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INTEGRATED APPROACH TO MICROWAVE DESIGN

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Abstract A new, integrated approach to microwave design is presented involving concepts such as optimal design centering, optimal design tolerancing, optimal design tuning, parasitic effects, uncertainties in circuit models, and mismatched terminations. The approach is of the worst case type, and previously published design schemes fall out as particular cases of the ideas presented. The mathematical and computational complexity as well as the benefits realized by our approach is illustrated by transformer examples, including a realistic stripline circuit.

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I. INTRODUCTION

The use of nonlinear programming techniques for the realistic design of microwave circuits has been well established. Applications hitherto reported by the authors, for example, fall into two categories: (1) the improvement of a response in the presence of parasitics [1 - 2]. In this case the function to be minimized is of the error function type and the constraints, if any, are normally imposed on the design parameters. (2) design centering and tolerance assignment to yield a minimum cost circuit that satisfies certain specifications, usually imposed on the frequency response, for all possible values of the actual parameters [3]. The function to be minimized is of the cost function type and the constraints are due to the specifications.

Tuning elements may be introduced to further increase the tolerances and thus decrease the cost or make a circuit meet specifications [4]. No consideration, however, of optimal tolerancing or tuning of microwave circuits has been reported, where parasitic effects were taken into account. A major complication is introduced here, since the models available for common parasitic elements normally include uncertainties on the value of the model parameters. These uncertainties are due to the fact that the model is usually only approximate and that approximations have to be made in the implementation of existing model formulas. A typical example of the latter is the relationship between the characteristic impedance and width of a symmetric stripline, where the exact formula involves elliptic integrals.

The model uncertainties can well be of the same order of magnitude as the tolerances on the physical network parameters so that a realistic design, including tolerances, can only be found when allowance is made for them.

In the approach adopted, an attempt is made to deal with the model uncertainties in the same way as with the other tolerances. This involves, however, a complication in the formulation of the problem. The physical tolerances, which can be fixed or variable, affect the physical parameters, which can also be fixed or variable, whereas the model uncertainties affect a set of intermediate parameters (which will be called the model parameters) in the calculation of the response.

In the present paper we consider design of microwave circuits with the following concepts treated as an integral part of the design process: optimal design centering, optimal design tolerancing, optimal design tuning, parasitic effects, uncertainties in the circuit modeling, and mismatches at the source and the load.

II. THEORY

The Tolerance-Tuning Problem

In this section we introduce some of the notation and briefly review the parameters involved in the tolerance-tuning problem.

We consider first a vector of nominal design parameters $\underset{\sim}{\phi}^0$ and a corresponding vector containing the manufacturing tolerances $\underset{\sim}{\varepsilon}$. Thus, for k variables,

$$\underset{\sim}{\phi}^0 \triangleq \begin{bmatrix} \phi_1^0 \\ \phi_2^0 \\ \vdots \\ \phi_k^0 \end{bmatrix}, \qquad \underset{\sim}{\varepsilon} \triangleq \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \vdots \\ \varepsilon_k \end{bmatrix}. \tag{1}$$

A possible outcome of a design is then

$$\underset{\sim}{\phi} = \underset{\sim}{\phi}^0 + \underset{\sim}{E}\, \underset{\sim}{\mu}_\varepsilon, \tag{2}$$

where

$$\underset{\sim}{\mu}_\varepsilon \triangleq \begin{bmatrix} \mu_{\varepsilon_1} \\ \mu_{\varepsilon_2} \\ \vdots \\ \mu_{\varepsilon_k} \end{bmatrix} \tag{3}$$

and

$$\underset{\sim}{E} \triangleq \begin{bmatrix} \varepsilon_1 & & & \\ & \varepsilon_2 & & \\ & & \ddots & \\ & & & \varepsilon_k \end{bmatrix} . \tag{4}$$

The vector $\underset{\sim}{\mu}_\varepsilon$ determines the actual outcome and can, for example, be bounded by

$$-1 \leq \mu_{\varepsilon_i} \leq 1, \ i = 1,2,\ldots,k. \tag{5}$$

It is assumed that the designer has no control over $\underset{\sim}{\mu}_\varepsilon$. This leads to the concept of the tolerance region R_ε, namely, the set of points $\underset{\sim}{\phi}$ of (2) subject to (5). An untuned design implies $\underset{\sim}{\phi}$ as given by (2). Consider a vector $\underset{\sim}{t}$ containing tuning variables corresponding to (1). Thus

$$\underset{\sim}{t} \triangleq \begin{bmatrix} t_1 \\ t_2 \\ \vdots \\ t_k \end{bmatrix} . \tag{6}$$

A design outcome with tuning implies

$$\underset{\sim}{\phi} = \underset{\sim}{\phi}^0 + \underset{\sim}{E}\,\underset{\sim}{\mu}_\varepsilon + \underset{\sim}{T}\,\underset{\sim}{\mu}_t, \tag{7}$$

where

$$\underset{\sim}{\mu}_t \triangleq \begin{bmatrix} \mu_{t_1} \\ \mu_{t_2} \\ \vdots \\ \mu_{t_k} \end{bmatrix} \tag{8}$$

and

$$\underset{\sim}{T} \triangleq \begin{bmatrix} t_1 & & & \\ & t_2 & & \\ & & \ddots & \\ & & & t_k \end{bmatrix} . \tag{9}$$

The vector $\underset{\sim}{\mu}_t$ controls the tuning and we consider, for convenience,

$$-1 \leq \mu_{t_i} \leq 1 \ , \ i = 1,2,\ldots,k. \tag{10}$$

Hence, we have a tuning region R_t centered at $\underset{\sim}{\phi}^0 + \underset{\sim}{E}\underset{\sim}{\mu}_\varepsilon$.

The worst-case tolerance-tuning problem is to obtain an optimal set $\{\underset{\sim}{\phi}^0, \underset{\sim}{\varepsilon}, \underset{\sim}{t}\}$ such that all possible outcomes (controlled by $\underset{\sim}{\mu}_\varepsilon$) can be tuned so as to satisfy the design specifications (by adjusting $\underset{\sim}{\mu}_t$) if tuning is available. If tuning is not available all outcomes must satisfy the design specifications.

<u>Model Uncertainties</u>

Taking $\underset{\sim}{\phi}$ as the vector of physical design parameters, we may consider an n-dimensional vector $\underset{\sim}{p}$ containing the model parameters, e.g., the parameters appearing in an electrical equivalent circuit. In general, $n \neq k$. We have an associated vector of nominal model parameters $\underset{\sim}{p}^0$ and a vector of model uncertainties $\underset{\sim}{\delta}$, where

$$\underset{\sim}{p}^0 \triangleq \begin{bmatrix} p_1^0 \\ p_2^0 \\ \vdots \\ p_n^0 \end{bmatrix} , \qquad \underset{\sim}{\delta} \triangleq \begin{bmatrix} \delta_1 \\ \delta_2 \\ \vdots \\ \delta_n \end{bmatrix} . \tag{11}$$

A possible model can then be described by

$$\underset{\sim}{p} = \underset{\sim}{p}^0 + \underset{\sim}{\Delta}\, \underset{\sim}{\mu}_\delta, \tag{12}$$

where

$$\underset{\sim}{\mu}_\delta \triangleq \begin{bmatrix} \mu_{\delta_1} \\ \mu_{\delta_2} \\ \vdots \\ \mu_{\delta_n} \end{bmatrix} \tag{13}$$

and

$$\underset{\sim}{\Delta} \triangleq \begin{bmatrix} \delta_1 & & & \\ & \delta_2 & & \\ & & \ddots & \\ & & & \delta_n \end{bmatrix} . \tag{14}$$

Thus, $\underset{\sim}{\mu}_\delta$ determines the particular model under consideration. We will assume

$$-1 \leq \mu_{\delta_i} \leq 1, \; i = 1,2,\ldots,n, \tag{15}$$

and also the functional dependence on $\underset{\sim}{\phi}$ implied by

$$\underset{\sim}{p} = \underset{\sim}{p}^0(\underset{\sim}{\phi}) + \underset{\sim}{\Delta}(\underset{\sim}{\phi}) \, \underset{\sim}{\mu}_\delta . \tag{16}$$

Given a tolerance region in the $\underset{\sim}{\phi}$ space it would be hard, in general, to envisage its effect in the $\underset{\sim}{p}$ space, even if $\underset{\sim}{\delta} = \underset{\sim}{0}$. The selection of worst-case $\underset{\sim}{\phi}$ is complicated by the modeling uncertainties. The whole design problem is inevitably becoming more abstract. In selecting candidates for worst case, however, we will assume, for simplicity, that

$$\mu_{\varepsilon_i}, \mu_{\delta_j} = \pm 1 \;, \; i = 1,2,\ldots,k, j = 1,2,\ldots,n. \tag{17}$$

Mismatch Considerations

At this point, the discussion needs to become more concrete. We consider environmental influences on the behavior of a circuit in the form of mismatches at the source and load ends. The situation is depicted in Fig. 1.

Fig. 1 (a) shows the ideal situation of matched resistive terminations R_I and R_0. Assume that the actual complex terminations as seen by the circuit are Z_S and Z_L, as shown in Fig. 1 (b). Then

$$\rho_S = \frac{Z_S - R_I}{Z_S + R_I} \tag{18}$$

at the source, and

$$\rho_L = \frac{Z_L - R_0}{Z_L + R_0} \tag{19}$$

at the load. The actual reflection coefficient ρ at the source is given by

$$\rho = \frac{Z - Z_S^*}{Z + Z_S} \quad , \tag{20}$$

using the notation of Fig. 1(b). *denotes the complex conjugate.

Consider the situation depicted in Fig. 1(c). We have, for a matched source and input impedance Z

$$\rho_a = \frac{Z - R_I}{Z + R_I} \tag{21}$$

and

$$\rho_b = \frac{Z_L - Z'^*}{Z_L + Z'} \quad , \tag{22}$$

where Z' is the impedance at the output when the input is matched. Associated with the latter situation is the parameter s_{22} given by (Fig. 1 (a))

$$s_{22} = \frac{Z' - R_0}{Z' + R_0} \quad . \tag{23}$$

Using Carlin and Giordano [5] we may readily derive the following expressions. For all possible phases,

$$\frac{||\rho_a| - |\rho_S||}{1-|\rho_a|\ |\rho_S|} \leq |\rho| \leq \frac{|\rho_a| + |\rho_S|}{1+|\rho_a|\ |\rho_S|} \quad , \tag{24}$$

where, assuming a lossless circuit, $|\rho_a| = |\rho_b|$ and

$$\frac{||\rho_L| - |s_{22}||}{1 - |\rho_L|\ |s_{22}|} \leq |\rho_b| \leq \frac{|\rho_L| + |s_{22}|}{1 + |\rho_L|\ |s_{22}|} \quad . \tag{25}$$

A particular example showing the extreme values of $|\rho_a|$ and $|\rho|$ is shown in Fig. 2.

Explicit upper and lower bounds on $|\rho|$ may be derived. Simplest is the upper bound, given by

$$\max |\rho| = \frac{K_p + |s_{22}|}{1 + K_p\ |s_{22}|} \quad , \tag{26}$$

where

$$K_p = \frac{|\rho_L| + |\rho_S|}{1 + |\rho_L|\ |\rho_S|} \quad . \tag{27}$$

Let

$$K_q = \frac{|\rho_L| - |\rho_S|}{1 - |\rho_L|\ |\rho_S|} \tag{28}$$

and

$$K_r = -K_q . \tag{29}$$

Assuming all possible phases of ρ_S and ρ_L, but constant amplitude we obtain the following lower bounds.

$$\min|\rho| = \begin{cases} \dfrac{|s_{22}| - K_p}{1 - K_p|s_{22}|} & \text{if } K_p < |s_{22}| \\[2ex] \dfrac{K_q - |s_{22}|}{1 - K_q|s_{22}|} & \text{if } K_p > |s_{22}|,\ |\rho_L| > |\rho_S|,\ K_q > |s_{22}| \\[2ex] \dfrac{K_r - |s_{22}|}{1 - K_r|s_{22}|} & \text{if } K_p > |s_{22}|,\ |\rho_L| < |\rho_S|,\ K_r > |s_{22}| \\[2ex] 0 & \text{otherwise} \end{cases} \tag{30}$$

Fig. 3 shows a comparison of these relations with the results of a Monte Carlo analysis with 1000 uniformly distributed values for the phases of ρ_S and ρ_L on $[0, 2\pi]$ for a particular example of an ideal one-section transformer from 50 Ω to 20 Ω with $|\rho_S| = 0.05$ and $|\rho_L| = 0.03$.

Assume all possible amplitudes up to $|\rho_S|$ and $|\rho_L|$ in addition to all possible phases. The upper bound remains the same as (26) but the lower bound becomes

$$\min|\rho| = \begin{cases} \dfrac{|s_{22}| - K_p}{1 - K_p|s_{22}|} & \text{if } K_p < |s_{22}| \\ 0 & \text{if } K_p \geq |s_{22}| \end{cases} \tag{31}$$

An illustration for $|\rho_S| = |\rho_L|$ is shown in Fig. 4. We note that under this restriction, the results are not affected by whether all possible amplitudes are considered or not.

Design Specifications

Let all the performance specifications and constraints be expressed in the form

$$g_i \geq 0 \tag{32}$$

where g_i is, in general, an ith nonlinear function of $\underset{\sim}{p}$ or $\underset{\sim}{\phi}$. Thus, we may consider mismatches by an expression of the form

$$g_i = g_i^0(\underset{\sim}{p}) + \mu_{\rho_i}(\underset{\sim}{p}, \rho_{S_i}, \rho_{L_i}) , \tag{33}$$

where subscript i may denote a sample point and where ρ_S represents the source mismatch and ρ_L the load mismatch. The function μ_{ρ_i} has the effect of shifting the constraint.

Given mismatches, model uncertainties and so on obviously influence the nominal design parameters and manufacturing tolerances. An objective, for example, is to find an optimal set $\{\phi^0, \epsilon, t\}$ such that all possible outcomes (controlled by μ_ϵ), all possible models (controlled by μ_δ) and all possible mismatches (controlled by μ_ρ) are accommodated in satisfying the design specifications.

III. EXAMPLES

To illustrate some of the ideas presented, we consider two simple circuits. The first includes tuning, the second considers possible model uncertainties, parasitic effects and mismatched terminations.

Two-Section Transformer

Specifications and sample points for a two-section lossless transmission-line transformer with quarter-wave length sections and impedance ratio of 10:1 and 100% relative bandwidth are given in Table I.

Table II shows some results of minimizing certain objective (cost) functions of relative tolerances and tuning ranges. The functions tend to discourage small tolerances and large tuning ranges. The design parameters are the normalized characteristic impedances of the two sections, namely, Z_1 and Z_2. The problem has already been considered from the purely tolerance point of view [3]. The parameter ϵ_i' is the effective tolerance [4] of the ith parameter, i.e.,

$$\epsilon_i' \triangleq \epsilon_i - t_i \text{ for } \epsilon_i > t_i \; . \tag{34}$$

A number of interesting, but not unexpected, features may be noted. Column 2 of Table II shows results for no tuning [3]. Columns 3 and 4 show results when Z_1 and Z_2 are tunable, respectively, by 10%. Note that the nominal points move and the tolerances increase. Figure 5 illustrates the optimal solution corresponding to Column 3. The remaining results indicate solutions when the tuning ranges are variables and included in the objective functions. Observe that the results in the final two columns are essentially the same as those in Column 2. The last column shows how the tuning ranges are automatically set to 0 when they are heavily weighted in the cost function, i.e., they are assumed to be expensive. Figure 6 corresponds to the situation of Column 7.

Tuning of any component enhances all the tolerances, as expected. Furthermore, if tuning is expensive it is rejected by the general formulation, which is useful if the designer has a number of possible alternative tunable components and is not sure which components should be effectively tuned and which should be effectively toleranced.

One-Section Stripline Transformer

A more realistic example of a one-section transformer on stripline from 50 Ω to 20 Ω is now considered. The physical circuit and its equivalent are depicted in Fig. 7. The specifications are listed in Table III. Also shown are source and load mismatches to be accounted for as well as fixed tolerances on certain fixed nominal parameters and assumed uncertainties in model parameters.

Thirteen physical parameters implying 2^{13} extreme points are

$$\phi = \begin{bmatrix} w_1 \\ w_2 \\ w_3 \\ \ell \\ \sqrt{\varepsilon_{r1}} \\ \sqrt{\varepsilon_{r2}} \\ \sqrt{\varepsilon_{r3}} \\ b_1 \\ b_2 \\ b_3 \\ t_{s1} \\ t_{s2} \\ t_{s3} \end{bmatrix} \begin{matrix} \left.\begin{matrix} \\ \\ \\ \\ \end{matrix}\right\} \text{variable nominal and variable tolerances} \\ \left.\begin{matrix} \\ \\ \\ \\ \\ \\ \\ \\ \\ \end{matrix}\right\} \text{fixed nominal and fixed tolerances} \end{matrix} \qquad (35)$$

where w denotes strip width, ℓ the length of the middle section, ε_r the dielectric constant, t_s the strip thickness and b the substrate thickness. For computational convenience, independent tolerances on ε_r, b and t_s were imposed for the three lines, but the nominal values were the same throughout.

Six model parameters implying 2^6 extreme points are

$$\underset{\sim}{p} = \begin{bmatrix} D_1 \\ D_2 \\ D_3 \\ L_1 \\ L_2 \\ \ell_t \end{bmatrix} , \tag{36}$$

where D denotes effective line width, L the junction parasitic inductance and ℓ_t the effective section length.

The formula for D_i used is [6]

$$D_i = w_i + \frac{2b_i}{\pi} \ln 2 + \frac{t_i}{\pi} \left[1 - \ln \frac{2t_i}{b_i} \right], \quad i = 1,2,3. \tag{37}$$

The formula is claimed to be good for $w_i/b_i > 0.5$. A 1% uncertainty was rather arbitrarily chosen for D_i. The characteristic impedance Z_i is then found as

$$Z_i = \frac{30\pi(b_i - t_i)}{D_i} . \tag{38}$$

The values of L_i were calculated as [7]

$$L_i = \frac{30\bar{b}_i}{c} K_i , \quad i = 1,2, \tag{39}$$

where

$$K_i = \ln \left[\left(\frac{1-\alpha_i^2}{4\alpha_i} \right) \left(\frac{1+\alpha_i}{1-\alpha_i} \right)^{\frac{\alpha_i + \frac{1}{\alpha_i}}{2}} \right] + \frac{2}{A_i} ,$$

$$\alpha_i = \frac{D_i}{D_{i+1}} < 1,$$

$$A_i = \left(\frac{1+\alpha_i}{1-\alpha_i}\right)^{2\alpha_i} \frac{1+S_i}{1-S_i} - \frac{1+3\alpha_i^2}{1-\alpha_i^2} ,$$

$$S_i = \sqrt{1 - \frac{D_{i+1}^2}{\bar{\lambda}_{gi}^2}} ,$$

$$\bar{\lambda}_{gi} = \frac{c}{f\sqrt{\bar{\varepsilon}_{ri}}} ,$$

$$\bar{b}_i = 0.5(b_i + b_{i+1}) ,$$

$$\sqrt{\bar{\varepsilon}_{ri}} = 0.5(\sqrt{\varepsilon_{ri}} + \sqrt{\varepsilon_{r(i+1)}}) .$$

Mean values across the junctions of adjacent sections of $\sqrt{\varepsilon_r}$ and b are taken since independent tolerances on these parameters are assumed. Data for estimating the uncertainties on L_i is available [6,7]. Other approximations have, however, been introduced due to the tolerancing. A 3% uncertainty on L_i was adopted.

The length ℓ_t is nominally the same as ℓ. Experimental results [6] indicate possibly large inaccuracies in d (see Fig. 7) and that it depends at least on α, so that it is actually different for the two junctions. A rather pessimistic estimated error of 1 mm on ℓ_t was chosen.

Maximum mismatch reflection coefficients of 0.025 were chosen for the source and load. Note that these values are assumed with respect to 50Ω and 20Ω, respectively. The relevant formulas developed in Section II can not be applied directly, since Z_1 and Z_3, which are affected by tolerances, must be considered for normalization. We take

$$|\rho_S| = \frac{0.025 + |\rho_1|}{1 + 0.025\,|\rho_1|} , \tag{40}$$

where

$$\rho_1 = \frac{50 - Z_1}{50 + Z_1} ,$$

and

$$|\rho_L| = \frac{0.025 + |\rho_3|}{1 + 0.025|\rho_3|} , \tag{41}$$

where

$$\rho_3 = \frac{25 - Z_3}{25 + Z_3} .$$

Figure 8 summarizes some of the results obtained from worst-case analyses. Depicted are curves of the ideal design with discontinuity (parasitic) effects taken into account; upper and lower bounds on the response with source and load mismatches also added; finally, upper and lower responses with model uncertainties further deteriorating the situation.

A worst-case study was made to select a reasonable number of constraints from the possible $2^{19} = 2^{13} \times 2^6$, since 2^{19} would have required about 5000s of CDC 6400 computing time per frequency point. The vertex selection procedure for the 13 physical parameters follows Bandler et al. [3]. From each of the selected vertices the worst values of the modeling parameters are chosen. Only the band edges are used during optimization. After each optimization the selection procedure is repeated, new constraints being added, if necessary.

Results on centering and tolerancing using DISOPT [8] are shown in Table IV. The final number of constraints used is 21 after 9 optimizations required to identify the final constraints. Less than 4 minutes on the CDC 6400 was altogether required. An intermediate, less accurate, solution is obtained using 18 constraints after 7 optimizations requiring 2 minutes on the CDC 6400. To verify that the solution meets the specification, the constraint selection procedure was repeated at 21 points in the band.

Figure 9 presents final results for this example. The reason for the discrepancy between the worst cases when vertices are used and when the Monte Carlo analysis is used is that the Monte Carlo analysis does not employ the pessimistic approximations of (40) and (41).

CONCLUSIONS

The concepts we have described and the results obtained are promising. Our approach is the most direct way of currently obtaining minimum cost designs under practical situations, at least in the worst case sense. It is felt that this work is a significant advance in the art of computer-aided design since the approach permits the inclusion of all realistic degrees of freedom of a design and all physical phenomena that influence the subsequent performance. The general problem is both mathematically and computationally far more complicated than the conventional computer-aided design process which seeks a single nominal optimal design. The latter approach, however, would normally be used to find a starting point for the work we have in mind.

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TABLE I

TWO-SECTION 10:1 QUARTER-WAVE TRANSFORMER

Relative Bandwidth	Sample Points (GHz)	Reflection Coefficient Specification	Type
100%	0.5, 0.6, ..., 1.5	0.55	upper
	Minimax solution (no tolerances)	$\lvert\rho\rvert = 0.4286$	

TABLE II

TWO-SECTION 10:1 QUARTER-WAVE TRANSFORMER
DESIGN CENTERING, TOLERANCING AND TUNING

Cost Function*	C_1	C_1	C_1	C_2	C_3	C_4	C_5
Z_1^0	2.1487	2.0340	2.2754	2.5025	1.8748	2.1487	2.1487
Z_2^0	4.7307	4.5355	4.9467	5.3337	4.2642	4.7307	4.7307
$\varepsilon_1/Z_1^0 \times 100\%$	12.74	17.83	17.60	25.08	31.62	31.62	12.74
$\varepsilon_2/Z_2^0 \times 100\%$	12.74	17.60	17.83	31.62	25.08	31.62	12.74
$t_1/Z_1^0 \times 100\%$	-	10.00	-	-	31.62	18.88	0.00
$t_2/Z_2^0 \times 100\%$	-	-	10.00	31.62	-	18.88	0.00
$\varepsilon_1'/Z_1^0 \times 100\%$	-	7.83	-	-	0.00	12.74	12.74
$\varepsilon_2'/Z_2^0 \times 100\%$	-	-	7.83	0.00	-	12.74	12.74

*$C_1 = Z_1^0/\varepsilon_1 + Z_2^0/\varepsilon_2$

$C_2 = Z_1^0/\varepsilon_1 + Z_2^0/\varepsilon_2 + 10(t_2/Z_2^0)$

$C_3 = Z_1^0/\varepsilon_1 + Z_2^0/\varepsilon_2 + 10(t_1/Z_1^0)$

$C_4 = Z_1^0/\varepsilon_1 + Z_2^0/\varepsilon_2 + 10(t_1/Z_1^0 + t_2/Z_2^0)$

$C_5 = Z_1^0/\varepsilon_1 + Z_2^0/\varepsilon_2 + 500(t_1/Z_1^0 + t_2/Z_2^0)$

TABLE III

ONE-SECTION STRIPLINE TRANSFORMER

Center Frequency	5 GHz
Frequency Band	4.5 - 5.5 GHz
Reflection Coefficient Specification	0.25 (upper)
Source Impedance	50 Ω (nominal)
Load Impedance	20 Ω (nominal)
Source Mismatch	0.025 (reflection coeff.)
Load Mismatch	0.025 (reflection coeff.)
ε_r	2.54 ± 1%
b	6.35 mm ± 1%
t_s	0.051 mm ± 5%
Uncertainty on L_1, L_2	3%
D_1, D_2, D_3	1%
ℓ_t	1 mm

TABLE IV

RESULTS FOR ONE-SECTION STRIPLINE TRANSFORMER

Cost Function	$\frac{1}{100}\left(\frac{w_1^0}{\varepsilon_{w_1}} + \frac{w_2^0}{\varepsilon_{w_2}} + \frac{w_3^0}{\varepsilon_{w_3}} + \frac{\ell^0}{\varepsilon_\ell}\right)$		
Sample Points	4.5, 5.5		GHz
Number of Variables	8		
Number of Final Constraints	18	21	
Number of Optimizations	7	9	
CDC 6400 Time	2	4	min
Minimal Cost	4.82	4.93	
w_1^0	4.660	4.642	mm
w_2^0	8.968	8.910	mm
w_3^0	15.463	15.442	mm
ℓ^0	8.457	8.437	mm
$\varepsilon_{w_1}/w_1^0 \times 100$	0.94	0.92	%
$\varepsilon_{w_2}/w_2^0 \times 100$	1.20	1.13	%
$\varepsilon_{w_3}/w_3^0 \times 100$	0.74	0.70	%
$\varepsilon_\ell/\ell^0 \times 100$	0.64	0.65	%

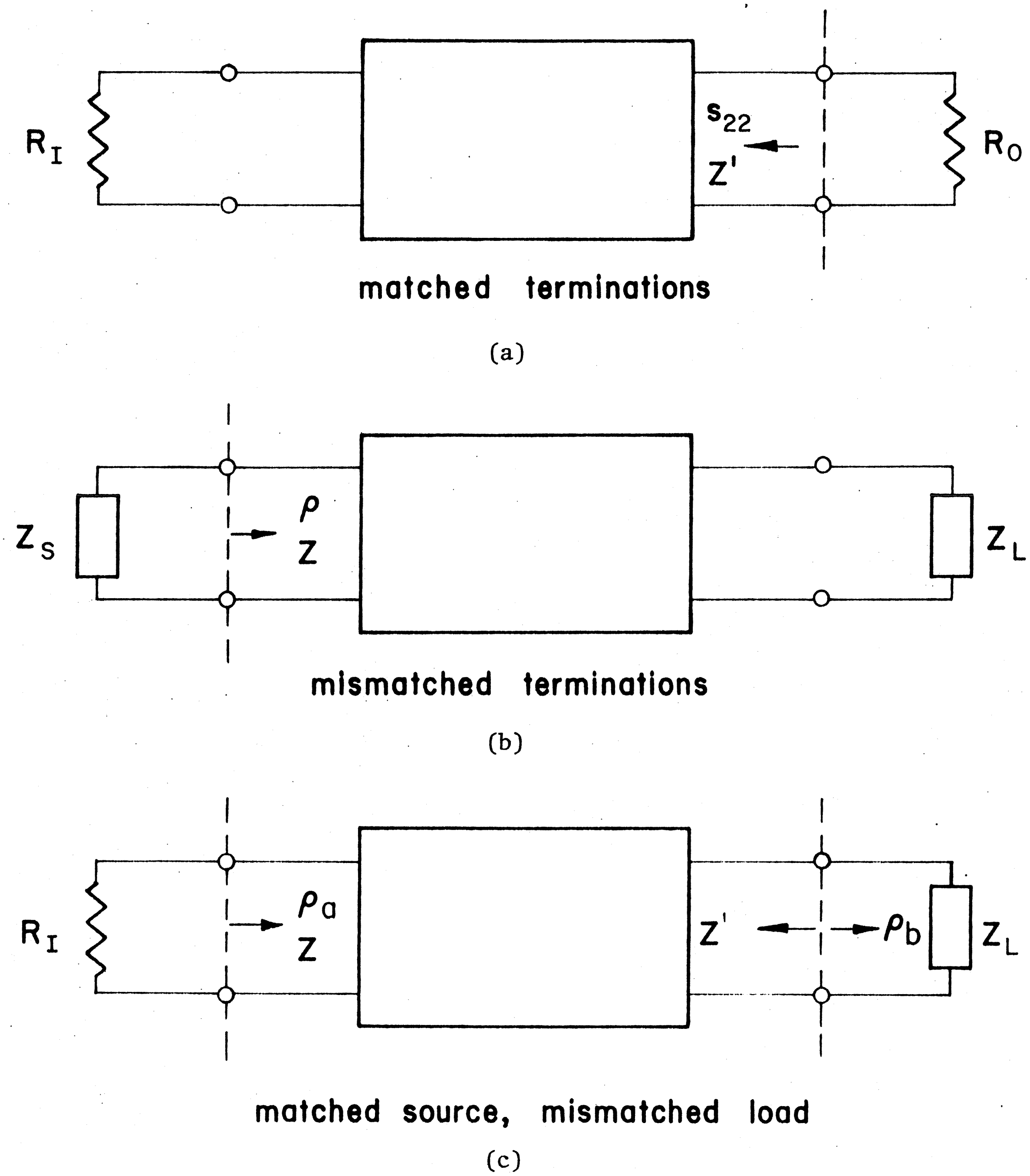

Fig. 1 Two-port circuit viewed with respect to three sets of terminations.

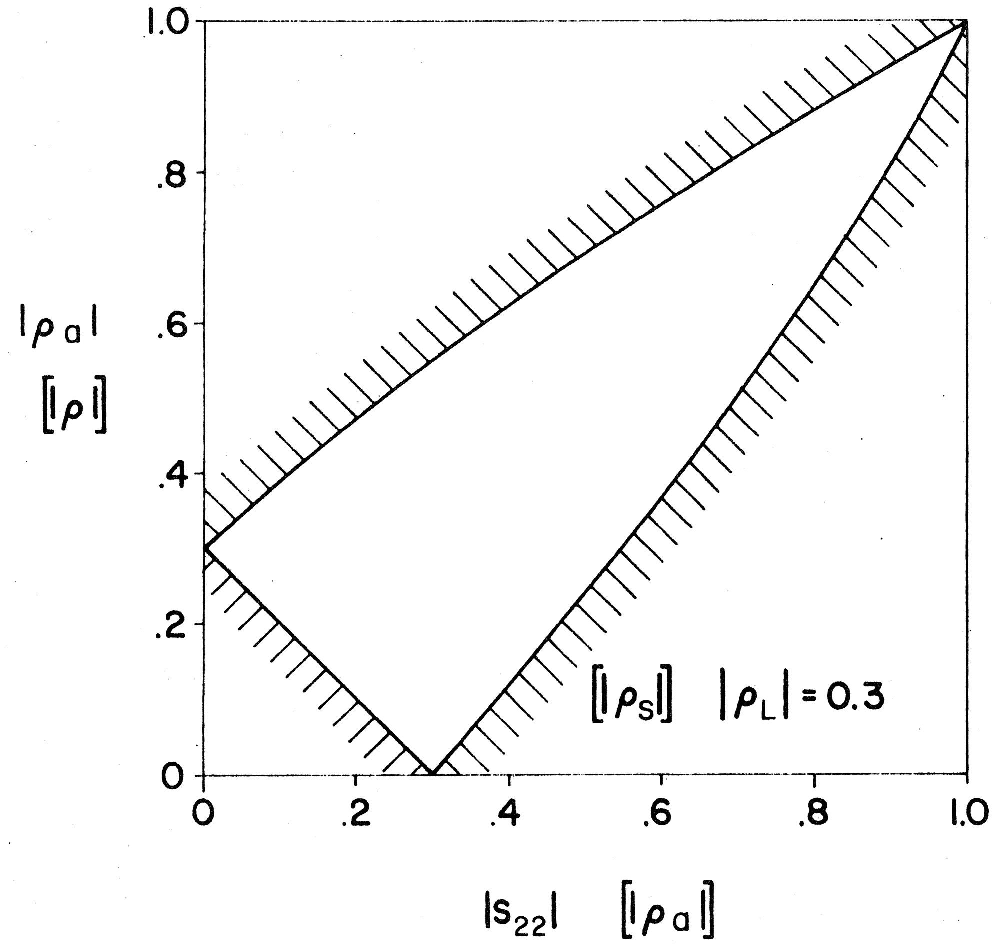

Fig. 2 Feasible region of reflection coefficients given that $|\rho_S| = |\rho_L| = 0.3$.

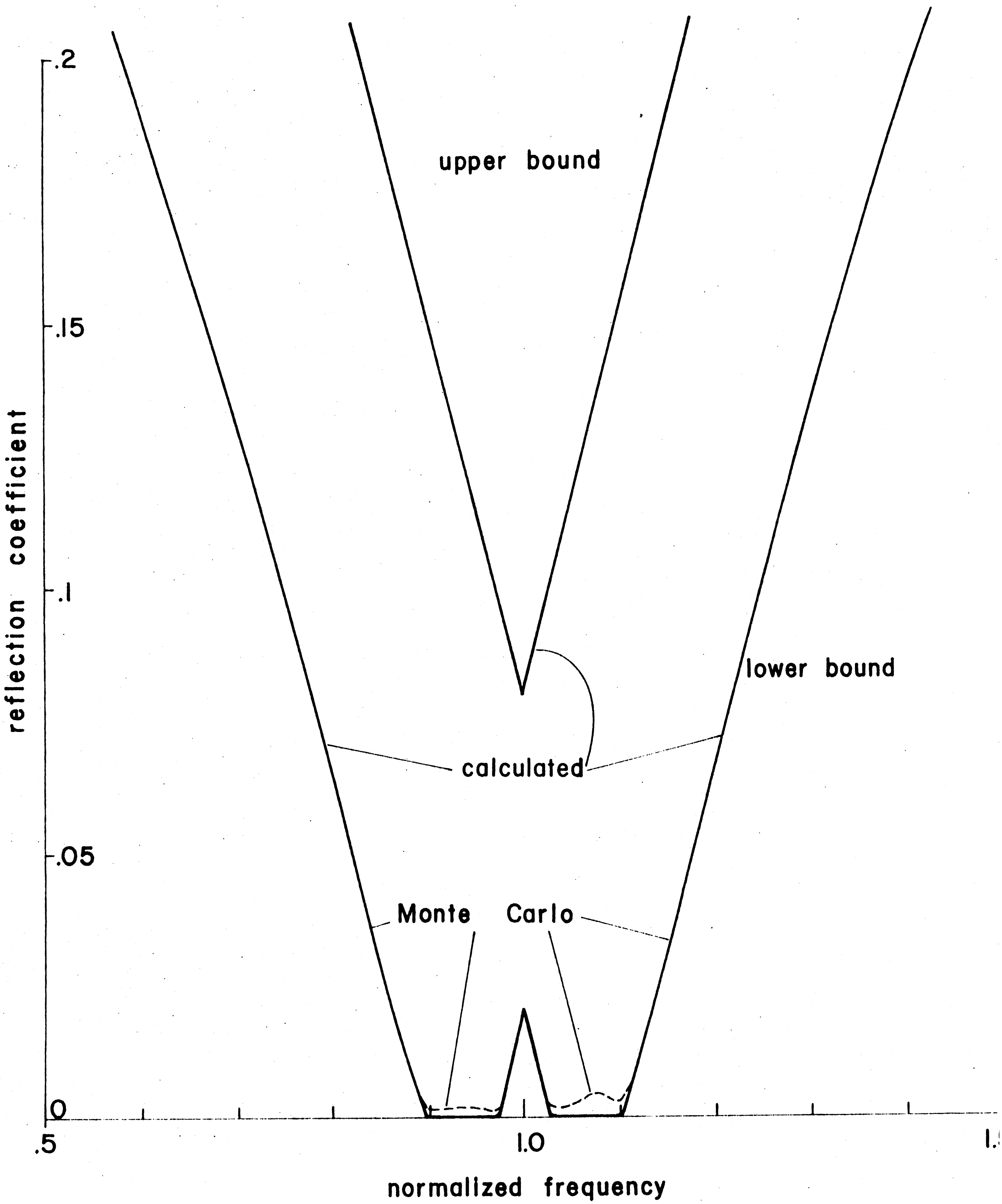

Fig. 3 Upper and lower bounds on reflection coefficient calculated from (26) and (30 and checked by a Monte Carlo analysis (1000 points) for an ideal one-section transformer from 50 to 20 Ω with $|\rho_S| = 0.05$ and $|\rho_L| = 0.03$.

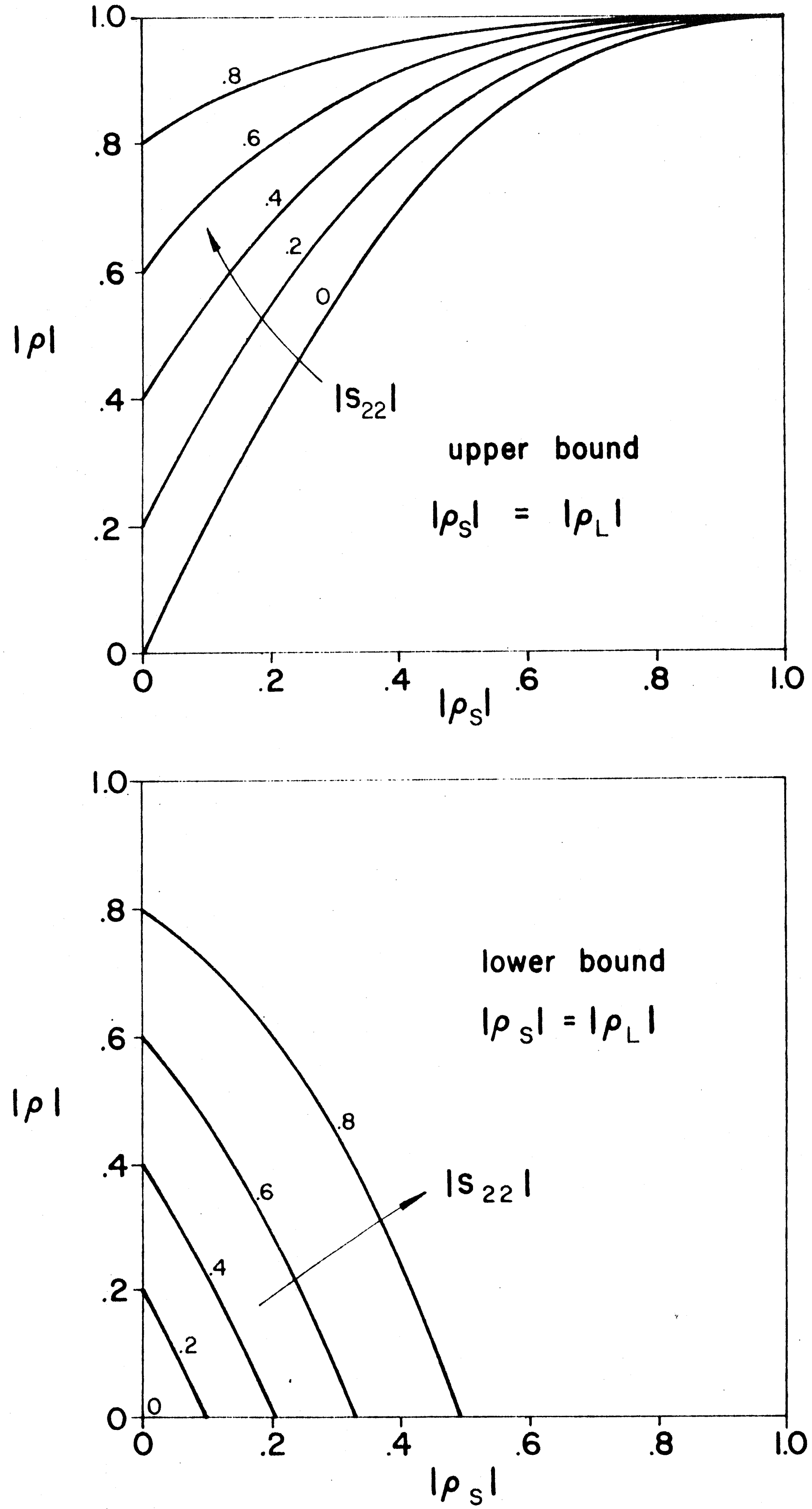

Fig. 4 Upper and lower bounds on $|\rho|$ for $|\rho_S| = |\rho_L|$.

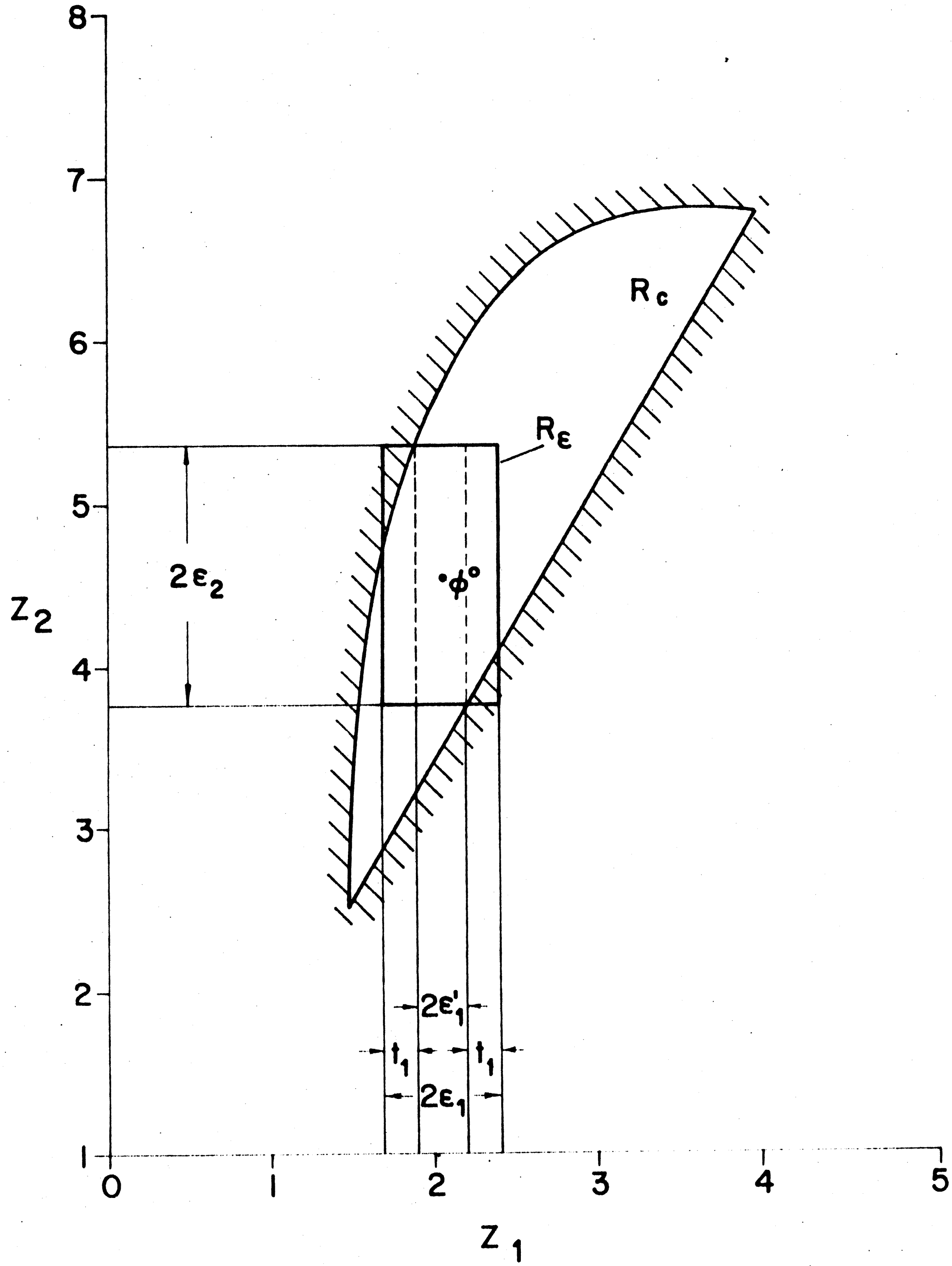

Fig. 5 Optimal solution corresponding to Column 3 of Table II. R_c is the constraint region, i.e., the region for which $|\rho| \leq 0.55$.

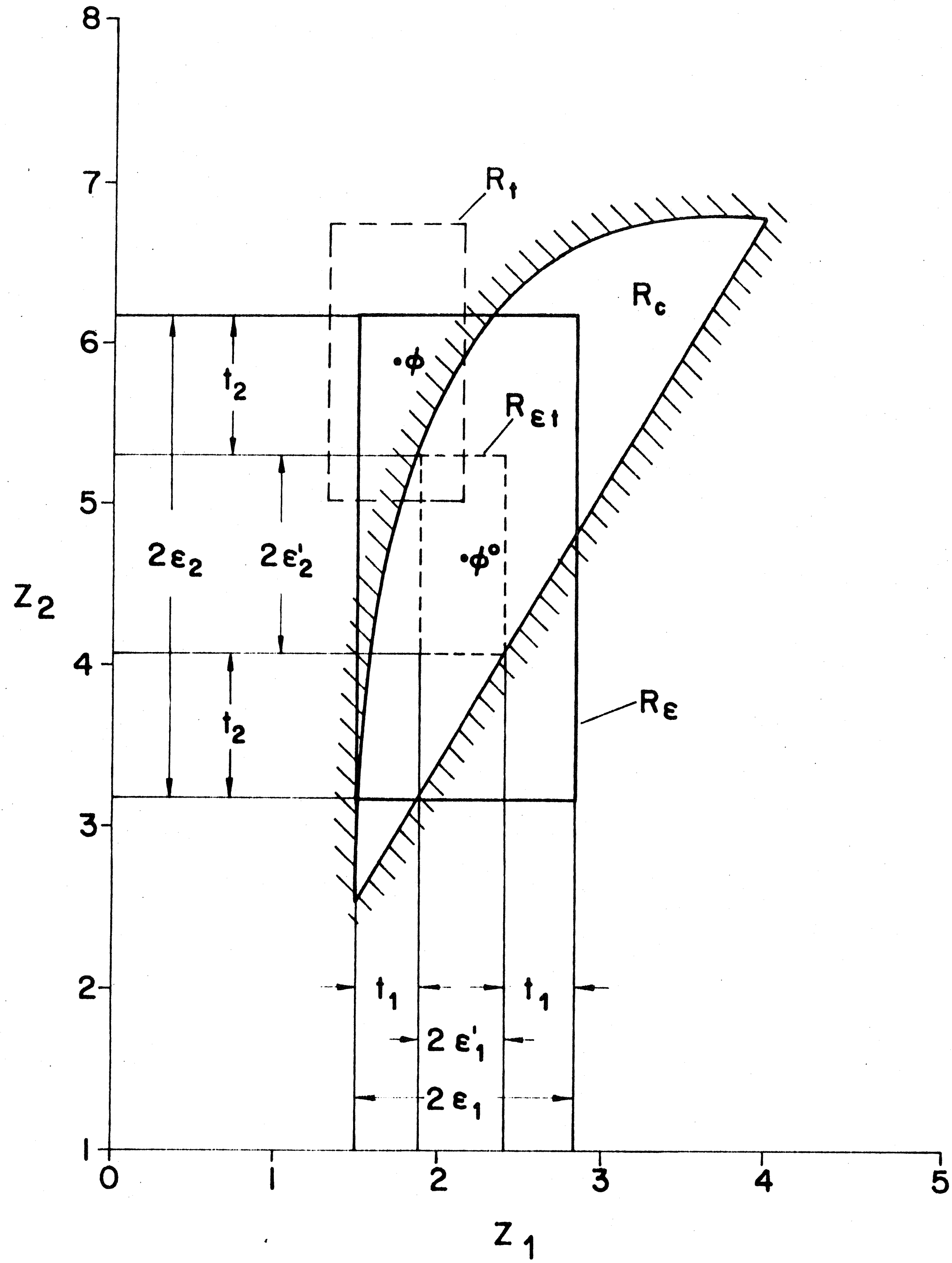

Fig. 6 Optimal solution corresponding to Column 7 of Table II. $R_{\varepsilon t}$ is the <u>effective</u> tolerance region.

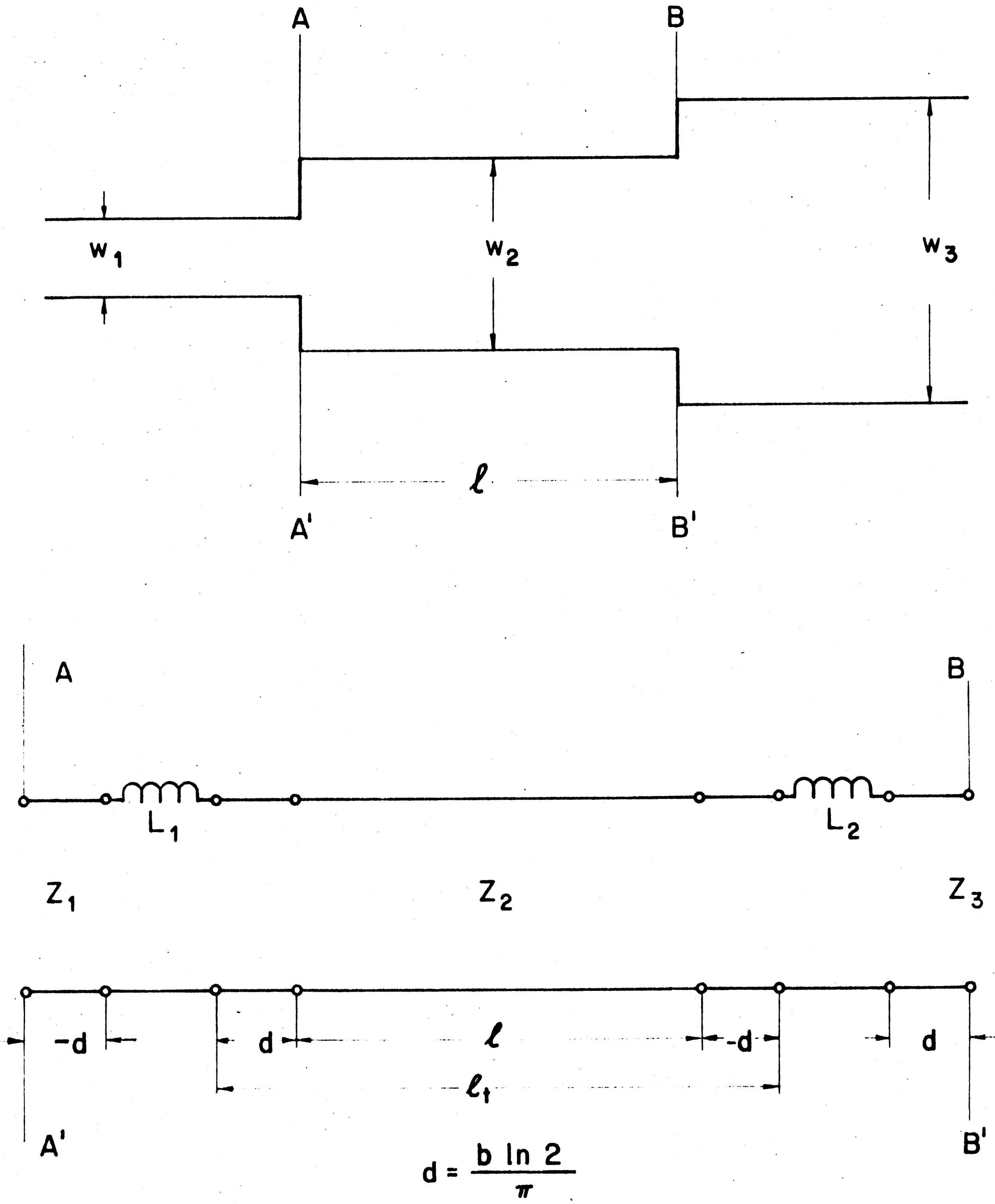

Fig. 7 Stripline transformer and equivalent circuit.

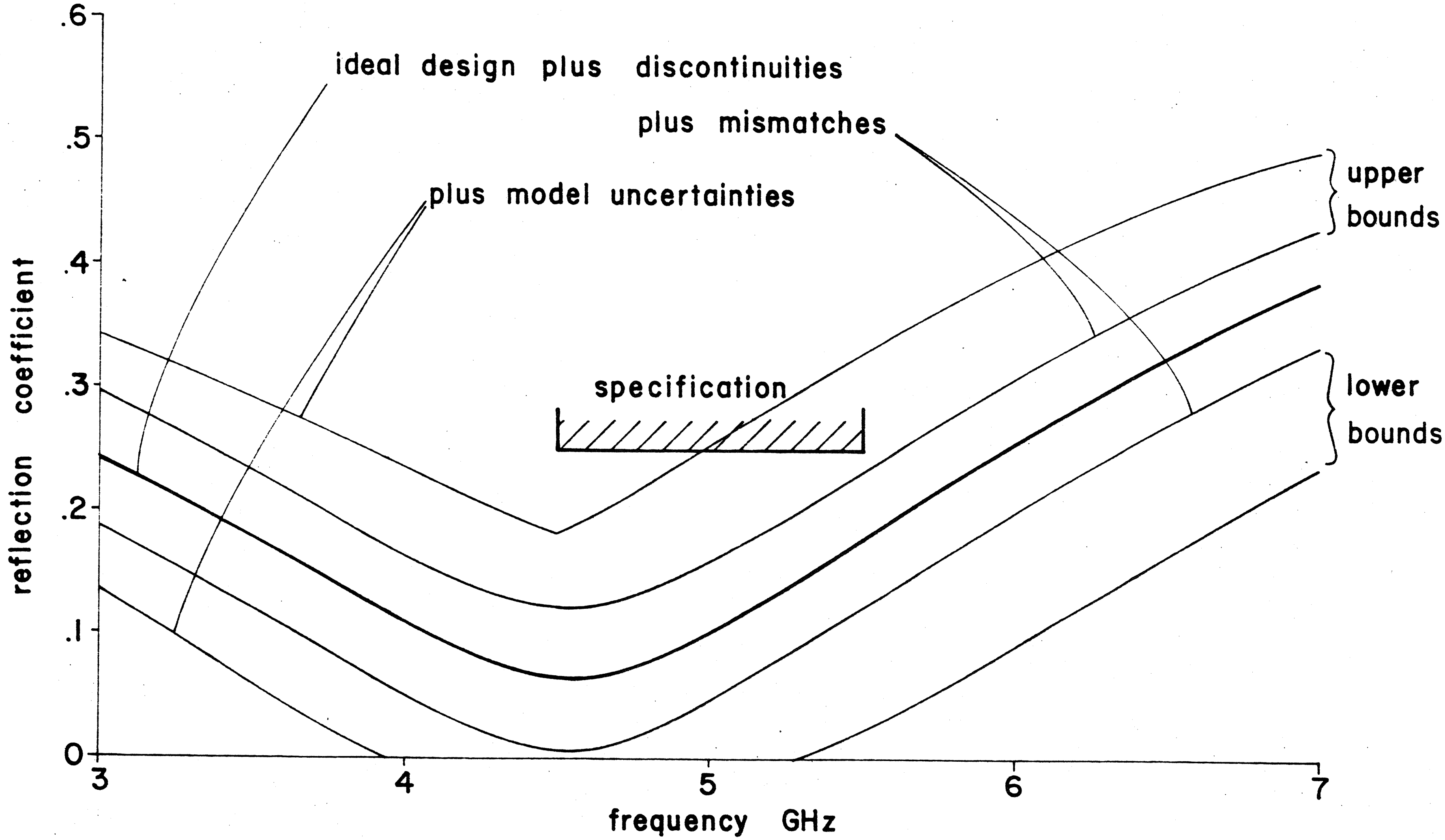

Fig. 8 Worst-case analyses for the stripline transformer.
Note that physical parameter tolerances are not included.

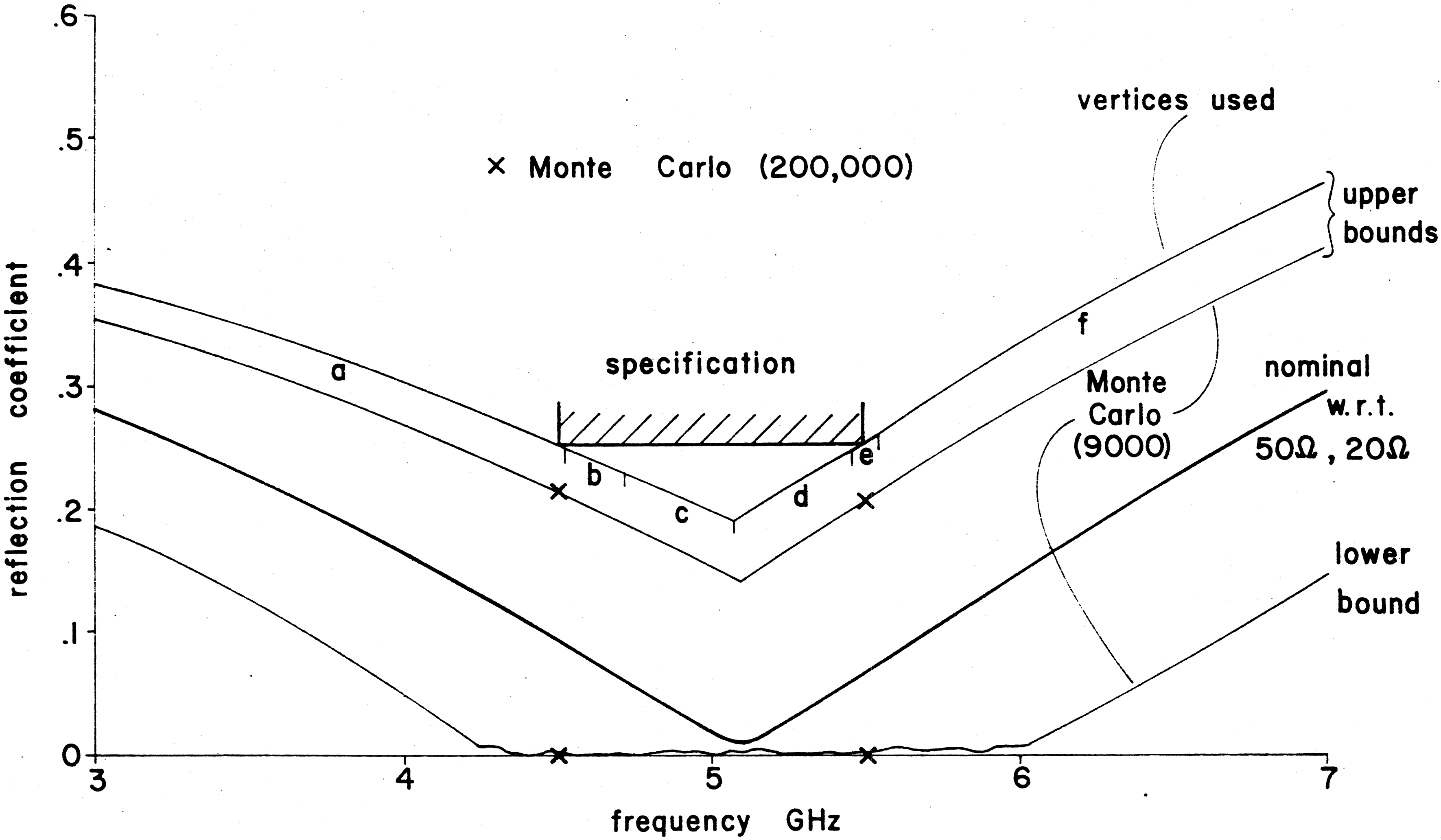

Fig. 9 Final results for the stripline transformer. The letters a,b,...,f indicate different vertices (designs) determining the worst case in different frequency bands.