

$$\boldsymbol{B}^{(j)} = (\boldsymbol{J}_c^{(j)T}\boldsymbol{J}_c^{(j)})^{-1}\boldsymbol{J}_c^{(j)T}\boldsymbol{J}_f^{(j)}$$

otherwise update $\boldsymbol{B}^{(j)}$ using a Broyden formula

Proposed Algorithm (continued)

Step 11 set $j = j+1$ and go to *Step* 6

the result is the solution $\bar{\boldsymbol{x}}_f$ and mapping matrix $\boldsymbol{B}$

Original Rosenbrock Function (Coarse Model)

(*Bandler et al., 1999*)

$$R_c(\boldsymbol{x}_c) = 100(x_2 - x_1^2)^2 + (1 - x_1)^2$$

where $\boldsymbol{x}_c = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ and $\boldsymbol{x}_c^* = \begin{bmatrix} 1.0 \\ 1.0 \end{bmatrix}$

Shifted Rosenbrock Function (Fine Model)

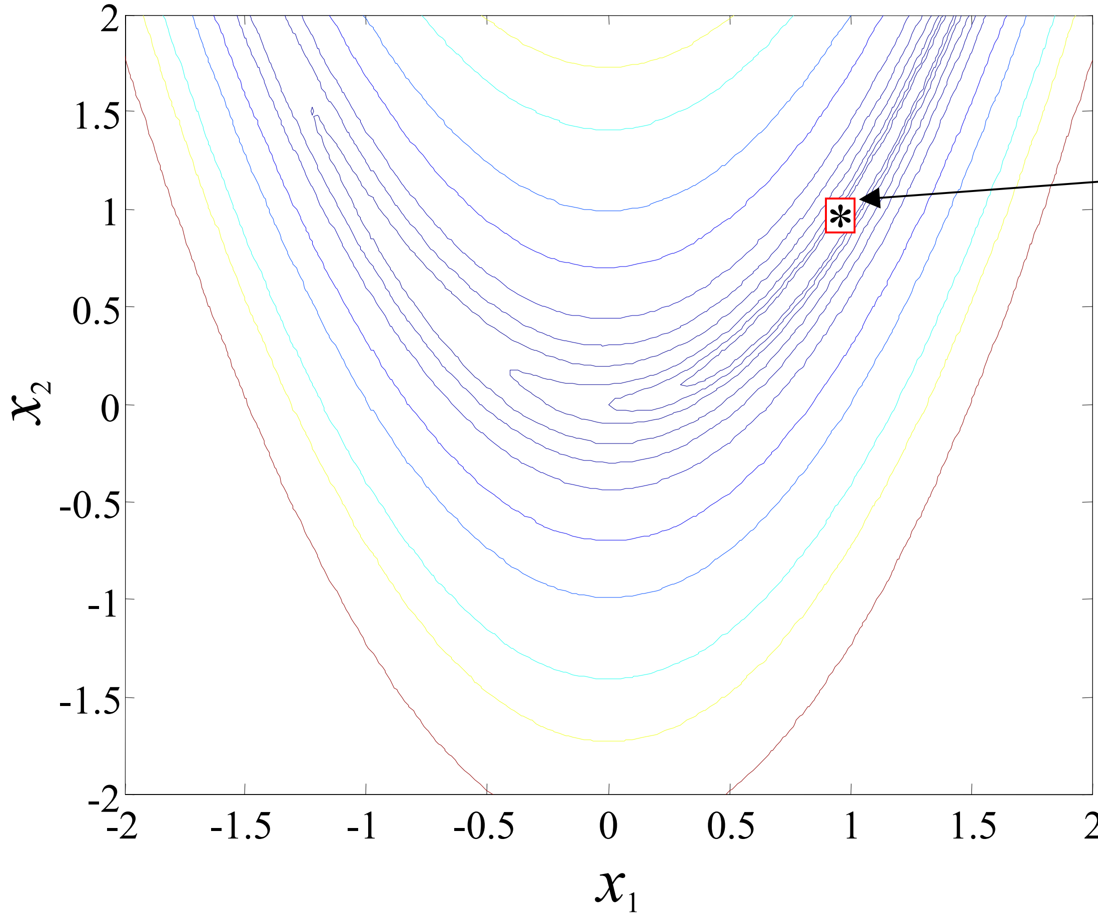
(*Bandler et al., 1999*)

$$R_f(\boldsymbol{x}_f) = 100\left((x_2 + \alpha_2) - (x_1 + \alpha_1)^2\right)^2 + \left(1 - (x_1 + \alpha_1)\right)^2$$

where $\boldsymbol{x}_f = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$, $\begin{bmatrix} \alpha_1 \\ \alpha_2 \end{bmatrix} = \begin{bmatrix} -0.2 \\ 0.2 \end{bmatrix}$ hence $\boldsymbol{x}_f^* = \begin{bmatrix} 1.2 \\ 0.8 \end{bmatrix}$

Original Rosenbrock Function (Coarse Model Contour Plot)

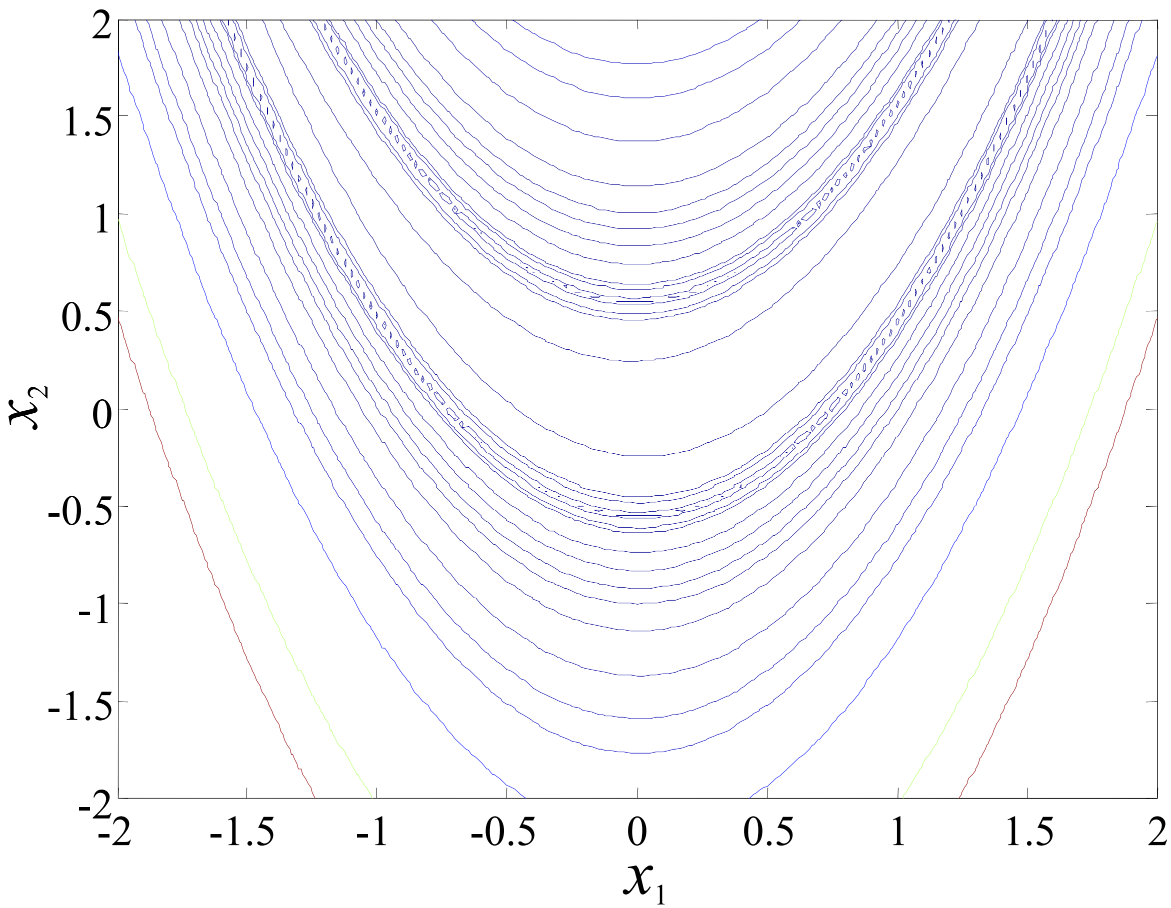
(*Bandler et al., 1999*)



$$\boldsymbol{x}_c^* = \begin{bmatrix} 1.0 \\ 1.0 \end{bmatrix}$$

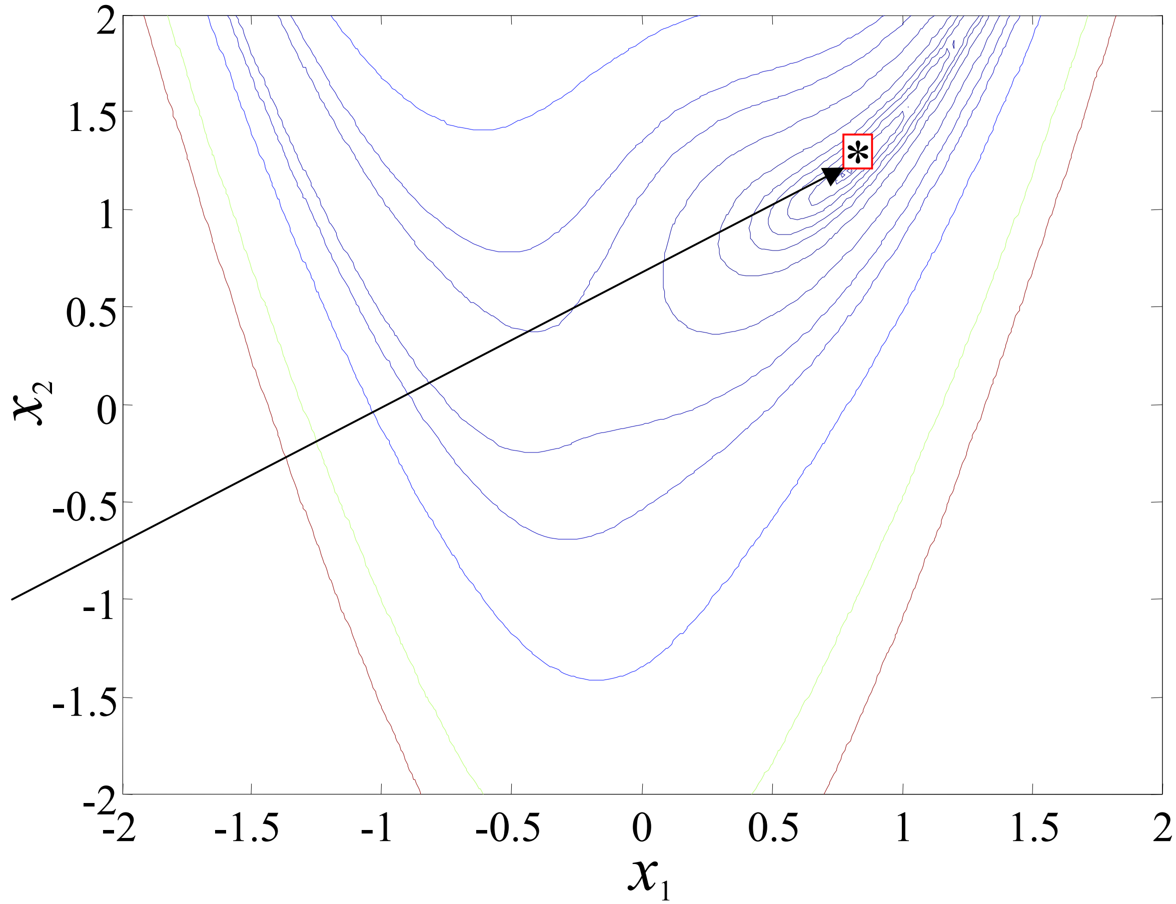
$$R_c(\boldsymbol{x}_c^*) = 0$$

Shifted Rosenbrock Function (*Bandler et al., 2002*)
Single point PE (SPE): nonuniqueness exists



Shifted Rosenbrock Function (*Bandler et al., 2002*)

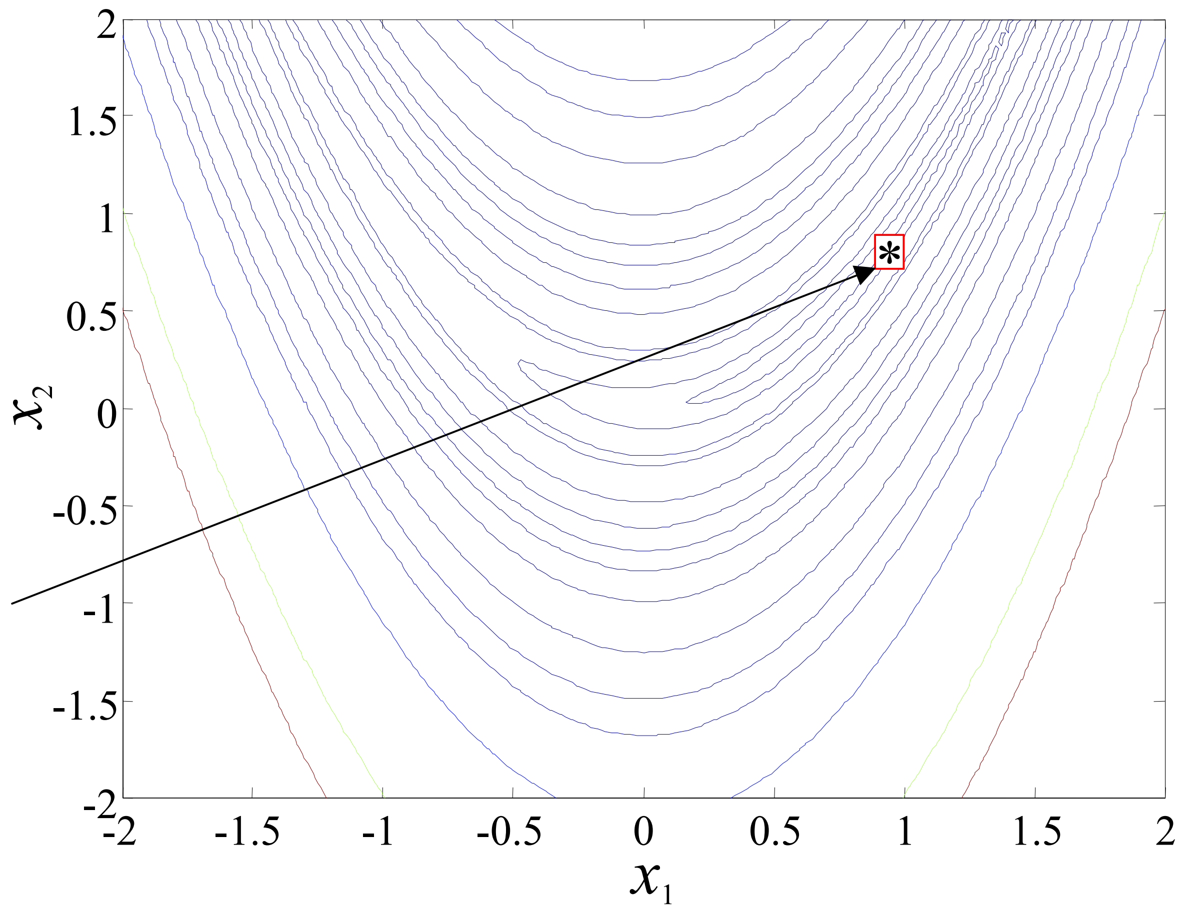
Gradient PE (1st iteration)



$$\boldsymbol{x}_c^{(1)} = \begin{bmatrix} 0.8 \\ 1.2 \end{bmatrix}$$

Shifted Rosenbrock Function (*Bandler et al., 2002*)

Gradient PE (2nd iteration)



$$\boldsymbol{x}_c^{(2)} = \begin{bmatrix} 1.0 \\ 1.0 \end{bmatrix}$$

Shifted Rosenbrock Function Results

(*Bandler et al., 2002*)

iteration	$\boldsymbol{x}_c^{(j)}$	$\boldsymbol{f}^{(j)}$	$\boldsymbol{B}^{(j)}$	$\boldsymbol{h}^{(j)}$	$\boldsymbol{x}_f^{(j)}$	R_f
0	$\begin{bmatrix}1.0\\1.0\end{bmatrix}$	---	---	---	$\begin{bmatrix}1.0\\1.0\end{bmatrix}$	31.4
1	$\begin{bmatrix}0.8\\1.2\end{bmatrix}$	$\begin{bmatrix}-0.2\\0.2\end{bmatrix}$	$\begin{bmatrix}1.0 & 0.0\\0.0 & 1.0\end{bmatrix}$	$\begin{bmatrix}0.2\\-0.2\end{bmatrix}$	$\begin{bmatrix}1.2\\0.8\end{bmatrix}$	0
	$\begin{bmatrix}1.0\\1.0\end{bmatrix}$	$\begin{bmatrix}0\\0\end{bmatrix}$				

Transformed Rosenbrock Function (Fine Model)

(*Bandler et al., 2002*)

linear transformation of the original Rosenbrock function

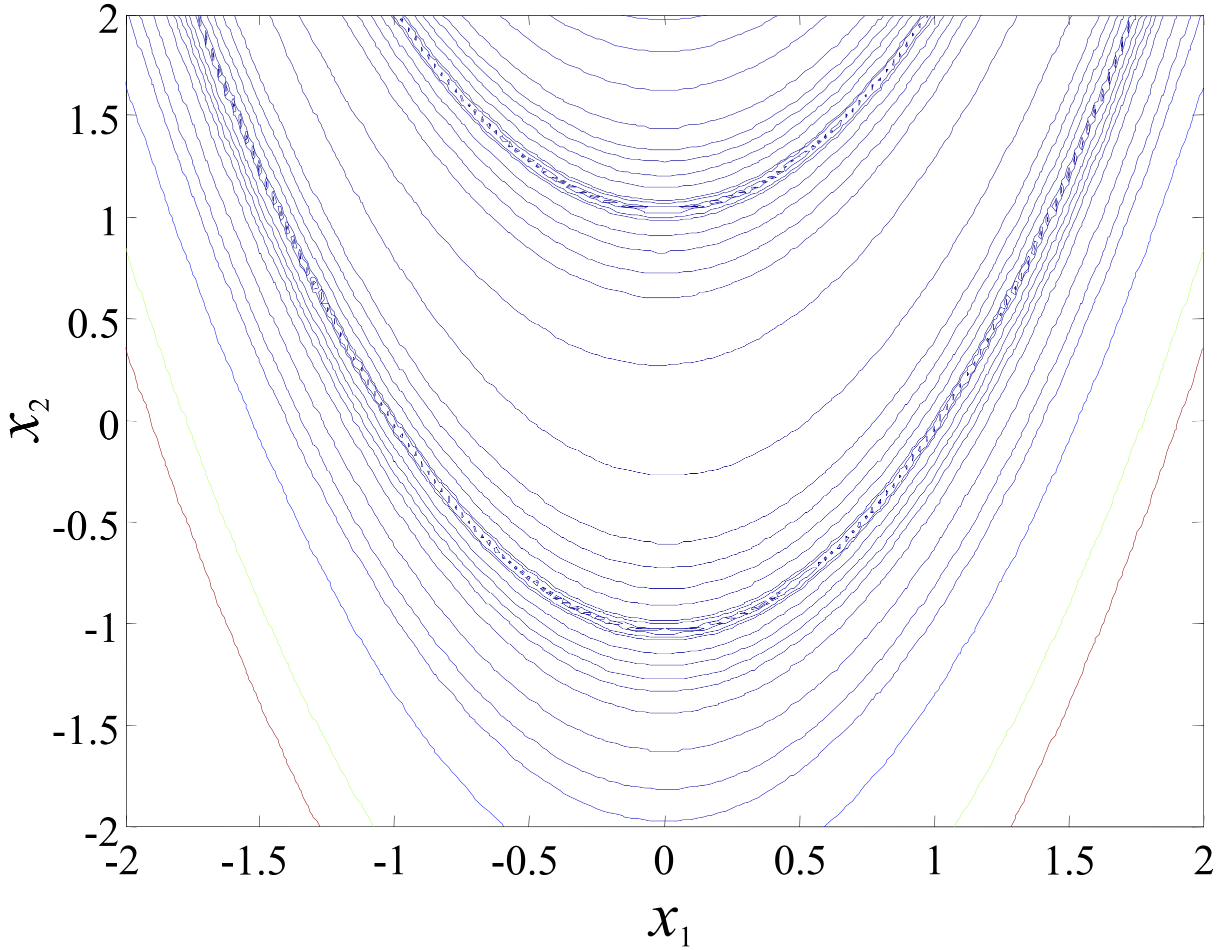
$$R_f(\boldsymbol{x}_f) = 100(u_2 - u_1^2)^2 + (1 - u_1)^2$$

$$\text{where } \boldsymbol{u} = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} 1.1 & -0.2 \\ 0.2 & 0.9 \end{bmatrix} \boldsymbol{x}_f + \begin{bmatrix} -0.3 \\ 0.3 \end{bmatrix}$$

$$\boldsymbol{x}_f^* = \begin{bmatrix} 1.2718447 \\ 0.4951456 \end{bmatrix}$$

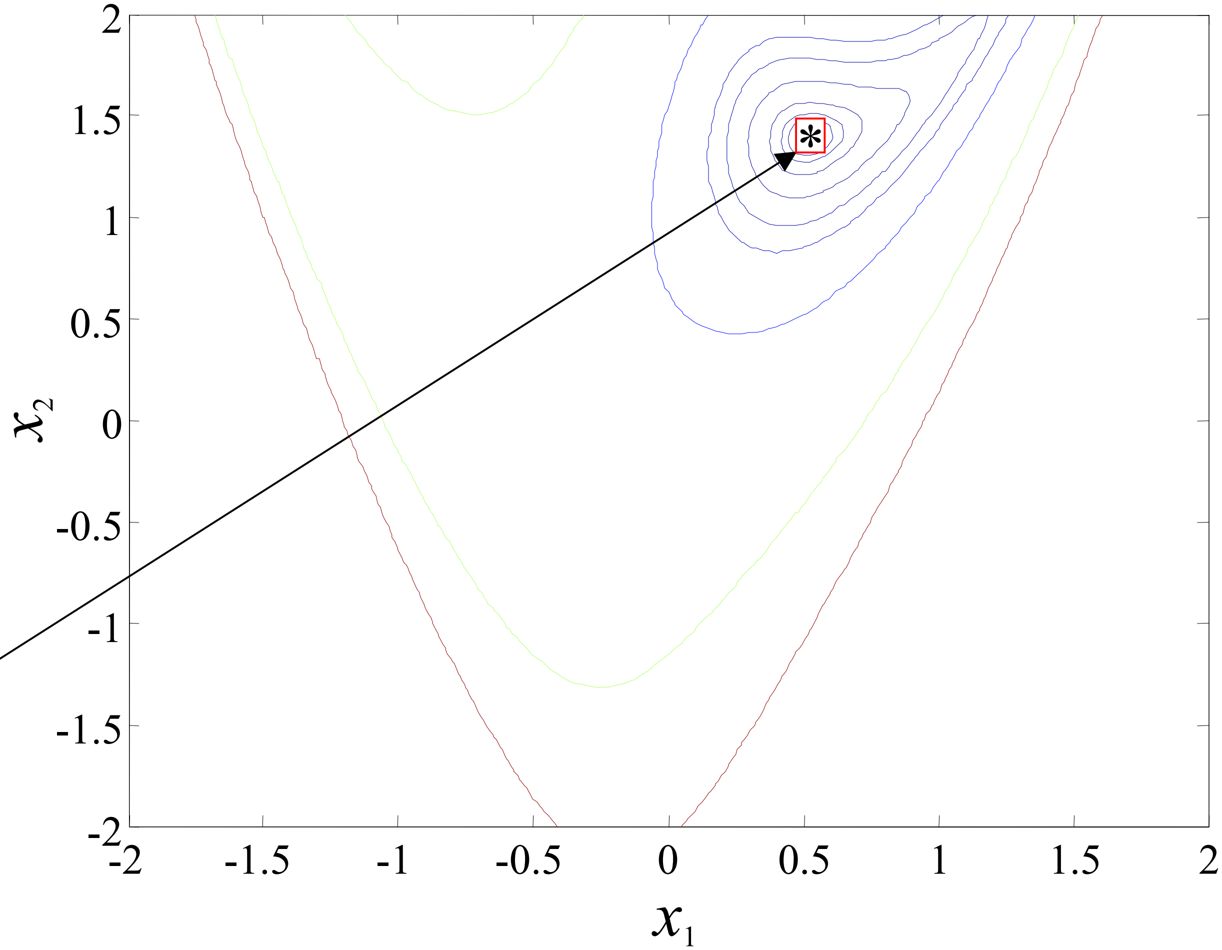
Transformed Rosenbrock Function (*Bandler et al., 2002*)

Single point PE (SPE): nonuniqueness exists



Transformed Rosenbrock Function (*Bandler et al., 2002*)

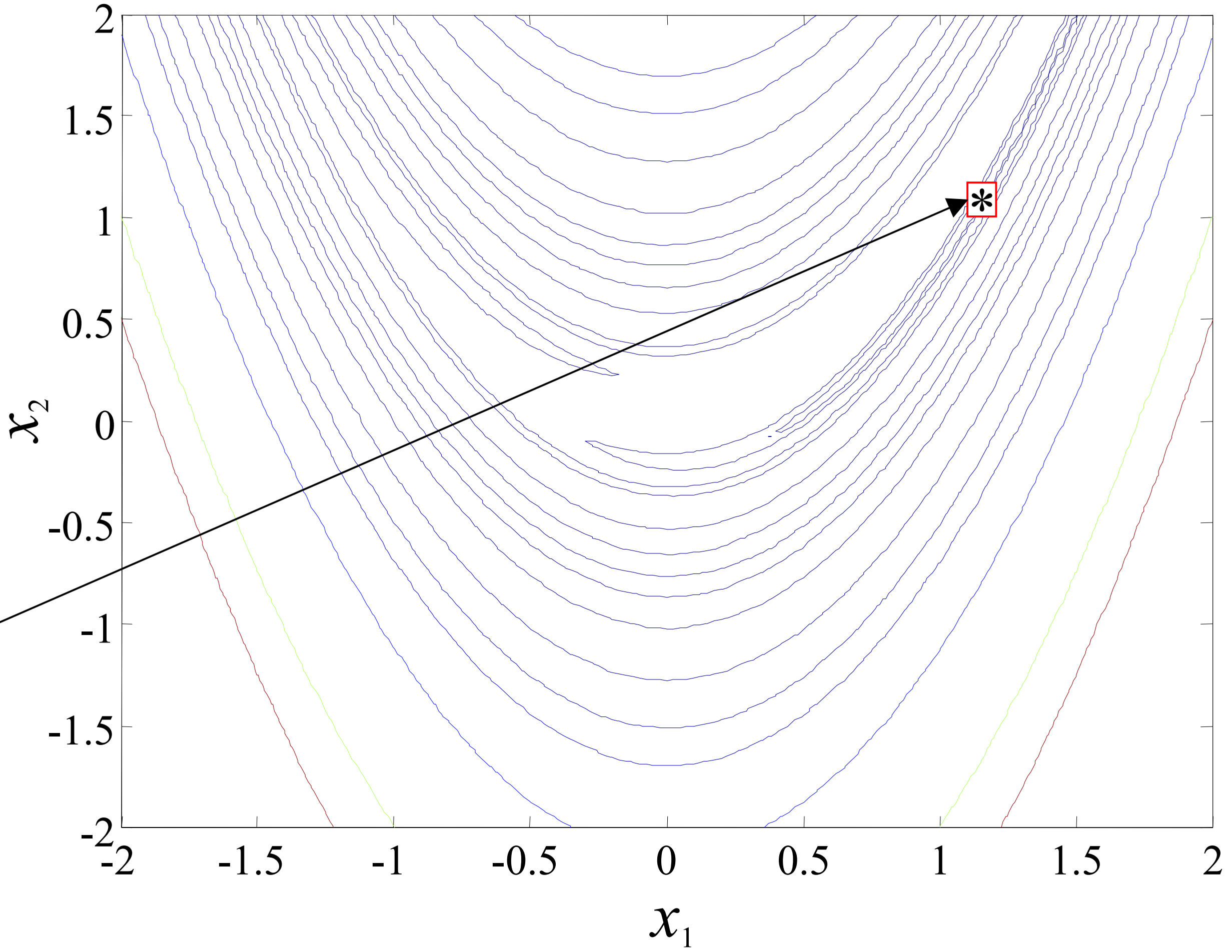
GPE (1st PE iteration)



$$\boldsymbol{x}_c^{(1)} = \begin{bmatrix} 0.53 \\ 1.38 \end{bmatrix}$$

Transformed Rosenbrock Function (*Bandler et al., 2002*)

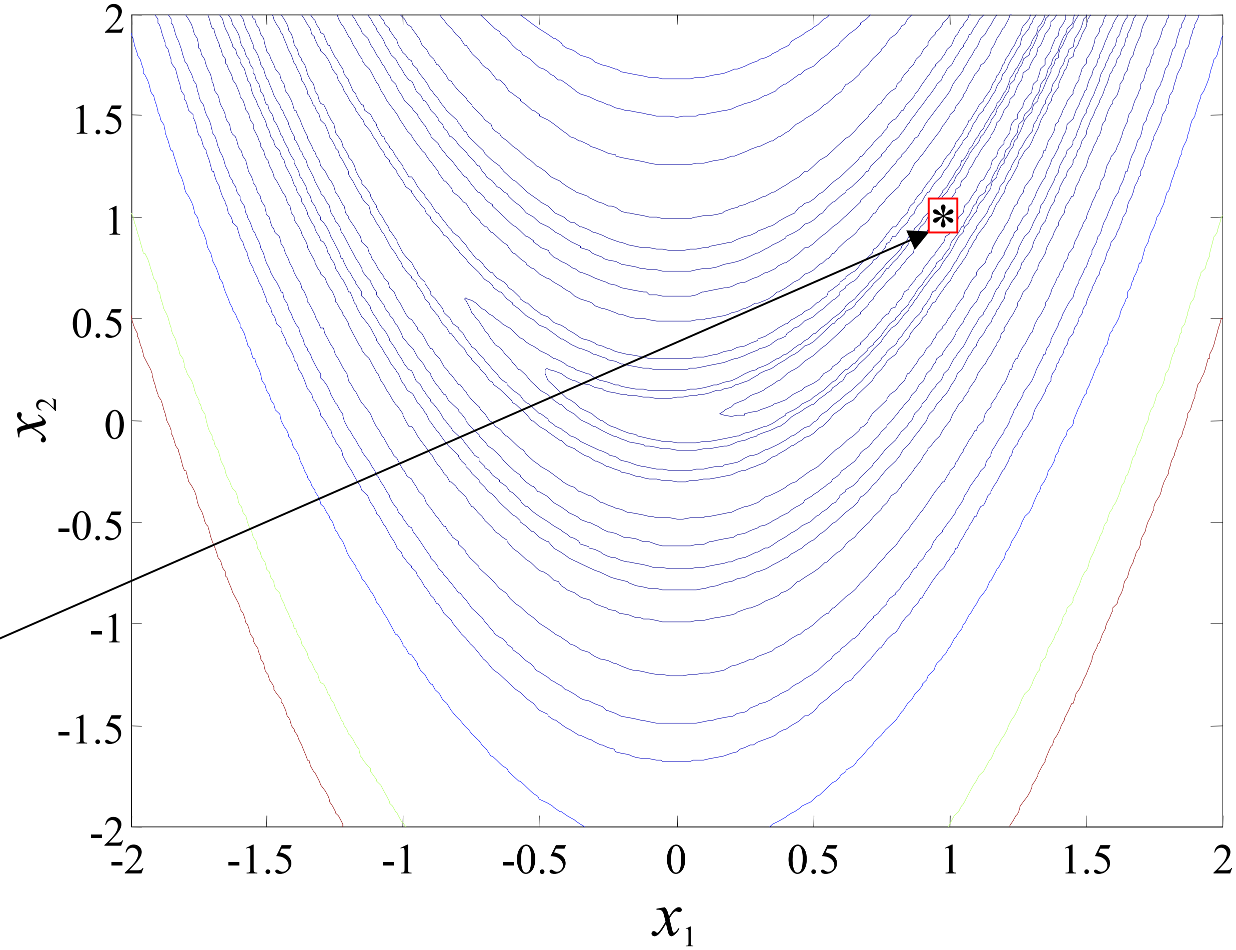
GPE (2nd PE iteration)



$$\boldsymbol{x}_c^{(2)} = \begin{bmatrix} 1.18 \\ 1.18 \end{bmatrix}$$

Transformed Rosenbrock Function (*Bandler et al., 2002*)

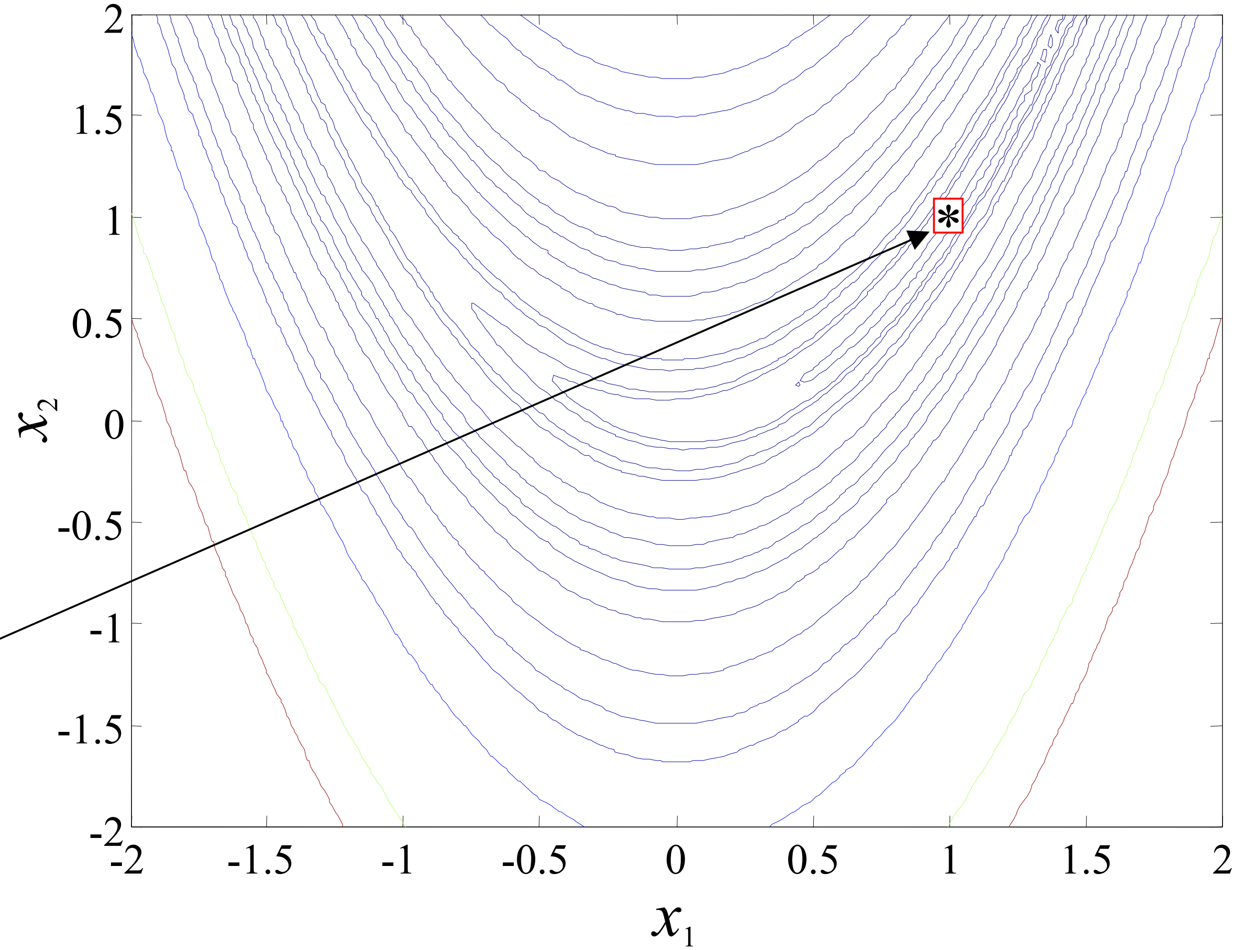
GPE (3rd PE iteration)



$$\boldsymbol{x}_c^{(3)} = \begin{bmatrix} 0.967 \\ 0.929 \end{bmatrix}$$

Transformed Rosenbrock Function (*Bandler et al., 2002*)

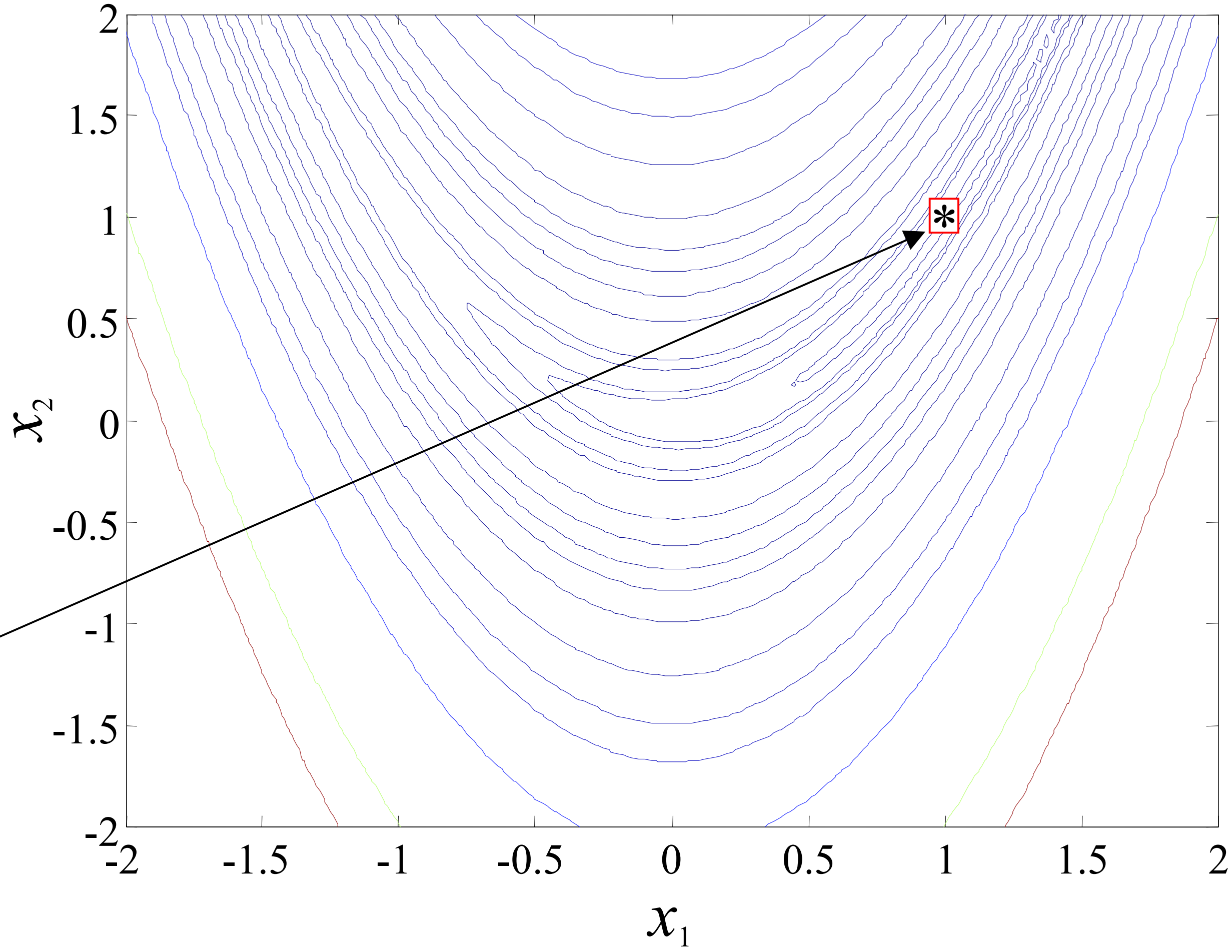
GPE (4th PE iteration)



$$\boldsymbol{x}_c^{(4)} = \begin{bmatrix} 1.0009 \\ 1.0018 \end{bmatrix}$$

Transformed Rosenbrock Function (*Bandler et al., 2002*)

GPE (5th and 6th PE iteration)



$$\boldsymbol{x}_c^{(5)} = \begin{bmatrix} 1.00002 \\ 1.00004 \end{bmatrix}$$

$$\boldsymbol{x}_c^{(6)} = \begin{bmatrix} 1.0 \\ 1.0 \end{bmatrix}$$

Transformed Rosenbrock Results (*Bandler et al., 2002*)

iteration	$\boldsymbol{x}_c^{(j)}$	$\boldsymbol{f}^{(j)}$	$\boldsymbol{B}^{(j)}$	$\boldsymbol{h}^{(j)}$	$\boldsymbol{x}_f^{(j)}$	R_f
0	$\begin{bmatrix}1.0\\1.0\end{bmatrix}$	---	---	---	$\begin{bmatrix}1.0\\1.0\end{bmatrix}$	108.3
1	$\begin{bmatrix}0.526\\1.384\end{bmatrix}$	$\begin{bmatrix}-0.474\\0.384\end{bmatrix}$	$\begin{bmatrix}1.01 & -0.05\\0.01 & 1.01\end{bmatrix}$	$\begin{bmatrix}0.447\\-0.385\end{bmatrix}$	$\begin{bmatrix}1.447\\0.615\end{bmatrix}$	5.119
2	$\begin{bmatrix}1.185\\1.178\end{bmatrix}$	$\begin{bmatrix}0.185\\0.178\end{bmatrix}$	$\begin{bmatrix}0.96 & -0.12\\-0.096 & 1.06\end{bmatrix}$	$\begin{bmatrix}-0.218\\-0.187\end{bmatrix}$	$\begin{bmatrix}1.23\\0.427\end{bmatrix}$	4.4E–3
3	$\begin{bmatrix}0.967\\0.929\end{bmatrix}$	$\begin{bmatrix}-0.033\\-0.071\end{bmatrix}$	$\begin{bmatrix}1.09 & -0.19\\0.168 & 0.92\end{bmatrix}$	$\begin{bmatrix}0.0429\\0.0697\end{bmatrix}$	$\begin{bmatrix}1.273\\0.4970\end{bmatrix}$	1.8E–6
4	$\begin{bmatrix}1.001\\1.001\end{bmatrix}$	$\begin{bmatrix}0.001\\0.001\end{bmatrix}$	$\begin{bmatrix}1.10001 & -0.1999\\0.1999 & 0.9001\end{bmatrix}$	$\begin{bmatrix}-0.001\\-0.002\end{bmatrix}$	$\begin{bmatrix}1.2719\\0.4952\end{bmatrix}$	5E–10

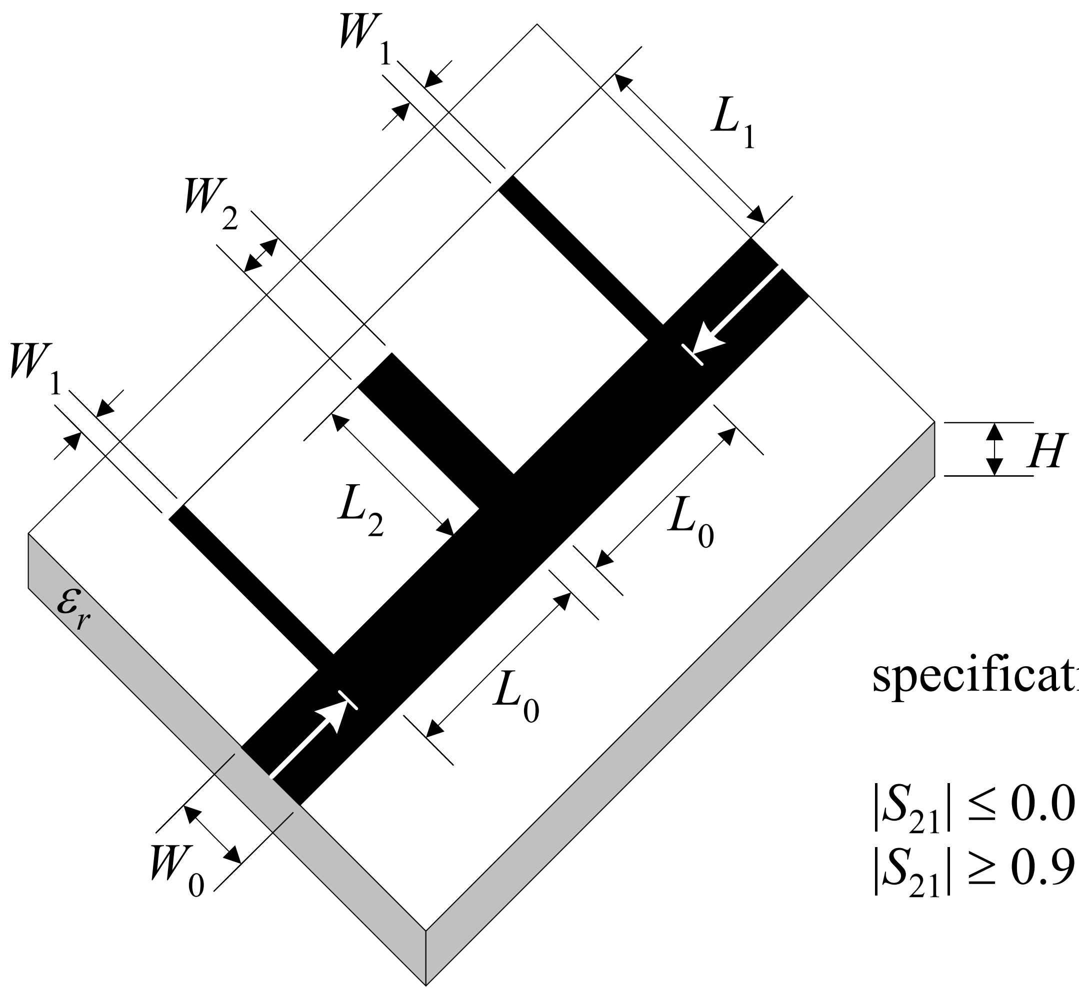
Transformed Rosenbrock Results (*Bandler et al., 2002*)

iteration	$\boldsymbol{x}_c^{(j)}$	$\boldsymbol{f}^{(j)}$	$\boldsymbol{B}^{(j)}$	$\boldsymbol{h}^{(j)}$	$\boldsymbol{x}_f^{(j)}$	R_f
5	$\begin{bmatrix}1.00002\\1.00004\end{bmatrix}$	$1\text{E}-4\times\begin{bmatrix}0.2\\0.4\end{bmatrix}$	$\begin{bmatrix}1.1 & -0.2\\0.2 & 0.9\end{bmatrix}$	$1\text{E}-4\times\begin{bmatrix}0.3\\0.5\end{bmatrix}$	$\begin{bmatrix}1.2718\\0.4951\end{bmatrix}$	3E–17
6	$\begin{bmatrix}1.0\\1.0\end{bmatrix}$	$1\text{E}-8\times\begin{bmatrix}0.1\\0.3\end{bmatrix}$	$\begin{bmatrix}1.1 & -0.2\\0.2 & 0.9\end{bmatrix}$	$1\text{E}-8\times\begin{bmatrix}0.2\\0.3\end{bmatrix}$	$\boldsymbol{x}_f^*$	9E–29

$$\boldsymbol{x}_f^* = \begin{bmatrix}1.27184466\\0.49514563\end{bmatrix}$$

Bandstop Microstrip Filter with Quarter-Wave Open Stubs

(*Bakr et al., 2000*)



H = 25 mil, W_0 = 25 mil,
ε_r = 9.4 (alumina)

the design parameters are
$\boldsymbol{x}_f = [W_1 \; W_2 \; L_0 \; L_1 \; L_2]^T$

specifications

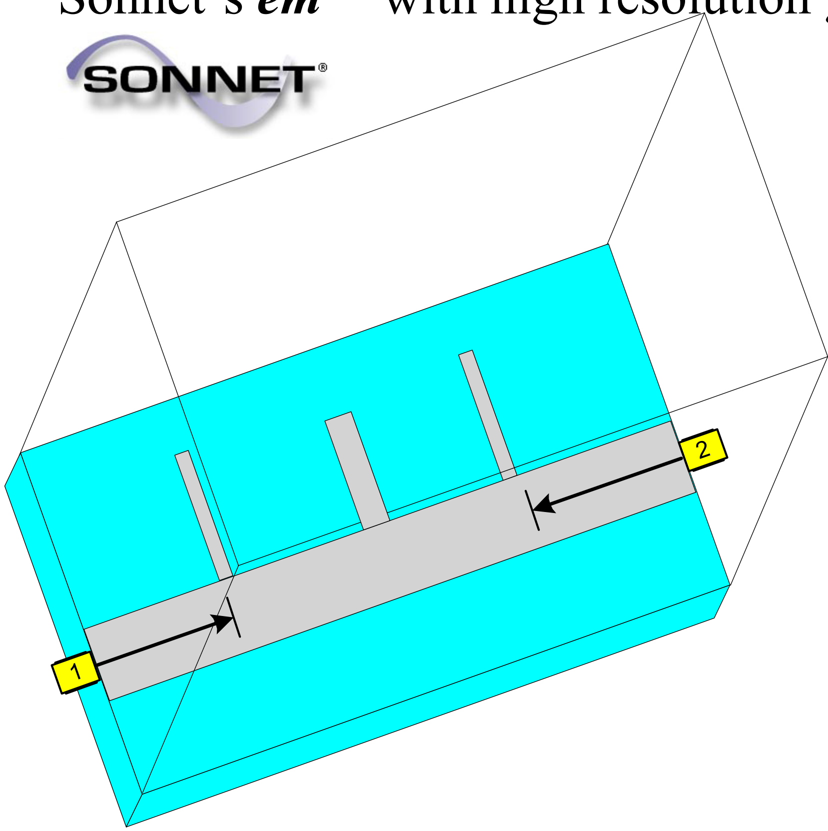
$|S_{21}| \leq 0.05$ for 9.3 GHz $\leq \omega \leq$ 10.7 GHz
$|S_{21}| \geq 0.9$ for $\omega \leq$ 8 GHz and $\omega \geq$ 12 GHz

Bandstop Microstrip Filter: Fine and Coarse Models

(*Bakr et al., 2000*)

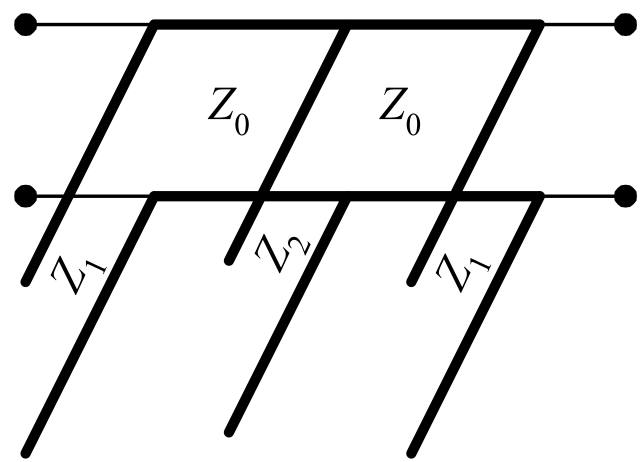
fine model:

Sonnet's ***em***™ with high resolution grid



coarse model:

OSA90/hope™ ideal transmission line sections and empirical formulas



Optimization of the Bandstop Filter
(*Bandler et al., 2002*)

finite differences estimate the fine and coarse Jacobians

use hybrid approach to update mapping

the final mapping is

$$\boldsymbol{B} = \begin{bmatrix} 0.532 & -0.037 & 0.026 & 0.017 & -0.006 \\ -0.051 & 0.543 & 0.022 & -0.032 & 0.026 \\ 0.415 & 0.251 & 1.024 & 0.073 & 0.011 \\ 0.169 & -0.001 & -0.022 & 0.963 & 0.008 \\ -0.213 & -0.003 & -0.045 & -0.052 & 0.958 \end{bmatrix}$$

Optimization of the Bandstop Filter (continued)

(*Bandler et al., 2002*)

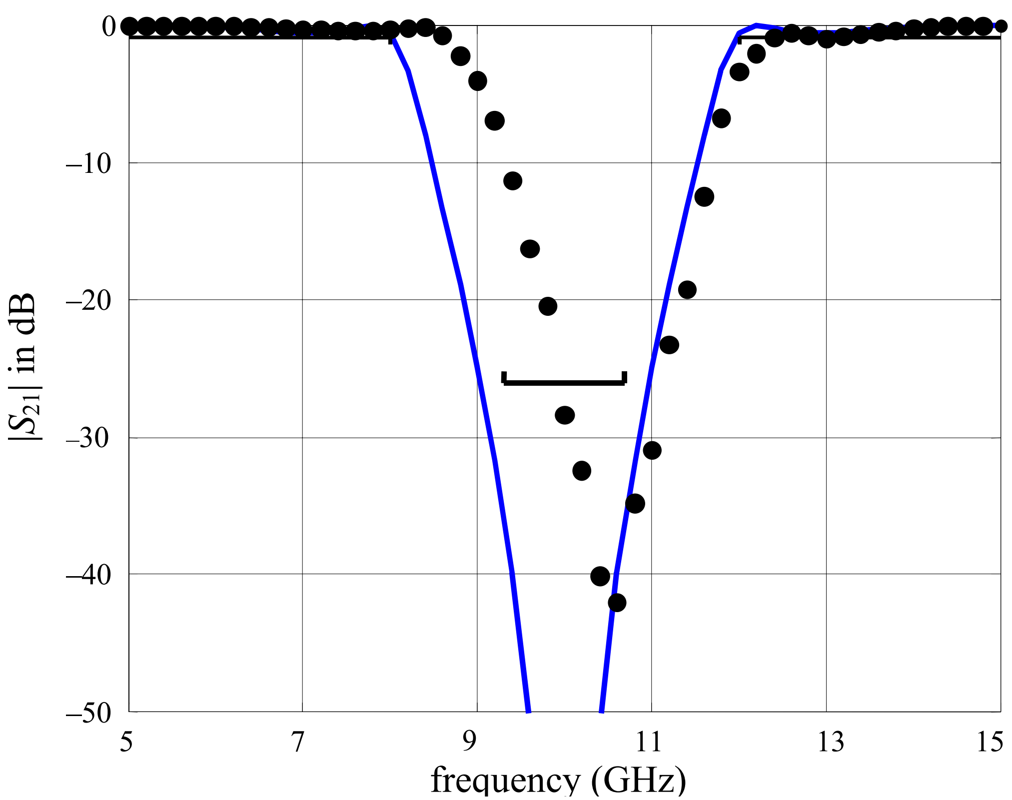
initial and final designs

Parameter	$x_f^{(0)}$	$x_f^{(5)}$
W_1	4.560	8.7464
W_2	9.351	19.623
L_0	107.80	97.206
L_1	111.03	116.13
L_2	108.75	113.99

All values are in mils

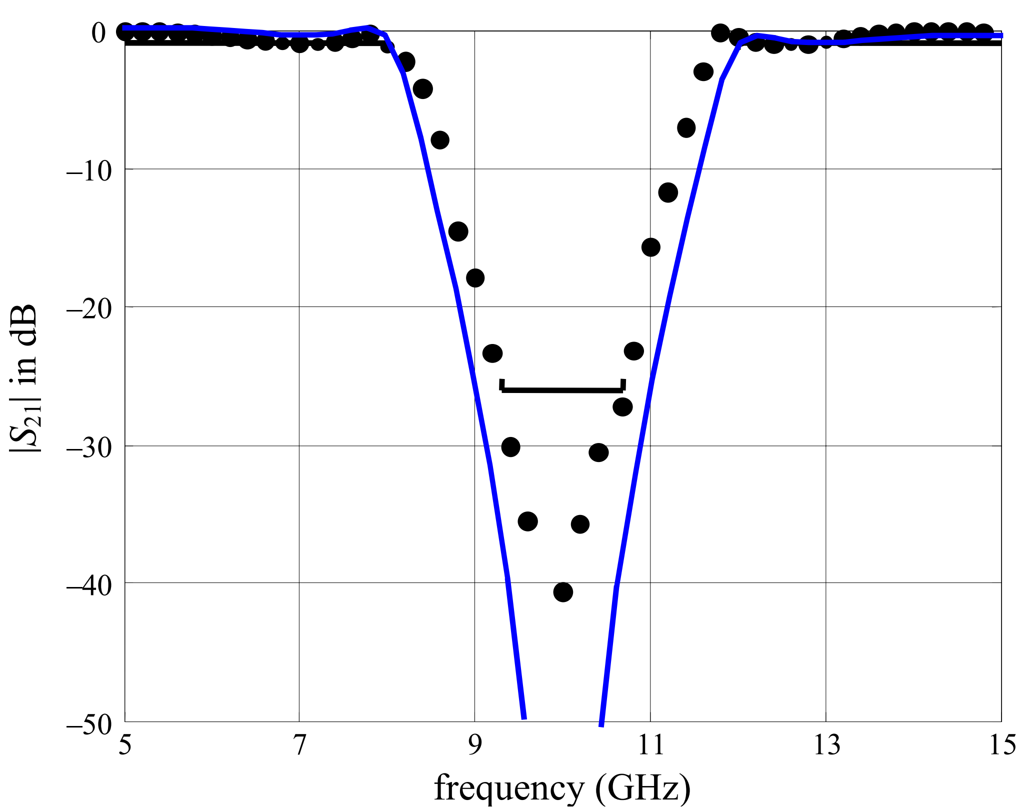
Optimization of the Bandstop Filter (*Bandler et al., 2002*)

initial coarse model OSA90™ response (–)
initial fine response ***em***™ (•)



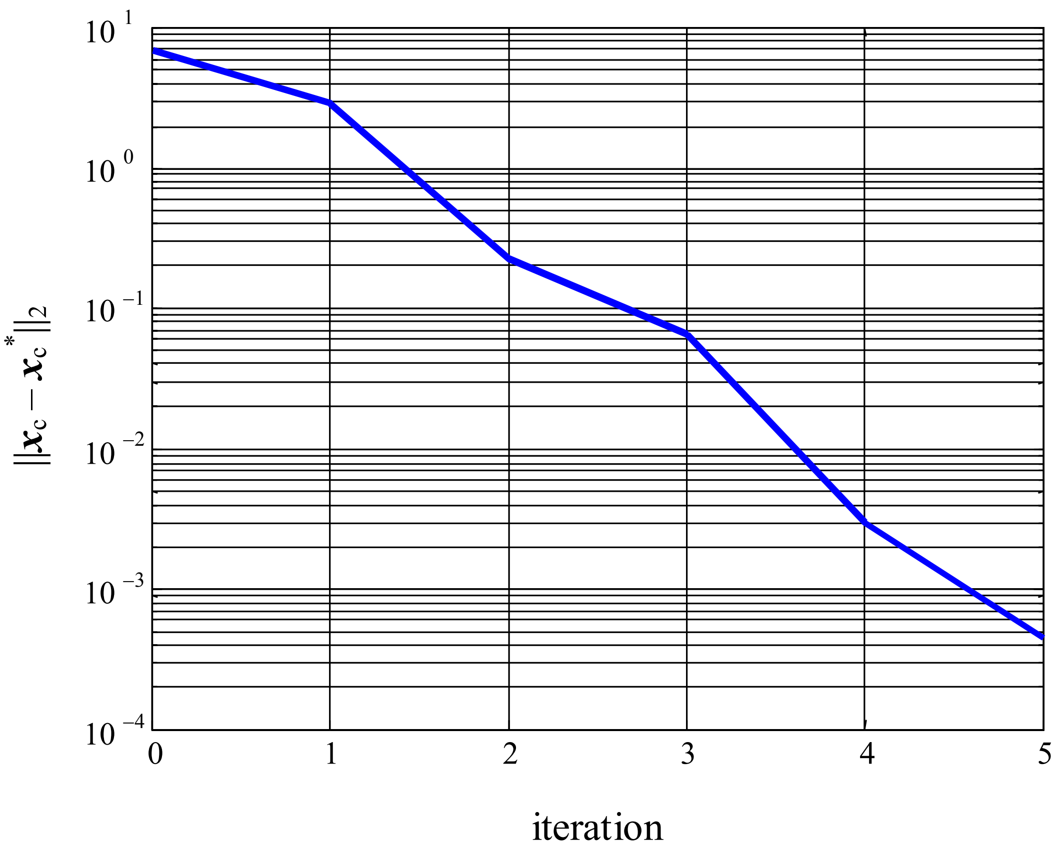
Optimization of the Bandstop Filter (*Bandler et al., 2002*)

initial coarse model OSA90™ response (–)
final fine response ***em***™ (●)



Optimization of the Bandstop Filter (*Bandler et al., 2002*)

$\|\boldsymbol{x}_c - \boldsymbol{x}_c^*\|_2$ versus iteration for the bandstop microstrip filter



Conclusions

new Aggressive Space Mapping techniques

Gradient Parameter Extraction (GPE) exploiting available Jacobians (exact or approximate)

consideration of mapping updates

available Jacobians can be used to build the mapping

Reference

[1] J.W. Bandler, A.S. Mohamed, M.H. Bakr, K. Madsen and J. Søndergaard, "EM-based optimization exploiting partial space mapping and exact sensitivities," *IEEE MTT-S Int. Microwave Symp*. (Seattle, WA, June 2002), pp. 2101-2104.

Mapping Update By Constraining *B*
(Bakr et al., 2000)

to constrain the mapping matrix to be close to ***I***

$$\boldsymbol{B} = \arg\min_{\boldsymbol{B}} \left\| [\boldsymbol{e}_1^T \cdots \boldsymbol{e}_n^T \; \eta\Delta\boldsymbol{b}_1^T \cdots \eta\Delta\boldsymbol{b}_n^T]^T \right\|_2^2$$

where η is a weighting factor, $\boldsymbol{e}_i$ and $\Delta\boldsymbol{b}_i$ are the ith columns of $\boldsymbol{E}$ and $\Delta\boldsymbol{B}$

$$\boldsymbol{E} = \boldsymbol{J}_f - \boldsymbol{J}_c\boldsymbol{B}$$

$$\Delta\boldsymbol{B} = \boldsymbol{B} - \boldsymbol{I}$$

analytical solution is

$$\boldsymbol{B} = (\boldsymbol{J}_c^T\boldsymbol{J}_c + \eta^2\boldsymbol{I})^{-1}(\boldsymbol{J}_c^T\boldsymbol{J}_f + \eta^2\boldsymbol{I})$$