INTERNAL REPORTS IN

SIMULATION, OPTIMIZATION AND CONTROL

No. SOC-214

ENGINEERING MODELLING AND DESIGN SUBJECT TO MODEL UNCERTAINTIES AND MANUFACTURING TOLERANCES

J.W. Bandler

July 1978

(Revised November 1978)

FACULTY OF ENGINEERING

McMASTER UNIVERSITY

HAMILTON, ONTARIO, CANADA

ENGINEERING MODELLING AND DESIGN SUBJECT TO MODEL UNCERTAINTIES AND MANUFACTURING TOLERANCES

John W. Bandler
Group on Simulation, Optimization and Control
Faculty of Engineering
McMaster University, Hamilton, Canada L8S 4L7

This paper deals with engineering design problems in which, for example, either a large volume of production is envisaged or in which only a few units are to be custom made. Designs are considered subject to manufacturing tolerances, material uncertainties, environmental uncertainties and model uncertainties. The reduction of cost by increasing tolerances, the determination and optimization of production yield, the problem of design centering and various aspects of tuning are discussed. Nonlinear programming approaches are considered. The important problem of searching for candidates for worst case solutions is briefly mentioned. Simplicial approximation, quadratic modelling, linear cuts and space regionalization are reviewed. A fairly extensive bibliography to relevant work in the modelling and design of electrical circuits is provided.

INTRODUCTION

This paper deals principally with those engineering design problems in which either a large volume of production is envisaged, e.g., integrated circuits, or in which only a few units are to be custom made, e.g., filters for satellite applications. In the latter case, reliability and high performance are essential, whereas in the former, low production cost at the expense of performance is more typical. All designs are considered to be subjected to manufacturing tolerances (e.g., on physical dimensions of components), uncertainties on the materials used in fabrication (e.g., on dielectric constants of insulators), uncertainties on the environment in which the product is to operate (e.g., on temperature), uncertainties in the computational models used in the design process (e.g., on equivalent circuits purporting to represent the actual circuits) and so on. Attention is directed towards the relevant modelling, the design and the manufacture of electrical circuits such as filters, amplifiers and switching circuits. Hence, certain undesirable effects which deteriorate performance over and above those already indicated may be due to electromagnetic coupling between (usually adjacent or closely located) components and to terminations which, inevitably, are not ideally matched to the circuit under consideration. The effects are particularly acute at high frequencies of operation.

The main objective is to discuss some approaches to the reduction of cost as effected by increasing or optimally assigning tolerances, maximizing production yield with respect to an assumed probability distribution function, utilizing any available design margins and the interrelated optimization problem called design centering which, as the term implies, involves the optimal location of a set of

This work was supported by the Natural Sciences and Engineering Research Council of Canada under Grant A7239.

nominal design parameter values. Post-production tuning is also highly relevant. Such tuning is customary in engineering production as a means of improving or repairing actual outcomes in an attempt to improve performance. Production yield in the present context may be simply defined by the ratio of number of outcomes which satisfy the specifications to the total number of outcomes. A potential yield may be similarly defined by replacing the number of outcomes by the number of outcomes which satisfy the specifications after tuning if necessary [13].

An extensive bibliography of relevant material is appended [1-48].

REVIEW OF COMPUTER-AIDED DESIGN

In order of general complexity, Fig. 1 attempts to highlight typical problem formulations in modern computer-aided engineering design, in particular using the nomenclature appropriate to the optimal design of electrical circuits and systems [1,10]. Fig. 2 shows some typical design situations. The concept of upper and lower specifications or desired bounds on a response function of an independent variable ψ, e.g., frequency or time, implies a constraint region in the k-dimensional space of designable variables $\underset{\sim}{\phi}$, viz.,

$$\underset{\sim}{\phi} \triangleq \begin{bmatrix} \phi_1 \\ \phi_2 \\ \cdot \\ \cdot \\ \cdot \\ \phi_k \end{bmatrix} . \qquad (1)$$

This concept is easily generalized to response functions of a number of independent variables similarly assembled in the vector $\underset{\sim}{\psi}$. Error functions involved in a minimax or Chebyshev approximation problem expressed along a sampled ψ axis can be represented in terms of $\underset{\sim}{\phi}$ by contour diagrams of the maximum, with its distinctive discontinuous derivatives. A family of possible responses, involving, e.g., a whole production line of circuits with independent parameters lying within a tolerance region of a nominal design is shown also. Since all the points in the tolerance region are also in the constraint region the envelope of responses lies within the upper and lower specifications.

The ideas of Figs. 1 and 2 may be amplified as follows. Nominal design: here we seek a single point in the space of designable variables which best meets a given set of performance specifications and design constraints. A suitable measure of deviation such as least squares, least pth or minimax [27] is typically chosen as the objective function to be minimized. Sensitivity minimization: here it is recognized that the solution point is subject to fluctuation or change due to any phenomenon associated with design parameters. An overall measure of this sensitivity, usually involving first-order sensitivities with respect to the parameters, is often included in the objective function [45]. The aim is, of course, to trade off some performance by gaining greater insensitivity of the possible outcome.

When undesirable effects such as model uncertainties and manufacturing tolerances are considered explicitly, two important classes of problems emerge: statistical design and worst-case design. In statistical design it is recognized a priori that a production yield of less than 100% is likely and there are, consequently, two principal aims. We attempt to minimize the cost or, alternatively, maximize the yield. Worst-case design requires that all units meet the performance specifications under all circumstances [7]. Typically, we either attempt to center the design with fixed assumed absolute or relative tolerances (analogous to maximizing yield) or we attempt to optimally assign tolerances to reduce production cost. What distinguishes all these problems from nominal designs or sensitivity minimization is the fact that a single point is no longer of

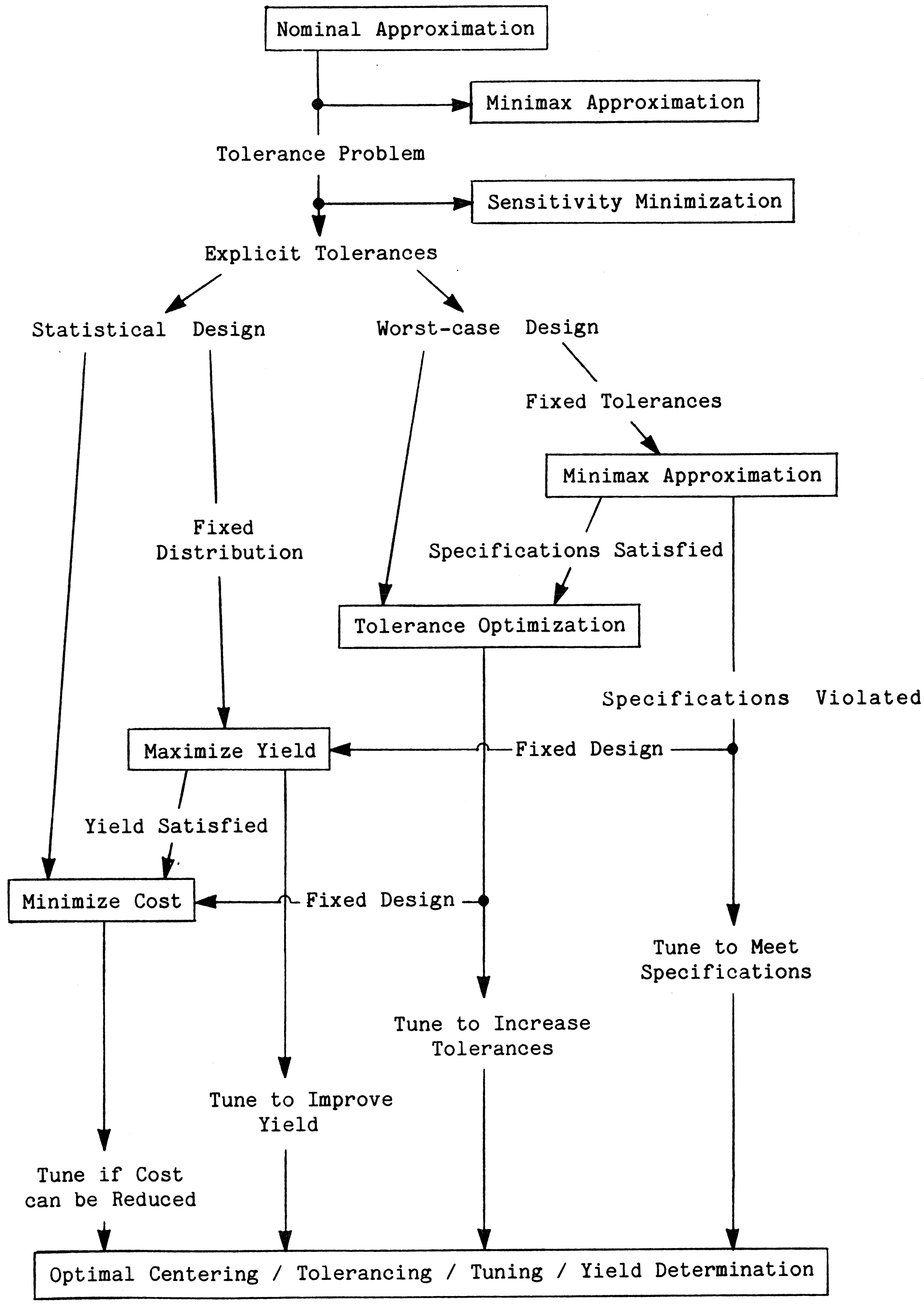


Fig. 1 A typical sequence of computer-aided design problems. As one proceeds down the diagram the problems tend to increase in complexity [1,10].

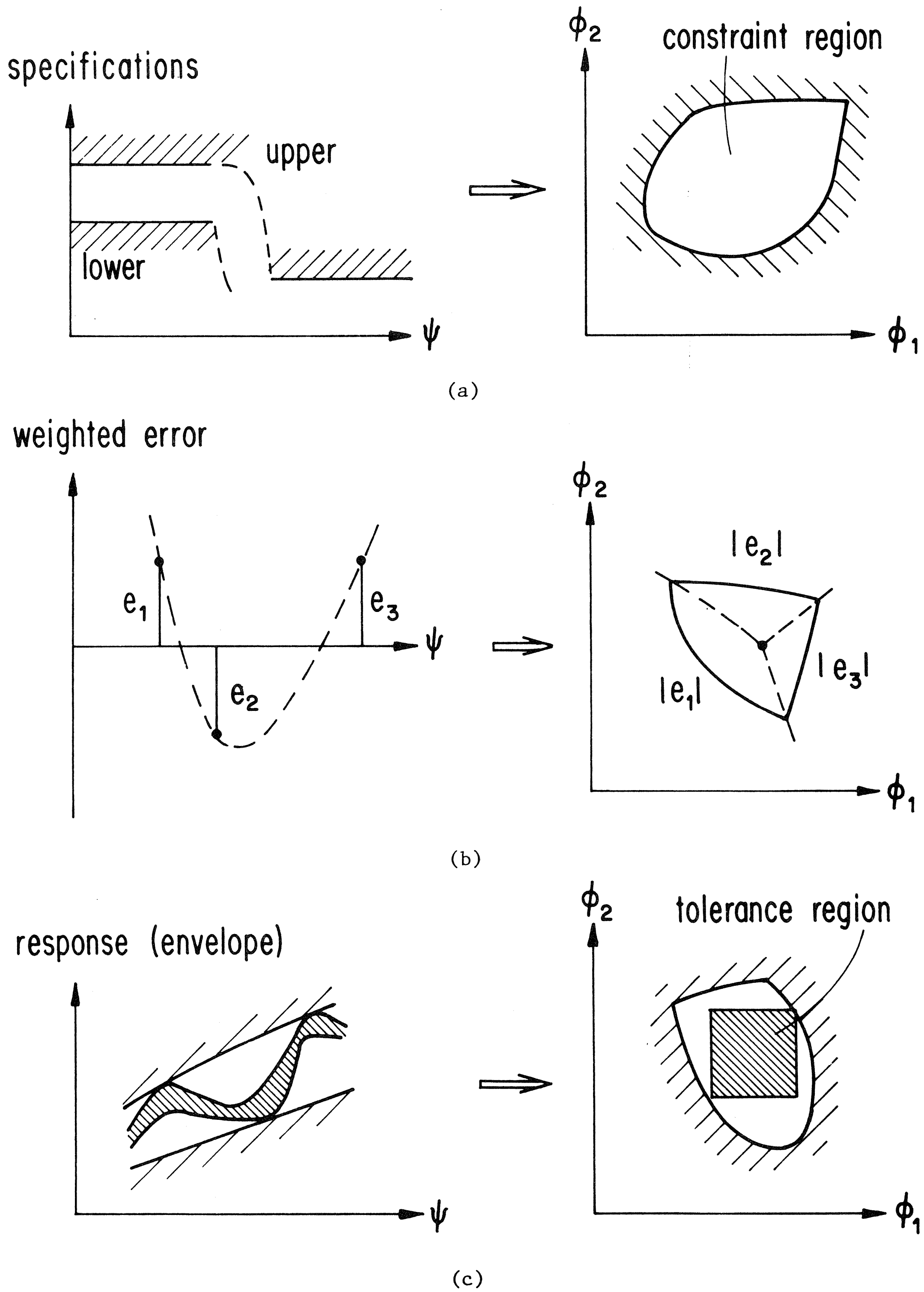


Fig. 2 Some typical design situations. In (a) we have upper and lower specifications on the response function of one independent variable implying a constraint region with respect to the designable parameters ϕ. In (b) is shown a weighted error function sampled at three points giving rise to contours of $\max_{1 \le i \le 3} |e_i|$. In (c) we have a toleranced design satisfying performance specifications.

interest: a (tolerance) region of possible outcomes is to be optimally located with respect to the feasible (acceptable, constraint) region.

We can deal with fixed designs or tunable designs. A fixed design or model implies that no subsequent tuning or adjustment is available to correct a posteriori for unacceptable performance. Thus, if a worst-case design is sought, then every unit has to be individually tested and each violating outcome discarded. On the other hand, a tunable design implies that at least one variable can be adjusted after such testing in an effort to meet the specifications [19]. There is no guarantee that the specifications will be met, of course, even after tuning unless it was properly accommodated in the original design.

The presence of tuning tends to increase cost, not only in making the tunable component available but possibly in having to pay a skilled person to carry out the tuning [19,41]. Repairing seems closely related to tuning in this respect. Tuning (repairing) may often be a design feature to permit the customer to alter his unit to meet different specifications. This leads to the concept of tunable constraint regions [11,19], which can be handled readily mathematically albeit computationally at greater expense.

Fig. 3 is a representation in the space of designable parameters of the concepts of the manufacturing tolerance over which it is presumed there is no control (by definition), the manufacturer's tuning variable, which is usually designed to be inaccessible to the customer and the tuning variable designed specifically for the customer which is exteriorized and often embodied in a large knob with attendant scale. It may be remarked that Fig. 3 applies equally in a descriptive sense whether the tolerance and tuning effects are all associated with one physical variable or whether they relate to different physical variables.

Setting any or all of the aforementioned problems up as nonlinear programs poses numerous difficulties [7,19]. There is the problem of identifying a suitable objective (cost) function to be minimized. Very little is known, in general, about production cost as a function of the variables entering into a design. This observation probably applies as much to most manufacturers as to outsiders. Hence, highly simplified points of view are usually taken in an attempt to render the problem tractable: functions which force the expansion of a tolerance orthotope within the constraint region accompanied by its optimal location, the optimal location of an expandable sphere within the region and so on. The matter is complicated by generally unknown correlations between variables, empirical assumptions about models and model parameter uncertainties and unreliable or unknown distributions of outcomes of component values between tolerance extremes. The number of constraints and even variables that could be chosen for an otherwise deceptively simple design problem is virtually unlimited.

One of the most challenging problems, therefore, is the development of general rules or methodology at a high level, procedures or algorithms at a lower level, for rapidly determining candidates for active constraints [47,48]. Various obvious mathematical assumptions have been proposed to simplify the selection problem: linearity of the constraints with respect to the variables [41], convexity of the constraint region [29,30], one-dimensional convexity of the constraint region [7,17,25].

The most frequently made assumption (virtually axiomatic in the light of the current state of the art) is that extremes of performance correspond to extremes in designable variables. Hence, most approaches to optimal design subject to tolerances concentrate on extreme points of parameter ranges.

The direct use of the Monte Carlo method of tolerance analysis within the optimization loop is extremely expensive [32]. Many approaches have therefore been suggested, either to avoid repeated use of the Monte Carlo method for

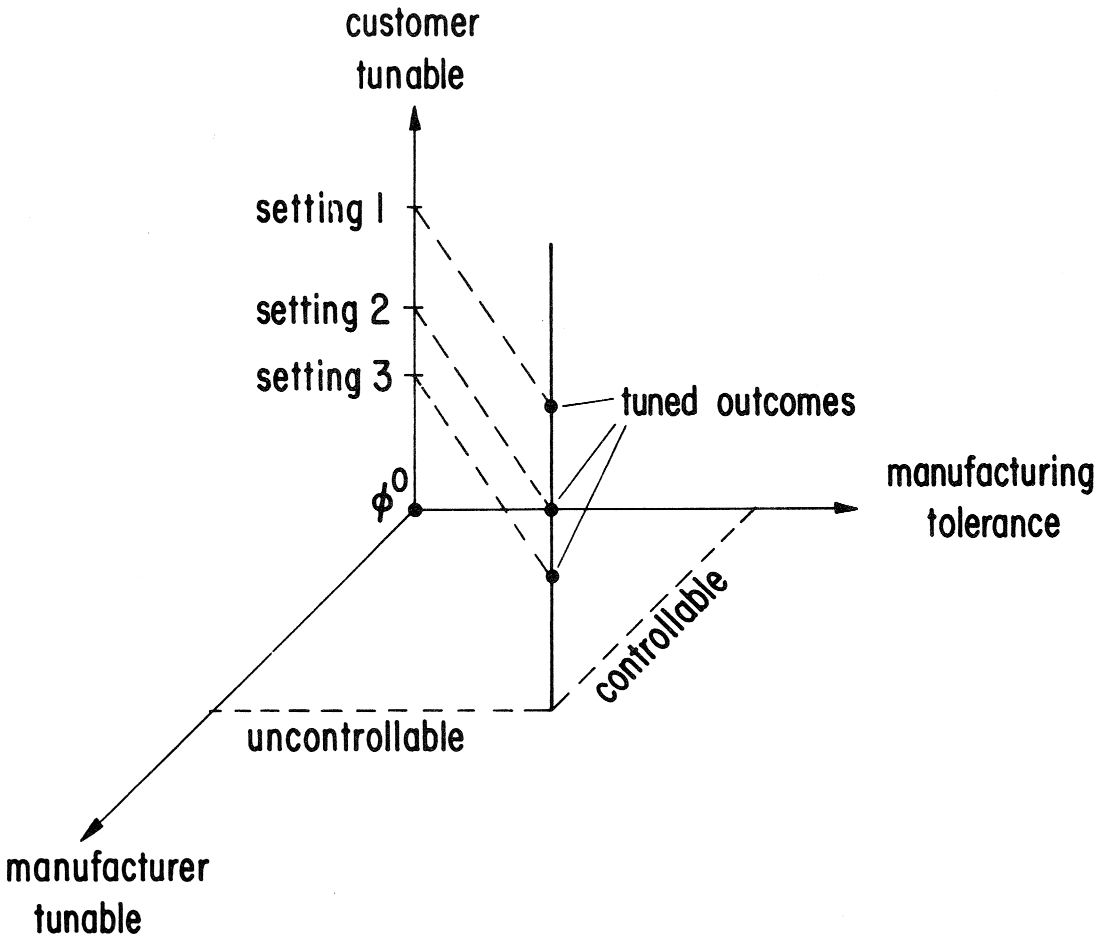


Fig. 3 A representation of the concepts of tolerance and independent tuning both by manufacturer and customer, with respect to a nominal design ϕ^0.

estimating yield, or to employ multidimensional approximations of the design constraints [1-4,9-14,29-31]. However, the need to optimize yield in the context of a huge production has sometimes dictated the computational impetuosity of uniting a Monte Carlo analysis with a general purpose simulator. That such extemporaneousness is extant only serves to underline the importance of the anticipated results.

SOME OPTIMIZATION APPROACHES

Bounding the Constraint Region R_c

The constraint region R_c is the set of all points ϕ for which all performance specifications and design constraints are satisfied. Thus

$$R_c \triangleq \{\phi \mid g(\phi) \geq 0\}, \tag{2}$$

where $\underset{\sim}{g}$ is the vector of constraint functions. Upper and lower bounds on each parameter ϕ_i (the ith component) for which $\phi \in R_c$ provide useful design information [28]. In a statistical analysis, for example, constraints can be stacked in order of increasing computational effort, suitable upper and lower bounds appearing at the top of the stack. If any constraint is violated further testing of the candidate solutions becomes unnecessary.

Fig. 4 illustrates the bounding of R_c. In general, 2k optimizations, where k is the dimensionality of ϕ, are required. In practice, however, since common solutions are likely to exist [8], significantly fewer optimizations would be expected. Upper bounds on tolerances for fixed designs can, following such an analysis, be immediately calculated. Similarly, upper bounds on yield can be calculated for fixed designs.

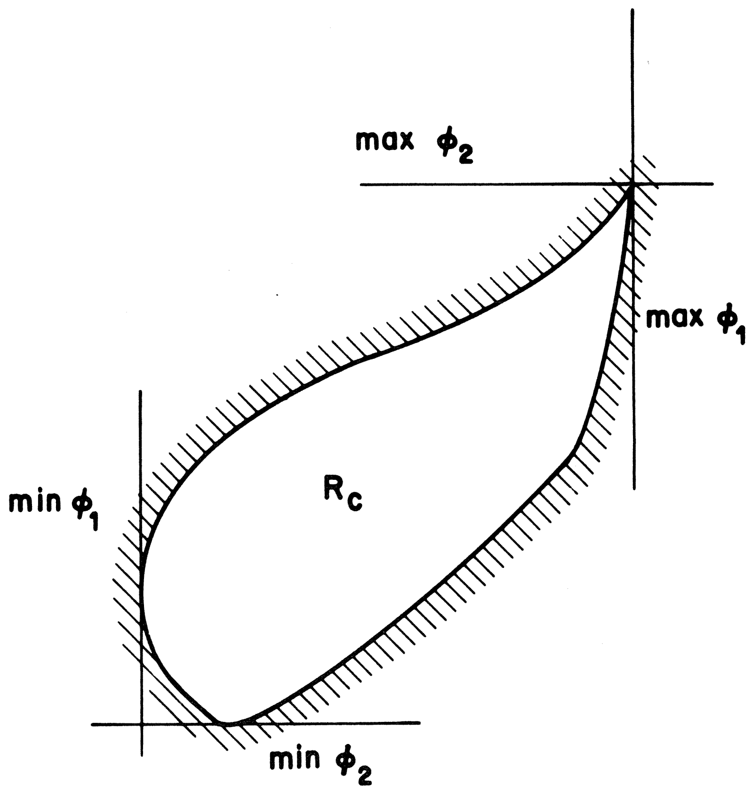


Fig. 4 Bounds on the constraint region obtained by independently maximizing or minimizing the parameters subject to the constraints [8].

Minimax and Least pth Objectives M and U

Often, in engineering design, we define a function of the form

$$M(\underset{\sim}{\phi}) \triangleq -\min_{i \in I_c} g_i(\underset{\sim}{\phi}), \tag{3}$$

where I_c is an index set, each element of which identifies a particular constraint defining R_c. More generally, the so-called generalized least pth objective [15,27] can be defined, for example, as

$$U(\underset{\sim}{\phi}) \triangleq \begin{cases} 0 & \text{if } M(\underset{\sim}{\phi}) = 0, \\ M(\underset{\sim}{\phi}) \left[\sum\limits_{i \in J(\underset{\sim}{\phi})} \left[\dfrac{-g_i(\underset{\sim}{\phi})}{M(\underset{\sim}{\phi})} \right]^q \right]^{1/q} & \text{if } M(\underset{\sim}{\phi}) \neq 0, \end{cases} \tag{4}$$

where

$$q = p \operatorname{sgn} M(\underset{\sim}{\phi}) \ , \ p \geq 1 \tag{5}$$

and

$$J(\underset{\sim}{\phi}) \triangleq \begin{cases} \{i \mid i \in I_c, \ g_i(\underset{\sim}{\phi}) < 0\} \text{ for } M > 0, \\ I_c \text{ for } M < 0. \end{cases} \tag{6}$$

The important feature of $U(\underset{\sim}{\phi})$ is that it coincides with $M(\underset{\sim}{\phi})$ when $M = 0$, it shares the same sign as $M(\underset{\sim}{\phi})$, yet under mild restrictions on p it is differentiable everywhere except when $M = 0$. The role of the M in the definition of U in (4) is twofold. Firstly, it scales the functions automatically ameliorating ill-conditioning attributable to the exponent q. Secondly, it facilitates the matching of the two otherwise discontinuous least pth objectives, namely, the objective for $M > 0$ and the one for $M < 0$. A discussion of the properties of this function is available in the literature [15,27].

The problem

$$\underset{\underset{\sim}{\phi}}{\text{minimize}} \ U(\underset{\sim}{\phi})$$

where $U(\underset{\sim}{\phi})$ is given by (4)-(6), is, consequently, not only a feasibility check but is also a centering process if R_c is not empty, since $\underset{\sim}{\phi}$ then tends to move away from the constraint boundary to the interior of R_c. Tolerances are not explicitly optimized by this formulation. However, this kind of optimization is virtually mandatory prior to introducing explicit tolerances. Active or near active constraints can then, for example, usually be identified.

Interior Approximation R_I and Exterior Approximation R_E

In accordance with the foregoing concepts we can now let

$$R_c \triangleq \{\underset{\sim}{\phi} \mid U(\underset{\sim}{\phi}) \leq 0\}, \tag{7}$$

where $U(\underset{\sim}{\phi})$ is given by (4)-(6). Under assumptions of convexity of R_c or one-dimensional convexity [7] we can eliminate the corresponding regions indicated in Fig. 5. This allows further refinement, for example, of finding an upper bound to production yield over and above that outlined earlier.

Interior or exterior approximations to R_c can be conceived [8] as illustrated by Fig. 6. A best exterior approximation may be found by deflation of a suitable

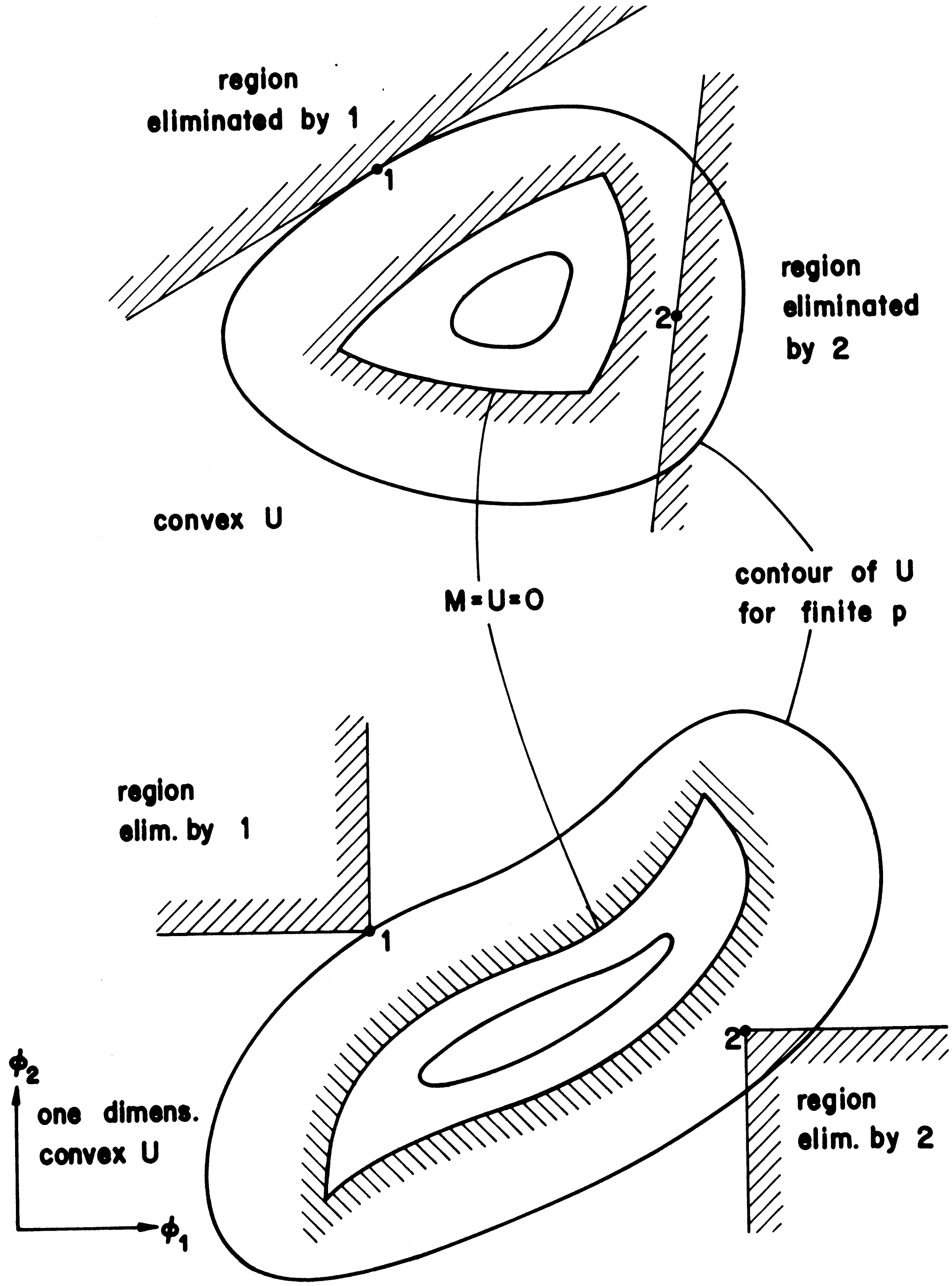


Fig. 5 Elimination of regions during an analysis process exploiting assumptions of (a) convexity and (b) one-dimensional convexity [8]. The condition U > 0 implies specification violated, U < 0 specification satisfied.

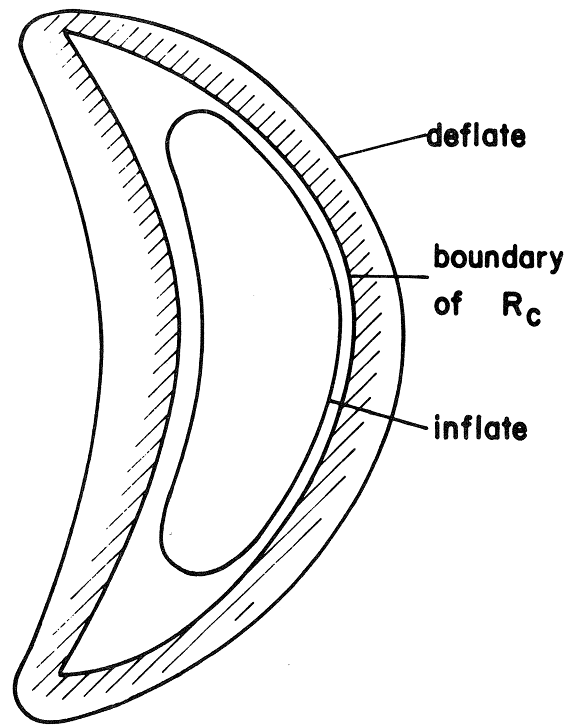


Fig. 6 Interior approximation R_I and exterior approximation R_E to the constraint region R_c. Optimal approximations are obtained by inflating the interior approximation and deflating the exterior approximation [8].

region R_E and a best interior approximation by inflation of a suitable region R_I, while retaining

$$R_I \subset R_c \subset R_E. \tag{8}$$

Under these circumstances, the calculation of the original functions describing R_c would only be necessary for $\phi \in R_E - R_I$, which is a region of uncertainty as to the location of the boundary.

While the exterior approximation may be exploited computationally to provide an upper bound, the interior approximation obviously leads to a lower bound.

The Tolerance Region R_ε

The interior or exterior approximation could be used in design centering procedures. Consider the tolerance region [7]

$$R_\varepsilon \triangleq \{\phi \mid \phi^0 - \varepsilon \leq \phi \leq \phi^0 + \varepsilon\}, \tag{9}$$

where ϕ^0 is called the nominal point and $\varepsilon \geq 0$ is a vector of associated tolerances. Thus,

$R_c \subset R_\varepsilon$ <==> R_ε is an exterior approximation,

$R_\varepsilon \subset R_c$ <==> R_ε is an interior approximation.

A serious problem, in general, in the above expressions is the implication that all points $\phi \in R_\varepsilon$ for the interior approximation and all $\phi \notin R_\varepsilon$ for the exterior approximation must be accounted for, i.e., we have to deal with an infinite number of constraints.

The Outcome Region R_μ

Consider, for example, the region

$$R_\mu \stackrel{\Delta}{=} \{\underset{\sim}{\mu} \mid -1 \leq \mu_i \leq 1\}, \tag{10}$$

where $\underset{\sim}{\mu}$ is a k-vector. In this case, we could define an alternative expression for the tolerance region of (9), namely, the orthotope

$$R_\varepsilon = \{\underset{\sim}{\phi} \mid \underset{\sim}{\phi} = \underset{\sim}{\phi}^0 + \underset{\sim}{E}\,\underset{\sim}{\mu},\ \underset{\sim}{\mu} \in R_\mu\}\ , \tag{11}$$

where

$$\underset{\sim}{E} \stackrel{\Delta}{=} \begin{bmatrix} \varepsilon_1 & & & \\ & \varepsilon_2 & & \\ & & \ddots & \\ & & & \varepsilon_k \end{bmatrix}. \tag{12}$$

Suppose also that a discrete set of $\underset{\sim}{\mu}$ is available, say $R_{\mu v}$, and let

$$R_v \stackrel{\Delta}{=} \{\underset{\sim}{\phi} \mid \underset{\sim}{\phi} = \underset{\sim}{\phi}^0 + \underset{\sim}{E}\,\underset{\sim}{\mu},\ \underset{\sim}{\mu} \in R_{\mu v}\}. \tag{13}$$

In practice, we are forced to consider a discrete set of $\underset{\sim}{\mu}$ out of computational necessity. What would distinguish one algorithm from another is the strategy for discarding elements of the set and/or adding new elements.

Candidates for Worst Case

If

$$R_v \subset R_c \implies R_\varepsilon \subset R_c \tag{14}$$

then R_v or $R_{\mu v}$ are said to provide candidates for a worst-case design. In practice [18-20], it is usual to consider vertices of R_ε or R_μ as candidates so that

$$R_{\mu v} = \{\underset{\sim}{\mu} \mid \mu_i \in \{-1,\ 1\}\}. \tag{15}$$

This corresponds to intuition, which suggests that extremes of performance correspond to extremes of designable variables. Mathematics, fortunately, was invented to harness intuition and avoid its pitfalls, consequently the assumption that $R_{\mu v}$ yields candidates for worst case requires justification in particular problems.

Optimization with Fixed Tolerances

The extension of the generalized least pth objective to handle all $\underset{\sim}{\mu} \in R_{\mu v}$ and the minimization of the resulting function $U(\underset{\sim}{\phi}^0)$ with respect to $\underset{\sim}{\phi}^0$ for fixed absolute or relative tolerances is rather simple. Difficulties occur essentially in housekeeping of arrays. Of course, depending on the sizes of the tolerances and the assumption (14) the solution obtained is not necessarily an acceptable worst case [21]. It may be that violations still occur at vertices and in the neighborhood of violating vertices.

Optimizing the Tolerance Orthotope

If we have a cost function $C(\underset{\sim}{\phi}^0,\underset{\sim}{\varepsilon})$ with the properties [7]

$$
\begin{aligned}
C(\underset{\sim}{\phi}^0,\underset{\sim}{\varepsilon}) &\rightarrow c \text{ as } \underset{\sim}{\varepsilon} \rightarrow \underset{\sim}{\infty} \\
C(\underset{\sim}{\phi}^0,\underset{\sim}{\varepsilon}) &\rightarrow \infty \text{ for any } \varepsilon_i \rightarrow \infty
\end{aligned}
\qquad (16)
$$

then the minimization of $C(\phi^0,\varepsilon)$ with respect to ϕ^0 and ε subject to $R_\varepsilon \subset R_c$ can be used to optimize (maximize) the tolerances. Again, candidates for worst case R_v are considered in practice to reduce computational effort.

Typical objective functions which fit the requirements of (16) and which have some physical justification [7,18,19,35,40,44,45] can take the form

$$C = \sum_{i=1}^{k} \frac{c_i}{\varepsilon_i} \qquad (17)$$

or

$$C = \sum_{i=1}^{k} c_i \frac{\phi_i^0}{\varepsilon_i}, \qquad (18)$$

where c_i are constants which reflect the cost or importance of the corresponding term.

Tolerancing and Tuning

Tuning is the post-production process which permits a manufacturer to correct for unavoidably large deviations from the specifications or the customer to optimize his unit under operating conditions. We can take into account a simple situation where one or more variables are tunable by considering the slack region

$$R_\rho \triangleq \{\underset{\sim}{\rho} \mid -1 \leq \rho_i \leq 1\}, \qquad (19)$$

where $\underset{\sim}{\rho}$ is a k-vector. A tuning region $R_t(\underset{\sim}{\mu})$ may then be defined as

$$R_t(\underset{\sim}{\mu}) \triangleq \{\underset{\sim}{\phi} \mid \underset{\sim}{\phi} = \underset{\sim}{\phi}^0 + \underset{\sim}{E}\,\underset{\sim}{\mu} + \underset{\sim}{T}\,\underset{\sim}{\rho},\ \underset{\sim}{\rho} \in R_\rho\}, \qquad (20)$$

where

$$\underset{\sim}{T} \triangleq \begin{bmatrix} t_1 & & & \\ & t_2 & & \\ & & \ddots & \\ & & & t_k \end{bmatrix} \qquad (21)$$

and a suitable cost function $C(\underset{\sim}{\phi}^0, \underset{\sim}{\varepsilon}, \underset{\sim}{t})$ minimized. Such a cost function should probably have the properties

$$\begin{aligned} &C(\underset{\sim}{\phi}^0, \underset{\sim}{\varepsilon}, \underset{\sim}{t}) \to c \text{ as } \underset{\sim}{\varepsilon} \to \underset{\sim}{\infty} \\ &C(\underset{\sim}{\phi}^0, \underset{\sim}{\varepsilon}, \underset{\sim}{t}) \to \infty \text{ for any } \varepsilon_i \to \infty \\ &C(\underset{\sim}{\phi}^0, \underset{\sim}{\varepsilon}, \underset{\sim}{t}) \to C(\underset{\sim}{\phi}^0, \underset{\sim}{\varepsilon}) \text{ as } \underset{\sim}{t} \to \underset{\sim}{0} \\ &C(\underset{\sim}{\phi}^0, \underset{\sim}{\varepsilon}, \underset{\sim}{t}) \to \infty \text{ for any } t_i \to \infty. \end{aligned} \tag{22}$$

See, for example, Bandler et al. [19]. The cost function might also involve $\underset{\sim}{\mu}$ [13], since the cost of tuning may well depend on a statistical distribution to outcomes.

The important concept to note in any worst-case optimization program, for example, that is set up to solve a centering, tolerancing and tuning problem is that for <u>all</u> selected sets of $\underset{\sim}{\mu}$ taken, for example, from $R_{\mu v}$ of (15) there must exist <u>some</u> $\underset{\sim}{\rho}$ (i.e., one independent ρ vector for each μ vector) which permits a corresponding point $\underset{\sim}{\phi}$ to be in R_c (i.e., the intersection of the $R_t(\underset{\sim}{\mu})$ of (20) and R_c must not be empty).

Vertex Selection

An efficient vertex selection scheme in a tolerance assignment or centering problem would involve finding local or global solutions $\check{\underset{\sim}{\mu}}$ to

$$\min_{\underset{\sim}{\mu} \in R_\mu} \quad g_i(\underset{\sim}{\phi}^0 + \underset{\sim}{E}\, \underset{\sim}{\mu}) \,.$$

It is easily shown that the components of $\check{\underset{\sim}{\mu}}$ satisfy [21,46,47]

$$\check{\mu}_j = -\operatorname{sgn} \frac{\partial g_i(\check{\underset{\sim}{\mu}})}{\partial \mu_j} \tag{23}$$

for $\check{\underset{\sim}{\mu}} \in R_{\mu v}$ of (15). Iterative approaches for solving the associated nonlinear system of equations

$$\underset{\sim}{\mu} = -\operatorname{sgn} \underset{\sim}{\nabla}_{\mu} g_i(\underset{\sim}{\mu}) \tag{24}$$

for all $i \in I_c$, where

$$\underset{\sim}{\nabla}_{\mu} \triangleq \begin{bmatrix} \partial/\partial\mu_1 \\ \partial/\partial\mu_2 \\ \cdot \\ \cdot \\ \cdot \\ \partial/\partial\mu_k \end{bmatrix} . \tag{25}$$

is the partial derivative operator with respect to the $\underset{\sim}{\mu}$, have been suggested [21,47,48].

The Generalized Tolerance Problem

Tromp [47,48] has carried out extensive research which has permitted the generalization of the tolerance problem to accommodate physical tolerances, model uncertainties, external disturbing effects and dependently toleranced parameters in a completely unified manner. In essence, his approach begins by defining the k_{0i}-dimensional vector $\tilde{\phi}^{0i}$, the k_i-dimensional vector $\tilde{\phi}^i$ and the $k_{\mu i}$-dimensional vector $\tilde{\mu}^i$ so that $\tilde{\phi}^i$ is a function of $\tilde{\phi}^{0i}$ and $\tilde{\mu}^i$ for all i = 1, 2, ... n, and $\tilde{\phi}^{0i}$ itself depends on all $\tilde{\phi}^{i-1}$ for i = 2, 3, ..., n.

Input parameters, for example, the physical parameters (dimensions, constants of materials used, etc.) available to the manufacturer might be identified as $\tilde{\phi}^1$, whereas $\tilde{\phi}^n$ would be the output vector, for example, the sampled response of a system or the constraint vector $\tilde{g}$, introduced earlier. The quantities $\tilde{\phi}^2$, ..., $\tilde{\phi}^{n-1}$ can be identified, for example, as appropriate intermediate or model parameters. The variables $\tilde{\mu}^i$, i = 1, 2, ..., n, embody unavoidable, undesirable or unknown fluctuations generally. Hence, we may assemble the vectors

$$\tilde{\phi}^0 \triangleq \begin{bmatrix} \tilde{\phi}^{01} \\ \tilde{\phi}^{02} \\ \cdot \\ \cdot \\ \cdot \\ \tilde{\phi}^{0n} \end{bmatrix}, \quad \tilde{\phi} \triangleq \begin{bmatrix} \tilde{\phi}^{1} \\ \tilde{\phi}^{2} \\ \cdot \\ \cdot \\ \cdot \\ \tilde{\phi}^{n} \end{bmatrix}, \quad \tilde{\mu} \triangleq \begin{bmatrix} \tilde{\mu}^{1} \\ \tilde{\mu}^{2} \\ \cdot \\ \cdot \\ \cdot \\ \tilde{\mu}^{n} \end{bmatrix}. \qquad (26)$$

The preceding discussions involving the μ variables and the μ-space generalize in an obvious manner. However, the tolerance region in the $\tilde{\phi}$-space need no longer turn out to be an orthotope [47,48].

This kind of analysis permits more than strictly design or manufacturing problems to be simulated and optimized. Planning problems, anticipated system operation, aging, measurement errors and so on may be embodied into the original simulation, and the design solution accordingly optimized to reflect these phenomena. It is obvious that uncertainty can enter into the problem at any stage: at a high (conceptual) level or at a low (computational) level. Distributions of and correlations between parameters exacerbate the situation, but one may even conceive of building desirable correlations into a manufacturing process.

Centering via Large-change Sensitivities and Performance Contours

The design centering approach of Butler involves large-change sensitivities in conjunction with pairwise changes in parameter values with respect to chosen performance contours [26]. A scalar continuous function of design parameters which reflects the goodness of a design is chosen as a performance criterion. A nominal design which satisfies this performance criterion is assumed to exist. The concept of large-change sensitivities is that of finding changes in function values due to significant deviations in designable parameters. This concept is used to draw contours of the performance criterion changing parameters in a pairwise manner for each contour. The design center is obtained by inspection, i.e., by choosing a nominal value which is well centered for all contours. As an example of a performance criterion, we might use $-M(\underset{\sim}{\phi})$ of (3). The method is illustrated in Fig. 7.

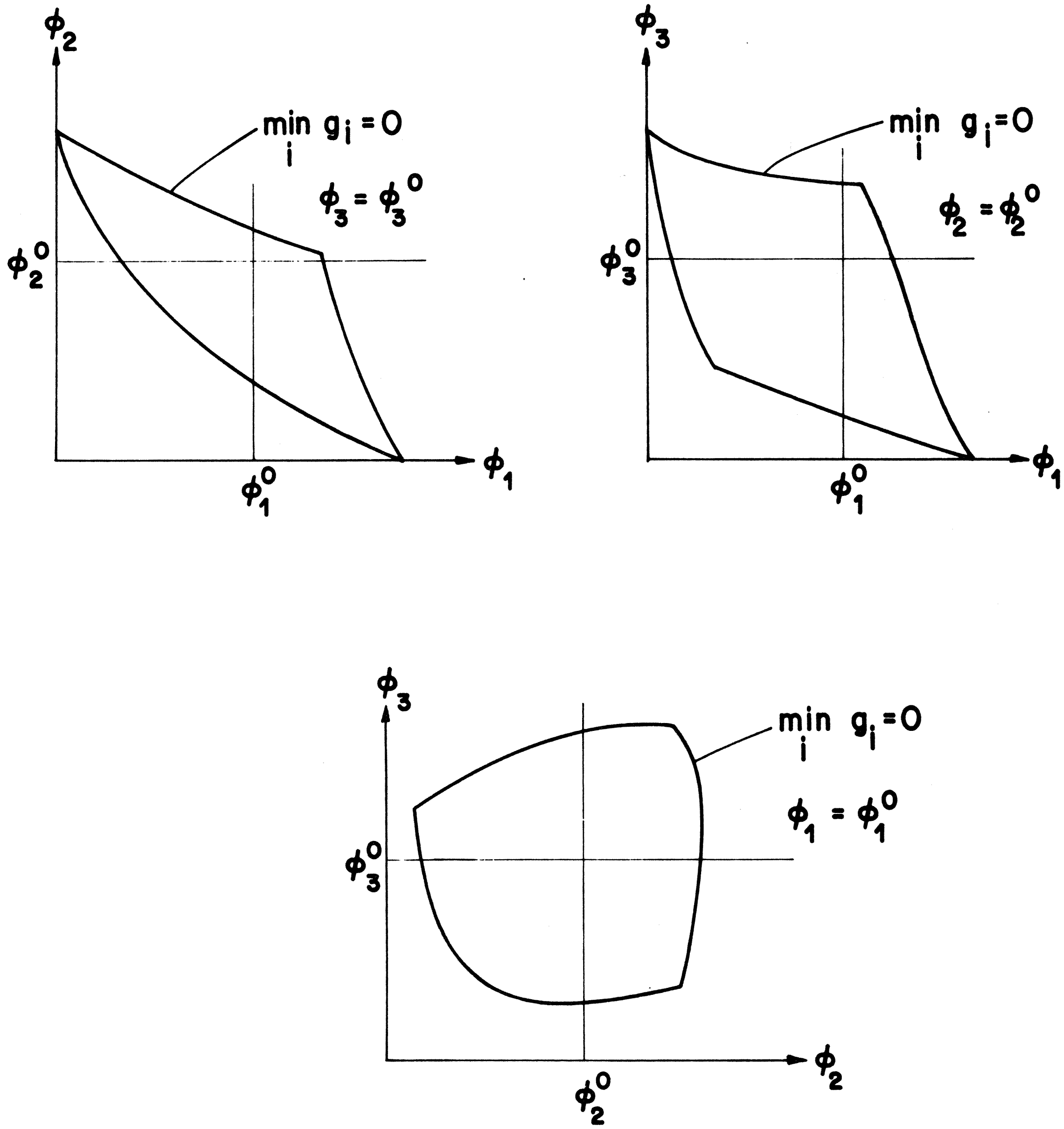
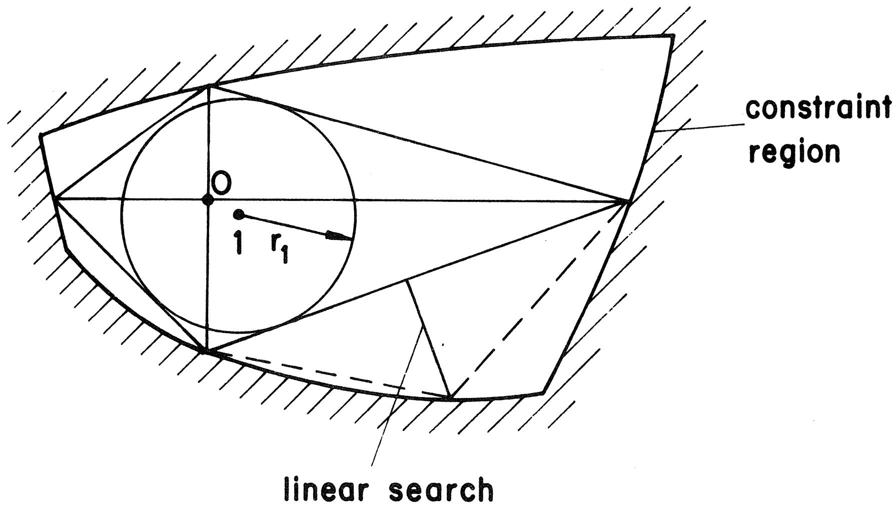


Fig. 7 Performance contours for pairwise changes in parameters [26]. Reducing ϕ_1^0 will result in a better centered nominal design [1].

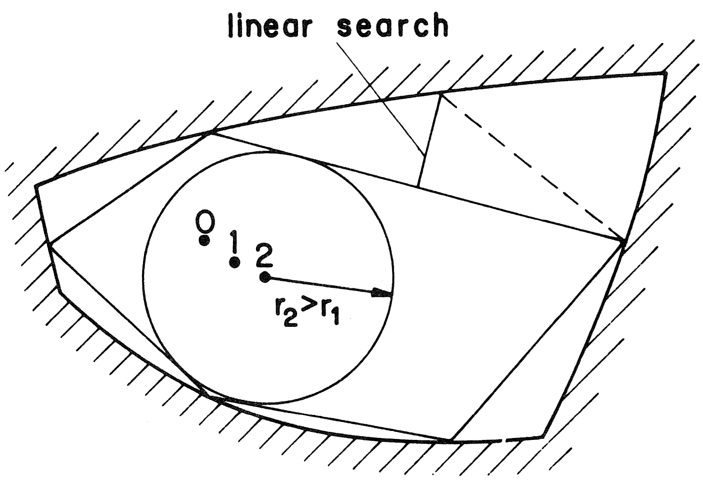
Centering via Simplicial Approximation

The simplicial approximation approach of Director and Hachtel [29,30] involves linear programming and one-dimensional search techniques. Their approach is to inscribe (inflate) a hypersphere inside the constraint region. During the process of enlarging this hypersphere a polytope which approximates the boundary of the constraint region is constructed. Fig. 8 illustrates the procedure.

The algorithm initially searches for points on the constraint boundary in both positive and negative directions for each parameter from a feasible point (a point within the constraint region). The convex hull described by these boundary



(a)



(b)

Fig. 8 An illustration of the simplicial approximation approach due to Director and Hachtel [29]. In (a) we show the results of an initial search for boundary points. In (b) is depicted the polytope approximating the boundary of the constraint region after two iterations [1].

points provides the initial polytope approximating the boundary of the constraint region. This polytope will be an interior approximation only if the constraint region is convex. Using linear programming a hypersphere is to be inflated inside this polytope in a k-dimensional space. The tangent hyperplanes are determined. These hyperplanes, faces of the polytope, are simplices in a space of k-1 dimensions. The largest simplex, i.e., the one which contains the largest hypersphere, is to be broken and replaced by k simplices. This is performed by adding a new vertex to the polytope obtained by searching for a boundary point along the normal direction to the largest simplex from the center of the corresponding hypersphere.

The inflation of an orthotope, as distinct from a sphere, describing the tolerance region is the essence of the work of Bandler et al. [1-21].

Quadratic Modelling of the Constraints

A nonlinear programming approach but employing approximations to the design constraints has been presented [1-5,8-14]. An interpolation region centered at the initial guess to the nominal design is chosen. The simulation program is used to provide the value of the response functions (constraints) at a certain set of base points. The base points are points within the interpolation region and defined in terms of values of the designable parameters. Based upon the corresponding values of the resulting responses, multidimensional quadratic polynomials are constructed. These quadratic polynomials have the general form

$$P(\underset{\sim}{\phi}) = a_0 + \underset{\sim}{a}^T(\underset{\sim}{\phi} - \overline{\underset{\sim}{\phi}}) + \frac{1}{2}(\underset{\sim}{\phi} - \overline{\underset{\sim}{\phi}})^T \underset{\sim}{H}(\underset{\sim}{\phi} - \overline{\underset{\sim}{\phi}}), \quad (27)$$

where a_0 and $\underset{\sim}{a}$ are, respectively, a constant scalar and a constant vector, $\underset{\sim}{H}$ is a constant symmetric Hessian matrix of the quadratic and $\overline{\underset{\sim}{\phi}}$ is the center of the chosen interpolation region.

The base points are simply those points where the approximated response function and the quadratic polynomial coincide. A system of simultaneous linear equations has to be solved to obtain the polynomial. The number of base points (exactly equal to the number of simulations required) is the minimum necessary to fully describe the responses and is given by

$$N = (k+1)(k+2)/2, \quad (28)$$

where k is the number of designable parameters. The number N is the number of the unknown coefficients. An arrangement of the base points is depicted by Fig. 9.

Space Regionalization for Statistical Analysis

Space regionalization was suggested by Scott and Walker [43]. Based upon the probability of having an outcome to fall within a region, a weight is assigned to this region and the center of the region is checked against the nonlinear constraints to determine whether this whole weight will contribute to the yield or not. See Fig. 10. The number of required analyses, however, increases exponentially with the number of variables subject to statistical variations, since the response at the center of each region is to be evaluated.

Regionalization was also used by Leung and Spence [36-38] in conjunction with systematic exploration. The centers of the regions are scanned systematically by changing one parameter at a time and employing efficient matrix inverse modification methods for the (linear) circuit analyses required. Leung and Spence also suggested checking the worst outcome in each region, instead of the center of the region, if a lower bound on yield is required.

Yield Determination and Optimization

Elias applied the Monte Carlo method directly to the constraints [32]. Director and Hachtel suggested applying the Monte Carlo method in conjunction with their simplicial approximation [30]. Their polytope could be updated according to the points which fall within the constraint region but not in the polytope. Pinel and Singhal used importance sampling, concentrating the distribution of sample points at critical regions to reduce computational effort [42].

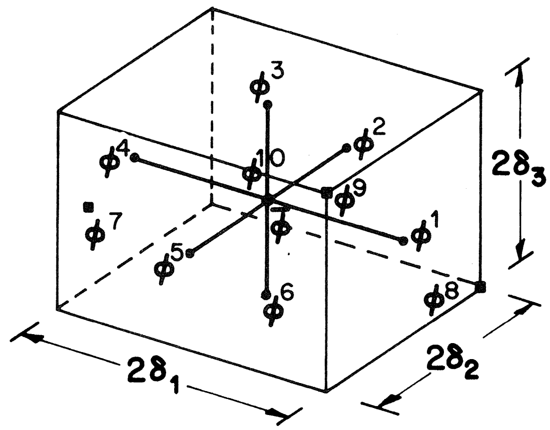


Fig. 9 Arrangement of base points for computing the quadratic interpolating polynomial in 3 dimensions [3]. In order to exploit sparsity in the associated system of linear equations ϕ^7, ϕ^8 and ϕ^9 should be, respectively, placed in the planes containing $\{\bar{\phi}, \phi^1, \phi^2\}$, $\{\bar{\phi}, \phi^1, \phi^3\}$ and $\{\bar{\phi}, \phi^2, \phi^3\}$, where $\bar{\phi}$ is the center of interpolation [3].

Bandler and Abdel-Malek [1-4,9,14] dealt with the mass of calculations involved in determining and optimizing yield as follows. Multidimensional quadratic polynomials are fitted to the constraints and updated periodically during the optimization process. An analytical approach is used to calculating yield and its sensitivities with respect to all the variables employing linear cuts of the tolerance region. The sensitivities permit the use of gradient methods of optimization.

The basic idea is to use weighted hypervolumes. Evaluating hypervolumes, in general, is expensive because it involves a multidimensional integration. For the special case of cutting an orthotope by a linear constraint, however, a simple formula can be found [2,14,46]. See, for example, Fig. 11.

The method of Bandler and Abdel-Malek does not assume that these linear cuts are fixed in the parameter space. It is possible for these linear cuts be continuously updated to follow the generally nonlinear constraints. This facilitates a good approximation to the boundary of the constraint region as the tolerance region is allowed to move in the parameter space during, for example, an optimization process. Methods for continuously updating the linear cuts have been given [3,14].

CONCLUSIONS

Most of the approaches in current use for design centering, tolerance assignment and yield optimization employed by electrical circuit designers appear quite general. While many applications of the techniques are directed towards design problems involving linear systems of equations, nonlinear circuits are of

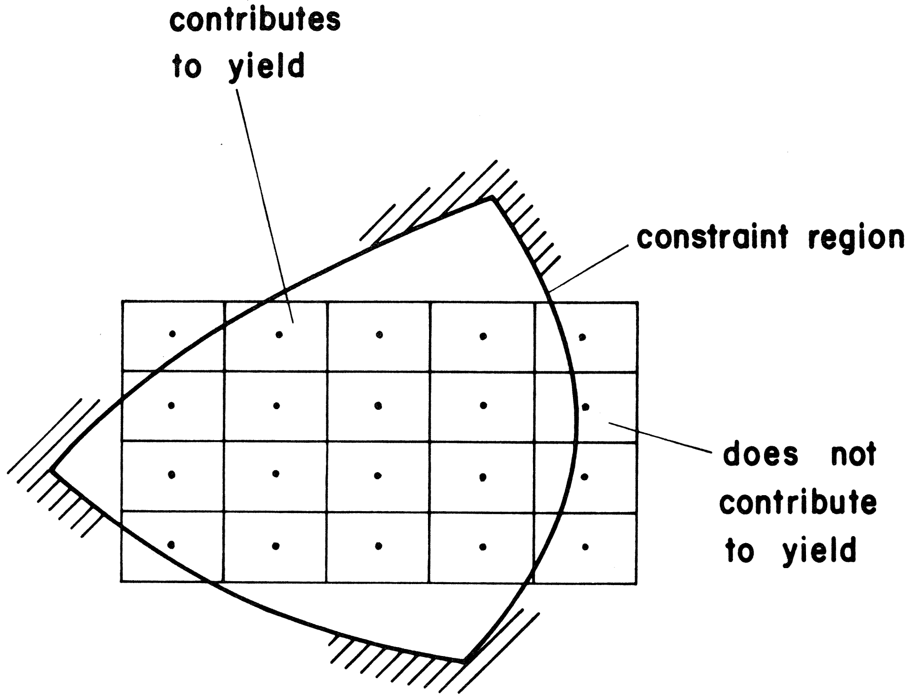


Fig. 10 Regionalization [43] of the parameter space for estimating production yield [1].

particular interest to circuit designers [3,4,11] and it is expected that considerable effort will be devoted to such systems in the future [22].

Some of the formulations described here, if expedited intact for problems involving masses of nonlinear constraints, for example, are formidable beyond our present computational means. With the anticipated breakthrough of computers with massive parallel processing capabilities, however, some of these problems are certain to be dealt with by engineers on an almost routine basis in the near future. Even at this time relevant and enormous computational problems are begging to be solved, not the least of which involve optimal topology and reliability of large systems. Of course, feasible solutions to such problems can be and are currently obtained suboptimally. It is the integration of all the concepts mentioned in the foregoing pages into a unified design methodology that is called for whether or not, for computational expedience, the superproblem thus created is subsequently partitioned or decomposed.

ACKNOWLEDGEMENT

The author would like to thank Dr. H.L. Abdel-Malek, now with the Department of Engineering Physics and Mathematics, Faculty of Engineering, Cairo University, Giza, Egypt, whose excellent Ph.D. thesis is the source of some of the material reported here. Furthermore, the author must acknowledge extensive and rewarding discussions with Dr. H. Tromp, Faculty of Engineering, University of Ghent, Ghent, Belgium, and Dr. Abdel-Malek, which helped clarify some of the finer points of the presentation. N.M. Sine of the Faculty of Engineering, McMaster University, Hamilton, Canada, is thanked for her efforts in the preparation of this manuscript.

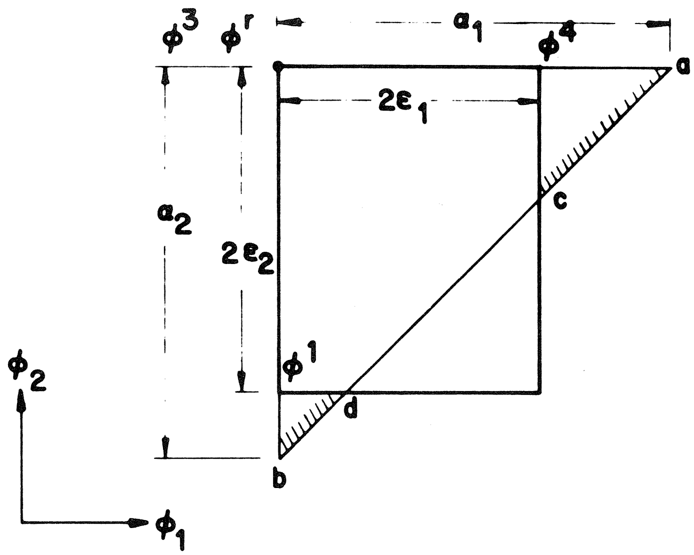


Fig. 11 A two-dimensional example illustrating the calculation of nonfeasible hypervolumes [1,2]. The nonfeasible area V is given by subtracting the areas of triangle $\underset{\sim}{\phi}^1$bd and $\underset{\sim}{\phi}^4$ac from $\underset{\sim}{\phi}^r$ab where $\underset{\sim}{\phi}^r$ is termed a reference vertex. Hence, it is easy to show that

$$V = 0.5\, \alpha_1 \alpha_2 \left(1 - \left(1 - \frac{2\varepsilon_1}{\alpha_1}\right)^2 - \left(1 - \frac{2\varepsilon_2}{\alpha_2}\right)^2\right) .$$

REFERENCES

[1] H.L. Abdel-Malek: A unified treatment of yield analysis, worst-case design and yield optimization, Ph.D. Thesis (1977) McMaster University, Hamilton, Canada.

[2] H.L. Abdel-Malek and J.W. Bandler: Yield estimation for efficient design centering assuming arbitrary statistical distributions, Int. J. Circuit Theory and Applications 6 (1978) 289-303.

[3] H.L. Abdel-Malek and J.W. Bandler: Yield optimization for arbitrary statistical distributions, Part I: theory, Proc. IEEE Int. Symp. Circuits and Systems (New York, 1978) 664-669.

[4] H.L. Abdel-Malek and J.W. Bandler: Yield optimization for arbitrary statistical distributions, Part II: implementation, Proc. IEEE Int. Symp. Circuits and Systems (New York, 1978) 670-674.

[5] H.L. Abdel-Malek and J.W. Bandler: Subroutines for implementing quadratic models of surfaces in optimal design, Report SOC-191 (1978) Faculty of Engineering, McMaster University, Hamilton, Canada.

[6] H.L. Abdel-Malek and J.W. Bandler: Centering, tolerancing, tuning and minimax design employing biquadratic models, Report SOC-211 (1978) Faculty of Engineering, McMaster University, Hamilton, Canada.

[7] J.W. Bandler: Optimization of design tolerances using nonlinear programming, J. Optimization Theory and Applications 14 (1974) 99-114.

[8] J.W. Bandler, H.L. Abdel-Malek, P.B. Johns and M.R.M. Rizk: Optimal design via modeling and approximation, Proc. IEEE Int. Symp. Circuits and Systems (Munich, 1976) 767-770.

[9] J.W. Bandler and H.L. Abdel-Malek: Optimal centering, tolerancing and yield determination using multidimensional approximations, Proc. IEEE Int. Symp. Circuits and Systems (Phoenix, AZ, 1977) 219-222.

[10] J.W. Bandler and H.L. Abdel-Malek: Modeling and approximation for statistical evaluation and optimization of microwave designs, Proc. 7th European Microwave Conf. (Copenhagen, 1977) 153-157.

[11] J.W. Bandler, H.L. Abdel-Malek, P. Dalsgaard, Z.S. El-Razaz and M.R.M. Rizk: Optimization and design centering of active and nonlinear circuits including component tolerances and model uncertainties, Proc. Int. Symp. Large Engineering Systems (Waterloo, Canada, 1978) 127-132.

[12] J.W. Bandler and H.L. Abdel-Malek: Algorithms for design centering involving yield and its sensitivities, Proc. 21st Midwest Symp. on Circuits and Systems (Ames, Iowa, 1978) 242-248.

[13] J.W. Bandler and H.L. Abdel-Malek: Advances in the mathematical programming approach to design centering, tolerancing and tuning, Joint Automatic Control Conf. (Philadelphia, PA, 1978) 329-344.

[14] J.W. Bandler and H.L. Abdel-Malek: Optimal centering, tolerancing and yield determination via updated approximations and cuts, IEEE Trans. Circuits and Systems CAS-25 (1978) 853-871.

[15] J.W. Bandler and C. Charalambous: Practical least pth optimization of networks, IEEE Trans. Microwave Theory Tech. MTT-20 (1972) 834-840.

[16] J.W. Bandler, J.H.K. Chen, P. Dalsgaard and P.C. Liu: TOLOPT - a program for optimal, continuous or discrete, design centering and tolerancing, Report SOC-105 (1975) Faculty of Engineering, McMaster University, Hamilton, Canada.

[17] J.W. Bandler and P.C. Liu: Some implications of biquadratic functions in the tolerance problem, IEEE Trans. Circuits and Systems CAS-22 (1975) 385-390.

[18] J.W. Bandler, P.C. Liu and J.H.K. Chen: Worst case network tolerance optimization, IEEE Trans. Microwave Theory Tech. MTT-23 (1975) 630-641.

[19] J.W. Bandler, P.C. Liu and H. Tromp: A nonlinear programming approach to optimal design centering, tolerancing and tuning, IEEE Trans. Circuits and Systems CAS-23 (1976) 155-165.

[20] J.W. Bandler, P.C. Liu and H. Tromp: Integrated approach to microwave design, IEEE Trans. Microwave Theory Tech. MTT-24 (1976) 584-591.

[21] J.W. Bandler, P.C. Liu and H. Tromp: Efficient, automated design centering and tolerancing, Proc. IEEE Int. Symp. Circuits and Systems (Munich, 1976) 710-713.

[22] J.W. Bandler and M.R.M. Rizk: Optimization of electrical circuits, Report SOC-183 (1977) Faculty of Engineering, McMaster University, Hamilton, Canada.

[23] P.W. Becker and F. Jensen: Design of Systems and Circuits for Maximum Reliability or Maximum Production Yield (Polyteknisk Forlag, Lyngby, Denmark, 1974).

[24] R.K. Brayton, G.D. Hachtel and S.W. Director: Arbitrary norms for statistical design via linear programming, Proc. IEEE Int. Symp. Circuits and Systems (New York, 1978) 161-164.

[25] R.K. Brayton, A.J. Hoffman and T.R. Scott: A theorem of inverses of convex sets of real matrices with application to the worst-case D.C. problem, IEEE Trans. Circuits and Systems CAS-24 (1977) 409-415.

[26] E.M. Butler: Realistic design using large-change sensitivities and performance contours, IEEE Trans. Circuit Theory CT-18 (1971) 58-66.

[27] C. Charalambous: A unified review of optimization, IEEE Trans. Microwave Theory Tech. MTT-22 (1974) 289-300.

[28] C. Charalambous: Discrete optimization, Int. J. Systems Science 5 (1974) 889-894.

[29] S.W. Director and G.D. Hachtel: The simplicial approximation approach to design centering, IEEE Trans. Circuits and Systems CAS-24 (1977) 363-372.

[30] S.W. Director and G.D. Hachtel: Yield estimation using simplicial approximation, Proc. IEEE Int. Symp. Circuits and Systems (Phoenix, AZ, 1977) 579-582.

[31] S.W. Director, G.D. Hachtel and L.M. Vidigal: Computationally efficient yield estimation procedures based on simplicial approximation, IEEE Trans. Circuits and Systems CAS-25 (1978) 121-130.

[32] N.J. Elias: New statistical methods for assigning device tolerances, Proc. IEEE Int. Symp. Circuits and Systems (Newton, MA, 1975) 329-332.

[33] G.D. Hachtel and S.W. Director: A point basis for statistical design, Proc. IEEE Int. Symp. Circuits and Systems (New York, 1978) 165-169.

[34] B.J. Karafin: The optimum assignment of component tolerances for electrical networks, BSTJ 50 (1971) 1225-1242.

[35] B.J. Karafin: The general component tolerance assignment problem in electrical networks, Ph.D. Thesis (1974) Univ. of Pennsylvania, Philadelphia, PA.

[36] K.H. Leung and R. Spence: Multiparameter large-change sensitivity analysis and systematic exploration, IEEE Trans. Circuits and Systems CAS-22 (1975) 796-804.

[37] K.H. Leung and R. Spence: Efficient frequency domain statistical circuit analysis, Proc. IEEE Int. Symp. Circuits and Systems (Munich, 1976) 197-200.

[38] K.H. Leung and R. Spence: Idealized statistical models for low-cost linear circuit yield analysis, IEEE Trans. Circuits and Systems CAS-24 (1977) 62-66.

[39] K. Madsen and H. Schjaer-Jacobsen: New algorithms for worst case tolerance optimization, Proc. IEEE Int. Symp. Circuits and Systems (New York, 1978) 681-685.

[40] J.F. Pinel and K.A. Roberts: Tolerance assignment in linear networks using nonlinear programming, IEEE Trans. Circuit Theory CT-19 (1972) 475-479.

[41] J.F. Pinel, K.A. Roberts and K. Singhal: Tolerance assignment in network design, Proc. IEEE Int. Symp. Circuits and Systems (Newton, MA, 1975) 317-320.

[42] J.F. Pinel and K. Singhal: Efficient Monte Carlo computation of circuit yield using importance sampling, Proc. IEEE Int. Symp. Circuits and Systems (Phoenix, AZ, 1977) 575-578.

[43] T.R. Scott and T.P. Walker, Jr.: Regionalization: a method for generating joint density estimates, IEEE Trans. Circuits and Systems CAS-23 (1976) 229-234.

[44] A.K. Seth: Electrical network tolerance optimization, Ph.D. Thesis (1972) University of Waterloo, Waterloo, Canada.

[45] M. Styblinski: Sensitivity minimization with an optimal assignment of network element tolerances, Proc. IEEE Int. Symp. Circuits and Systems (Phoenix, AZ, 1977) 223-226.

[46] H. Tromp: unpublished formula (1975) Faculty of Engineering, University of Ghent, Ghent, Belgium.

[47] H. Tromp: The generalized tolerance problem and worst case search, Conf. Computer-aided Design of Electronic and Microwave Circuits and Systems (Hull, England, 1977) 72-77.

[48] H. Tromp: Generalized worst case design, with applications to microwave networks, Doctoral Thesis (in Dutch) (1978) Faculty of Engineering, University of Ghent, Ghent, Belgium.

SOC-214

ENGINEERING MODELLING AND DESIGN SUBJECT TO MODEL UNCERTAINTIES AND MANUFACTURING TOLERANCES

J.W. Bandler

July 1978, No. of Pages: 23

Revised: November 1978

Key Words: Engineering modelling, optimal design, model uncertainties, manufacturing tolerances, production yield, design centering, Monte Carlo analysis, tuning

Abstract: This paper deals with engineering design problems in which, for example, either a large volume of production is envisaged or in which only a few units are to be custom made. Designs are considered subject to manufacturing tolerances, material uncertainties, environmental uncertainties and model uncertainties. The reduction of cost by increasing tolerances, the determination and optimization of production yield, the problem of design centering and various aspects of tuning are discussed. Nonlinear programming approaches are considered. The important problem of searching for candidates for worst case solutions is briefly mentioned. Simplicial approximation, quadratic modelling, linear cuts and space regionalization are reviewed. A fairly extensive bibliography to relevant work in the modelling and design of electrical circuits is provided.

Description: Presented at Symposium on Modelling and Simulation Methodology (Rehovot, Israel, Aug. 1978). To appear in Symposium Book, B.P. Zeigler, Ed. North-Holland, 1979.

Related Work: SOC-1, SOC-18, SOC-24, SOC-29, SOC-37, SOC-49, SOC-62, SOC-65, SOC-87, SOC-93, SOC-105, SOC-110, SOC-111, SOC-113, SOC-118, SOC-119, SOC-120, SOC-124, SOC-132, SOC-142, SOC-155, SOC-167, SOC-173, SOC-177, SOC-179, SOC-182, SOC-183, SOC-184, SOC-185, SOC-188, SOC-190, SOC-191, SOC-192, SOC-193, SOC-194, SOC-209, SOC-210, SOC-211.

Price: $ 6.00.