INTERNAL REPORTS IN

SIMULATION, OPTIMIZATION AND CONTROL

No. SOC-190

NEW RESULTS IN NETWORK SIMULATION, SENSITIVITY AND TOLERANCE ANALYSIS FOR CASCADED STRUCTURES

J.W. Bandler, M.R.M. Rizk and H.L. Abdel-Malek

March 1978

(Revised June 1978)

FACULTY OF ENGINEERING
McMASTER UNIVERSITY
HAMILTON, ONTARIO, CANADA

Group on Simulation, Optimization and Control
FACULTY OF ENGINEERING
McMASTER UNIVERSITY, HAMILTON, CANADA L8S 4L7

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Coordinator: Dr. J. W. Bandler, Department of Electrical Engineering

NEW RESULTS IN NETWORK SIMULATION, SENSITIVITY AND TOLERANCE ANALYSIS FOR CASCADED STRUCTURES

J.W. Bandler, Fellow, IEEE, M.R.M. Rizk, Student Member, IEEE,
and H.L. Abdel-Malek, Student Member, IEEE

Abstract An attractive, exact and efficient approach to network analysis for cascaded structures is presented. It is useful for sensitivity and tolerance analyses, in particular, for a multiple of simultaneous large changes in design parameter values. It also facilitates the exploitation of symmetry to reduce computational effort for the analysis. Responses at different loads in branched networks, which may be connected in series or in parallel with the main cascade, can be obtained analytically in terms of the variable elements. Sensitivity and large-change effects w.r.t. these variables can be easily evaluated. The approach is not confined to 2-port elements but can be generalized to 2p-port cascaded elements.

This work was supported by the National Research Council of Canada under Grant A7239 and by a Postdoctoral Fellowship to H.L. Abdel-Malek. This paper was presented at the 1978 IEEE International Microwave Symposium, Ottawa, Canada, June 27-29, 1978.

The authors are with the Group on Simulation, Optimization and Control and Department of Electrical Engineering, McMaster University, Hamilton, Canada L8S 4L7.

I. INTRODUCTION

This paper presents a new and comprehensive treatment of computer-oriented cascaded network analysis. The analysis of cascaded networks plays a very important role in the design and optimization of microwave circuits, so that an attractive approach which facilitates efficient analytical and numerical investigations of response, first- and higher-order sensitivities of response, simultaneous and arbitrary large-change sensitivity evaluation is highly desirable. As is well known, first-order sensitivities, for example, are useful in network optimization by gradient methods. Large-change sensitivities are important in tolerance analysis and design centering.

The approach we have developed permits efficient

(a) exact analysis of cascaded networks in any direction,

(b) exact evaluation of first-order response sensitivities at any location,

(c) exact evaluation of the effects of any number of simultaneous large changes in any elements,

(d) the exploitation of network structure: branches, symmetry, reciprocity, etc.,

(e) evaluation of the exact effect due to simultaneously growing elements in appropriate locations,

(f) exact response and response sensitivity evaluation for branches connected in series or parallel with the main cascade.

The conceptual advantages enjoyed by our approach and applicable to 2-port elements are

(a) all calculations are applied directly to the given network: no auxiliary or adjoint network is defined,

(b) all calculations involve at most the premultiplication of two by two matrices by row vectors or postmultiplications by column vectors: no explicit matrix inversion is ever required,

(c) response functions, sensitivities or large-change effects are represented analytically in terms of the parameters to be investigated: all parts of the network to be kept constant are reduced numerically to a few two-element vectors appearing as constants in the formulas,

(d) calculations can be carried out easily by hand, if appropriate, or are readily programmed.

II. THEORETICAL FOUNDATION

Consider the two-port element depicted in Fig. 1. The basic iteration, also summarized by Table I, is $\bar{\underset{\sim}{y}} = \underset{\sim}{A}\ \underset{\sim}{y}$, where $\underset{\sim}{A}$ is the transmission or chain matrix, $\underset{\sim}{y}$ contains the output voltage and current and $\bar{\underset{\sim}{y}}$ the corresponding input quantities.

Forward analysis (see Fig. 2 and Table I) consists of initializing a $\bar{\underset{\sim}{u}}^T$ row vector as either [1 0], [0 1] or a suitable linear combination and successively premultiplying each constant chain matrix by the resulting row vector until an element of interest or a termination is reached.

Reverse analysis, which is similar to conventional analysis of cascaded networks, proceeds by initializing a $\underset{\sim}{v}$ column vector as either $[1\ 0]^T$ or $[0\ 1]^T$ or a suitable linear combination and successively postmultiplying each constant matrix by the resulting column vector, again until either an element of interest, or a termination is reached.

In summary, assuming a cascade of n two-ports we have

$$\bar{\underset{\sim}{y}}^1 = \underset{\sim}{y}^0 = \underset{\sim}{A}^1\underset{\sim}{A}^2 \ldots \underset{\sim}{A}^i \ldots \underset{\sim}{A}^n\underset{\sim}{y}^n \tag{1}$$

and, applying forward and reverse analyses up to $\underset{\sim}{A}^i$, this reduces to an expression of the form

$$d = \bar{\underset{\sim}{u}}^{1^T} \bar{\underset{\sim}{y}}^1 = c\ \bar{\underset{\sim}{u}}^{i^T} \underset{\sim}{A}^i \underset{\sim}{v}^i , \tag{2}$$

where

$$\underset{\sim}{y}^n = c\ \underset{\sim}{v}^n \tag{3}$$

and c and d relate selected output and input variables of interest explicitly with $\underset{\sim}{A}^i$.

The typical formula will, therefore, contain factors of the form

function evaluation: $\bar{\underset{\sim}{u}}^T\underset{\sim}{A}\ \underset{\sim}{v} \Longrightarrow Q$, (4)

first-order sensitivity: $\bar{\underset{\sim}{u}}^T\delta\underset{\sim}{A}\ \underset{\sim}{v} \Longrightarrow \delta Q$, (5)

partial derivative: $\bar{\underset{\sim}{u}}^T \dfrac{\partial \underset{\sim}{A}}{\partial \phi}\ \underset{\sim}{v} \Longrightarrow Q'$, (6)

large-change sensitivity: $\bar{\underset{\sim}{u}}^T\ \Delta\underset{\sim}{A}\ \underset{\sim}{v} \Longrightarrow \Delta Q$, (7)

where the parameter ϕ is contained in $\underset{\sim}{A}$. A full reverse analysis taking

$$[\underset{\sim}{v}_1^n\ \ \underset{\sim}{v}_2^n] = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

yields

$$[\underset{\sim}{v}_1^i\ \ \underset{\sim}{v}_2^i] = \underset{\sim}{A}^{i+1}\ \underset{\sim}{A}^{i+2} \ldots \underset{\sim}{A}^n \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

and a corresponding full forward analysis taking

$$[\bar{u}_1^1 \;\; \bar{u}_2^1]^T = [u_1^0 \;\; u_2^0]^T = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

yields

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A^1 A^2 \ldots A^{i-1} = [\bar{u}_1^i \;\; \bar{u}_2^i]^T .$$

Symmetrical Networks Consisting of Symmetrical Elements

In many practical cases we encounter symmetrical networks (around a central plane) which consist of reciprocal and symmetrical elements. Series impedances, shunt admittances, transmission lines and RC lines are examples of such elements. The properties possessed by these elements are

$$a_{11} = a_{22}$$

and

$$\det A = a_{11} a_{22} - a_{12} a_{21} = 1,$$

where

$$A \triangleq \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}, \tag{8}$$

which lead to the transformation

$$[e_1 \;\; -e_2] \, A \, [e_1 \;\; -e_2] = A^{-1}$$

or

$$[e_1 \;\; -e_2] \, A^{-1} \, [e_1 \;\; -e_2] = A \; .$$

Using this transformation it may be shown, for such networks, that

$$[v_1^i \;\; v_2^i] = [\bar{u}_1^{n-i+1} \;\; \bar{u}_2^{n-i+1}]^T .$$

This equality can be used to reduce computational effort.

Reference Planes

In considering more than one element in the cascade we divide the network into subnetworks by reference planes. These in turn are chosen so that no more than one element is to be explicitly considered between any pair of reference planes. In Fig. 2 the element $\underset{\sim}{A}$ is the only element whose effect is to be considered. In Fig. 3 the elements $\underset{\sim}{A}^k$, $\underset{\sim}{A}^i$ and $\underset{\sim}{A}^j$ are considered in the kth, the ith and the jth subnetworks, respectively. Note that the superscripts of $\underset{\sim}{A}$ here, and from now on, denote the subnetwork and not the element. Forward and reverse analyses are initiated at the reference planes. A forward iteration of the structure of Fig. 3 is illustrated in Fig. 4, where equivalent (Thevenin) sources are iteratively determined. Reverse iteration is shown in Fig. 5, where equivalent (Norton) sources are iteratively determined.

III. NETWORK FUNCTIONS IN TERMS OF ELEMENTS UNDER CONSIDERATION

Performing forward analysis from the source of the ith subnetwork to the input of $\underset{\sim}{A}^i$ and reverse analysis from the load to the output of $\underset{\sim}{A}^i$ we have

$$V_S^i = (\bar{\underset{\sim}{u}}_1 + Z_S^i \bar{\underset{\sim}{u}}_2)^T \underset{\sim}{A}^i (V_L^i \underset{\sim}{v}_1 + (Y_L^i V_L^i - I_L^i)\underset{\sim}{v}_2) = V_L^k + Z_S^i I_S^i \tag{9}$$

and the current through the voltage source of the ith subnetwork

$$I_S^i = \bar{\underset{\sim}{u}}_2^T \underset{\sim}{A}^i (V_L^i \underset{\sim}{v}_1 + (Y_L^i V_L^i - I_L^i)\underset{\sim}{v}_2) = V_L^k Y_L^k - I_L^k . \tag{10}$$

From (9), letting $I_L^i = 0$ and $Y_L^i = 0$, we have $I_S^j = 0$ and the <u>Thevenin</u>

<u>voltage</u>

$$V_S^j = V_L^i = \frac{V_S^i}{(\bar{\underset{\sim}{u}}_1 + Z_S^i \bar{\underset{\sim}{u}}_2)^T \underset{\sim}{A}^i \underset{\sim}{v}_1} = \frac{V_S^i}{Q_{11}^i + Z_S^i Q_{21}^i} , \tag{11}$$

where the Q terms have been defined in (4). Letting $V_S^i = 0$ and $Y_L^i = 0$, we have $I_S^j = -I_L^i$ and the <u>output impedance</u>

$$Z_S^j = \frac{V_L^i}{I_L^i} = \frac{(\bar{\underset{\sim}{u}}_1 + Z_S^i \bar{\underset{\sim}{u}}_2)^T \underset{\sim}{A}^i \underset{\sim}{v}_2}{(\bar{\underset{\sim}{u}}_1 + Z_S^i \bar{\underset{\sim}{u}}_2)^T \underset{\sim}{A}^i \underset{\sim}{v}_1} = \frac{Q_{12}^i + Z_S^i Q_{22}^i}{Q_{11}^i + Z_S^i Q_{21}^i} , \tag{12}$$

where, again, the Q terms of (4) are used to obtain a compact expression. These expressions for V_S^j and Z_S^j permit equivalent Thevenin sources to be moved in a forward iteration.

From (9) and (10), letting $I_L^i = 0$ and $Z_S^i = 0$ we have $I_L^k = 0$ and the <u>input admittance</u>

$$Y_L^k = \frac{I_S^i}{V_S^i} = \frac{\bar{\underset{\sim}{u}}_2^T \underset{\sim}{A}^i (\underset{\sim}{v}_1 + Y_L^i \underset{\sim}{v}_2)}{\bar{\underset{\sim}{u}}_1^T \underset{\sim}{A}^i (\underset{\sim}{v}_1 + Y_L^i \underset{\sim}{v}_2)} = \frac{Q_{21}^i + Y_L^i Q_{22}^i}{Q_{11}^i + Y_L^i Q_{12}^i} . \tag{13}$$

Letting $V_S^i = 0$ and $Z_S^i = 0$, we have $V_L^k = 0$ and the <u>Norton current</u>

$$I_L^k = -I_S^i = -I_L^i (Y_L^k \bar{\underset{\sim}{u}}_1 - \bar{\underset{\sim}{u}}_2)^T \underset{\sim}{A}^i \underset{\sim}{v}_2 = -I_L^i (Y_L^k Q_{12}^i - Q_{22}^i) . \tag{14}$$

These expressions for I_L^k and Y_L^k permit equivalent Norton sources to be moved (if desired) in a reverse iteration.

The input current I_S^i for $I_L^i = 0$ is obtained via (13) as

$$I_S^i = V_S^i \Bigg/ \left[Z_S^i + \frac{\bar{\underset{\sim}{u}}_1^T \underset{\sim}{A}^i (\underset{\sim}{v}_1 + Y_L^i \underset{\sim}{v}_2)}{\bar{\underset{\sim}{u}}_2^T \underset{\sim}{A}^i (\underset{\sim}{v}_1 + Y_L^i \underset{\sim}{v}_2)} \right]$$

$$= \frac{V_S^i \bar{\underset{\sim}{u}}_2^{T} \underset{\sim}{A}^i (\underset{\sim}{v}_1 + Y_L^i \underset{\sim}{v}_2)}{(\bar{\underset{\sim}{u}}_1 + Z_S^i \bar{\underset{\sim}{u}}_2)^T \underset{\sim}{A}^i (\underset{\sim}{v}_1 + Y_L^i \underset{\sim}{v}_2)} = \frac{V_S^i (Q_{21}^i - Y_L^i Q_{22}^i)}{Q_{11}^i + Y_L^i Q_{12}^i + Z_S^i Q_{21}^i + Z_S^i Y_L^i Q_{22}^i} . \quad (15)$$

Tables II and III summarize the procedures and the effort required in evaluating the different factors in the derived equations.

Useful special cases of these formulas for I_S and V_L in Fig. 2 are, from (15) and (11), respectively,

$$I_S = V_S \frac{\bar{\underset{\sim}{u}}_2^T \underset{\sim}{A} \underset{\sim}{v}_1}{\bar{\underset{\sim}{u}}_1^T \underset{\sim}{A} \underset{\sim}{v}_1} = V_S \frac{Q_{21}}{Q_{11}} \quad (16)$$

and

$$V_L = \frac{V_S}{\bar{\underset{\sim}{u}}_1^T \underset{\sim}{A} \underset{\sim}{v}_1} = \frac{V_S}{Q_{11}} . \quad (17)$$

Table IV gives some useful formulas which can be obtained for variations in a particular element $\underset{\sim}{A}$. We note, for example, that, since $\underset{\sim}{A}$ is arbitrary and at most only one full analysis yields all Q_{11}, δQ_{11}, Q_{11}' and ΔQ_{11}, the corresponding V_L, δV_L, $\partial V_L / \partial \phi$ and ΔV_L w.r.t. all possible parameters anywhere in the cascade can be evaluated exactly for one network analysis. This particular special case is equivalent to the results of previous researchers [1,2].

IV. NUMERICAL EXAMPLE

The cascaded seven-section bandpass filter shown in Fig. 6 [3, 4] serves as a numerical example. All sections are quarter-wave at 2.175 GHz. The normalized minimax characteristic impedances are [4]

$$Z_1^0 = Z_7^0 = 0.606463$$
$$Z_2^0 = Z_6^0 = 0.303051$$
$$Z_3^0 = Z_5^0 = 0.722061$$
$$Z_4^0 = 0.235593$$

The output voltage V_L at a normalized frequency of 0.7 is $0.49740790 - j3.9011594 \times 10^{-3}$, verified twice using (11): once associating $\underset{\sim}{A}^i$ with Z_3 and once with Z_4. Furthermore, one analysis yielded

$$V_L(Z_4^0+0.03) = 0.49838950 - j\ 0.034901610$$
$$V_L(Z_4^0-0.03) = 0.49062912 + j\ 0.034959186$$

The open-circuit voltage at the load end was calculated using (11) as

$$V_{OC} = 0.98624507 + j\ 0.092266904$$

and the Thevenin impedance using (12) is

$$Z_{TH} = 0.98119253 + j\ 0.20103391$$

which further verified V_L.

One analysis taking $\varepsilon_2 = 0.021$, $\varepsilon_5 = 0.024$ yielded

$$V_L(Z_2^0-\varepsilon_2, Z_5^0-\varepsilon_5) = 0.49719716 + j\ 2.2191360 \times 10^{-3}$$
$$V_L(Z_2^0+\varepsilon_2, Z_5^0-\varepsilon_5) = 0.49583538 - j\ 2.3636314 \times 10^{-2}$$
$$V_L(Z_2^0-\varepsilon_2, Z_5^0+\varepsilon_5) = 0.49732462 + j\ 1.7909912 \times 10^{-2}$$
$$V_L(Z_2^0+\varepsilon_2, Z_5^0+\varepsilon_5) = 0.49751427 - j\ 8.3726470 \times 10^{-3}$$

A multidimensional quadratic approximation was carried out for V_L following the approach of Bandler and Abdel-Malek [5]. The variables for the approximation were the characteristic impedances as well as the normalized frequency. The circuit responses at 45 base points (which is equal to $(k+1)(k+2)/2$, where k is 8) were needed to evaluate the coefficients of the quadratic polynomial approximating the response function [5]. A base point is a point where the approximation and the actual function coincide. The center base point, which is the center of the interpolation region in which the approximation is assumed to be valid, had the characteristic impedances given before and a normalized frequency of 0.7. 16 base points were determined by varying one parameter at a time by $\pm\delta$ w.r.t. its value at the center of interpolation. For the characteristic impedances δ was chosen to be 0.03 and for the normalized frequency it was 0.01. At the remaining 28 base points only two parameters were perturbed at a time from their values at the center of interpolation by a percentage of their δ.

The symmetry of the structure was taken into consideration in choosing these base points. Letting $\bar{\underset{\sim}{\phi}}$ be the center of the interpolation region, the base points can be expressed by [6]

$$[\underset{\sim}{\phi}^1 \;\; \underset{\sim}{\phi}^2 \; \cdots \; \underset{\sim}{\phi}^N] = \underset{\sim}{D}\,[\underset{\sim}{1}_k \;\; -\underset{\sim}{1}_k \;\; \underset{\sim}{B} \;\; \underset{\sim}{0}_k] + [\bar{\underset{\sim}{\phi}} \;\; \bar{\underset{\sim}{\phi}} \; \cdots \; \bar{\underset{\sim}{\phi}}] \,, \qquad (18)$$

where

N is equal to 45 in our case,

$\underset{\sim}{1}_k$ is a k-dimensional identity matrix,

$\underset{\sim}{0}_k$ is a zero vector of dimension k,

$$\mathbf{D} = \begin{bmatrix} 0.03 & & & & \\ & 0.03 & & & \\ & & \ddots & & \\ & & & 0.03 & \\ & & & & 0.01 \end{bmatrix},$$

and

$\mathbf{B}$ is a k x [k(k-1)/2] matrix given on page 12.

Examining this $\mathbf{B}$ matrix we note that the entries for perturbing two parameters at a time are the same as for their corresponding symmetrical parameters. The choice of base points given by (18) preserves symmetry in the appropriate coefficients of the multidimensional polynomials.

Taking the optimal minimax characteristic impedances [4]:

$$Z_1 = Z_7 = 0.606595$$

$$Z_2 = Z_6 = 0.303547$$

$$Z_3 = Z_5 = 0.722287$$

$$Z_4 = 0.235183$$

and calculating the group delay using the derivative of V_L w.r.t. ω obtained from the quadratic approximation yielded

$$T_G = 0.893 \text{ ns},$$

while the exact group delay is [7]

$$T_{G_{exact}} = 0.895 \text{ ns}.$$

$$
\underset{\sim}{B} = \left[\begin{array}{cccccccccccccccccccccccccccc}
.5 & -.2 & .9 & -.2 & .5 & .3 & .7 & & & & & & & & & & & & & & & & & & & & & \\
.7 & & & & & & & .7 & .5 & .7 & .9 & .7 & -.3 & & & & & & & & & & & & & & & \\
 & .8 & & & & & & .8 & & & & & & .7 & -.4 & .8 & .8 & .8 & & & & & & & & & & \\
 & & -.5 & & & & & & .9 & & & & & -.9 & & & & & -.9 & .9 & .5 & .8 & & & & & & \\
 & & & .8 & & & & & & .8 & & & & & .8 & & & & .7 & & & & .8 & .8 & .8 & & & \\
 & & & & .7 & & & & & & -.2 & & & & & .7 & & & & .5 & & & .7 & & & .7 & -.3 & \\
 & & & & & -.9 & & & & & & .5 & & & & & -.2 & & & & .9 & & & -.2 & & .5 & & .7 \\
 & & & & & & .6 & & & & & & .4 & & & & & .9 & & & & .4 & & & .9 & & .4 & .6
\end{array}\right]
$$

V. TWO ALGORITHMS FOR EVALUATION OF LARGE CHANGES

The two following algorithms were used to obtain responses at the base points for the interpolation performed in the previous section. The first was used when one parameter at a time was perturbed and the second was used when pairs of parameters were perturbed simultaneously. Note that when the normalized frequency was perturbed a whole new analysis had to be performed.

<u>Algorithm 1 Multiple One-at-a-time Changes</u>

<u>Step 1</u> Initialize $\underset{\sim}{\bar{u}}$ and $\underset{\sim}{v}$.

Set $i \leftarrow 1$, $m \leftarrow 1$, $j \leftarrow n$.

<u>Comment</u> n is the total number of elements in the cascade and m is a counter for the variable elements.

<u>Step 2</u> If $i = \ell_m$ go to Step 5.

<u>Comment</u> ℓ_m is an element of L, an index set containing superscripts of the k matrices containing the k variable parameters and ordered consecutively. It is assumed that each matrix contains only one variable.

<u>Step 3</u> $\underset{\sim}{\bar{u}}^T \leftarrow \underset{\sim}{\bar{u}}^T \underset{\sim}{A}^i$.

$i \leftarrow i + 1$.

<u>Step 4</u> If $i = \ell_m$ go to Step 5.

Go to Step 3.

<u>Step 5</u> Let $\underset{\sim}{x}^m \leftarrow \underset{\sim}{\bar{u}}$.

If $i = \ell_k$ go to Step 7.

<u>Comment</u> $\underset{\sim}{x}^1, \underset{\sim}{x}^2, \ldots, \underset{\sim}{x}^k$ are working arrays to store the $\underset{\sim}{\bar{u}}$ vectors required in the evaluation of the large changes taking place.

Step 6 $m \leftarrow m + 1$. Go to Step 3.

Step 7 If $n = \ell_k$ go to Step 10.

Step 8 $\underset{\sim}{v} = \underset{\sim}{A}^j \underset{\sim}{v}$.

$j \leftarrow j - 1$.

Step 9 If $j = \ell_m$ go to Step 10.

Go to Step 8.

Step 10 Evaluate Q using the stored $\underset{\sim}{x}^m$, $\underset{\sim}{v}$ and the perturbed $\underset{\sim}{A}^j$.

If $j = \ell_1$ stop.

Comment Positive and negative extremes of the variable in $\underset{\sim}{A}^j$ are considered simultaneously.

Step 11 $m \leftarrow m - 1$. Go to Step 8.

Algorithm 2 Multiple Pairwise Changes

This algorithm is for evaluating the response at the k(k-1)/2 base points where two parameters are perturbed at a time. At the first k-1 points following those considered in Algorithm 1 the parameters indicated by the subscripts

1,2 1,3 ... 1,k

are changed; at the next k-2 points the parameters indicated by the subscripts

2,3 2,4 ... 2,k

are changed, and so on, until the final point at which parameters k-1 and k are perturbed. Fig. 7 serves to illustrate the analyses involved.

Step 1 Initialize $\underset{\sim}{u}_1^0$, $\underset{\sim}{u}_2^0$, $\underset{\sim}{u}_1^1$ and $\underset{\sim}{u}_2^1$.

Set $i \leftarrow 1$, $m \leftarrow 1$, $q \leftarrow 0$, $r \leftarrow 1$ and $s \leftarrow k - 1$.

Comment $\underset{\sim}{u}_1^1$ and $\underset{\sim}{u}_2^1$ are vectors to be initialized as $\underset{\sim}{u}_1^0$ and $\underset{\sim}{u}_2^0$, respectively. They have the same role as $\underset{\sim}{u}_1^0$ and $\underset{\sim}{u}_2^0$ in the forward analysis initiated at a reference plane immediately following the first variable element.

Step 2 If $i = \ell_m$ go to Step 4.

Comment ℓ_m is an element of L, an index set containing superscripts of the k matrices containing the k variable parameters as indicated in Algorithm 1.

Step 3 $\underset{\sim}{u}_1^{0T} \leftarrow \underset{\sim}{u}_1^{0T} \underset{\sim}{A}^i$.

$\underset{\sim}{u}_2^{0T} \leftarrow \underset{\sim}{u}_2^{0T} \underset{\sim}{A}^i$.

Step 4 If m = 1 go to Step 6.

$\underset{\sim}{u}_1^{1T} \leftarrow \underset{\sim}{u}_1^{1T} \underset{\sim}{A}^i$.

$\underset{\sim}{u}_2^{1T} \leftarrow \underset{\sim}{u}_2^{1T} \underset{\sim}{A}^i$.

.
.
.

$\underset{\sim}{u}_1^{qT} \leftarrow \underset{\sim}{u}_1^{qT} \underset{\sim}{A}^i$.

$\underset{\sim}{u}_2^{qT} \leftarrow \underset{\sim}{u}_2^{qT} \underset{\sim}{A}^i$.

Comment This step is not performed until we reach a variable element, since the analyses involving the $\underset{\sim}{u}^j$ do not begin until the jth variable element has been considered.

Step 5 Set $i \leftarrow i + 1$.

Step 6 If $i = \ell_m$ go to Step 7.

Go to Step 3.

Step 7 If m = k go to Step 9.

Calculate the Thevenin impedances and voltages

$Z_S(m,1), \ldots, Z_S(m,s)$,

$V_S(m,1), \ldots, V_S(m,s)$.

$s \leftarrow s - 1$.

Comment For the first variable element k-1 sets of Z_S and V_S have to be evaluated since changes in this element will be coupled one at a time with changes in the next k-1 variable elements. For the second variable element k-2 sets of Z_S and V_S are calculated and so on. See Fig. 7.

Step 8   If m = 1 go to Step 13.

Step 9   Set p ← 1.

Comment   p is an internal counter.

Step 10 $\underset{\sim}{x}^r \leftarrow \underset{\sim}{u}^p$.

If p = q go to Step 12.

Comment   When the analysis has reached a reference plane immediately preceding an element containing a variable whose change is to be associated with any previously encountered variable a snapshot of the appropriate $\underset{\sim}{u}$ vectors is taken and stored in the $\underset{\sim}{x}$ arrays. See Fig. 7.

Step 11   Set r ← r + 1.

p ← p + 1.

Go to Step 10.

Step 12   Set r ← r + 1.

Step 13   If m = k go to Step 16.

$\underset{\sim}{u}_1^{0T} \leftarrow \underset{\sim}{u}_1^{0T} \underset{\sim}{A}^i$.

$\underset{\sim}{u}_2^{0T} \leftarrow \underset{\sim}{u}_2^{0T} \underset{\sim}{A}^i$.

Step 14   If m = 1 go to Step 15.

$\underset{\sim}{u}_1^{1T} \leftarrow \underset{\sim}{u}_1^{1T} \underset{\sim}{A}^i$.

$\underset{\sim}{u}_2^{1T} \leftarrow \underset{\sim}{u}_2^{1T} \underset{\sim}{A}^i$.

.

.

.

$\underset{\sim}{u}_1^{qT} \leftarrow \underset{\sim}{u}_1^{qT} \underset{\sim}{A}^i$.

$\underset{\sim}{u}_2^{qT} \leftarrow \underset{\sim}{u}_2^{qT} \underset{\sim}{A}^i$.

Comment — In Step 7 we calculated sets of Z_S and V_S accounting for variations in $\tilde{A}^i$. In Steps 13 and 14, however, we carry forward the analyses for which $\tilde{A}^i$ is considered fixed.

Step 15 — Set $i \leftarrow i + 1$.

$m \leftarrow m + 1$.

$q \leftarrow q + 1$.

Initialize $\tilde{u}_1^{qT}$ and $\tilde{u}_2^{qT}$ and go to Step 6.

Comment — $\tilde{u}_1^{qT}$ and $\tilde{u}_2^{qT}$ are initialized to start a forward analysis at a reference plane immediately following a variable element $\tilde{A}^i$.

Step 16 — Set $r \leftarrow r - 1$.

$m \leftarrow m - 1$.

Initialize $\tilde{v}_1$ and $\tilde{v}_2$.

Comment — At this step we start the analysis from the load end.

Step 17 — If $n = \ell_k$ go to Step 20.

Set $j \leftarrow n$.

Comment — n is the total number of elements in the cascade.

Step 18 — $\tilde{v}_1 \leftarrow \tilde{A}^j \tilde{v}_1$.

$\tilde{v}_2 \leftarrow \tilde{A}^j \tilde{v}_2$.

$j \leftarrow j - 1$.

Step 19 — If $j = \ell_m$ go to Step 20.

Go to Step 18.

Step 20 — $p \leftarrow 1$.

Step 21 — Calculate Q using V_S, Z_S, $\tilde{A}^j$ and $\tilde{v}$ and the appropriate $\tilde{x}$.

Comment — When we reach the kth variable element we calculate k-1 values of Q, and when the variable element k-1 is reached we calculate k-2 values of Q and so on as illustrated in Fig. 7.

<u>Step 22</u> If $p = q$ go to Step 23.

Set $r \leftarrow r - 1$.

$p \leftarrow p + 1$.

Go to Step 21.

<u>Step 23</u> If $m = 1$ Stop.

Set $q \leftarrow q - 1$.

$m \leftarrow m - 1$.

Go to Step 18.

VI. BRANCHED CIRCUITS

Consider, as an example, the cascaded circuit shown in Fig. 8, which has two branches, one connected in series and one in parallel. In the series and parallel branches we highlight, for example, the elements $\underset{\sim}{B}$ and $\underset{\sim}{C}$, respectively. The series branch can be thought of equivalently as an element consisting of a series impedance connected in cascade with the main circuit as shown in Fig. 8. This impedance Z may be taken as the inverse of the input admittance derived in (13) and is given by

$$Z = \frac{\bar{\underset{\sim}{u}}_{1B}^{T} \; \underset{\sim}{B} \; \underset{\sim}{v}_{1B}}{\bar{\underset{\sim}{u}}_{2B}^{T} \; \underset{\sim}{B} \; \underset{\sim}{v}_{1B}} , \qquad (19)$$

where the subscript $\underset{\sim}{B}$ distinguishes terms associated with the branch from that of the cascaded main circuit. The forward analysis is initiated at reference plane d and the reverse analysis is initiated at reference plane b. (See Fig. 8.)

Similarly, the parallel branch can be thought of equivalently as an admittance Y connected in shunt in the cascade. The admittance Y (as in

(13)) is given by

$$Y = \frac{\bar{\underset{\sim}{u}}{}_{2C}^{T} \; \underset{\sim}{C} \; \underset{\sim}{v}_{1C}}{\bar{\underset{\sim}{u}}{}_{1C}^{T} \; \underset{\sim}{C} \; \underset{\sim}{v}_{1C}} , \tag{20}$$

where the forward analysis is initiated at reference plane e and the reverse analysis is initiated at reference plane c.

Different formulas relating the load voltages of the branches to the variables can be derived. The load voltage of the series branch can be derived (Appendix A1) as a function of $\underset{\sim}{B}$ as

$$V_{BL}(\underset{\sim}{B}) = \frac{\underset{\sim}{e}_2^{T} \; \underset{\sim}{v}_{1Z} \; V_S}{\bar{\underset{\sim}{u}}{}_{2B}^{T} \; \underset{\sim}{B} \; \underset{\sim}{v}_{1B} \; \bar{\underset{\sim}{u}}{}_{1Z}^{T} \begin{bmatrix} 1 & Z \\ 0 & 1 \end{bmatrix} \underset{\sim}{v}_{1Z}} , \tag{21}$$

where

$\underset{\sim}{u}_{1Z}^{T}$ is the result at reference plane f of a forward analysis initiated at the source,

$\underset{\sim}{v}_{1Z}$ is the result at reference plane g of a reverse analysis initiated at the load reference plane a.

It can also be obtained (Appendix A2) as a function of $\underset{\sim}{C}$ as

$$V_{BL}(\underset{\sim}{C}) = \frac{[\bar{\underset{\sim}{u}}{}_{1Yf}^{T} - \bar{\underset{\sim}{u}}{}_{1Yg}^{T}] \begin{bmatrix} 1 & 0 \\ Y & 1 \end{bmatrix} \underset{\sim}{v}_{1Y} \; V_S}{\bar{\underset{\sim}{u}}{}_{1B}^{T} \; \underset{\sim}{B} \; \underset{\sim}{v}_{1B} \; \bar{\underset{\sim}{u}}{}_{1Y}^{T} \begin{bmatrix} 1 & 0 \\ Y & 1 \end{bmatrix} \underset{\sim}{v}_{1Y}} , \tag{22}$$

where

$\bar{\underset{\sim}{u}}{}_{1Y}^{T}$ is the result at reference plane h of a forward analysis,

$\underset{\sim}{v}_{1Y}$ is the result at reference plane k of a reverse analysis,

$\bar{\underset{\sim}{u}}{}_{1Yf}^{T}$ is the result at reference plane h of a forward analysis initiated at reference plane f,

$\bar{u}^T_{1Yg}$ is the result at reference plane h of a forward analysis initiated at reference plane g.

The load voltage of the parallel branch can also be derived (Appendix A3) as a function of C as

$$V_{CL}(C) = \frac{e_1^T \; v_{1Y} \; V_S}{\bar{u}^T_{1C} \; C \; v_{1C} \; \bar{u}^T_{1Y} \begin{bmatrix} 1 & 0 \\ Y & 1 \end{bmatrix} v_{1Y}} , \tag{23}$$

and (Appendix A4) as a function of B as

$$V_{CL}(B) = \frac{e_1^T \; v_{1Y} \; V_S}{\bar{u}^T_{1C} \; C \; v_{1C} \; \bar{u}^T_{1Z} \begin{bmatrix} 1 & Z \\ 0 & 1 \end{bmatrix} v_{1Z}} . \tag{24}$$

VII. CASCADED NETWORKS OF 2p-PORT ELEMENTS

The approach we have developed can also be utilized in the analysis and design of cascaded networks consisting of 2p-port elements. Consider the 2p-port element shown in Fig. 9, possessing p input ports and p output ports. Its transmission matrix is given by

$$A \triangleq \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} ,$$

where A_{11}, A_{12}, A_{21} and A_{22} are p x p matrices. The input quantities in this case are

$$\bar{\underset{\sim}{y}} = \begin{bmatrix} \bar{y}_1 \\ \bar{y}_2 \\ \vdots \\ \bar{y}_p \\ \bar{y}_{p+1} \\ \bar{y}_{p+2} \\ \vdots \\ \bar{y}_{2p} \end{bmatrix},$$

and the output quantities are

$$\underset{\sim}{y} = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_p \\ y_{p+1} \\ y_{p+2} \\ \vdots \\ y_{2p} \end{bmatrix},$$

where the element with subscripts 1 to p denote voltages and from p+1 to 2p denote currents.

For the forward and reverse analyses the matrices $\bar{\underset{\sim}{U}}_1$, $\bar{\underset{\sim}{U}}_2$, $\underset{\sim}{V}_1$ and $\underset{\sim}{V}_2$ are initialized such that

$$\underset{\sim}{E}_1 \Longrightarrow \underset{\sim}{U}_1 \text{ or } \underset{\sim}{V}_1 ,$$

$$\underset{\sim}{E}_2 \Longrightarrow \underset{\sim}{U}_2 \text{ or } \underset{\sim}{V}_2 ,$$

where

$$\underset{\sim}{E}_1 \triangleq \begin{bmatrix} \underset{\sim}{1}_p \\ \\ \underset{\sim}{0}_p \end{bmatrix},$$

and

$$\underset{\sim}{E}_2 \triangleq \begin{bmatrix} \underset{\sim}{0}_p \\ \\ \underset{\sim}{1}_p \end{bmatrix},$$

and where

$\underset{\sim}{1}_p$ is the unit matrix of order p,

$\underset{\sim}{0}_p$ is the null matrix of order p.

We can now derive in an analogous manner to the derivation of (9)

$$\underset{\sim}{V}_S = (\overline{\underset{\sim}{U}}_1^T + \underset{\sim}{Z}_S\, \overline{\underset{\sim}{U}}_2^T)\ \underset{\sim}{A}\ (\underset{\sim}{V}_1\, \underset{\sim}{V}_L + \underset{\sim}{V}_2\, (\underset{\sim}{Y}_L\, \underset{\sim}{V}_L - \underset{\sim}{I}_L)), \qquad (25)$$

where

$\overline{\underset{\sim}{U}}_1$, $\overline{\underset{\sim}{U}}_2$, $\underset{\sim}{V}_1$ and $\underset{\sim}{V}_2$ are the matrices obtained from forward and reverse analyses,

$\underset{\sim}{V}_S$ is the vector containing the p source voltages,

$\underset{\sim}{V}_L$ is the vector of load voltages,

$\underset{\sim}{I}_L$ is the vector of current sources at the loads (if any),

$\underset{\sim}{Z}_S$ is a diagonal matrix containing the impedances of the sources,

$\underset{\sim}{Y}_L$ is a diagonal matrix containing the load admittances.

To evaluate the unknowns $\underset{\sim}{V}_L$, having obtained numerical values for (25),a system of p linear equations is solved. When $\underset{\sim}{A}$ is perturbed or when derivatives are required, only $6p^3$ additional multiplications and the solution of a p-system of linear equations are needed and not a whole reanalysis of the entire cascaded circuit.

VIII. CONCLUSIONS

An important claim we make in this paper is that equations (9) - (15) can be used to generate in a straightforward manner, following differencing or differentiating (as appropriate), any desired exact formulas for multiple network analyses, sensitivity and tolerance analysis with simultaneous large changes. All calculations are carried forward simultaneously and redundant calculations are obviated as demonstrated by the examples and algorithms presented.

Symmetry of the networks analyzed can be exploited leading to saving of computational effort. Branched circuits can be handled readily. Formulas, similar to (21)-(24), can be derived for other branched structures using the same concepts so as to render the sensitivity analysis and design of these circuits as simple as possible. The approach should prove to be very suitable for computer-aided design of cascaded microwave circuits and systems consisting of 2-ports. It appears to be readily extendable to 2p-port networks.

APPENDIX

A1 To Obtain V_{BL} as a Function of V_S and $\underset{\sim}{B}$

The voltage across the impedance Z, representing the branched circuit, in terms of V_{BL} is given by

$$V_Z = \bar{\underset{\sim}{u}}_{1B}^T \underset{\sim}{B} \underset{\sim}{v}_{1B} V_{BL} \quad , \tag{A1}$$

and it can be expressed in terms of voltages in the main cascaded circuit as

$$V_Z = \underset{\sim}{e}_1^T [\bar{\underset{\sim}{v}}_{1Z} - \underset{\sim}{v}_{1Z}] V_L \ , \tag{A2}$$

where $\bar{\underset{\sim}{v}}_{1Z}$ is the result of the reverse analysis at reference plane f. So (A2) can be written, substituting for the chain matrix of the element representing the branch, as

$$V_Z = \underset{\sim}{e}_1^T [\begin{bmatrix} 1 & Z \\ 0 & 1 \end{bmatrix} \underset{\sim}{v}_{1Z} - \underset{\sim}{v}_{1Z}] V_L \tag{A3}$$

$$= [1 \quad 0] \left(\begin{bmatrix} 1 & Z \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right) \underset{\sim}{v}_{1Z} V_L \tag{A4}$$

$$= \underset{\sim}{e}_2^T \underset{\sim}{v}_{1Z} Z V_L \ . \tag{A5}$$

The load voltage of the main cascade V_L can be expressed by

$$V_L = \frac{V_S}{\bar{\underset{\sim}{u}}_{1Z}^T \begin{bmatrix} 1 & Z \\ 0 & 1 \end{bmatrix} \underset{\sim}{v}_{1Z}} \tag{A6}$$

and (A1) can be rewritten as

$$V_{BL} = \frac{V_Z}{\bar{u}_{1B}^T \, B \, v_{1B}} \quad . \tag{A7}$$

Substituting for V_Z of (A5) we have

$$V_{BL} = \frac{e_2^T \, v_{1Z} \; Z \; V_L}{\bar{u}_{1B}^T \, B \, v_{1B}} \tag{A8}$$

and substituting for V_L from (A6) and Z from (19), we get

$$V_{BL} = \frac{e_2^T \, v_{1Z} \, \dfrac{\bar{u}_{1B}^T \, B \, v_{1B}}{\bar{u}_{2B}^T \, B \, v_{1B}} \, V_S}{\bar{u}_{1B}^T \, B \, v_{1B} \, \bar{u}_{1Z}^T \begin{bmatrix} 1 & Z \\ 0 & 1 \end{bmatrix} v_{1Z}} \, , \tag{A9}$$

hence

$$V_{BL}(B) = \frac{e_2^T \, v_{1Z} \, V_S}{\bar{u}_{2B}^T \, B \, v_{1B} \, \bar{u}_{1Z}^T \begin{bmatrix} 1 & Z \\ 0 & 1 \end{bmatrix} v_{1Z}} \; .$$

A2 To Obtain V_{BL} as a Function of V_S and C

From (A7) and (A2) we can write V_{BL} as

$$V_{BL} = \frac{e_1^T \, [\bar{v}_{1Z} - v_{1Z}] \, V_L}{\bar{u}_{1B}^T \, B \, v_{1B}} \; . \tag{A10}$$

The load voltage V_L can be expressed (compare with (A6)) by

$$V_L = \frac{V_S}{\bar{u}_{1Y}^T \begin{bmatrix} 1 & 0 \\ Y & 1 \end{bmatrix} v_{1Y}} \quad . \tag{A11}$$

We can write, using notation defined for (22),

$$\underset{\sim}{e}_1^T \, \underset{\sim}{\bar{v}}_{1Z} = \underset{\sim}{\bar{u}}_{1Yf}^T \begin{bmatrix} 1 & 0 \\ Y & 1 \end{bmatrix} \underset{\sim}{v}_{1Y} \, . \tag{A12}$$

Similarly,

$$\underset{\sim}{e}_1^T \, \underset{\sim}{v}_{1Z} = \underset{\sim}{\bar{u}}_{1Yg}^T \begin{bmatrix} 1 & 0 \\ Y & 1 \end{bmatrix} \underset{\sim}{v}_{1Y} \, . \tag{A13}$$

Substituting these terms and V_L of (A11) into (A10) we obtain

$$V_{BL}(\underset{\sim}{C}) = \frac{[\underset{\sim}{\bar{u}}_{1Yf}^T - \underset{\sim}{\bar{u}}_{1Yg}^T] \begin{bmatrix} 1 & 0 \\ Y & 1 \end{bmatrix} \underset{\sim}{v}_{1Y} V_S}{\underset{\sim}{\bar{u}}_{1B}^T \, \underset{\sim}{B} \, \underset{\sim}{v}_{1B} \, \underset{\sim}{\bar{u}}_{1Y}^T \begin{bmatrix} 1 & 0 \\ Y & 1 \end{bmatrix} \underset{\sim}{v}_{1Y}} \, . \tag{A14}$$

A3 To Obtain V_{CL} as a Function of V_S and $\underset{\sim}{C}$

The voltage across Y in terms of V_{CL} is given by

$$V_Y = \underset{\sim}{\bar{u}}_{1C}^T \, \underset{\sim}{C} \, \underset{\sim}{v}_{1C} \, V_{CL} \, , \tag{A15}$$

and in terms of V_L, as

$$V_Y = \underset{\sim}{e}_1^T \, \underset{\sim}{v}_{1Y} \, V_L \, . \tag{A16}$$

But V_L is also given by

$$V_L = \frac{V_S}{\underset{\sim}{\bar{u}}_{1Y}^T \begin{bmatrix} 1 & 0 \\ Y & 1 \end{bmatrix} \underset{\sim}{v}_{1Y}} \, . \tag{A17}$$

So, substituting this V_L into (A16) and the resulting V_Y into (A15) we get

$$V_{CL}(\underset{\sim}{C}) = \frac{\underset{\sim}{e}_1^T \, \underset{\sim}{v}_{1Y} \, V_S}{\underset{\sim}{\bar{u}}_{1C}^T \, \underset{\sim}{C} \, \underset{\sim}{v}_{1C} \, \underset{\sim}{\bar{u}}_{1Y}^T \begin{bmatrix} 1 & 0 \\ Y & 1 \end{bmatrix} \underset{\sim}{v}_{1Y}} \, . \tag{A18}$$

A4 To Obtain V_{CL} as a Function of V_S and $\underset{\sim}{B}$

From (A15), (A16) and (A6) we can write V_{CL} as

$$V_{CL}(\underset{\sim}{B}) = \frac{\underset{\sim}{e}_1^T \; \underset{\sim}{v}_{1Y} \; V_S}{\bar{\underset{\sim}{u}}_{1C}^T \; \underset{\sim}{C} \; \underset{\sim}{v}_{1C} \; \bar{\underset{\sim}{u}}_{1Z}^T \begin{bmatrix} 1 & Z \\ 0 & 1 \end{bmatrix} \underset{\sim}{v}_{1Z}} . \qquad \text{(A19)}$$

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TABLE I

PRINCIPAL CONCEPTS INVOLVED IN THE ANALYSES

Concept	Definition	Implication
Basic iteration	$\bar{\underset{\sim}{y}} = \underset{\sim}{A}\,\underset{\sim}{y}$	$\underset{\sim}{y} \Longrightarrow \bar{\underset{\sim}{y}}$
Forward operation	$\bar{\underset{\sim}{u}}^T \underset{\sim}{A} = \underset{\sim}{u}^T$	$\bar{\underset{\sim}{u}}^T \bar{\underset{\sim}{y}} = \bar{\underset{\sim}{u}}^T \underset{\sim}{A}\underset{\sim}{y} = \underset{\sim}{u}^T \underset{\sim}{y}$
Reverse operation	$\bar{\underset{\sim}{v}} = \underset{\sim}{A}\underset{\sim}{v}$	$\underset{\sim}{y} = c\underset{\sim}{v} \Longrightarrow \bar{\underset{\sim}{y}} = c\bar{\underset{\sim}{v}}$
Voltage selector	$\underset{\sim}{e}_1 \triangleq \begin{bmatrix} 1 \\ 0 \end{bmatrix}$	$\underset{\sim}{e}_1 \Longrightarrow \underset{\sim}{u}_1$ or $\underset{\sim}{v}_1$
Current selector	$\underset{\sim}{e}_2 \triangleq \begin{bmatrix} 0 \\ 1 \end{bmatrix}$	$\underset{\sim}{e}_2 \Longrightarrow \underset{\sim}{u}_2$ or $\underset{\sim}{v}_2$
Equivalent source	$\underset{\sim}{y} = \begin{bmatrix} V_S - Z_S I_S \\ I_S \end{bmatrix}$	$\underset{\sim}{e}_1^T \underset{\sim}{y} = V_S - Z_S I_S$, $\underset{\sim}{e}_2^T \underset{\sim}{y} = I_S$
Equivalent load	$\underset{\sim}{y} = \begin{bmatrix} V_L \\ Y_L V_L - I_L \end{bmatrix}$	$\underset{\sim}{y} = V_L \underset{\sim}{e}_1 + (Y_L V_L - I_L)\underset{\sim}{e}_2$

TABLE II

NOTATION AND IMPLIED INITIAL CONDITIONS

Factor	Identification	Initial Conditions Forward	Initial Conditions Reverse
$\bar{\underset{\sim}{u}}_1^T\ (*)\ \underset{\sim}{v}_1$	$(\dagger)_{11}$	voltage	voltage
$\bar{\underset{\sim}{u}}_1^T\ (*)\ \underset{\sim}{v}_2$	$(\dagger)_{12}$	voltage	current
$\bar{\underset{\sim}{u}}_2^T\ (*)\ \underset{\sim}{v}_1$	$(\dagger)_{21}$	current	voltage
$\bar{\underset{\sim}{u}}_2^T\ (*)\ \underset{\sim}{v}_2$	$(\dagger)_{22}$	current	current

(*) denotes either $\underset{\sim}{A}$, $\delta\underset{\sim}{A}$, $\partial\underset{\sim}{A}/\partial\phi$ or $\Delta\underset{\sim}{A}$

(†) denotes Q, δQ, Q' or ΔQ, as taken from (4), (5), (6) or (7), respectively

TABLE III

ANALYSES REQUIRED BY CERTAIN TERMS

Term	Analysis Required
$\underset{\sim}{u}^T \underset{\sim}{v}$	Forward and reverse (<u>conventional</u>) cascade analysis to <u>any</u> corresponding reference plane, whichever is convenient
$\underset{\sim 1}{u}^T \underset{\sim}{v}$, $\underset{\sim 2}{u}^T \underset{\sim}{v}$	Preferably one <u>reverse</u> analysis to source reference plane (avoiding calculation of $\underset{\sim 1}{u}$ and $\underset{\sim 2}{u}$)
$\underset{\sim}{u}^T \underset{\sim 1}{v}$, $\underset{\sim}{u}^T \underset{\sim 2}{v}$	Preferably one <u>forward</u> analysis to load reference plane (avoiding calculation of $\underset{\sim 1}{v}$ and $\underset{\sim 2}{v}$)
$\overline{\underset{\sim}{u}}^T \cdot \underset{\sim}{v}$	One forward analysis to input of $\underset{\sim}{A}$ and one reverse analysis to output of $\underset{\sim}{A}$
$\overline{\underset{\sim 1}{u}}^T \cdot \underset{\sim}{v}$, $\overline{\underset{\sim 2}{u}}^T \cdot \underset{\sim}{v}$	One full forward analysis to input of $\underset{\sim}{A}$ and one reverse analysis to output of $\underset{\sim}{A}$
$\overline{\underset{\sim}{u}}^T \cdot \underset{\sim 1}{v}$, $\overline{\underset{\sim}{u}}^T \cdot \underset{\sim 2}{v}$	One full reverse analysis to output of $\underset{\sim}{A}$ and one forward analysis to input of $\underset{\sim}{A}$
$\overline{\underset{\sim 1}{u}}^T \cdot \underset{\sim 1}{v}$, $\overline{\underset{\sim 1}{u}}^T \cdot \underset{\sim 2}{v}$ $\overline{\underset{\sim 2}{u}}^T \cdot \underset{\sim 1}{v}$, $\overline{\underset{\sim 2}{u}}^T \cdot \underset{\sim 2}{v}$	One full forward analysis to input of $\underset{\sim}{A}$ and one full reverse analysis to output of $\underset{\sim}{A}$

TABLE IV

FUNCTIONS OF INPUT CURRENT I_S AND OUTPUT VOLTAGE V_L FOR

CHANGES IN $\underset{\sim}{A}$ ONLY

Variable	Input	Output
$\underset{\sim}{A}$	$I_S = V_S \dfrac{Q_{21}}{Q_{11}}$	$V_L = \dfrac{V_S}{Q_{11}}$
$\delta \underset{\sim}{A}$	$\delta I_S = \dfrac{V_S \delta Q_{21} - I_S \delta Q_{11}}{Q_{11}}$	$\delta V_L = - \dfrac{V_L^2}{V_S} \delta Q_{11}$
$\dfrac{\partial \underset{\sim}{A}}{\partial \phi}$	$\dfrac{\partial I_S}{\partial \phi} = \dfrac{V_S Q_{21}' - I_S Q_{11}'}{Q_{11}}$	$\dfrac{\partial V_L}{\partial \phi} = - \dfrac{V_L^2}{V_S} Q_{11}'$
$\Delta \underset{\sim}{A}$	$\Delta I_S = \dfrac{V_S \Delta Q_{21} - I_S \Delta Q_{11}}{Q_{11} + \Delta Q_{11}}$	$\Delta V_L = - \dfrac{V_L^2}{V_L + V_S / \Delta Q_{11}}$

Figure Captions

Fig. 1 Notation for an element in the chain, indicating reference directions and voltage and current variables.

Fig. 2 Forward and reverse analyses of a cascaded network with source and load impedances assumed constant.

Fig. 3 Subnetwork i cascaded with subnetworks k (at source end) and j (at load end).

Fig. 4 Forward iteration for Fig. 3, transferring an equivalent source accounting for design variables from subnetwork k from one reference plane to the other.

Fig. 5 Reverse iteration for Fig. 3, transferring an equivalent source accounting for design variables from subnetwork j from one reference plane to the other.

Fig. 6 Seven-section filter containing unit elements and stubs [3]. All sections are quarter-wave at 2.175 GHz.

Fig. 7 Illustration for a cascade of 6 two-ports of the principal stages in the calculations involved in the multiple pairwise changes algorithm. Three variable elements are considered, hence three sets of simultaneous analyses are effectively performed.

Fig. 8 An example of a cascaded circuit with a branch connected in series and a branch connected in parallel. Branches are represented in the cascade by their equivalents. Reference planes where different analyses are initiated are labelled.

Fig. 9 A 2p-port element: a generalization of Fig. 1.

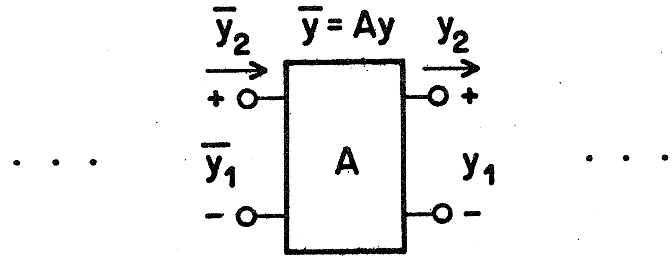
$\bar{y}_2$
$\bar{y} = Ay$
y_2
+
+
. . .
$\bar{y}_1$
A
y_1
. . .
−
−

Fig. 1

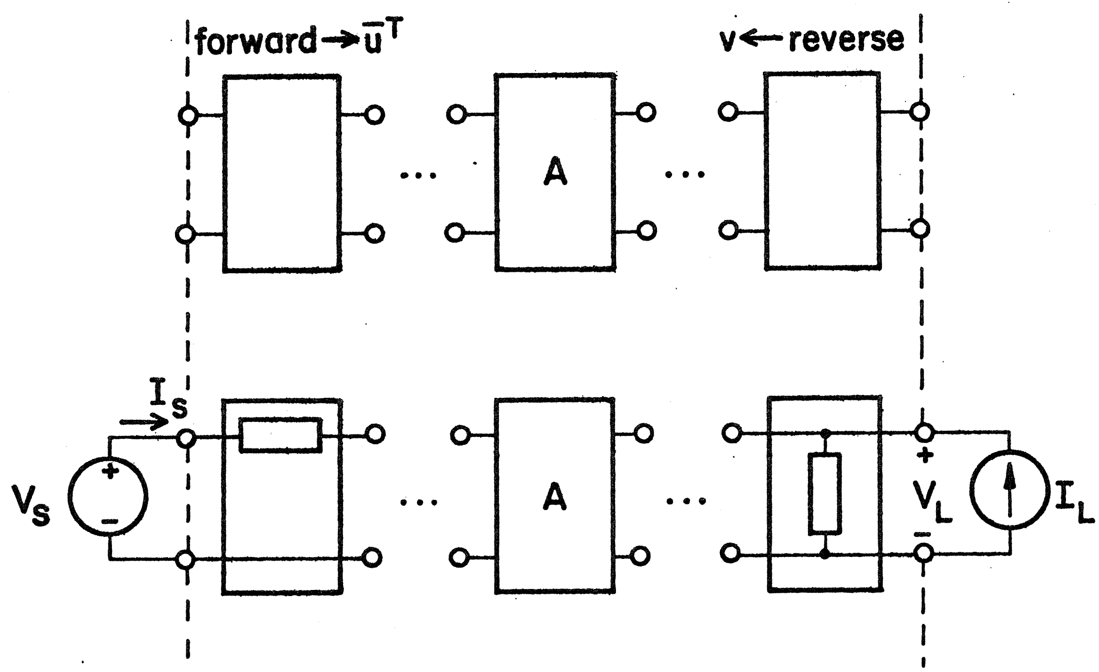
forward → $\bar{u}^T$
v ← reverse
...
A
...
I_S
V_S
+
−
...
A
...
+
V_L
−
I_L

Fig. 2

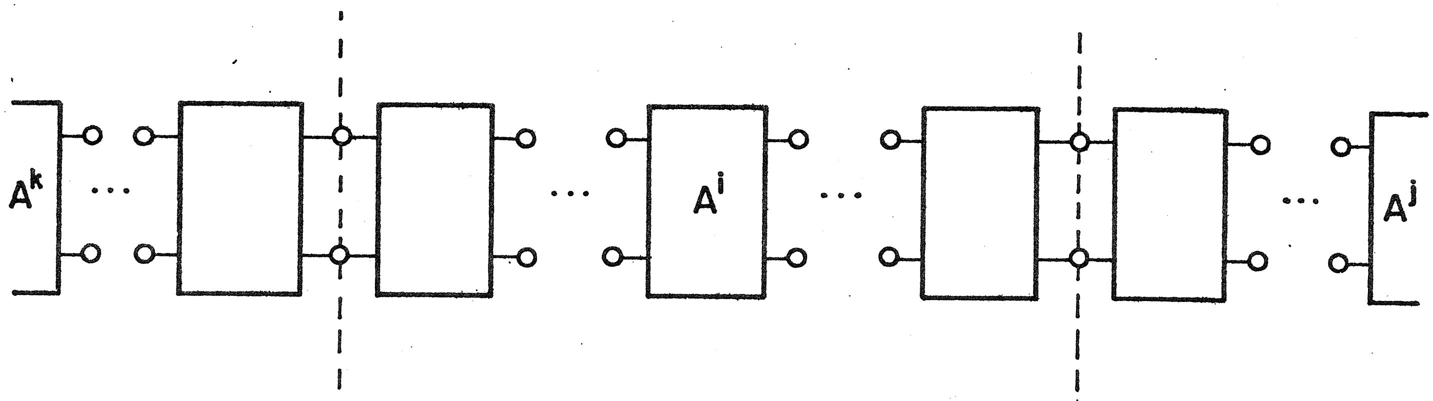

Fig. 3

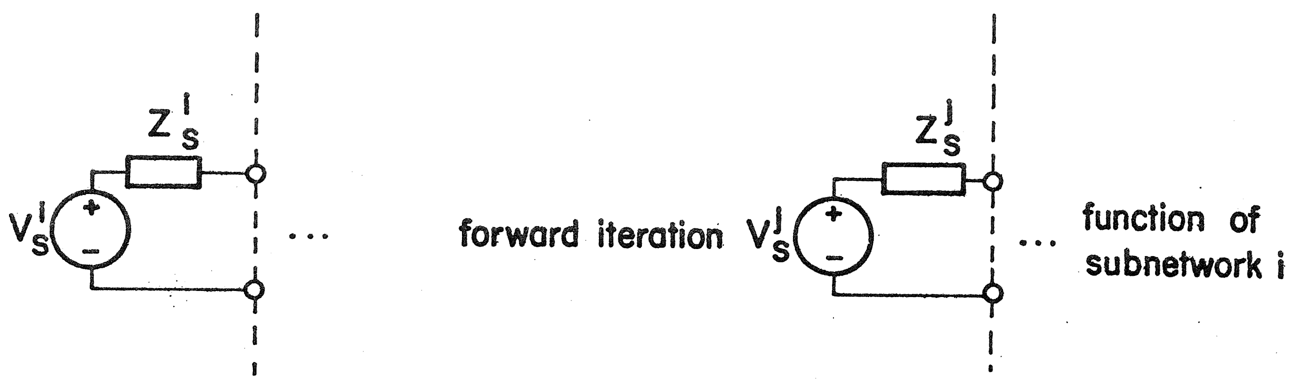

Fig. 4

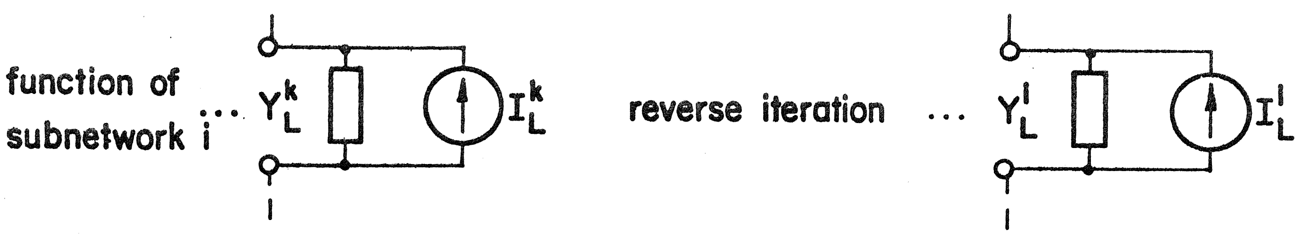

Fig. 5

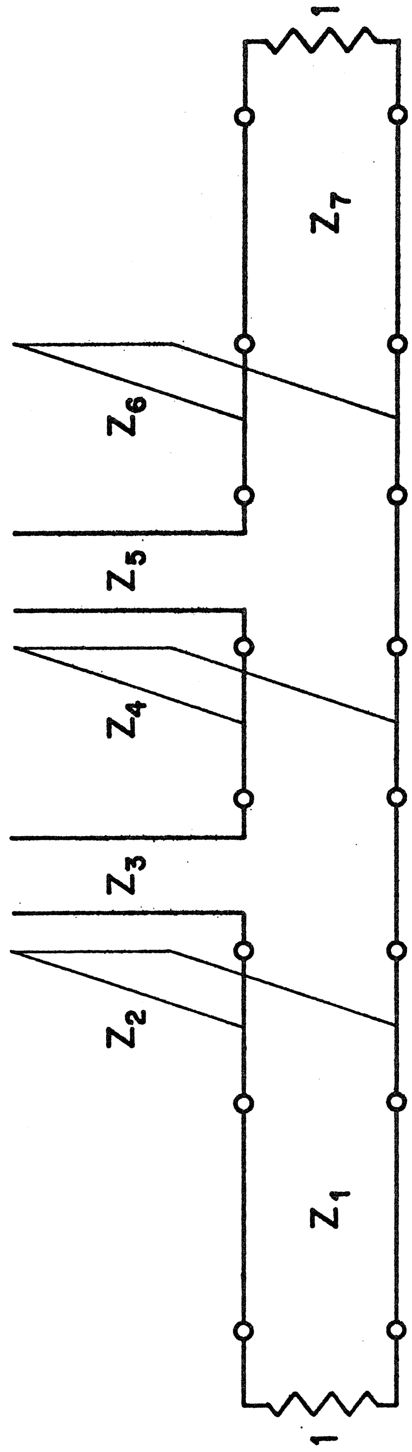

Fig. 6

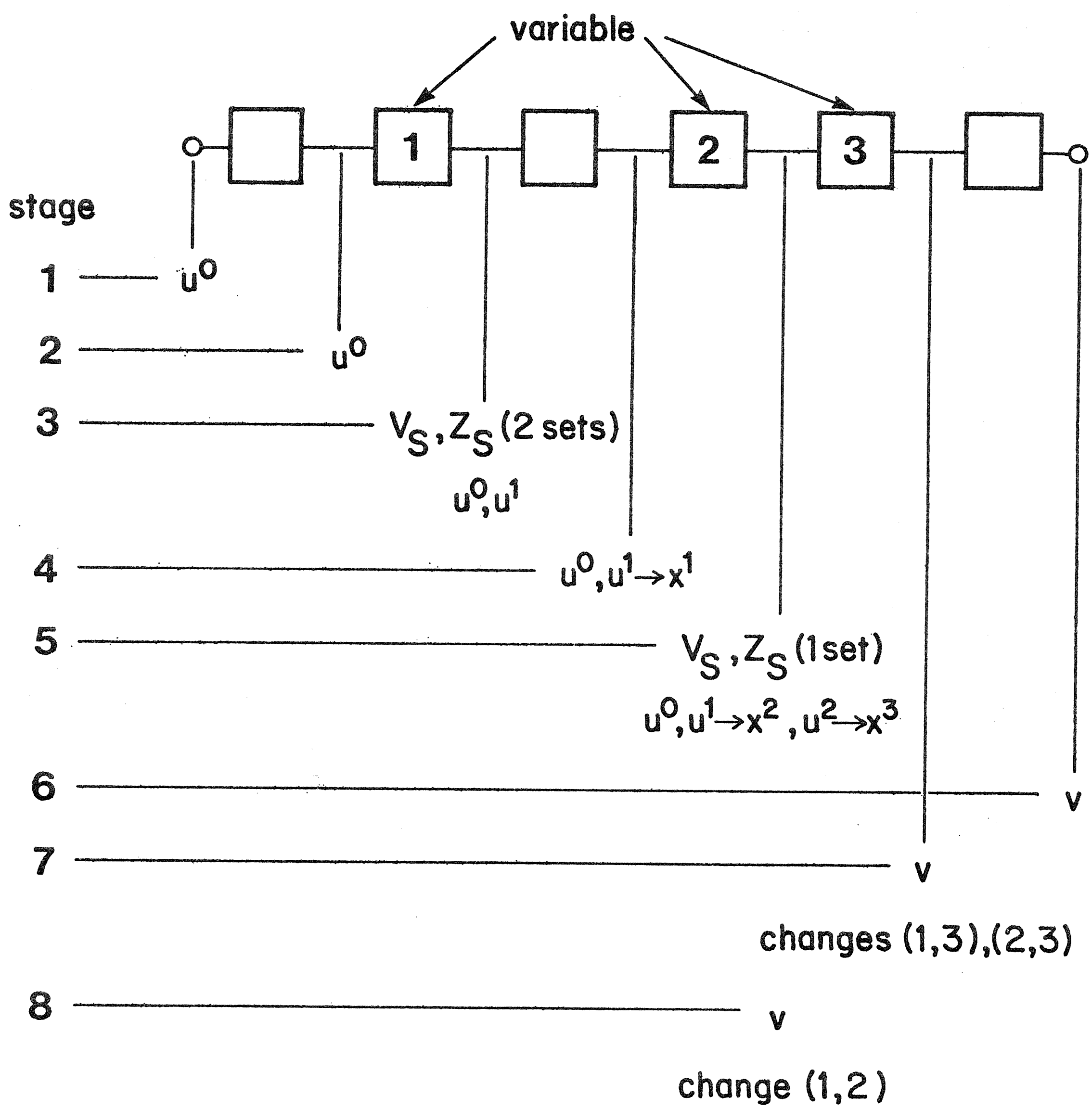
variable
1
2
3
stage
1
2
3
4
5
6
7
8
u⁰
u⁰
V_S, Z_S (2 sets)
u^0, u^1
$u^0, u^1 \rightarrow x^1$
V_S, Z_S (1 set)
$u^0, u^1 \rightarrow x^2, u^2 \rightarrow x^3$
v
v
changes (1,3),(2,3)
v
change (1,2)

Fig. 7

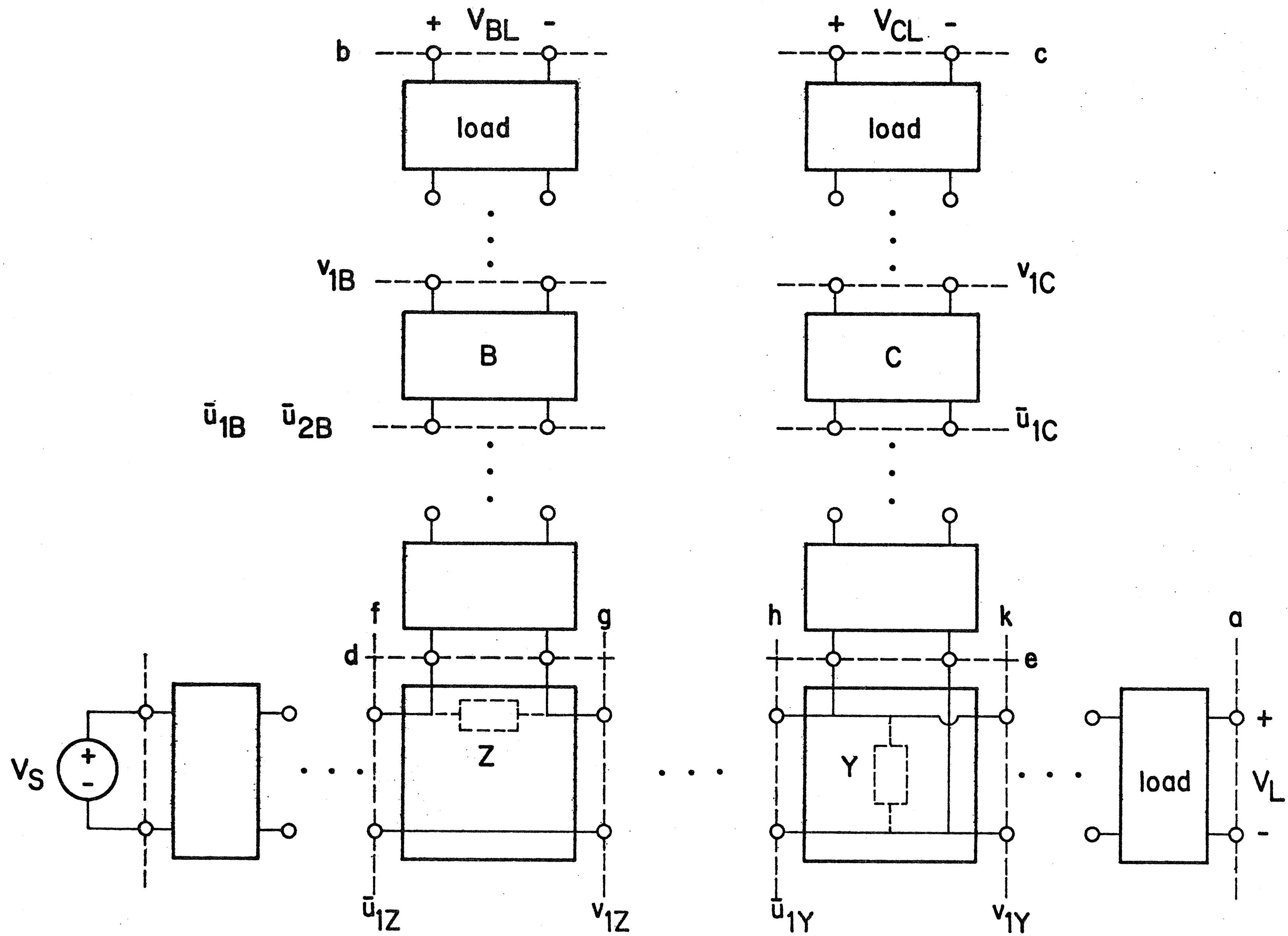
+ VBL -
b
load
v1B
B
ū1B ū2B
+ VCL -
c
load
v1C
C
ū1C
f
g
d
Z
ū1Z
v1Z
h
k
e
Y
ū1Y
v1Y
a
load
+
VL
-
VS
+
-

Fig. 8

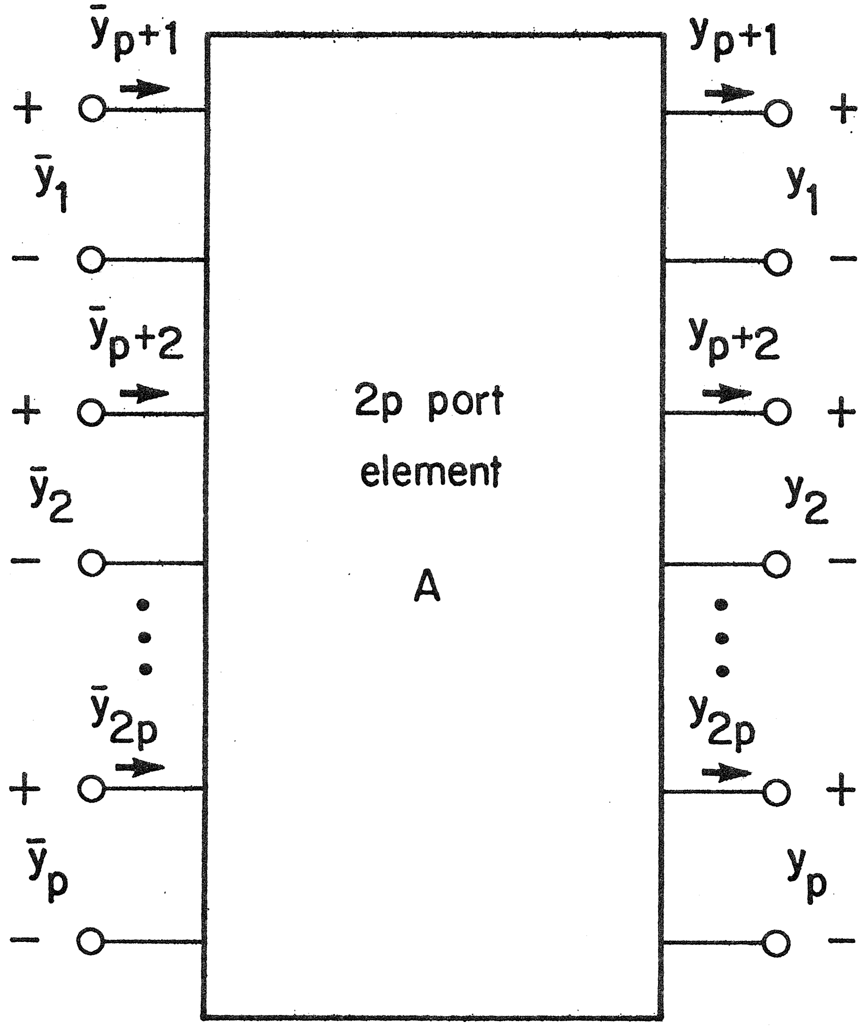
$\bar{y}_{p+1}$
+
$\bar{y}_1$
−
$\bar{y}_{p+2}$
+
$\bar{y}_2$
−
$\bar{y}_{2p}$
+
$\bar{y}_p$
−
2p port
element
A
y_{p+1}
+
y_1
−
y_{p+2}
+
y_2
−
y_{2p}
+
y_p
−

Fig. 9

SOC-190

NEW RESULTS IN NETWORK SIMULATION, SENSITIVITY AND TOLERANCE ANALYSIS FOR CASCADED STRUCTURES

J.W. Bandler, M.R.M. Rizk and H.L. Abdel-Malek

March 1978, No. of Pages: 40

Revised: June 1978

Key Words: Sensitivity analysis, tolerance analysis, cascaded networks

Abstract: An attractive, exact and efficient approach to network analysis for cascaded structures is presented. It is useful for sensitivity and tolerance analyses, in particular, for a multiple of simultaneous large changes in design parameter values. It also facilitates the exploitation of symmetry to reduce computational effort for the analysis. Responses at different loads in branched networks, which may be connected in series or in parallel with the main cascade, can be obtained analytically in terms of the variable elements. Sensitivity and large-change effects with respect to these variables can be easily evaluated. The approach is not confined to 2-port elements but can be generalized to 2p-port cascaded elements.

Description: Presented at the IEEE International Microwave Symposium (Ottawa, Canada, June 1978). See the symposium digest.

Related Work: SOC-184.

Price: $ 6.00.