INTERNAL REPORTS IN

SIMULATION, OPTIMIZATION AND CONTROL

No. SOC-234

A NEW, A.C. APPROACH TO POWER SYSTEM SENSITIVITY ANALYSIS AND PLANNING

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September 1979

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Abstract - This paper presents an approach for sensitivity analysis and gradient evaluation required in power system analysis and planning. The approach utilizes Tellegen's theorem in an augmented form which allows different power system problems to be handled based on the a.c. power flow model in general and without any approximations.

The approach provides the flexibility of including line responses directly while preserving the advantages of compactness, sparsity and simplicity of the adjoint system. Numerical results are presented for illustration and comparison.

INTRODUCTION

Many power system problems have been successfully solved by optimization techniques [1]. For effective application of these optimization techniques [2] to analysis and design the gradient vector of performance functions and constraints w.r.t. the design parameters are required to be calculated in an efficient way. Relevant problems include optimal power flow studies, contingency analysis and planning of electric power systems.

Tellegen's theorem [3] with extensions [4] has been successfully exploited in electronic circuit analysis and design [5] as a powerful tool for calculating the required gradients using one additional linear network analysis.

Fischl and Puntel [6] and Puntel et al. [7] described the use of the adjoint network in the transmission system planning problem based on the linear d.c. power flow model. The d.c. model may be considered of sufficient accuracy for some applications. However, it is characterized by the restrictive assumptions of neglecting transmission losses, excluding reactive power flows and considering flat voltage profiles which make it inadequate for other studies requiring a more accurate model and more information.

In the work of Wu and Sullivan [8], the a.c. power flow model has been recognized with some approximations. Two coupled models have been developed to supply the gradient information. The tableau approach [9] presented by Director and Sullivan utilizes the Lagrange multiplier method to provide gradient information based on the a.c. power model. In this approach the total number of variables of a particular problem dictates the problem dimensionality.

The Lagrange multiplier method handles the power flow constraints as general equality constraints. These constraints characterize a particular electrical network. We can utilize this fact to achieve a great deal of dimensionality reduction and scheme compactness. Indeed, Tellegen's theorem facilitates such achievement. Tellegen's theorem has been successfully used by Püttgen and Sullivan [10] in a suitably extended form to provide gradient information. The a.c. model was used with some approximations regarding generator modelling. Their approach provides bus voltage gradients w.r.t. transmission-line parameters.

Fischl and Wasley [11] have theoretically outlined an approach for calculating power flow sensitivities. It is based on the a.c. power flow model and effectively utilizes an adjoint method to provide the gradients for several classes of functions. The system states are bus quantities so that the adjoint matrix of coefficients is the transpose of the original Jacobian matrix. State transformations are required to handle functions of line quantities.

In this paper, we present an approach which allows Tellegen's theorem in an augmented form to be directly used for efficient sensitivity analysis and gradient calculations. The a.c. power flow model is used without any restrictive assumptions. Several classes of functions can be considered. Line quantities are directly handled. The adjoint matrix of coefficients is of the same size and sparsity as the Jacobian matrix of the original network.

Numerical examples are presented based on a 6-bus sample power system. Several types of functions are considered for the purpose of illustration and comparison.

TELLEGEN'S THEOREM FOR THE A.C. POWER MODEL

Assume V and I represent the complex voltage and current variables associated with the given network. We will use ^ to distinguish the corresponding variables associated with the topologically similar adjoint network. Tellegen's theorem in the form usually used for electronic circuit analysis states that

$$\sum_b V_b \hat{I}_b = 0, \tag{1a}$$

$$\sum_b I_b \hat{V}_b = 0, \tag{1b}$$

where the summation is taken over all branches, subscript b denoting the bth branch.

To exploit the a.c. power model we introduce the following valid variations of Tellegen's theorem. First, we have a pair of complex conjugate terms corresponding to (1a) and (1b), namely,

$$\sum_b V_b^* \hat{I}_b^* = 0 \tag{2a}$$

and

$$\sum_b I_b^* \hat{V}_b^* = 0, \tag{2b}$$

where * denotes the complex conjugate. These terms have been considered by Püttgen and Sullivan [10]. Next, we consider the pair of power terms

$$\sum_b S_b^* = \sum_b (P_b - jQ_b) = \sum_b V_b^* I_b = 0 \tag{3a}$$

and

$$\sum_b S_b = \sum_b (P_b + jQ_b) = \sum_b V_b I_b^* = 0. \tag{3b}$$

These terms provide the key to the generality we are seeking. Note, incidentally, that the directions of power and current are the same.

Equations (1)-(3) can be written in terms, for example, of first-order changes in the voltage and current variables in the given network. Doing this and collecting the terms in an appropriate manner, we have

$$\sum_b [\hat{I}_b \,\delta V_b + \hat{I}_b^* \,\delta V_b^* - \hat{V}_b \,\delta I_b - \hat{V}_b^* \,\delta I_b^* + \delta(V_b^* I_b) + \delta(V_b I_b^*)] = 0 \tag{4a}$$

or

$$\sum_b [(\hat{I}_b + I_b^*)\,\delta V_b + (\hat{I}_b^* + I_b)\,\delta V_b^* + (V_b^* - \hat{V}_b)\,\delta I_b + (V_b - \hat{V}_b^*)\,\delta I_b^*] = 0. \tag{4b}$$

TERMS FOR NETWORK ELEMENTS

We consider the number of loads to be n_L and the number of generators to be n_G. The buses are ordered such that subscripts $\ell = 1, 2, \ldots, n_L$ identify load branches, $g = n_L + 1, \ldots, n_L + n_G$ identify generator branches and $n = n_L + n_G + 1$ identifies the slack generator branch.

A term of (4) associated with a load is now considered, namely,

$$\hat{I}_\ell \,\delta V_\ell + \hat{I}_\ell^* \,\delta V_\ell^* - \hat{V}_\ell \,\delta I_\ell - \hat{V}_\ell^* \,\delta I_\ell^* + \delta S_\ell^* + \delta S_\ell .$$

Since

$$\delta S_\ell = \delta(V_\ell I_\ell^*) = V_\ell \,\delta I_\ell^* + I_\ell^* \delta V_\ell$$

we have

$$\delta I_\ell^* = [\delta S_\ell - I_\ell^* \,\delta V_\ell]/V_\ell \tag{5a}$$

hence

$$\delta I_\ell = [\delta S_\ell^* - I_\ell \,\delta V_\ell^*]/V_\ell^*. \tag{5b}$$

Substituting for δI_ℓ^* from (5a) and δI_ℓ from (5b) we obtain

$$[\hat{I}_\ell + \hat{V}_\ell^* I_\ell^*/V_\ell]\,\delta V_\ell + [\hat{I}_\ell^* + \hat{V}_\ell I_\ell/V_\ell^*]\delta V_\ell^* + [1 - \hat{V}_\ell^*/V_\ell]\delta S_\ell + [1 - \hat{V}_\ell/V_\ell^*]\delta S_\ell^*. \tag{6}$$

A term of (4) associated with a generator is considered as

$$\hat{I}_g \,\delta V_g + \hat{I}_g^* \,\delta V_g^* - \hat{V}_g \,\delta I_g - \hat{V}_g^* \,\delta I_g^* + \delta S_g^* + \delta S_g .$$

Note that

$$\delta |V_g|^2 = \delta(V_g V_g^*) = V_g \,\delta V_g^* + V_g^* \,\delta V_g, \tag{7}$$

from which

$$\delta V_g^* = \delta(V_g V_g^*)/V_g - V_g^* \,\delta V_g/V_g. \tag{8}$$

Note also that the real part of S_g is expressed by

$$2\delta P_g = \delta(S_g + S_g^*) = V_g \delta I_g^* + I_g^* \delta V_g + V_g^* \delta I_g + I_g \delta V_g^* \tag{9}$$

from which, and using (8), we obtain

$$\delta I_g^* = \delta(S_g + S_g^*)/V_g - I_g \,\delta(V_g V_g^*)/V_g^2 - [I_g^* - I_g V_g^*/V_g]\delta V_g/V_g - V_g^* \delta I_g/V_g .$$

Substituting for δV_g^* and δI_g^* the term associated with the generator becomes

$$[\hat{I}_g - \hat{I}_g^* V_g^*/V_g + \{I_g^* - (I_g V_g^*/V_g)\}\,\hat{V}_g^*/V_g]\,\delta V_g - [\hat{V}_g - (\hat{V}_g^* V_g^*/V_g)]\delta I_g + [\hat{I}_g^*/V_g + \hat{V}_g^* I_g/V_g^2]\,\delta(V_g V_g^*) + [1 - \hat{V}_g^*/V_g]\,\delta(S_g + S_g^*). \tag{10}$$

The term of (4) corresponding to the slack bus is, for

$$\delta V_n = 0, \tag{11}$$

given by

$$(V_n^* - \hat{V}_n)\;\delta I_n + (V_n - \hat{V}_n^*)\;\delta I_n^*. \tag{12}$$

Other elements, e.g., transmission-line elements, characterized by

$$I_t = Y_t V_t \tag{13}$$

lead to the first-order expression

$$\delta I_t = Y_t \;\delta V_t + V_t \;\delta Y_t$$

from which

$$\delta V_t = (\delta I_t - V_t \;\delta Y_t)/Y_t, \tag{14a}$$

hence

$$\delta V_t^* = (\delta I_t^* - V_t^* \;\delta Y_t^*)/Y_t^*. \tag{14b}$$

Substituting (14) into the appropriate term of (4) we get

$$[V_t^* - \hat{V}_t + (\hat{I}_t + I_t^*)/Y_t]\,\delta I_t + [V_t - \hat{V}_t^* + (\hat{I}_t^* + I_t)/Y_t^*]\delta I_t^* - (\hat{I}_t + I_t^*)(V_t/Y_t)\,\delta Y_t - (\hat{I}_t^* + I_t)(V_t^*/Y_t^*)\;\delta Y_t^*. \tag{15}$$

ADJOINT NETWORK AND NETWORK SENSITIVITIES

Consider an ith (real) design variable ϕ_{ti} which may appear as an argument of admittance Y_t. Note that ϕ_{ti} can represent, for example, the parameters of shunt control elements and phase shifting transformers. Then

$$\delta Y_t = \sum_i \frac{\partial Y_t}{\partial \phi_{ti}}\;\Delta\phi_{ti}, \tag{16a}$$

hence

$$\delta Y_t^* = \sum_i \frac{\partial Y_t^*}{\partial \phi_{ti}}\;\Delta\phi_{ti}. \tag{16b}$$

Rewriting the Tellegen summation (4) we have

$$\sum_\ell [\hat{I}_\ell + \hat{V}_\ell^* I_\ell^*/V_\ell]\delta V_\ell + \sum_\ell [\hat{I}_\ell^* + \hat{V}_\ell I_\ell/V_\ell^*]\,\delta V_\ell^* + \sum_g [\hat{I}_g - \hat{I}_g^* V_g^*/V_g + \{I_g^* - I_g V_g^*/V_g\}\;\hat{V}_g^*/V_g]\;\delta V_g - \sum_g [\hat{V}_g - (\hat{V}_g^* V_g^*/V_g)]\,\delta I_g + (V_n^* - \hat{V}_n)\,\delta I_n + (V_n - \hat{V}_n^*)\,\delta I_n^*$$

$$+\sum_t [V_t^* - \hat{V}_t + (\hat{I}_t + I_t^*)/Y_t]\delta I_t$$

$$+\sum_t [V_t - \hat{V}_t^* + (\hat{I}_t^* + I_t)/Y_t^*]\delta I_t^* + \sum_\ell [1 - \hat{V}_\ell^*/V_\ell]\delta S_\ell$$

$$+\sum_\ell [1 - \hat{V}_\ell/V_\ell^*]\delta S_\ell^* + \sum_g [\hat{I}_g^*/V_g + \hat{V}_g^* I_g/V_g^2]\delta(V_g V_g^*)$$

$$+\sum_g [1 - \hat{V}_g^*/V_g]\delta(S_g + S_g^*) - \sum_t\sum_i [(\hat{I}_t + I_t^*)(V_t/Y_t)\frac{\partial Y_t}{\partial \phi_{ti}}$$

$$+ (\hat{I}_t^* + I_t)(V_t^*/Y_t^*)\frac{\partial Y_t^*}{\partial \phi_{ti}}]\,\Delta\phi_{ti} = 0. \quad (17)$$

Obviously, if we have an explicit performance or constraint function, for example, of the responses V_ℓ, V_ℓ^*, V_g, I_g, I_t and I_t^* as $f(V_\ell, V_\ell^*, V_g, I_g, I_t, I_t^*)$ then we are at liberty to define the adjoint element

$$\hat{V}_n = V_n^*, \quad (18)$$

which eliminates the expressions involving δI_n and δI_n^*. We then rewrite the remaining components of (17) as

$$\delta f = \sum_\ell (\frac{\partial f}{\partial V_\ell}\delta V_\ell + \frac{\partial f}{\partial V_\ell^*}\delta V_\ell^*) + \sum_g (\frac{\partial f}{\partial V_g}\delta V_g + \frac{\partial f}{\partial I_g}\delta I_g)$$

$$+ \sum_t (\frac{\partial f}{\partial I_t}\delta I_t + \frac{\partial f}{\partial I_t^*}\delta I_t^*)$$

$$= \sum_\ell (\frac{df}{dS_\ell}\delta S_\ell + \frac{df}{dS_\ell^*}\delta S_\ell^*) + \sum_g (\frac{df}{d(V_g V_g^*)}\delta(V_g V_g^*)$$

$$+ \frac{df}{d(S_g + S_g^*)}\delta(S_g + S_g^*)) + \sum_t\sum_i \frac{df}{d\phi_{ti}}\Delta\phi_{ti}, \quad (19)$$

where we have defined the adjoint elements

$$\hat{I}_\ell = \frac{\partial f}{\partial V_\ell} - \hat{V}_\ell^* I_\ell^*/V_\ell, \quad (20)$$

$$\hat{V}_g - \hat{V}_g^* V_g^*/V_g = -\frac{\partial f}{\partial I_g}, \quad (21)$$

$$\hat{I}_g - \hat{I}_g^* V_g^*/V_g = \frac{\partial f}{\partial V_g} - (I_g^* - I_g V_g^*/V_g)(\hat{V}_g^*/V_g), \quad (22)$$

$$V_t^* - \hat{V}_t = \frac{\partial f}{\partial I_t} - (\hat{I}_t + I_t^*)/Y_t, \quad (23a)$$

or

$$\hat{I}_t = Y_t \hat{V}_t + Y_t \frac{\partial f}{\partial I_t} - V_t^*(Y_t + Y_t^*). \quad (23b)$$

Note that each of (20) and (23) represents two conditions while each of (21) and (22) represents only one condition.

Note also that since f is real

$$\frac{\partial f}{\partial V_\ell} = (\frac{\partial f}{\partial V_\ell^*})^* \quad (24)$$

and

$$\frac{\partial f}{\partial I_t} = (\frac{\partial f}{\partial I_t^*})^*. \quad (25)$$

INTERPRETATION OF THE ADJOINT NETWORK

Consider Fig. 1 and the equation associated with a load bus, namely, (20).

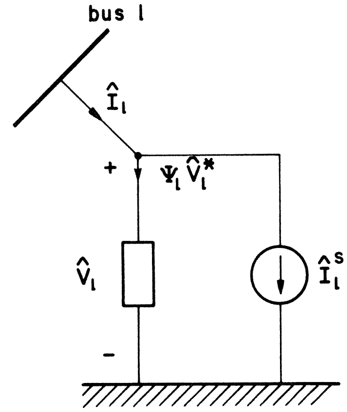

Fig. 1 Adjoint element model for a load bus

For convenience, we write (20) as

$$\hat{I}_\ell = \hat{I}_\ell^S + \Psi_\ell \hat{V}_\ell^*, \quad (26)$$

where

$$\hat{I}_\ell^S \triangleq \partial f/\partial V_\ell \quad (27)$$

and

$$\Psi_\ell \triangleq -S_\ell/V_\ell^2. \quad (28)$$

Fig. 1 shows the independent source $\hat{I}_\ell^S$ and the element Ψ_ℓ.

Now consider Fig. 2 and the equations associated with a generator bus.

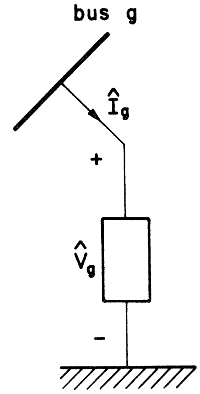

Fig. 2 Adjoint element model for a generator bus, requiring the solution of (29) and (33)

Equation (22) is rewritten as

$$\hat{V}_g = \Phi_g \hat{I}_g + \bar{\Phi}_g \hat{I}_g^* + \hat{V}_g^S, \quad (29)$$

where

$$\Phi_g \triangleq -V_g V_g^*/(j\,2Q_g), \quad (30)$$

$$\bar{\Phi}_g \triangleq (V_g^*)^2/(j\,2Q_g), \quad (31)$$

$$\hat{V}_g^S \triangleq -(V_g^*)^2(\partial f/\partial V_g)^*/(j\,2Q_g), \quad (32)$$

and where

$$j\,2Q_g = I_g^* V_g - I_g V_g^*.$$

Equation (21) is also rewritten (see Fig. 2) in the form

$$V_g \hat{V}_g - V_g^* \hat{V}_g^* = -V_g \frac{\partial f}{\partial I_g}\,. \tag{33}$$

Observe that the linear system (29) and (33) must be solved to define the adjoint element corresponding to the generator in the given network.

The slack bus constraint (18) is illustrated by Fig. 3.

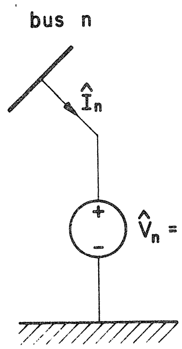

Fig. 3 Adjoint element model for the slack bus

Equation (23) for the remaining elements becomes

$$\hat{I}_t = Y_t \hat{V}_t + \hat{I}_t^S, \tag{34}$$

where

$$\hat{I}_t^S \triangleq Y_t \frac{\partial f}{\partial I_t} - V_t^*(Y_t + Y_t^*). \tag{35}$$

Independent sources associated with each branch are summed, as shown in Fig. 4, as

$$\hat{J}_m \triangleq \sum_{t \in T_m} \hat{I}_t^{Sm} \tag{36}$$

for any m (= ℓ, g or n), where T_m identifies those branches connected to bus m.

Observe that the transmission elements of the adjoint network are identical to those in the original network as is the case for the d.c. power flow model [7].

THE ADJOINT EQUATIONS

The derivation of the adjoint equations are outlined in this section. In general, they take the complex form

$$\begin{bmatrix} \underset{\sim}{Y}_{LL} & \underset{\sim}{Y}_{LG} & \underset{\sim}{Y}_{LN} \\ \underset{\sim}{Y}_{GL} & \underset{\sim}{Y}_{GG} & \underset{\sim}{Y}_{GN} \\ \underset{\sim}{Y}_{NL} & \underset{\sim}{Y}_{NG} & Y_{nn} \end{bmatrix} \begin{bmatrix} \hat{\underset{\sim}{V}}_L \\ \hat{\underset{\sim}{V}}_G \\ \hat{V}_n \end{bmatrix} = - \begin{bmatrix} \hat{\underset{\sim}{I}}_L + \hat{\underset{\sim}{J}}_L \\ \hat{\underset{\sim}{I}}_G + \hat{\underset{\sim}{J}}_G \\ \hat{I}_n + \hat{J}_n \end{bmatrix}, \tag{37}$$

where the nxn bus admittance matrix has been partitioned into blocks associated with the sets of load, generator and slack buses of appropriate dimension. Note that Y_{nn} and associated variables are scalars.

For load buses we let

$$\hat{\underset{\sim}{I}}_L = \underset{\sim}{\Psi}_L \hat{\underset{\sim}{V}}_L^* + \hat{\underset{\sim}{I}}_L^S, \tag{38}$$

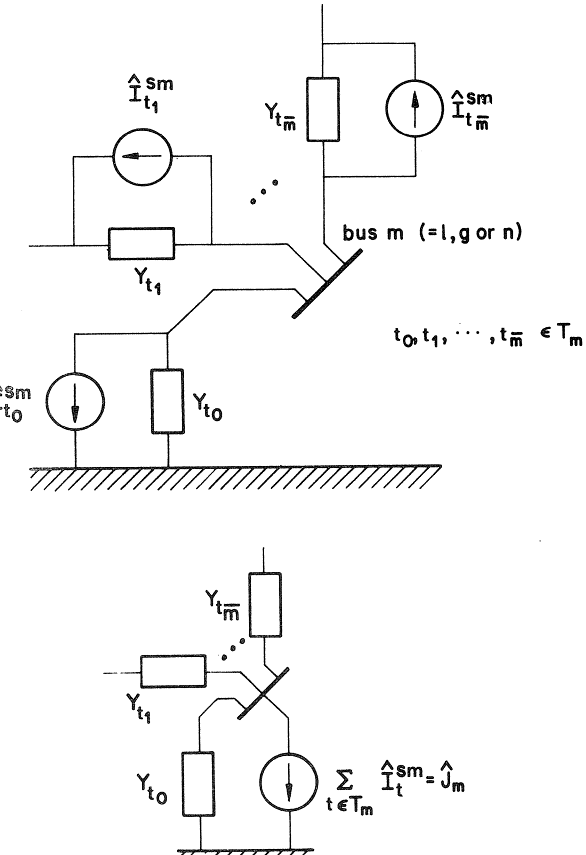

Fig. 4 Equivalent adjoint elements at bus m

where $\hat{\underset{\sim}{I}}_L$, $\hat{\underset{\sim}{V}}_L$ and $\hat{\underset{\sim}{I}}_L^S$ are vectors of dimension n_L consisting of the $\hat{I}_\ell$, $\hat{V}_\ell$ and $\hat{I}_\ell^S$, respectively, and $\underset{\sim}{\Psi}_L$ is a diagonal matrix whose diagonal elements are the corresponding Ψ_ℓ of (28). For generator buses we let

$$\hat{\underset{\sim}{V}}_G = \underset{\sim}{\Phi}_G \hat{\underset{\sim}{I}}_G + \underset{\sim}{\Phi}_G \hat{\underset{\sim}{I}}_G^* + \hat{\underset{\sim}{V}}_G^S, \tag{39}$$

$$\underset{\sim}{R}_G \hat{\underset{\sim}{V}}_G - \underset{\sim}{R}_G^* \hat{\underset{\sim}{V}}_G^* = \underset{\sim}{F}_G, \tag{40}$$

where $\hat{\underset{\sim}{V}}_G$, $\hat{\underset{\sim}{I}}_G$ and $\hat{\underset{\sim}{V}}_G^S$ are vectors of dimension n_G consisting of the $\hat{V}_g$, $\hat{I}_g$ and $\hat{V}_g^S$, respectively, and $\underset{\sim}{\Phi}_G$, $\underset{\sim}{\Phi}_G$, $\underset{\sim}{R}_G$ and $\underset{\sim}{F}_G$ are diagonal matrices whose diagonal elements are taken from (30), (31) and (33).

Let

$$\underset{\sim}{Y}_{RS} = \underset{\sim}{G}_{RS} + j\underset{\sim}{B}_{RS}, \tag{41a}$$

$$\hat{\underset{\sim}{V}}_M = \hat{\underset{\sim}{V}}_{M1} + j\hat{\underset{\sim}{V}}_{M2}, \tag{41b}$$

$$\hat{\underset{\sim}{J}}_M = \hat{\underset{\sim}{J}}_{M1} + j\hat{\underset{\sim}{J}}_{M2}, \tag{41c}$$

where R, S and M can be G, L or n. Further, let

$$\underset{\sim}{\Psi}_L = \underset{\sim}{\Psi}_{L1} + j\underset{\sim}{\Psi}_{L2}, \tag{42a}$$

$$\hat{\underset{\sim}{I}}_G = \hat{\underset{\sim}{I}}_{G1} + j\hat{\underset{\sim}{I}}_{G2}, \tag{42b}$$

$$\underset{\sim}{\Phi}_G = j\underset{\sim}{\Phi}_{G2}, \tag{42c}$$

$$\underset{\sim}{\Phi}_G = \underset{\sim}{\Phi}_{G1} + j\underset{\sim}{\Phi}_{G2}, \tag{42d}$$

$$\underset{\sim}{R}_G = \underset{\sim}{R}_{G1} + j\underset{\sim}{R}_{G2}, \tag{42e}$$

$$\underset{\sim}{F}_G = j\underset{\sim}{F}_{G2}. \tag{42f}$$

Using the foregoing notation we arrive at

$$\left[\begin{array}{c|c|c|c} \mathbf{G}_{LL} + \boldsymbol{\Psi}_{L1} & \mathbf{G}_{LG} & -\mathbf{B}_{LL}+\boldsymbol{\Psi}_{L2} & -\mathbf{B}_{LG} \\ \hline -\bar{\boldsymbol{\Phi}}_{G1}\mathbf{G}_{GL} + \tilde{\boldsymbol{\Phi}}_{G2}\mathbf{B}_{GL} & -\bar{\boldsymbol{\Phi}}_{G1}\mathbf{G}_{GG} + \tilde{\boldsymbol{\Phi}}_{G2}\mathbf{B}_{GG}-\mathbf{1} & \bar{\boldsymbol{\Phi}}_{G1}\mathbf{B}_{GL} + \tilde{\boldsymbol{\Phi}}_{G2}\mathbf{G}_{GL} & \bar{\boldsymbol{\Phi}}_{G1}\mathbf{B}_{GG} + \tilde{\boldsymbol{\Phi}}_{G2}\mathbf{G}_{GG} \\ \hline \mathbf{B}_{LL} + \boldsymbol{\Psi}_{L2} & \mathbf{B}_{LG} & \mathbf{G}_{LL} - \boldsymbol{\Psi}_{L1} & \mathbf{G}_{LG} \\ \hline \mathbf{0} & 2\mathbf{R}_{G2} & \mathbf{0} & 2\mathbf{R}_{G1} \end{array}\right] \left[\begin{array}{c} \hat{\mathbf{V}}_{L1} \\ \hline \hat{\mathbf{V}}_{G1} \\ \hline \hat{\mathbf{V}}_{L2} \\ \hline \hat{\mathbf{V}}_{G2} \end{array}\right] = \left[\begin{array}{c} -\hat{\mathbf{I}}^S_{L1} - \hat{\mathbf{J}}_{L1} - \mathbf{G}_{LN}\hat{V}_{n1} + \mathbf{B}_{LN}\hat{V}_{n2} \\ \hline -\hat{\mathbf{V}}^S_{G1} - \tilde{\boldsymbol{\Phi}}_{G2}\hat{\mathbf{J}}_{G2} + \bar{\boldsymbol{\Phi}}_{G1}\hat{\mathbf{J}}_{G1} + (\bar{\boldsymbol{\Phi}}_{G1}\mathbf{G}_{GN} \\ -\tilde{\boldsymbol{\Phi}}_{G2}\mathbf{B}_{GN})\hat{V}_{n1} - (\bar{\boldsymbol{\Phi}}_{G1}\mathbf{B}_{GN} + \tilde{\boldsymbol{\Phi}}_{G2}\mathbf{G}_{GN})\hat{V}_{n2} \\ \hline -\hat{\mathbf{I}}^S_{L2} - \hat{\mathbf{J}}_{L2} - \mathbf{G}_{LN}\hat{V}_{n2} - \mathbf{B}_{LN}\hat{V}_{n1} \\ \hline \mathbf{F}_{G2} \end{array}\right] \tag{43}$$

where

$$\tilde{\boldsymbol{\Phi}}_{G2} \triangleq \boldsymbol{\Phi}_{G2} - \bar{\boldsymbol{\Phi}}_{G2}. \tag{44}$$

The rows of (43) corresponding to the load buses are obtained in a straightforward manner by substituting the separated forms of $\mathbf{Y}_{LL}$, $\mathbf{Y}_{LG}$, $\mathbf{Y}_{LN}$, $\boldsymbol{\Psi}_L$, $\hat{\mathbf{I}}^S_L$ and $\hat{\mathbf{J}}_L$ into (37) and (38).

For the generator buses, consider the real part of (39) as

$$\hat{\mathbf{V}}_{G1} = -\boldsymbol{\Phi}_{G2}\hat{\mathbf{I}}_{G2} + \bar{\boldsymbol{\Phi}}_{G1}\hat{\mathbf{I}}_{G1} + \bar{\boldsymbol{\Phi}}_{G2}\hat{\mathbf{I}}_{G2} + \hat{\mathbf{V}}^S_{G1}. \tag{45}$$

The subset of equations (37) corresponding to the generator buses is

$$\boldsymbol{I}_G = -(\hat{\mathbf{I}}_G + \hat{\mathbf{J}}_G), \tag{46}$$

where

$$\boldsymbol{I}_G \triangleq \mathbf{Y}_{GL}\hat{\mathbf{V}}_L + \mathbf{Y}_{GG}\hat{\mathbf{V}}_G + \mathbf{Y}_{GN}\hat{V}_n. \tag{47}$$

Let

$$\boldsymbol{I}_G = \boldsymbol{I}_{G1} + j\boldsymbol{I}_{G2}. \tag{48}$$

Eliminating $\hat{\mathbf{I}}_{G1}$ and $\hat{\mathbf{I}}_{G2}$ from (45) and (46) we obtain

$$(\boldsymbol{\Phi}_{G2}-\bar{\boldsymbol{\Phi}}_{G2})\boldsymbol{I}_{G2}-\bar{\boldsymbol{\Phi}}_{G1}\boldsymbol{I}_{G1}-\hat{\mathbf{V}}_{G1} = -(\boldsymbol{\Phi}_{G2}-\bar{\boldsymbol{\Phi}}_{G2})\hat{\mathbf{J}}_{G2}+\bar{\boldsymbol{\Phi}}_{G1}\hat{\mathbf{J}}_{G1}-\hat{\mathbf{V}}^S_{G1}. \tag{49}$$

Equation (49) in conjunction with (40) separated into real and imaginary parts lead to the rows of (43) corresponding to the generator buses.

GRADIENT CALCULATIONS

Comparing (19) with (17) we derive the following.

Load Variables

$$\text{Re}\ \{\frac{df}{dS_\ell}\ \delta S_\ell\} = -\ \text{Re}\{[1 - \hat{V}^*_\ell/V_\ell]\delta S_\ell\}$$

$$= -\ \text{Re}\{1-\hat{V}^*_\ell/V_\ell\}\delta P_\ell + \text{Im}\{1-\hat{V}^*_\ell/V_\ell\}\delta Q_\ell$$

hence we can write

$$\frac{df}{dP_\ell} = -\ 2\text{Re}\{1 - \hat{V}^*_\ell/V_\ell\}, \tag{50}$$

$$\frac{df}{dQ_\ell} = 2\text{Im}\{1 - \hat{V}^*_\ell/V_\ell\}. \tag{51}$$

Generator Variables

$$\frac{df}{d(V_gV^*_g)} = -\ \hat{I}^*_g/V_g - \hat{V}^*_g I_g/V^2_g, \tag{52}$$

$$\frac{df}{d(S_g+S^*_g)} = -\ 1 + \hat{V}^*_g/V_g. \tag{53}$$

Other Variables

$$\frac{df}{d\phi_{ti}} = 2\text{Re}\ \{\frac{V_t}{Y_t}\ (\hat{I}_t + I^*_t)\ \frac{\partial Y_t}{\partial \phi_{ti}}\ \}$$

$$= 2\text{Re}\{V_t(\hat{V}_t + \frac{\partial f}{\partial I_t} - V^*_t)\ \frac{\partial Y_t}{\partial \phi_{ti}}\ \}. \tag{54}$$

Notice that the partial derivative depends on unperturbed currents and voltages in the original and adjoint networks. Consequently, the two analyses accommodate any number of variables ϕ_{ti}. The term $\partial f/\partial V_\ell$, $\partial f/\partial V^*_\ell$, $\partial f/\partial V_g$, $\partial f/\partial I_g$, $\partial f/\partial I_t$ or $\partial f/\partial I^*_t$ is zero if f is not explicitly a function of V_ℓ, V^*_ℓ, V_g, I_g, I_t or I^*_t, respectively.

EXAMPLES

We present in this section some numerical results to illustrate the practical use of the formulas we have derived and, in particular, to exhibit the structure of the adjoint system of equations which have to be solved. A 6-bus sample power system (Fig. 5) employed by Garver [12] serves our purpose.

Required data for the problem is shown in Tables I and II. Powers injected into buses are shown. Table III shows the corresponding a.c. load flow solution. Table IV shows the matrix for the adjoint system (43), which is common to all the sensitivity calculations.

So as not to be restricted to any particular application, we consider the following four examples where we take, without loss of generality, the parameters ϕ_{ti} to represent line conductances and susceptances of transmission lines. In this case (54) becomes

$$\frac{df}{dG_t} = 2\text{Re}\{(\alpha_t + j\ \beta_t)\} = 2\alpha_t$$

and

$$\frac{df}{dB_t} = 2\text{Re}\{j(\alpha_t + j\ \beta_t)\} = -\ 2\beta_t,$$

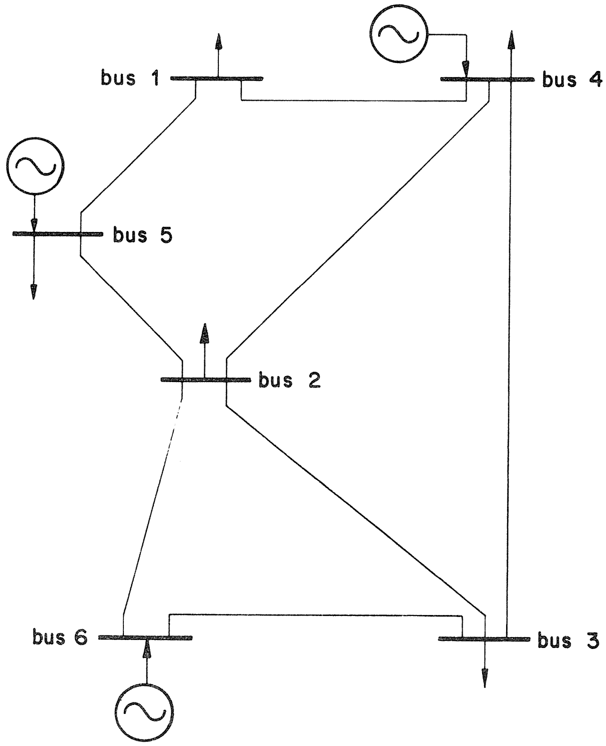

Fig. 5 6-bus sample power system

Table I
Bus Data

Bus Index, i	Bus Type	P_i (pu)	Q_i (pu)	$\lvert V_i \rvert \angle \delta_i$ (pu)
1	load	-2.40	0	- $\angle$-
2	load	-2.40	0	- $\angle$-
3	load	-1.60	-0.40	- $\angle$-
4	generator	-0.30	-	1.02 $\angle$-
5	generator	1.25	-	1.04 $\angle$-
6	slack	-	-	1.04 $\angle 0$

Table II
Line Data

Branch Index, i	Terminal Buses	Resistance R_i (pu)	Reactance X_i (pu)	Number of Lines
1	1,4	0.05	0.20	1
2	1,5	0.025	0.10	2
3	2,3	0.10	0.40	1
4	2,4	0.10	0.40	1
5	2,5	0.05	0.20	1
6	2,6	0.01875	0.075	4
7	3,4	0.15	0.60	1
8	3,6	0.0375	0.15	2

where

$$\alpha_t + j\,\beta_t \triangleq V_t\left(\hat{V}_t + \frac{\partial f}{\partial I_t} - V_t^*\right)$$

Table III
Load Flow Solution

Load Buses

$V_1 = 0.9787 \angle -0.6602$

$V_2 = 0.9633 \angle -0.2978$

$V_3 = 0.9032 \angle -0.3036$

Generator Buses

$Q_4 = 0.7866; \quad \delta_4 = -0.5566$

$Q_5 = 0.9780; \quad \delta_5 = -0.4740$

Slack Bus

$P_6 = 6.1298; \quad Q_6 = 1.3546$

and

$$Y_t \triangleq G_t + j\,B_t.$$

Example 1

Let

$$f = \sum_{t \in T_L} |I_t|^2 = \sum_{t \in T_L} I_t\, I_t^*,$$

where T_L indicates the set of transmission lines in the system.

For this function

$$\frac{\partial f}{\partial I_t} = I_t^*, \; \frac{\partial f}{\partial V_\ell} = \frac{\partial f}{\partial V_g} = \frac{\partial f}{\partial I_g} = 0.$$

The first part of Table V shows the RHS vector of the adjoint equations for this function and the adjoint voltages resulting from the solution of (43).

Table VI shows the derivatives calculated by our approach and by small perturbations around the base point for checking the formulas. Table VII shows the excitation and solution vectors of the original and adjoint systems and the calculated derivatives in the corresponding d.c. power flow study.

Example 2

Consider, for example,

$$f = |V_3|^2 = V_3\, V_3^*,$$

for which

$$\frac{\partial f}{\partial V_3} = V_3^*, \; \frac{\partial f}{\partial V_\ell} = 0 \text{ for } \ell \neq 3, \; \frac{\partial f}{\partial I_t} = 0,$$

$$\frac{\partial f}{\partial V_g} = 0 \text{ and } \frac{\partial f}{\partial I_g} = 0.$$

The second part of Table V shows the RHS vector of (43) and its solution vector for this function.

Table VIII shows the derivatives calculated by our approach and some checks by small perturbations.

Table IV
Adjoint Matrix of Coefficients

2.9085	0.0000	0.0000	-1.1765	-2.3529	11.6900	0.0000	0.0000	-4.7059	-9.4118
0.0000	3.3490	-0.5882	-0.5882	-1.1765	0.0000	20.5097	-2.3529	-2.3529	-4.7059
0.0000	-0.5882	1.2179	-0.3922	0.0000	0.0000	-2.3529	8.6744	-1.5686	0.0000
-5.1854	-2.5927	-1.7285	8.5065	0.0000	-1.6702	-0.8351	-0.5567	3.0620	0.0000
-9.2970	-4.6485	0.0000	0.0000	12.9455	-2.1677	-1.0839	0.0000	0.0000	3.2516
-16.5453	0.0000	0.0000	4.7059	9.4118	4.1503	0.0000	0.0000	-1.1765	-2.3529
0.0000	-23.4119	2.3529	2.3529	4.7059	0.0000	7.6314	-0.5882	-0.5882	-1.1765
0.0000	2.3529	-11.7178	1.5686	0.0000	0.0000	-0.5882	3.8802	-0.3922	0.0000
0.0000	0.0000	0.0000	-1.0777	0.0000	0.0000	0.0000	0.0000	1.7321	0.0000
0.0000	0.0000	0.0000	0.0000	-0.9495	0.0000	0.0000	0.0000	0.0000	1.8506

Table V
RHS and Solution Vectors of the Adjoint Networks

Element No.	$f = \sum_t \lvert I_t \rvert^2$		$f = \lvert V_3 \rvert^2$		$f = \sin \delta_5$	
	RHS Vector	Solution Vector	RHS Vector	Solution Vector	RHS Vector	Solution Vector
1	15.5879	6.5387	-0.9356	0.7547	-0.9356	0.6888
2	22.0101	4.9065	2.6385	0.9114	2.6385	0.8884
3	8.8078	3.7802	0.1381	0.8461	0.9999	0.8437
4	-0.6401	6.2411	-0.2573	0.8425	-0.2573	0.7997
5	-12.4168	5.8574	0.4477	0.9072	0.0268	0.8211
6	-12.5143	5.4964	0.7346	0.5854	0.7346	0.5339
7	-53.3547	2.3090	-12.0161	0.2661	-12.0161	0.2607
8	-16.4201	2.4281	-6.1743	0.1640	-5.9043	0.2539
9	0.0000	3.8832	0.0000	0.5242	0.0000	0.4976
10	0.0000	3.0053	0.0000	0.4655	0.0000	0.4213

Table VI
The Results for $f = \sum_t \lvert I_t \rvert^2$

Line	Derivatives w.r.t. G		Derivatives w.r.t. B	
	By Our Approach	By Small Perturbations $\pm 10^{-6}$	By Our Approach	By Small Perturbations $\pm 10^{-6}$
1,4	0.146915	0.146915	0.219814	0.219814
1,5	0.522770	0.522774	0.196945	0.196943
2,3	0.126807	0.126811	0.025452	0.025456
2,4	0.603831	0.603835	0.533374	0.533373
2,5	0.289047	0.289050	0.011592	0.011590
2,6	0.786006	0.786006	0.374631	0.374626
3,4	0.715027	0.715033	1.08442	1.08443
3,6	0.453939	0.453940	1.03172	1.03172

Example 3

Let

$$f = \sin \delta_5 = \frac{-j}{2} (V_5 - V_5^*)/\lvert V_5 \rvert ,$$

where δ_5 is the voltage angle of bus 5. Now,

$$f = - \frac{j}{2} [V_5 - (\lvert V_5 \rvert^2 / V_5)]/\lvert V_5 \rvert$$

hence

$$\frac{\partial f}{\partial V_5} = - \frac{j}{2} [1 + \lvert V_5 \rvert^2 / V_5^2]/\lvert V_5 \rvert .$$

Table VII
Results of d.c. Power Flow Analysis

Original Network		Adjoint Network	
RHS	Solution	RHS	Solution
-2.4000	-0.6148	40.1883	10.0237
-2.4000	-0.2844	81.1520	6.5346
-1.6000	-0.2998	22.7818	4.9685
-0.3000	-0.5022	- 1.5803	7.9698
1.2500	-0.4161	-29.3648	6.7807

Line	Derivative w.r.t. B	Line	Derivative w.r.t. B
1,4	0.112	2,5	-0.131
1,5	-0.099	2,6	-0.172
2,3	-0.025	3,4	0.479
2,4	0.089	3,6	0.362

Note that the partial derivative of f has been defined taking into consideration $\lvert V_5 \rvert$ as a control variable.

The last part of Table V shows both the RHS and the solution vector of (43) for this function. Table IX shows the calculated derivatives using our approach and also some checks by small perturbations.

Example 4

Here we investigate line removals by considering functions of the form

Table VIII
The Results for $f = |V_3|^2$

Line	Derivatives w.r.t. G	Derivatives w.r.t. B
1,4	-0.982894×10^{-3}	0.595004×10^{-3}
1,5	-0.131659×10^{-2}	-0.173856×10^{-2}
2,3	0.300576×10^{-2}	-0.103833×10^{-1}
2,4	0.254091×10^{-2}	-0.695912×10^{-2}
2,5	0.272289×10^{-2}	-0.337865×10^{-2}
2,6	-0.711205×10^{-2}	-0.932314×10^{-2}
3,4	0.490701×10^{-1}	-0.490622×10^{-2}
3,6	-0.516076×10^{-1}	-0.462822×10^{-1}

Checks by Small Perturbations $\pm 10^{-6}$

$$\Delta f/\Delta G_{23} = 0.300556 \times 10^{-2}$$

$$\Delta f/\Delta B_{15} = -0.173845 \times 10^{-2}$$

Table IX
The Results for $f = \sin \delta_5$

Line	Derivative w.r.t. G	Derivative w.r.t. B
1,4	0.257275×10^{-3}	-0.686200×10^{-2}
1,5	-0.930866×10^{-2}	0.108172×10^{-2}
2,3	-0.182742×10^{-2}	0.333698×10^{-3}
2,4	-0.619060×10^{-2}	-0.194655×10^{-1}
2,5	-0.127479×10^{-1}	-0.272739×10^{-1}
2,6	-0.466628×10^{-2}	-0.235790×10^{-1}
3,4	-0.139558×10^{-1}	-0.256494×10^{-1}
3,6	-0.217424×10^{-2}	-0.155615×10^{-1}

Checks by Small Perturbations $\pm 10^{-6}$

$$\Delta f/\Delta G_{25} = -0.127479 \times 10^{-1}$$

$$\Delta f/\Delta B_{15} = 0.108173 \times 10^{-2}$$

$$f = |I_t|^2 = I_t I_t^*.$$

Table X shows some results of different contingencies.

REMARKS

The expressions we have derived employ voltage and current variables. Performance and constraint functions can be formulated directly in terms of complex voltages and currents, bus or branch quantities as required for a particular problem. It should be obvious that we could equally well have derived our expressions in terms of other basic variables.

We have designated the control quantities S_ℓ, S_ℓ^*, $(V_g V_g^*)$ and $(S_g + S_g^*)$ as well as the parameters ϕ_{ti} as practical designable variables. We remark here that

$$P_\ell = (S_\ell + S_\ell^*)/2,$$

$$Q_\ell = -j(S_\ell - S_\ell^*)/2,$$

$$|V_g|^2 = V_g V_g^*,$$

$$P_g = (S_g + S_g^*)/2.$$

Any computer program featuring our approach would incorporate the following steps.

Table X
Contingency Results of Example 4

Function Line Index	Removed Line Index	Calculated Function Change	Exact Function Change
2,3	1,5*	0.002	0.005
2,3	2,3	-0.029	-0.021
2,4	2,4	-0.470	-0.404

* Only one line of branch 1,5 is removed.

Algorithm

Step 1 Obtain a base load flow solution.

Step 2 Evaluate partial derivatives of functions f_1, f_2, ..., f_m w.r.t. V_ℓ, V_ℓ^*, V_g, I_g, I_t, I_t^*.

Comment We have dealt with partial derivatives of any function f w.r.t. complex variables. This notation has facilitated the derivations and subsequent formulation of the equations to be solved. Two real quantities are assigned as independent control variables at a bus. Then the required partial derivatives can be easily obtained by expressing f in terms of the chosen controls and states. We observe that in Example 1 f is in terms of I_t, which is a chosen state. Similarly, f is in terms of a chosen state in Example 2. In Example 3, however, the V_g^* is neither chosen state nor control, hence it has to be replaced appropriately.

Step 3 Define the adjoint parameters required for equation (43).

Step 4 Solve the adjoint system (43).

Comment For each function f_1, f_2, ..., f_m only one forward and one backward substitution is required (LU factors are calculated once).

Step 5 Calculate the gradient vector using (50)-(54).

Comment If the effect of line additions or removals is to be determined appropriate first-order changes are calculated using the gradient information of Step 5.

CONCLUSIONS

Difficulties which undoubtedly prevented previous workers from applying Tellegen's theorem to the a.c. power flow model in general and without any approximations are overcome in this paper by using a suitably augmented form of the Tellegen sum in which power terms are included.

The addition of these power terms is the key for increasing the ability of Tellegen's theorem to handle more general classes of power system problems. The utilization of the same idea can lead to several forms of different sets of variables.

Our approach provides the flexibility of including line responses directly while preserving the advantages of compactness, sparsity and simplicity of the adjoint system. It preserves the usual advantages of easy handling of nonexisting elements in planning studies. In contingency analysis, multiple contingencies can be analyzed by summation as in the d.c. model case.

ACKNOWLEDGEMENT

This work was supported by the Natural Sciences and Engineering Research Council of Canada under Grant A7239.

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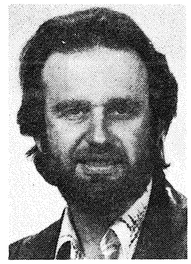

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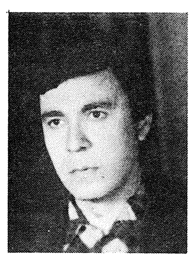

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SOC-234

A NEW, A.C. APPROACH TO POWER SYSTEM SENSITIVITY ANALYSIS AND PLANNING

J.W. Bandler and M.A. El-Kady

September 1979, No. of Pages: 9

Revised:

Key Words: Power system analysis, network planning, sensitivity analysis, optimal load flow, contingency analysis, Tellegen's theorem

Abstract: This paper presents an approach for sensitivity analysis and gradient evaluation required in power system analysis and planning. The approach utilizes Tellegen's theorem in an augmented form which allows different power system problems to be handled based on the a.c. power flow model in general and without any approximations.

The approach provides the flexibility of including line responses directly while preserving the advantages of compactness, sparsity and simplicity of the adjoint system. Numerical results are presented for illustration and comparison.

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