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OPTIMAL DESIGN VIA MODELING AND APPROXIMATION

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Recent ideas and results developed by the authors involving concepts of modeling and approximation are reviewed. The approaches taken include abstract ones as well as a physically meaningful one in the area of time domain circuit analysis involving transmission-line modeling of lumped circuits. Optimal centering and tolerancing is also considered.

Introduction

Recent ideas and results developed by the authors involving concepts of modeling and approximation are reviewed. Both an abstract approach to approximation of the response functions with respect to the design parameters as well as a physically meaningful approach to time domain circuit analysis are discussed. The optimal assignment of component tolerances and optimal centering is considered. Low-order multidimensional approximations of the functions concerned allow rapid and accurate yield estimation and optimization. Transmission-line modeling of lumped circuits is used for optimization in the time domain. The response evaluation is exact for the model and exact derivatives are easily obtained.

Bounding and Approximating R_c

The constraint region R_c is the set of points ϕ, the vector of design parameters, for which all performance specifications and design constraints are satisfied. Upper and lower bounds on the parameters ϕ_i for which points satisfying all the requirements can be found provide useful design information[1]. In a statistical analysis, for example, constraints can be stacked in the order of increasing computational effort, upper and lower bounds appearing at the top of the stack. If any is violated, further testing becomes unnecessary.

See Fig.1. In practice, 2k optimizations are not required, if the necessary conditions for optimality are taken advantage of in the k-space.

Fig.2 shows an alternative approach to eliminating regions unlikely to contain acceptable solutions. A generalized least pth objective $U(\phi)$ based on all the constraint functions is formed[2]. $M(\phi)$ is the corresponding maximum. A sequence ϕ is generated. If R_c is convex or one-dimensionally convex[3] we can eliminate from further consideration the regions shown in Fig.2.

We can employ interior and exterior approximations[4] to R_c as shown in Fig.3. A best exterior approximation may be found by deflation of a suitable region R_E and a best interior approximation by inflation of a suitable region R_I[4], keeping $R_I \subset R_c \subset R_E$. Thus, the original functions would be calculated only for $\phi \in R_E - R_I$.

The interior approximation could also be used in design centering[5]. The tolerance region for independent variables R_ε is an example of an interior approximation. The upper and lower bounds on R_c form an exterior approximation.

Optimization Utilizing Polynomial Approximations

Consider the approximation of the constraint functions using values at selected sets of points (base points)[5,6]. In conventional optimization: (1) The functions are approximated by quadratic polynomials using $0.5(k + 2)(k + 1)$ base points (number of coefficients), where k is the dimensionality. The base points lie in a neighbourhood of the starting point (current estimate of the solution) within a step $\pm\delta$ in each variable in the ϕ space. (2) Optimization is carried out with the approximate functions. The solution becomes the next starting point. (3) If the solution is close to the interpolation region, e.g., each parameter has not changed by more than 1.5δ the step size is reduced, e.g., by a factor of 0.25. (4) The procedure is repeated from (1) until an appropriate termination criterion is satisfied.

For solving centering and tolerancing problems consider the following. See Fig.4. (1) As previously with δ chosen greater than the starting or current values of the tolerances ε_i, $i = 1,2,\ldots,k$. (2) As previously but where the problem under consideration is the worst case tolerance problem. (3) If the nominal point ϕ^0 moves too far from the interpolation region (Fig.4), e.g., parameters change by more than 1.5δ, the procedure is repeated from (1). (4) If the nominal point has not moved too far δ is reduced. If δ is still greater than the tolerances approximation as in (1) is carried out. If δ is greater than only some of the toler-

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ances approximation is carried out separately for constraints corresponding to the active vertices of R_ε spaced by less than twice the step size around the center of the hyperface (Fig.4). When δ becomes less than all the tolerances each constraint is re-approximated around the appropriate active vertices. (5) δ is subsequently reduced only when all active vertices stay within the corresponding interpolation region. (6) The procedure is repeated as necessary until parameter changes satisfy an appropriate termination criterion when the step size is reduced.

The structure of the approximations permits efficient function and gradient evaluation even when all 2^k vertices are used. Yield can be estimated by enlarging the tolerance region and using the quadratic approximation to find the yield by Monte Carlo analysis.

Transmission-line Modeling

We can apply an approach called the transmission-line matrix (TLM) method extensively used for the solution of field problems[7,8] to the time domain analysis of lumped networks. Lumped components are represented by transmission-line elements with various terminations. The exact response of the model to an impulse can be found by a numerical procedure, with attendant advantages in the physical description of errors and stability for stiff systems.

Consider a ladder network of series inductors and shunt capacitors. Fig.5 depicts elements and continuized models for a cascaded transmission-line representation. Stubs, appropriately terminated, or a combination of stubs and interconnecting lines can also be employed.

In the cascade analysis (two-port junctions) an ideal delta function pulse is launched from the first junction.The pulse scatters on reaching the next junction, being partly reflected and partly transmitted.This scattering occurs at every junction,pulses racing to and fro between junctions. For simplicity, assume equal lengths and velocities of propagation for all the sections. If the velocity is 1 m/s, then the time h in seconds for a pulse to travel between sections is numerically equal to the length h in meters.

The TLM iteration process is

$$_{j}\underset{\sim}{V}^{r}(\ell) = \underset{\sim}{S}(\ell)\; _{j}\underset{\sim}{V}^{i}(\ell)$$

$$_{j+1}V_{1}^{i}(\ell) = {}_{j}V_{2}^{r}(\ell-1)$$

$$_{j+1}V_{2}^{i}(\ell) = {}_{j}V_{1}^{r}(\ell+1)$$

where j denotes the iteration, ℓ the junction number, S the junction scattering matrix, i incident and r reflected pulses, and subscripts 1 and 2 distinguish the two junction ports. Simple programming and simple calculation of exact sensitivities w.r.t. design variables is possible.

Examples

Consider the worst-case tolerance optimization of impedances Z_1 and Z_2 of a 2-section quarterwave lossless, 10:1,100% bandwidth, transmission-line transformer[2]. See Figs.6 and 7. The expected solutions[9] were obtained from ε_1=0.2 and ε_2=0.4 (11 sample points). About 7 sec (18 function evaluations (f.e.)) and 2.5 sec (12 f.e.)for the initial solutions of Figs.6 and 7,respectively,were required on a CDC 6400 using δ=0.4 with FLNLP2[10]. Setting δ=0.1 gave the final results shown in 9.5 sec (24 f.e.) and 3 sec (18 f.e.), respectively.

Minimizing $1/\varepsilon_1 + 1/\varepsilon_2$ (Fig.6) subject to a yield (uniform distribution) $Y \geq 90\%$ enlarged ε_i by about 50%.Yield and sensitivities were estimated from formulas by Tromp[11].

Symmetrical LC lowpass filter was optimized in the time domain. Fig.8 shows a "specified" impulse response for $L_1 = L_2 = 1.0$, $C = 2.0$. Taking 100 sample points, using TLM analysis, least 4th approximation yielded the solution in 21 sec (24 f.e.) and 17 sec (19 f.e.) from starting points a and b, respectively, with a maximum error of about 3×10^{-17}. The specifications of Fig.9 were met with a minimax error of 0.0021992 after 37 sec (46 f.e.) using 33 sample points for optimization.

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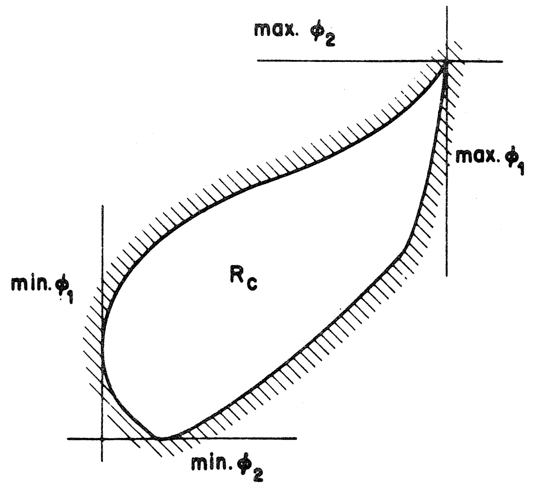

Fig.1 Bounding R_c.

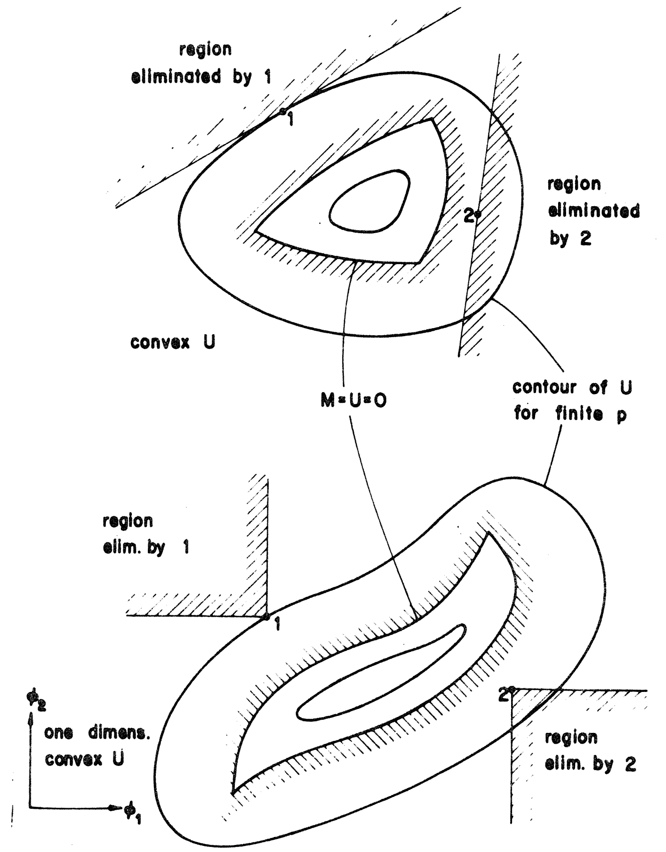

Fig.2 Elimination of regions under assumptions of convexity and one-dimensional convexity.
$U > 0 \rightarrow$ specification violated
$U < 0 \rightarrow$ specification satisfied

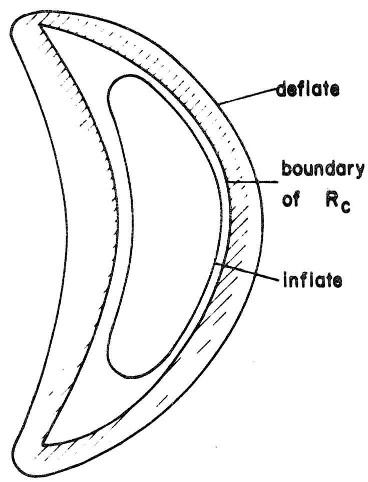

Fig.3 Interior and exterior approximations to R_c.

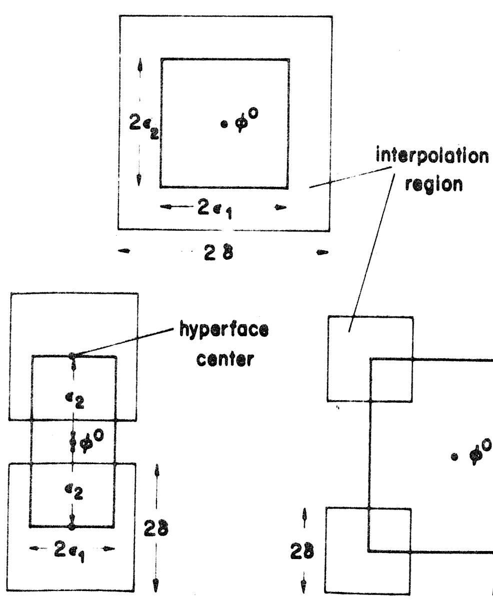

Fig.4 Tolerance regions and interpolation regions.

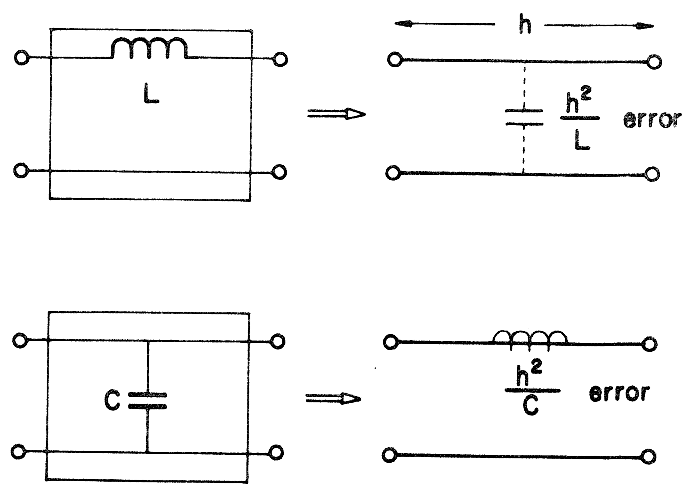

Fig.5 Lossless transmission-line models:an inductor and a capacitor become lines of characteristic impedance L/h and h/C,respectively.

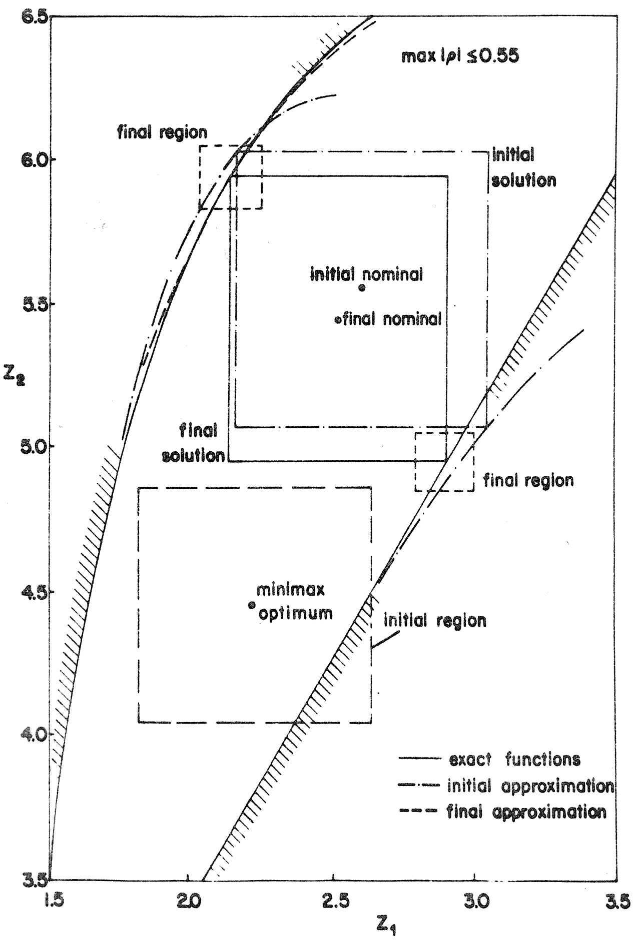

Fig.6 Minimization of $1/\varepsilon_1 + 1/\varepsilon_2$ for the two-section transformer.

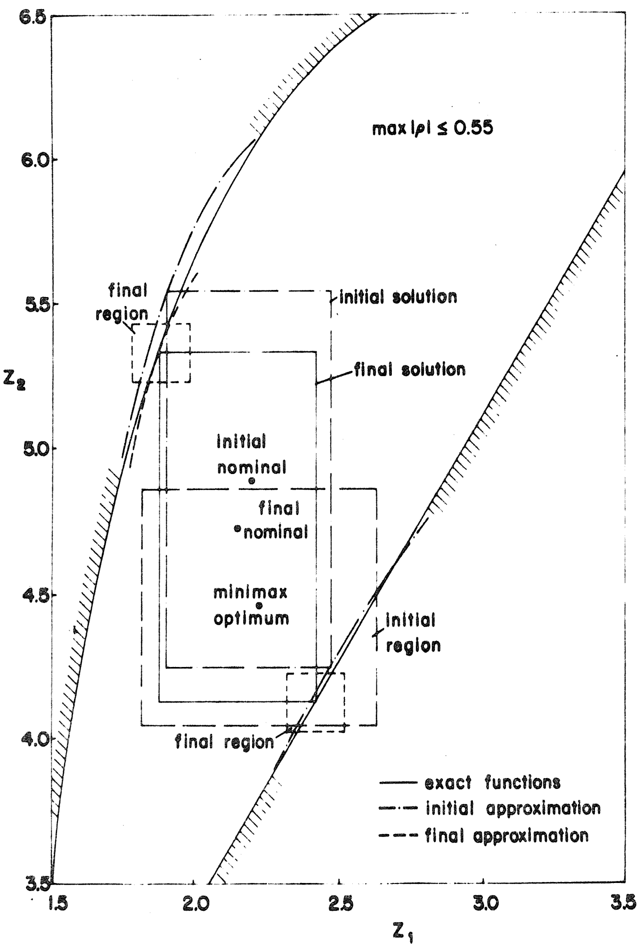

Fig.7 Minimization of $Z_1^0/\varepsilon_1 + Z_2^0/\varepsilon_2$ for the two-section transformer.

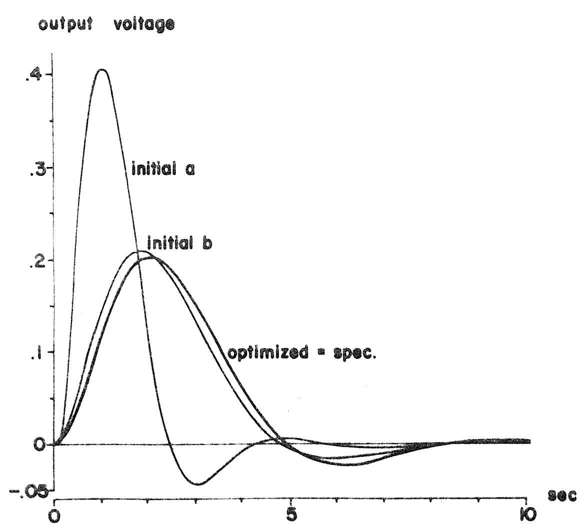

Fig. 8 Optimization using TLM analysis. Starting point a: $L_1 = L_2 = 0.5$, $C = 1.0$. Starting point b: $L_1 = L_2 = 0.8$, $C = 2.2$.

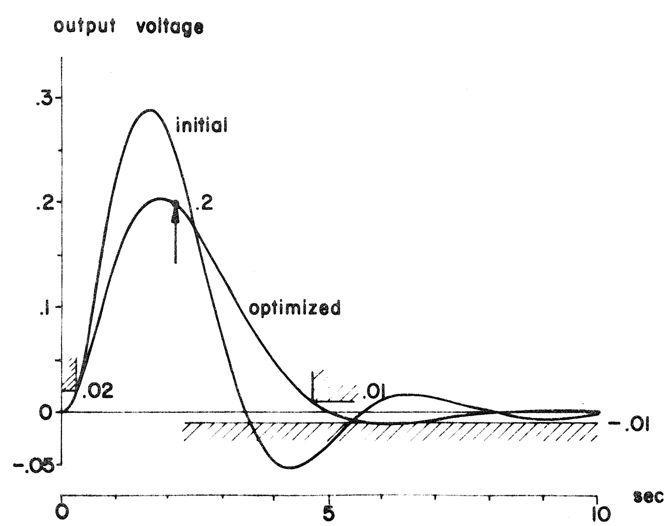

Fig.9 Optimization using TLM analysis. Starting point $L_1 = L_2 = C = 1.0$. Solution: $L_1 = L_2 = 0.76646$, $C = 2.3739$.

SOC-119

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