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J.W. Bandler

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FACULTY OF ENGINEERING
McMASTER UNIVERSITY
HAMILTON, ONTARIO, CANADA

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J.W. BANDLER[2]

Abstract A possible mathematical formulation of the practical problem of computer-aided design of, for example, electrical circuits and systems and engineering designs in general, subject to tolerances on the k independent parameters is proposed. An automated scheme is suggested starting from arbitrary initial acceptable or unacceptable designs and culminating in designs which under reasonable restrictions are acceptable in the worst case sense. It is proved, in particular, that if the region of points in the parameter space for which designs are both feasible and acceptable satisfies a certain condition (less restrictive than convexity) then no more than 2^k points, the vertices of the tolerance region, need to be considered during optimization.

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[2]Associate Professor of Electrical Engineering, Department of Electrical Engineering, McMaster University, Hamilton, Ontario, Canada.

1. Introduction

An extremely important practical problem is the problem of optimal design subject to tolerances. Recently published work (Refs. 1 to 6) has yielded some practical insight into the nature of the problem. Indeed, it immediately suggests the possibility of formulating the complete worst case design of circuits or systems as a nonlinear programming problem.

An automated scheme would start from an arbitrary acceptable or unacceptable design and under appropriate restrictions stop at an acceptable design which is optimum in the worst case sense for specified tolerances. The most suitable objective function to be minimized would also seem to be one that best describes the cost of fabrication of the circuit or system, as suggested by some authors (Refs. 1 to 6).

It is the purpose of this paper to propose possible formulations and to discuss this problem generally. It is not claimed that a complete solution has been obtained. However, a number of interesting objective functions (more appropriately, perhaps, cost functions) have been investigated.

Many types of objective functions can be formulated. A number of variations on the sum of the inverses of the absolute tolerances or the sum of the inverses of the tolerances relative to respective nominal parameter values can be obtained. Furthermore, the nominal parameter values may or may not be variable. The relative merits of these and other functions which attempt in some way to maximize the size of the region of possible designs, namely, the tolerance region, are discussed.

For the purposes of this paper, it is assumed that the parameter tolerances can be independently specified. Furthermore, it is assumed that the design parameters and tolerances can be continuously varied. The tolerance region, in this case, will be defined by simple upper and lower bounds on the parameters. The region will, of course, contain an infinite number of acceptable designs, assuming that it is a subregion of the intersection of regions of acceptable and feasible designs. It is proved that if this region satisfies a certain condition (less restrictive than convexity) then only the (finite) number of vertices of the tolerance region need at most to be investigated.

2. Feasible and Acceptable Designs

A wide range of design problems can be formulated as nonlinear programming problems. One usually defines a scalar objective function $U(\underset{\sim}{\phi})$, where

$$\underset{\sim}{\phi} \triangleq \begin{bmatrix} \phi_1 \\ \phi_2 \\ \vdots \\ \phi_k \end{bmatrix} \tag{1}$$

represents the k independent design parameters. Design constraints can be assembled into a column vector $\underset{\sim}{g}(\underset{\sim}{\phi})$ and the problem stated as finding $\check{\underset{\sim}{\phi}}$ such that

$$U(\check{\underset{\sim}{\phi}}) = \min_{\underset{\sim}{\phi} \varepsilon R_c} U(\underset{\sim}{\phi}) \tag{2}$$

where

$$R_c \triangleq \{\underset{\sim}{\phi} | \underset{\sim}{g}(\underset{\sim}{\phi}) \geq \underset{\sim}{0}\} \tag{3}$$

For the purposes of the present discussion let us assume that two kinds of constraint functions are present, ones that determine the <u>feasibility</u> of a design designated $\underset{\sim}{g}_f(\underset{\sim}{\phi})$ and ones that determine the <u>acceptability</u> of a design designated $\underset{\sim}{g}_a(\underset{\sim}{\phi})$. We will therefore define a feasible region of points R_f as

$$R_f \triangleq \{\underset{\sim}{\phi} | \underset{\sim}{g}_f \geq \underset{\sim}{0}\} \tag{4}$$

and an acceptable region of points R_a as

$$R_a \triangleq \{\underset{\sim}{\phi} | \underset{\sim}{g}_a \geq \underset{\sim}{0}\} \tag{5}$$

Thus, $R_c = R_f \cap R_a$. It is assumed that all sets are nonempty. Note that R_a not necessarily a subset of R_f.

The objective function is usually set up so that a feasible solution is obtained at an interior point of the acceptable region, and as far as possible, in some sense, from its boundary. The reasoning behind this is the hope that when the design is fabricated, inevitable errors in the design parameters might, nevertheless, yield an acceptable design. It is this flexibility which can be exploited in the optimization of tolerances. Often

$$U(\underset{\sim}{\phi}) = -\min_{i \epsilon I_a} g_i(\underset{\sim}{\phi}) \tag{6}$$

where the index set I_a relates to constraints defining R_a. It follows then that

$$R_a = \{\underset{\sim}{\phi} \mid U(\underset{\sim}{\phi}) \leq 0\} \tag{7}$$

3. <u>The Tolerance Region</u>

Given a <u>nominal</u> point $\underset{\sim}{\phi}^o$ and a set of nonnegative <u>tolerances</u> $\underset{\sim}{\varepsilon}$, where

$$\underset{\sim}{\varepsilon} \triangleq \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \vdots \\ \varepsilon_k \end{bmatrix} \geq \underset{\sim}{0} \tag{8}$$

we can define a region of possible designs R_t as

$$R_t \triangleq \{\underset{\sim}{\phi} | \phi_i^o - \varepsilon_i \leq \phi_i \leq \phi_i^o + \varepsilon_i, \quad i=1,2,\ldots,k\} \qquad (9)$$

or, equivalently,

$$R_t \triangleq \{\underset{\sim}{\phi} | \phi_i = \phi_i^o + t_i\varepsilon_i, \; -1 \leq t_i \leq 1, \quad i=1,2,\ldots,k\} \qquad (10)$$

Obviously, depending on the location of $\underset{\sim}{\phi}^o$ and the value of $\underset{\sim}{\varepsilon}$, R_t may or may not be a subset of R_c.

The tolerance problem is beginning to take shape - R_t should be placed inside R_c in some optimal manner by adjusting $\underset{\sim}{\phi}^o$ and $\underset{\sim}{\varepsilon}$ to optimal values $\check{\underset{\sim}{\phi}}^o$ and $\check{\underset{\sim}{\varepsilon}}$. A serious development, however, is that all points $\underset{\sim}{\phi} \in R_t$ must satisfy $\underset{\sim}{g} \geq \underset{\sim}{0}$. We have, effectively, to deal with an infinite number of constraints.

For any given point $\underset{\sim}{\phi}^o$ we can view the functions $\underset{\sim}{g}(\underset{\sim}{\phi})$ with respect to $\underset{\sim}{\varepsilon}$ as follows. We let the origin of the $\underset{\sim}{\varepsilon}$ space correspond to $\underset{\sim}{\phi}^o$ (translation). We then consider all the possible linear parameter transformations, from (10),

$$\underset{\sim}{\varepsilon} = \underset{\sim}{T}(\underset{\sim}{\phi} - \underset{\sim}{\phi}^o)$$

suggested by the transformation matrix (magnification and reflection)

$$\underset{\sim}{T} \triangleq \begin{bmatrix} \frac{1}{t_1} & 0 & \cdots & 0 \\ 0 & \frac{1}{t_2} & \cdots & 0 \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \cdots & \frac{1}{t_k} \end{bmatrix} \tag{11}$$

for $-1 \leq t_i \leq 1$, $i=1,2,\ldots,k$.

Two-dimensional examples of allowable tolerances in the tolerance space corresponding to particular constraints and particular nominal points in the parameter space are shown in Fig. 1.

4. Restrictions on R_c

For obvious reasons it is impractical to consider an infinite number of constraints. In order to make the problem tractable a number of simplifying assumptions could be made to try to obtain a solution to the problem with reasonable computational effort.

It can be shown that if R_c is convex then, from Refs. 7 or 8,

$$\underset{\sim}{\phi}^i \in R_c \text{ for } i = 1,2,\ldots,n \tag{12}$$

implies that

$$\underset{\sim}{\phi} = \sum_{i=1}^{n} \lambda_i \underset{\sim}{\phi}^i \in R_c \tag{13}$$

for all λ_i satisfying

$$\sum_{i=1}^{n} \lambda_i = 1$$
$$\lambda_i \geq 0 \quad i = 1,2,\ldots,n \tag{14}$$

Given, for example, a finite number of points $\underset{\sim}{\phi}^i$ in a finite-dimensional Euclidean space it is easy to visualize that the $\underset{\sim}{\phi}^i$ are vertices of a polytope (the intersection of a finite number of closed halfspaces) and that $\underset{\sim}{\phi}$ is any interior or boundary point. If R_c is itself a polytope (all constraints linear) it is clearly convex.

R_t is a polytope with 2^k vertices. Let the ith vertex be denoted $\underset{\sim}{\phi}^i$ and let

$$\underset{\sim}{\phi}^i = \underset{\sim}{\phi}^o - \underset{\sim}{\varepsilon} + 2\underset{\sim}{E}\,\underset{\sim}{v}_{i-1} \;\varepsilon\; R_c \text{ for } i = 1,2, \ldots, 2^k \tag{15}$$

where

$$\underset{\sim}{E} \triangleq \begin{bmatrix} \varepsilon_1 & 0 & \cdots & 0 \\ 0 & \varepsilon_2 & \cdots & 0 \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \cdots & \varepsilon_k \end{bmatrix} \tag{16}$$

and $\underset{\sim}{v}_i$ is a k-element vector whose elements reflect the subscript i in binary notation, i.e.,

$$\underset{\sim}{v}_0 \triangleq \begin{bmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \; \underset{\sim}{v}_1 \triangleq \begin{bmatrix} 1 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \; \underset{\sim}{v}_2 \triangleq \begin{bmatrix} 0 \\ 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \; \underset{\sim}{v}_3 \triangleq \begin{bmatrix} 1 \\ 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \; \ldots \tag{17}$$

$\underset{\sim}{v}_{i-1}$ may be formed

$$\underset{\sim}{v}_{i-1} = \sum_{j=1}^{k} \mu_j(i)\, \underset{\sim}{u}_j \tag{18}$$

where

$$\mu_1, \mu_2, \ldots, \mu_k \in \{0,1\} \tag{19}$$

must satisfy (see Table 1)

$$i = 1 + \sum_{j=1}^{k} \mu_j(i) 2^{j-1} \tag{20}$$

and where the k-element vectors $\underset{\sim}{u}_j$ are given by

$$\underset{\sim}{u}_1 \triangleq \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \quad \underset{\sim}{u}_2 \triangleq \begin{bmatrix} 0 \\ 1 \\ \vdots \\ 0 \end{bmatrix}, \ldots, \underset{\sim}{u}_k \triangleq \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 1 \end{bmatrix} \tag{21}$$

Fig. 2 illustrates an example in three dimensions. Observe that

$$\underset{\sim}{E}\, \underset{\sim}{v}_{i-1} = \sum_{j=1}^{k} \mu_j(i)\, \varepsilon_j\, \underset{\sim}{u}_j \tag{22}$$

Using (12)-(14),

$$\underset{\sim}{\phi} = \underset{\sim}{\phi}^{o} - \underset{\sim}{\varepsilon} + 2 \sum_{i=1}^{2^k} (\lambda_i \sum_{j=1}^{k} \mu_j(i) \varepsilon_j \underset{\sim}{u}_j) \ \varepsilon \ R_c \tag{23}$$

if R_c is convex and the vertices of R_t are elements of R_c. Equation (23) generates the set R_t. Therefore, $R_t \subset R_c$. See Fig. 3(a).

It will now be shown that the assumption that R_c is convex is unnecessarily restrictive.

Theorem 4.1 If the vertices of R_t are in R_c then $R_t \subset R_c$ if, for all $j=1,2,\ldots,k$,

$$\underset{\sim}{\phi}^{a}, \ \underset{\sim}{\phi}^{b(j)} = \underset{\sim}{\phi}^{a} + \alpha \underset{\sim}{u}_j \ \varepsilon \ R_c \tag{24}$$

where α is a scalar, implies that

$$\underset{\sim}{\phi} = \underset{\sim}{\phi}^{a} + \lambda(\underset{\sim}{\phi}^{b(j)} - \underset{\sim}{\phi}^{a}) \ \varepsilon \ R_c \tag{25}$$

for all λ satisfying

$$0 \leq \lambda \leq 1 \tag{26}$$

See, for example, Fig. 3(b).

Proof Let $\underset{\sim}{\phi}_{\ell}$ denote some point, in general, in an ℓ-dimensional linear manifold generated by the first 2^{ℓ} vertices as

$$\underset{\sim}{\phi}_{\ell} = \underset{\sim}{\phi}^{o} - \underset{\sim}{\varepsilon} + 2 \sum_{i=1}^{2^{\ell}} (p_i \sum_{j=1}^{\ell} \mu_j(i) \varepsilon_j \underset{\sim}{u}_j) \tag{27}$$

with p_i satisfying

$$\sum_{i=1}^{2^\ell} p_i = 1$$

$$p_i \geq 0 \quad i=1,2,\ldots,2^\ell \tag{28}$$

Note that since max i = 2^ℓ, we can deduce from (20) that

$$\mu_j = 0 \quad \text{for } j \geq \ell \tag{29}$$

in (22), so that the relevant summation need be taken only up to ℓ and not k.

Assume that $\underset{\sim}{\phi}_\ell \in R_c$ for all $\underset{\sim}{\phi}^i \in R_c$ given in (22). Now consider

$$\underset{\sim}{\phi}_{\ell+1} = \underset{\sim}{\phi}^o - \underset{\sim}{\varepsilon} + 2 \sum_{i=1}^{2^{\ell+1}} (q_i \sum_{j=1}^{\ell+1} \mu_j(i)\varepsilon_j \underset{\sim}{u}_j) \tag{30}$$

with q_i satisfying

$$\sum_{i=1}^{2^{\ell+1}} q_i = 1$$

$$q_i \geq 0 \quad i=1,2,\ldots,2^{\ell+1} \tag{31}$$

After some manipulation, we find that

$$\underset{\sim}{\phi}_{\ell+1} = \underset{\sim}{\phi}^o - \underset{\sim}{\varepsilon} + 2 \sum_{i=1}^{2^\ell} [(q_i + q_{2^\ell+i}) \sum_{j=1}^{\ell} \mu_j(i)\varepsilon_j \underset{\sim}{u}_j]$$

$$+ 2(\sum_{i=2^\ell+1}^{2^{\ell+1}} q_i)\varepsilon_{\ell+1} \underset{\sim}{u}_{\ell+1} \tag{32}$$

Let

$$\lambda = \sum_{i=2^\ell+1}^{2^{\ell+1}} q_i \tag{33}$$

and

$$p_i = q_i + q_{2^{\ell}+i} \qquad i=1,2,\ldots,2^{\ell} \tag{34}$$

Hence, (32) becomes

$$\underset{\sim}{\phi}_{\ell+1} = \underset{\sim}{\phi}_{\ell} + 2\lambda\,\varepsilon_{\ell+1}\,\underset{\sim}{u}_{\ell+1} \tag{35}$$

With $\lambda=0$, $\underset{\sim}{\phi}_{\ell+1} = \underset{\sim}{\phi}_{\ell} \varepsilon R_c$ by assumption. If $\lambda=1$, $\underset{\sim}{\phi}_{\ell+1} = \underset{\sim}{\phi}_{\ell} + 2\varepsilon_{\ell+1}\underset{\sim}{u}_{\ell+1}$, which represents a translation of the ℓ-dimensional manifold. Thus, $\underset{\sim}{\phi}_{\ell+1} \varepsilon R_c$ by assumption. For $0<\lambda<1$ we note $\underset{\sim}{\phi}_{\ell+1} \varepsilon R_c$ if (24) to (26) hold for $j = \ell+1$.

It is easy to verify that $\underset{\sim}{\phi}_1 \varepsilon R_c$ and, furthermore, that $\underset{\sim}{\phi}_2 \varepsilon R_c$ if (24) to (26) hold for j=1 and j=2, respectively. It follows by the foregoing inductive reasoning that $\underset{\sim}{\phi}_k = \underset{\sim}{\phi}$, as defined by (23), is in R_c under the conditions of the theorem.

The theorem allows both Fig. 3(a) and 3(b), but not Fig. 3(c).

5. Some Objective Functions

A number of potentially useful and fairly well-behaved objective functions which might be used to represent the cost of a design can be formulated. In practice, of course, a suitable modelling problem would first have to be solved to determine the significant parameters involved partially or totally in the actual cost. Here, we will assume that either absolute or relative tolerances are the main variables; furthermore, that the total cost $C(\underset{\sim}{\phi}^o, \underset{\sim}{\varepsilon})$ of the design is just the sum of the cost of the individual components.

It is intuitively reasonable to assume that

$$C(\underset{\sim}{\phi}^o, \underset{\sim}{\varepsilon}) \to c \geq 0 \quad \text{as} \quad \underset{\sim}{\varepsilon} \to \underset{\sim}{\infty} \tag{36}$$

$$C(\underset{\sim}{\phi}^o, \underset{\sim}{\varepsilon}) \to \infty \quad \text{for any } \varepsilon_i \to 0 \tag{37}$$

Two out of many possible functions which fulfil these requirements are, for c=0,

$$C_a = \sum_{i=1}^{k} \frac{c_i}{\varepsilon_i} \tag{38}$$

subject to $\underset{\sim}{\varepsilon} \geq \underset{\sim}{0}$ as stated in (8), and

$$C_r = \sum_{i=1}^{k} c_i \log_e \frac{\phi_i^o}{\varepsilon_i} \tag{39}$$

subject to

$$\underset{\sim}{\phi}^o \geq \underset{\sim}{\varepsilon} \geq \underset{\sim}{0} \tag{40}$$

In both cases

$$c_i \geq 0 \qquad i = 1,2,\ldots,k \tag{41}$$

6. Examples

It is interesting to consider C_a and C_r for the different regions R_c sketched in Fig. 4. We will let $c_1 = c_2 = 1$. Fig. 4(a) depicts a situation where $\check{\underset{\sim}{\phi}}^o$ will have relatively little variation in going from C_a to C_r. Fig. 4(b) will have $\check{\phi}_1^o > \check{\varepsilon}_1$ and $\check{\phi}_2^o = \check{\varepsilon}_2$; for C_a, $\check{\phi}_2^o > 0$ but for C_r, $\check{\phi}_2^o = 0$ which, physics permitting, indicates that one parameter may be "removed". It can be shown (See Fig. 5(a)) that min C_r is given by $\check{\phi}_2^o = 0$, at $\check{\phi}_1^o = 2\frac{1}{2}$, $\check{\varepsilon}_1 = 1\frac{1}{2}$. Fig. 4(c) allows the possibility of removing ϕ_1 if C_r is optimized. The minimum cost is then $\log_e 9$. It is easily shown, however, that to minimize cost ϕ_1 should not be removed. See, for example, Fig. 5(b). Using C_r in Fig. 4(d) would indicate that $\check{\phi}_1^o$ and $\check{\phi}_2^o$ may be zero. Using C_a in all the cases of Fig. 4 we would find $\check{\underset{\sim}{\phi}}^o$ to be an interior point of R_c.

A number of corresponding observations to those made above can be made if, for the cases sketched in Fig. 4, we took, for example, $\phi_1' = 1/\phi_1$ and $\phi_2' = \phi_2$ as parameters.

7. Conclusions

If, as is usual in the design of circuits or systems, the optimal design is obtained by solving an approximation problem, then a fairly large number of inequality constraints usually define the acceptable region. For any particular set of reasonable tolerances one could exploit the likelihood of the worst case (point most likely to violate a given constraint) being predictable by a local linearization or higher-order approximation of the constraints to greatly reduce the actual cost of the necessary computations than is implied by the 2^k vertices of the tolerance region. Further study of these ideas from a nonlinear programming point of view should yield more insight into the possible success or failure of particular tolerance optimization algorithms that might suggest themselves.

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Table 1. The numbering scheme for the vertices of R_t.

i	$\mu_1(i)$	$\mu_2(i)$	$\mu_3(i)$ ...	$\mu_k(i)$	$\sum_{j=1}^{k} \mu_j(i)\varepsilon_j \underset{\sim}{u}_j$
1	0	0	0	0	$\underset{\sim}{0}$
2	1	0	0	0	$\varepsilon_1 \underset{\sim}{u}_1$
3	0	1	0	0	$\varepsilon_2 \underset{\sim}{u}_2$
4	1	1	0	0	$\varepsilon_1 \underset{\sim}{u}_1 + \varepsilon_2 \underset{\sim}{u}_2$
5	0	0	1	0	$\varepsilon_3 \underset{\sim}{u}_3$
6	1	0	1	0	$\varepsilon_1 \underset{\sim}{u}_1 + \varepsilon_3 \underset{\sim}{u}_3$
7	0	1	1	0	$\varepsilon_2 \underset{\sim}{u}_2 + \varepsilon_3 \underset{\sim}{u}_3$
8	1	1	1	0	$\varepsilon_1 \underset{\sim}{u}_1 + \varepsilon_2 \underset{\sim}{u}_2 + \varepsilon_3 \underset{\sim}{u}_3$
⋮	⋮	⋮	⋮	⋮	⋮
2^k	1	1	1	1	$\underset{\sim}{\varepsilon}$

Figure Captions

Fig. 1. Allowable tolerances corresponding to particular constraints and particular nominal points.

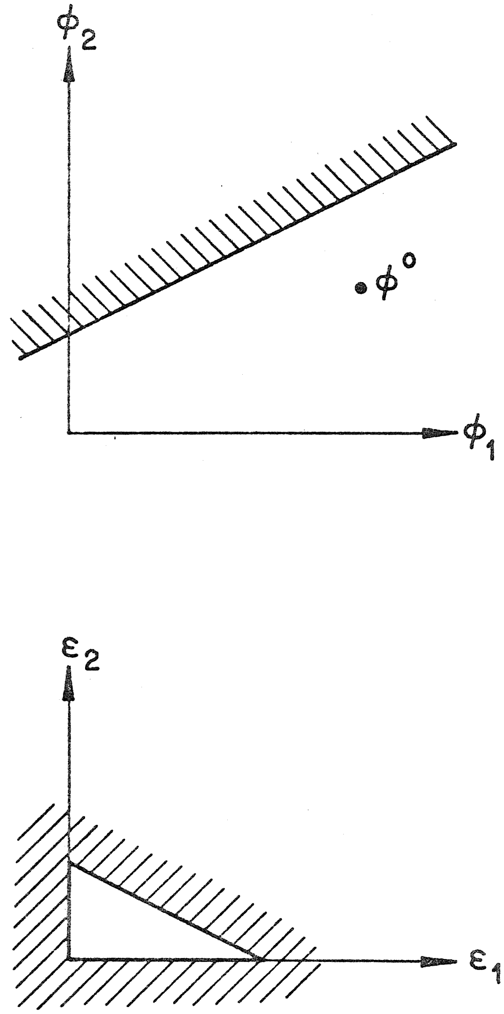
Fig. 2. A three-dimensional example of points defining the vertices of R_t.

Fig. 3. Possible regions R_c.

Fig. 4. Examples used in the discussion of objective functions.

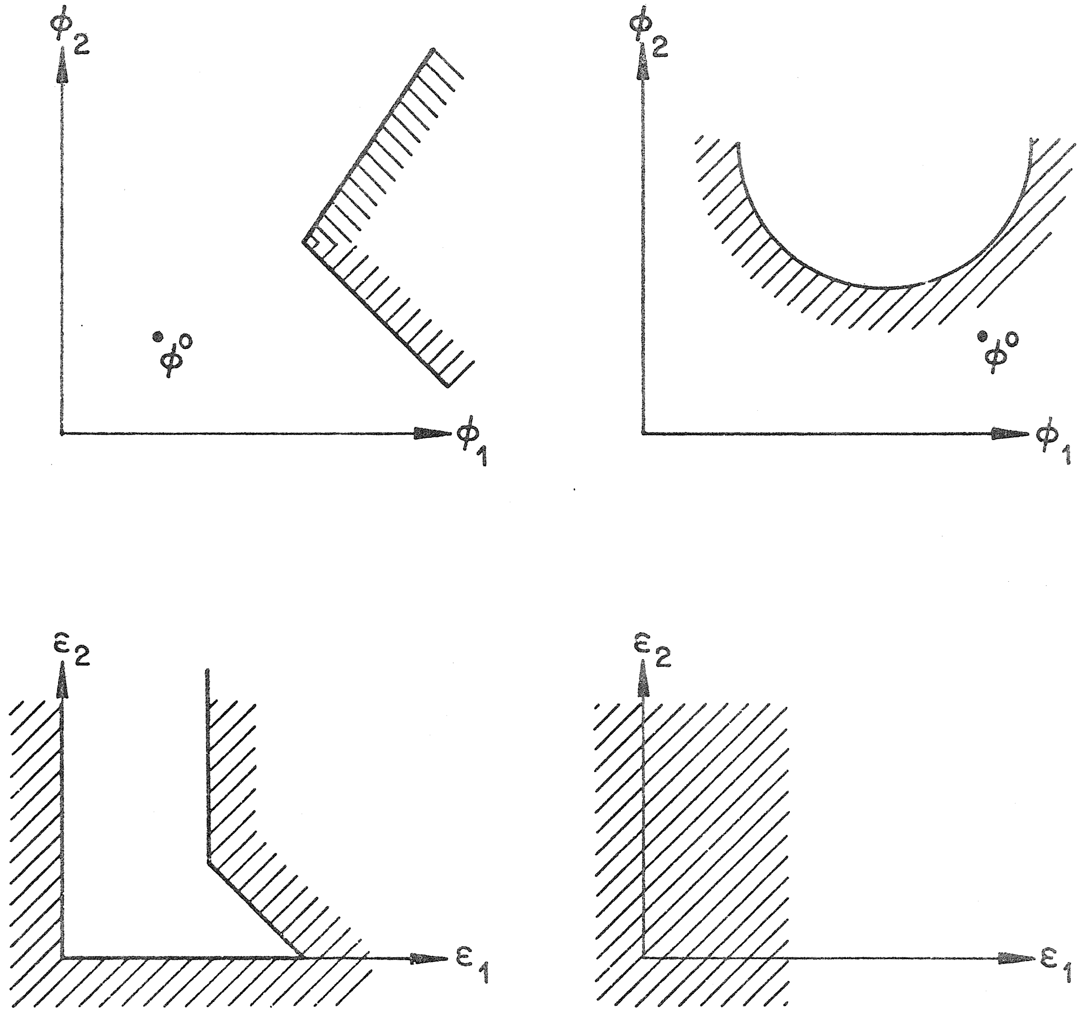
Fig. 5. (a) Example corresponding to Fig. 4(b) with $\check{\phi}_2^o = \check{\varepsilon}_2 = 0$.

(b) Example corresponding to Fig. 4(c) with $\phi_1^o = 1$ and $\varepsilon_1 = \frac{1}{2}$. The best value of C_r is, in this case, $\log_e 3$.



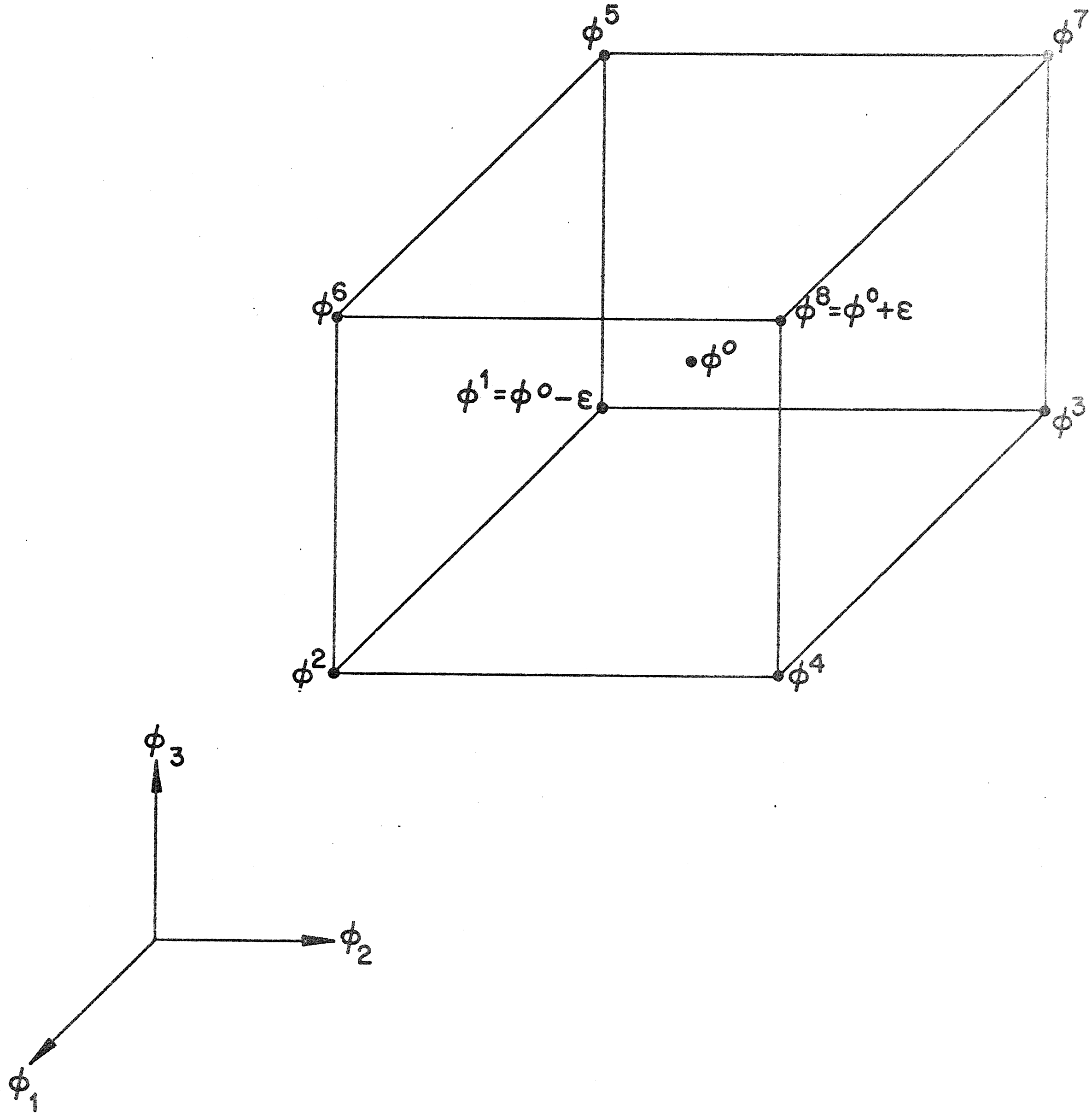
ϕ_2
ϕ^0
ϕ_1
ϕ_2
ϕ^0
ϕ_1
ε_2
ε_1
ε_2
ε_1

Fig. 1.



ϕ_2
ϕ^0
ϕ_1
ϕ_2
ϕ^0
ϕ_1
ε_2
ε_1
ε_2
ε_1

Fig. 1. [cont.]



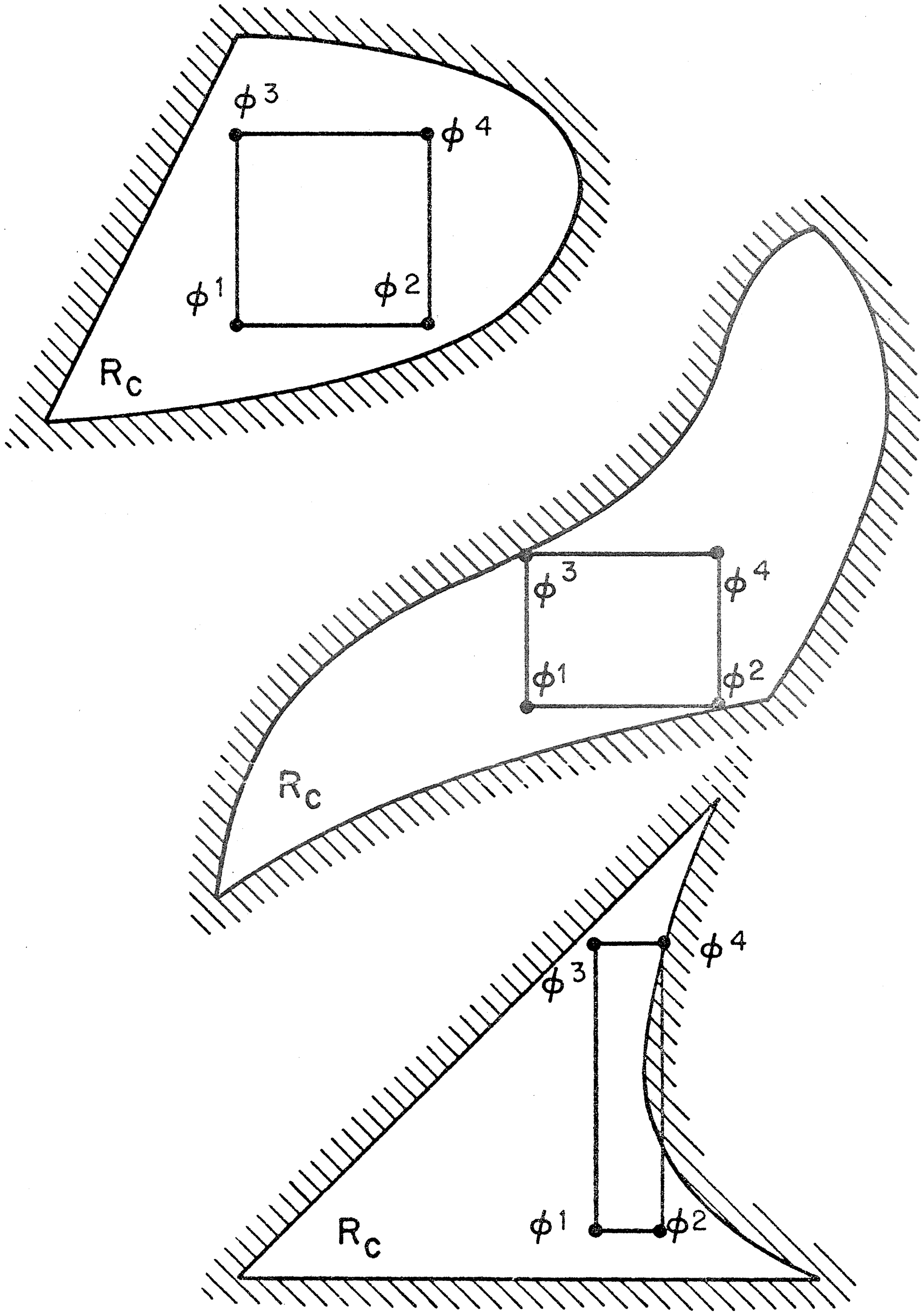
ϕ^5
ϕ^7
ϕ^6
$\phi^8 = \phi^0 + \varepsilon$
ϕ^0
$\phi^1 = \phi^0 - \varepsilon$
ϕ^3
ϕ^2
ϕ^4
ϕ_3
ϕ_2
ϕ_1

Fig. 2.

(a)

(b)

(c)



ϕ^3
ϕ^4
ϕ^1
ϕ^2
R_c
ϕ^3
ϕ^4
ϕ^1
ϕ^2
R_c
ϕ^4
ϕ^3
R_c
ϕ^1
ϕ^2

Fig. 3.

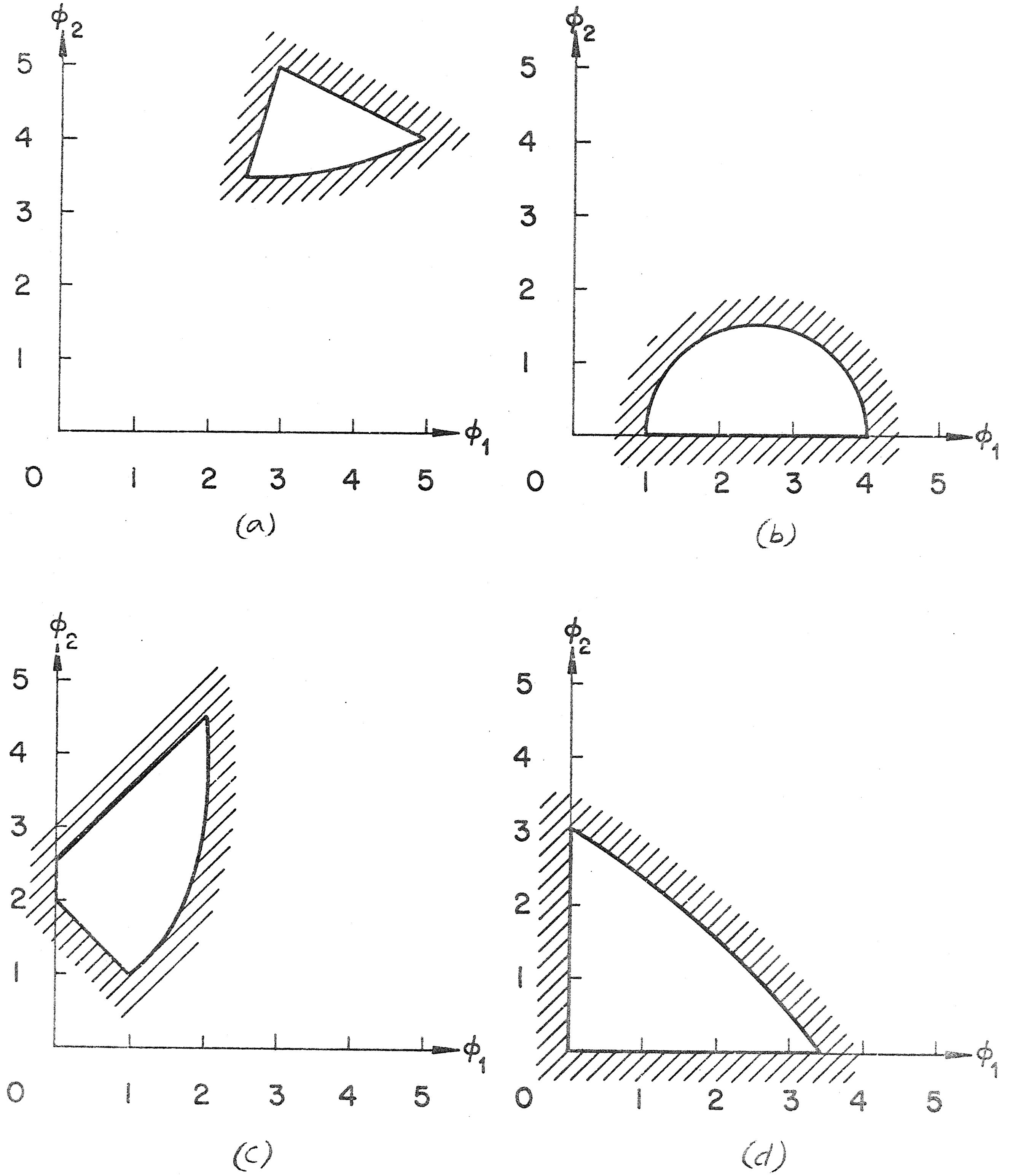
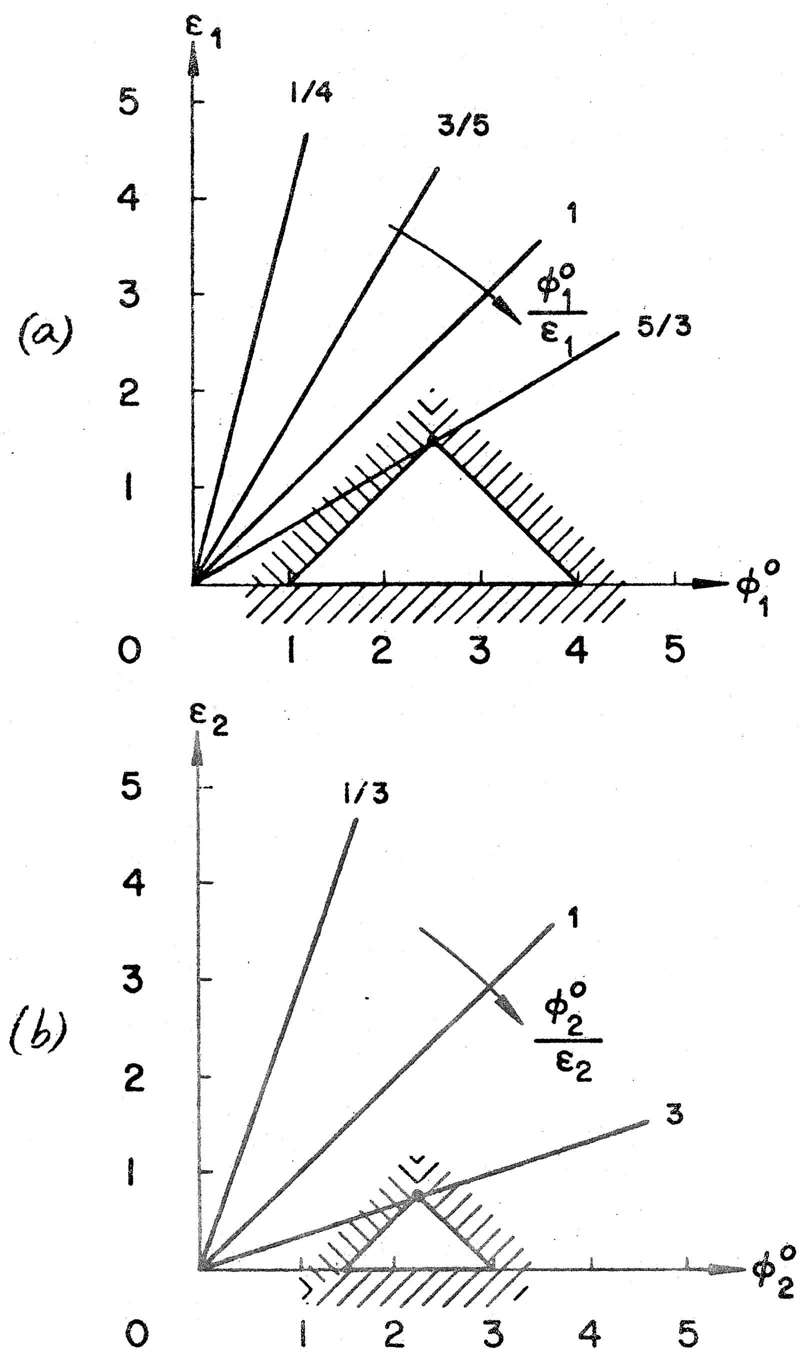


Fig. 4.



(a)
ε1
5
4
3
2
1
0
1/4
3/5
1
5/3
φ°1 / ε1
φ°1
1
2
3
4
5
(b)
ε2
5
4
3
2
1
0
1/3
1
3
φ°2 / ε2
φ°2
1
2
3
4
5

Fig. 5.