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POSTPRODUCTION PARAMETER IDENTIFICATION

R.M. Biernacki and J.W. Bandler

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FACULTY OF ENGINEERING
McMASTER UNIVERSITY
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Radoslaw M. Biernacki and John W. Bandler, Fellow, IEEE

Abstract

This paper deals with postproduction identification of network parameters for linear analog circuits. Methods for selected as well as for the identification of all parameters are discussed. Generalized hybrid equivalents are used to check whether identification of selected parameters can be carried out. The methods are based on measurements of voltage using mainly current excitations. Tests are assumed to be performed at a single frequency. The well known nodal approach is used to formulate the appropriate systems of equations for identification of all parameters for passive as well as active circuits. A ladder network example is studied in some detail. The capabilities and limitations of the approaches are investigated and partially solved. Some unsolved problems are also indicated.

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R.M. Biernacki is on leave from the Institute of Electronics Fundamentals, Technical University of Warsaw, Warsaw, Poland. J.W. Bandler is with the Group on Simulation, Optimization and Control and Department of Electrical and Computer Engineering, McMaster University, Hamilton, Canada, L8S 4L7.

I. INTRODUCTION

Computer-aided circuit design, which has become one of the most powerful tools in the design of analog electrical devices [1], enables us to deal with, for instance, manufacturing tolerance and tuning problems. Bandler, Liu and Tromp [2] formulated the design problem taking postproduction tuning into account. This work was extended by Polak and Sangiovanni-Vincentelli [8,9]. Although the algorithms proposed, for the time being, are computationally extravagant, there is hope for better and more efficient ones in the future. These methods are employed before a circuit is manufactured in order to assign the appropriate values of circuit parameters. A practical design, however, does not stop at that stage. The use of computer aids in further stages can also be helpful. Testing and tuning problems are of special interest [3-7, 10-14]. The main objective of testing is to check whether the circuit, which is already manufactured, meets the required specifications or not.

Another objective of testing is related to postproduction tuning. So-called deterministic tuning requires not only knowledge as to which elements have to be altered, but it is also necessary to know the actual values of network parameters in order to be able to calculate the amount of tuning to be carried out. This is the subject of the actual parameter identification which is based on measurements of the network already manufactured. Most authors dealing with tuning problems assume that actual values of network parameters are available. Nevertheless, since the elements can not usually be taken out of the network this can not be done directly. Therefore, appropriate methods of identification should exist.

The two objectives of testing are both subjects of fault analysis. However, it is felt that the term "fault analysis" is better suited to the situation when only a few elements are at fault and all remaining elements are correct. Then we want to locate the faulty elements. Thus, the situation when we are interested in actual values of all (or some) network elements can be better described by the term "parameter identification".

The solvability of the all parameter identification problem was first considered by Berkowitz [3]. He introduced the concept of accessible (and partly accessible) terminals where voltages and/or currents (or only voltages) can be applied and/or measured. From the theoretical point of view there is no difference as to which kind of excitation is used. However, from a practical point of view the use of current sources seems to be a little bit more reasonable. We will consider ideal current sources because for any nonideal source the source resistance can easily be treated as an additional element of the network.

We assume that no existing connection can be broken, hence current measurements are difficult to take. We may, however, consider that some ports can be shorted and the currents in these shorts measured. Therefore, voltage measurements are preferred over current measurements. We will try to consider voltage measurements only and as few of them as possible.

The solvability of the all parameter identification problem was later investigated by several other authors [6,7,14]. Mayeda and Peponides [6] gave a topological characterization of the problem. Navid and Wilson [7], using symbollic network functions, formulated sufficient conditions for this solvabilty. Trick et al. [12-14] considered the problem of identification and showed how to formulate an appropriate system of equations using the adjoint network concept. They proved the very important result that, for linear networks, the problem can be solved by means of linear equations. Their approach, however, seems to be unnecessarily complicated, because many simulations of the adjoint have to be performed in order to formulate the equations. They formulate the equations using changes w.r.t. nominal values as unknowns. Of course we can assume that the nominal values of network parameters are known. This assumption is essential if we want to locate one or more faults assuming that the other elements are at their nominal values. For the purpose of identification this assumption is not essential, i.e., there is no need to know these values if we are interested in finding actual values of all parameters. The knowledge of the nominal values is only a matter of formulation of an appropriate system of equations (either actual values or actual changes can be used).

Although it is known how to check whether chosen tests are sufficient for identification no paper solves the problem of how to choose these tests to be independent (except the situation when we measure everything possible as in [7]). There are some other papers which investigate this problem from the test point selection point of view [5,10,11].

Most papers on parameter identification assume tests to be performed at a single frequency [3,6,7,12-14]. This is quite a reasonable assumption since such identification provides the values of passive admittances and control coefficients of controlled sources.

Repeating the identification at different frequencies enables us to identify the component values provided that there is a unique dependence of element values on the frequency response (as for canonical structures). Moreover, testing at a single frequency is essentially the same as that for resistive networks.

As is known [3], parallel elements are not solvable, so we assume that there are no direct parallel connections of elements or, alternatively, we have to be satisfied with the knowledge of the admittance of the whole connection. For instance, we can not determine individual values of two parallel resistors (obviously, even if we use measurements at different frequency points), so we have to satisfy ourselves with the composite resistor.

This work deals with the postproduction identification of network parameters. Analog linear and lumped networks are considered. Methods for selected elements as well as for the identification of all parameters are discussed. The methods are based on measurements of voltage using mainly current excitations. Tests are assumed to be performed at a single frequency point. The well known nodal approach is used in order to formulate the appropriate systems of equations.

II. IDENTIFICATION OF SELECTED PARAMETERS

Consider the identification of a single two-terminal element within a known environment. The surrounding network can be replaced by the Thevenin equivalent as shown in Fig. 1. Assuming that the voltage V_x across the unknown element Z_x is known we find Z_x from

$$(V_{TH} - V_x)Z_x = V_x Z_{TH}. \tag{1}$$

Observe that the assumption

$$Z_{TH} \neq 0 \tag{2}$$

is crucial for the identification. We also note that the knowledge of a single voltage may be sufficient for identification of a single element.

The above approach can be generalized as follows. Consider n unknown elements of a network. The situation can be represented as an active n-port being terminated by unknown elements $Y_1, Y_2, \ldots, Y_n$ (Fig. 2(a)). Assume that there exists a hybrid equivalent of the active n-port shown in Fig. 2(b). The equivalent is described as follows.

The vector of port voltage sources is

$$\underset{\sim}{V}_a^S \triangleq [V_1^S \; V_2^S \; \ldots \; V_k^S]^T, \tag{3}$$

the vector of port current sources is

$$\underset{\sim}{I}_b^S \triangleq [I_{k+1}^S \; I_{k+1}^S \; \ldots \; I_n^S]^T, \tag{4}$$

and the hybrid matrix $\underset{\sim}{H}'$ of the n-port without independent sources is defined by

$$\begin{bmatrix} \underset{\sim}{V}_a' \\ \underset{\sim}{I}_b' \end{bmatrix} = \underset{\sim}{H}' \begin{bmatrix} \underset{\sim}{I}_a \\ \underset{\sim}{V}_b \end{bmatrix}, \tag{5}$$

where

$$\underset{\sim}{H}' = \begin{bmatrix} \underset{\sim}{H}_{aa} & \underset{\sim}{H}_{ab} \\ \underset{\sim}{H}_{ba} & \underset{\sim}{H}_{bb} \end{bmatrix}. \tag{6}$$

According to Fig. 2(b) we have

$$\begin{aligned} \underset{\sim}{V}_a' &= \underset{\sim}{V}_a - \underset{\sim}{V}_a^S, \\ \underset{\sim}{I}_b' &= \underset{\sim}{I}_b^S - \underset{\sim}{Y}_{bb}\, \underset{\sim}{V}_b, \\ \underset{\sim}{I}_a &= -\underset{\sim}{Y}_{aa}\, \underset{\sim}{V}_a, \end{aligned} \tag{7}$$

where

$$\underset{\sim}{Y}_{aa} \triangleq \mathrm{diag}(Y_1\ Y_2\ \dots\ Y_k)$$

and

$$\underset{\sim}{Y}_{bb} \triangleq \mathrm{diag}(Y_{k+1}\ Y_{k+2}\ \dots\ Y_n).$$

Substituting (7) into (5), and after some manipulations we obtain

$$-\underset{\sim}{I} \triangleq \begin{bmatrix} \underset{\sim}{Y}_{aa}\underset{\sim}{V}_a \\ \underset{\sim}{Y}_{bb}\underset{\sim}{V}_b \end{bmatrix} = \begin{bmatrix} \underset{\sim}{H}_{aa}^{-1} & \underset{\sim}{0} \\ \underset{\sim}{H}_{ba}\underset{\sim}{H}_{aa}^{-1} & \underset{\sim}{1} \end{bmatrix} \begin{bmatrix} \underset{\sim}{V}_a^S \\ \underset{\sim}{I}_b^S \end{bmatrix} + \begin{bmatrix} \underset{\sim}{H}_{aa}^{-1} & -\underset{\sim}{H}_{aa}^{-1}\underset{\sim}{H}_{ab} \\ \underset{\sim}{H}_{ba}\underset{\sim}{H}_{aa}^{-1} & \underset{\sim}{H}_{bb} - \underset{\sim}{H}_{ba}\underset{\sim}{H}_{aa}^{-1}\underset{\sim}{H}_{ab} \end{bmatrix} \begin{bmatrix} \underset{\sim}{V}_a \\ \underset{\sim}{V}_b \end{bmatrix}, \tag{8}$$

where $\underset{\sim}{1}$ is the identity matrix of an appropriate order and $\underset{\sim}{I}$ is the vector of currents through the unknown elements. From (8) it is seen that the existence of the inverse $\underset{\sim}{H}_{aa}^{-1}$ is necessary to obtain the solution. This is equivalent to the existence of the admittance matrix of the n-port (observe that the matrix in the last part of (8) is actually the admittance matrix). On the other hand we have considered another assumption, i.e., that the hybrid equivalent exists. It can be shown that the existence of a hybrid matrix is sufficient for the existence of the corresponding hybrid equivalent. Therefore, if we assume that the admittance matrix exists we can consider a Norton equivalent and, according to the above discussion, we can find the solution. This leads to the following theorem.

Theorem 1

Identification of n elements based on voltages across these elements (Fig. 2(a)) is possible if and only if there exists the admittance matrix of the corresponding n-port (after shorting independent voltage sources and open-circuiting independent current sources).

The solution can also be obtained directly without looking for a hybrid equivalent. According to Theorem 1 and using a representation of the network of Fig. 2(a), which is shown in Fig. 3, there exists a matrix $\underset{\sim}{H}$ such that

$$\begin{bmatrix} \underset{\sim}{I} \\ \underset{\sim}{I}_V \\ \underset{\sim}{V}_I \end{bmatrix} = \underset{\sim}{H} \begin{bmatrix} \underset{\sim}{V} \\ \underset{\sim}{V}_V \\ \underset{\sim}{I}_I \end{bmatrix}, \tag{9}$$

where $\underset{\sim}{H}_{11}$ is the admittance matrix of the n-port of Fig. 2(b). From (9) we have

$$\underset{\sim}{I} = \underset{\sim}{H}_{11}\underset{\sim}{V} + \underset{\sim}{H}_{12}\underset{\sim}{V}_V + \underset{\sim}{H}_{13}\underset{\sim}{I}_I. \tag{10}$$

Once we know $\underset{\sim}{I}$ then we can easily find the values of the unknown elements, since

$$Y_i = -I_i/V_i \quad \text{or} \quad Z_i = -V_i/I_i \tag{11}$$

for i = 1, 2, ..., n.

Now, consider that the measurement ports are different from the ports of the elements which are to be identified. Assume that there exists a hybrid equivalent as shown in Fig. 4. According to Fig. 4 we have (m and x identify the measurement and identification ports, respectively, and a, b, c, d identify ports of the same kind within the two groups)

$$\begin{bmatrix} \underset{\sim}{V}_a - \underset{\sim}{V}_a^S \\ \underset{\sim}{I}_b + \underset{\sim}{I}_b^S \\ \underset{\sim}{V}_c - \underset{\sim}{V}_c^S \\ \underset{\sim}{I}_d^S \end{bmatrix} = \underset{\sim}{H} \begin{bmatrix} \underset{\sim}{I}_a \\ \underset{\sim}{V}_b \\ \underset{\sim}{0} \\ \underset{\sim}{V}_d \end{bmatrix}, \tag{12}$$

where

$$\underset{\sim}{H} = \left[\begin{array}{cc|cc} \underset{\sim}{H}_{aa} & \underset{\sim}{H}_{ab} & \underset{\sim}{H}_{ac} & \underset{\sim}{H}_{ad} \\ \underset{\sim}{H}_{ba} & \underset{\sim}{H}_{bb} & \underset{\sim}{H}_{bc} & \underset{\sim}{H}_{bd} \\ \hline \underset{\sim}{H}_{ca} & \underset{\sim}{H}_{cb} & \underset{\sim}{H}_{cc} & \underset{\sim}{H}_{cd} \\ \underset{\sim}{H}_{da} & \underset{\sim}{H}_{db} & \underset{\sim}{H}_{dc} & \underset{\sim}{H}_{dd} \end{array}\right] = \begin{bmatrix} \underset{\sim}{H}_{xx} & \underset{\sim}{H}_{xm} \\ \underset{\sim}{H}_{mx} & \underset{\sim}{H}_{mm} \end{bmatrix} \tag{13}$$

and $\underset{\sim}{H}_{xx}$, $\underset{\sim}{H}_{xm}$, $\underset{\sim}{H}_{mx}$, $\underset{\sim}{H}_{mm}$ are n x n matrices. Observe that in order to solve the system (12) for unknown vectors $\underset{\sim}{I}_a$, $\underset{\sim}{V}_b$, $\underset{\sim}{V}_a$, $\underset{\sim}{I}_b$ we have to know the inverse $\underset{\sim}{H}_{mx}^{-1}$. This corresponds to the existence of the transmission matrix linking ports of identification as the input with ports of measurement as the output. Assuming that there exists a mixed "transmission-hybrid" representation of the network described by (see Fig. 5)

$$\begin{bmatrix} \underset{\sim}{V}^x \\ \underset{\sim}{I}^x \\ \underset{\sim}{I}_V \\ \underset{\sim}{V}_I \end{bmatrix} = \underset{\sim}{A} \begin{bmatrix} \underset{\sim}{V}^m \\ \underset{\sim}{I}^m \\ \underset{\sim}{V}_V \\ \underset{\sim}{I}_I \end{bmatrix}, \tag{14}$$

we find, for $\underset{\sim}{I}^m = \underset{\sim}{0}$,

$$\begin{bmatrix} \underset{\sim}{V}^x \\ \underset{\sim}{I}^x \end{bmatrix} = \begin{bmatrix} \underset{\sim}{A}_{11} \\ \underset{\sim}{A}_{21} \end{bmatrix} \underset{\sim}{V}^m + \begin{bmatrix} \underset{\sim}{A}_{13} & \underset{\sim}{A}_{14} \\ \underset{\sim}{A}_{23} & \underset{\sim}{A}_{24} \end{bmatrix} \begin{bmatrix} \underset{\sim}{V}_V \\ \underset{\sim}{I}_I \end{bmatrix}. \tag{15}$$

The above discussion gives us the following theorem.

Theorem 2

Existence of the transmission-type matrix defined by (14) is necessary and sufficient for identification of n unknown elements based upon n voltage measurements if ports of measurement are different from ports of identification.

The requirements of Theorem 1 can easily be verified. The admittance matrix exists if and only if no port can be shorted by shorting all the remaining ports. In contrast, verifying the conditions of Theorem 2 is more difficult. This is simply because the elements of a general transmission matrix of a 2n-port network (unlike a 2-port) cannot be defined as ratios of single input and single output in the presence of shorts and openings of other ports. Moreover, the existence of the transmission matrix is not related to the existence of any particular hybrid matrix. Hence, Theorem 2 is not very useful in practice and one should look for another and simpler criterion. Nevertheless, we observe that in both cases there exists a limit to the number of elements which can be identified. Usually, Theorems 1 and 2 are satisfied as far as the identification of one or two elements is concerned. The more elements we want to consider the more unlikely it is to satisfy the corresponding theorem. The number of elements which can still be identified strongly depends upon topology and elements chosen. But in any case, for a particular network, there exists a maximum number of elements which can be identified by methods described in this section and this number is less than the total number of elements in the network. In the next section we deal with the problem of identification of all elements since this cannot be done by the above methods.

III. IDENTIFICATION OF ALL PARAMETERS

We now consider the situation when all network elements are unknown. We assume that voltages across all elements are available. Since Kirchhoff's voltage law is satisfied (i.e., we assume that measurements are accurate enough) we can consider nodal voltages only. Using the preferable current excitations we have a generalized branch shown in Fig. 6. As is well known, a network with p branches and r nodes can be described by the branch-node incidence matrix

$$\underset{\sim}{\Lambda} = [\lambda_{ik}], \tag{16}$$

where

$$\lambda_{ik} = \begin{cases} +1, & \text{for the kth branch directed towards the ith node,} \\ -1, & \text{for the kth branch directed away from the ith node,} \\ 0, & \text{for the kth branch not incident with the ith node,} \end{cases} \tag{17}$$

$i = 1, 2, \ldots, r-1$ and $k = 1, 2, \ldots, p$.

Following the typical nodal approach we introduce the vector of nodal current excitations as

$$\underset{\sim}{I}^S = \underset{\sim}{\Lambda} \begin{bmatrix} J_1 \\ J_2 \\ \cdot \\ \cdot \\ \cdot \\ J_p \end{bmatrix}. \tag{18}$$

This enables us to write Kirchhoff's current law in the form

$$\underset{\sim}{\Lambda}\, \underset{\sim}{I} = - \underset{\sim}{I}^S, \tag{19}$$

where

$$\underset{\sim}{I} \triangleq [I_1\ I_2\ \ldots\ I_p]^T \tag{20}$$

is the vector of branch currents.

Using the notation

$$\underset{\sim}{Y} = [Y_1 \; Y_2 \; \ldots \; Y_p]^T \tag{21}$$

for the vector of branch admittances, and

$$\underset{\sim}{U} = \mathrm{diag}(U_1 \; U_2 \; \ldots \; U_p) \tag{22}$$

for the matrix of branch voltages, we can write Ohm's law for all branches of the network as

$$\underset{\sim}{I} = -\underset{\sim}{U}\,\underset{\sim}{Y}. \tag{23}$$

Since Kirchhoff's voltage law is satisfied automatically we note that equation (23) together with (19) are all the available equations for the network. The current vector $\underset{\sim}{I}$ is of no interest, so eliminating it from (23) and (19) we find

$$(\underset{\sim}{\Lambda}\,\underset{\sim}{U})\underset{\sim}{Y} = \underset{\sim}{I}^S. \tag{24}$$

This is simply the system of equations which has been sought. It contains r-1 equations with the p unknown values of Y_1, Y_2, ..., Y_p. Matrix $\underset{\sim}{\Lambda}$ consists of r-1 linearly independent rows, so if branch voltages are different from zero then the matrix $(\underset{\sim}{\Lambda}\underset{\sim}{U})$ also consists of r-1 linearly independent rows. Note that p can be equal to r-1 only if the network graph is a tree. In this case all network elements can easily be determined if the excitations chosen are such that there is a nonzero current in every branch of the tree. This is a rather obvious result since, knowing the excitations, we know immediately all branch currents. In other cases we always have $p > r-1$ and we are not able to identify all elements Y_1, Y_2, ..., Y_p based only on the equation (24). If some of these elements are known (at least $p-r+1$ of them) we can solve (24) for the remaining parameters provided that the resulting system contains an appropriate number of linearly independent equations. This is another approach to the problems considered in the foregoing section.

Now, we are interested in the identification of all parameters of the network. Since the number of equations in (24) is less than the number of unknowns we have to find additional equations based on other set(s) of measurements. According to (24) one set of measurements gives us at most $r-1$ independent equations. This means that we need at least m sets of measurements, where

$$m = \mathrm{int}\left(\frac{p}{r-1} \right) \tag{25}$$

and $\mathrm{int}(x)$ denotes the smallest integer x_o such that $x \leq x_o$. Because the number of branches p is between $r-1$ (for a tree-network) and $r(r-1)/2$ (for a complete-graph network), i.e.,

$$r-1 \leq p \leq \frac{r(r-1)}{2}, \tag{26}$$

we find that

$$1 \leq m \leq \mathrm{int}\left(\frac{r}{2} \right). \tag{27}$$

For typical networks m is expected to equal 2 or 3. Every set of measurements $\underset{\sim}{U}^i$ provides the appropriate system of equations (24) as

$$(\underset{\sim}{\Lambda}^i \; \underset{\sim}{U}^i)\underset{\sim}{Y} = \underset{\sim}{I}^{Si} \tag{28}$$

for i = 1, 2, ..., M where $M \geq m$.

All of those systems give us the final matrix equation

$$\begin{bmatrix} \underset{\sim}{\Lambda}^1 \underset{\sim}{U}^1 \\ \underset{\sim}{\Lambda}^2 \underset{\sim}{U}^2 \\ \cdot \\ \cdot \\ \cdot \\ \underset{\sim}{\Lambda}^M \underset{\sim}{U}^M \end{bmatrix} \underset{\sim}{Y} = \begin{bmatrix} \underset{\sim}{I}^{S1} \\ \underset{\sim}{I}^{S2} \\ \cdot \\ \cdot \\ \cdot \\ \underset{\sim}{I}^{SM} \end{bmatrix}. \tag{29}$$

Usually, different sets of measurements are obtained only under different excitations, while the network topology is not changed at all, so

$$\underset{\sim}{\Lambda}^1 = \underset{\sim}{\Lambda}^2 = \ldots = \underset{\sim}{\Lambda}^M. \tag{30}$$

However, if we can short certain nodes, different ones for different measurements, then in general, we should consider different matrices $\underset{\sim}{\Lambda}^i$.

The system (29) is required to contain exactly p independent equations. Roughly speaking, the systems (28) should be "independent" of each other. In other words, we have to arrange for M "independent" measurements. How to arrange for these independent measurements, however, is not known so far. Nevertheless, several directions can be proposed.

It would seem to be optimal if the subsequent measurements provided equations which formed an independent system along with all previously obtained equations and, furthermore, if the final system was not ill-conditioned. In other words, we want the rows of matrices

$$\underset{\sim}{\Lambda}^1 \; \underset{\sim}{U}^1, \quad \begin{bmatrix} \underset{\sim}{\Lambda}^1 & \underset{\sim}{U}^1 \\ \underset{\sim}{\Lambda}^2 & \underset{\sim}{U}^2 \end{bmatrix}, \quad \begin{bmatrix} \underset{\sim}{\Lambda}^1 & \underset{\sim}{U}^1 \\ \underset{\sim}{\Lambda}^2 & \underset{\sim}{U}^2 \\ \underset{\sim}{\Lambda}^3 & \underset{\sim}{U}^3 \end{bmatrix}, \ldots \tag{31}$$

to be linearly independent. Assuming (30) holds we have $M = m$. Of course if $p/(r-1)$ is not an integer then the last system (for $i = M$) contains a few more equations and the system (29) is overdetermined. Alternatively, for the last set of measurements, we can make an appropriate number of shorts such that the matrix $\underset{\sim}{\Lambda}^M$ consists of $p-(m-1)(r-1)$ rows and the system (29) has exactly p equations. For this approach, we would propose to use different locations for the excitations for the different measurements. These excitations should be as remote from one another as possible.

Now, as an important example, we apply the foregoing theory to ladder networks.

IV. METHODS FOR LADDER NETWORKS

Consider the ladder network shown in Fig. 7. The branch-node incidence matrix $\underset{\sim}{\Lambda}$ consists of $r-1 = n+1$ rows and $p = 2n+1$ columns and its structure is

$$\underset{\sim}{\Lambda}^{1} = \begin{bmatrix} 1 & 1 & & & & & & \\ & -1 & 1 & 1 & & & & \\ & & & -1 & 1 & 1 & & \\ & & & & & \cdot\ \cdot\ \cdot\ \cdot & & \\ & & & & & -1 & 1 & 1 \\ & & & & & & -1 & 1 \end{bmatrix} . \tag{32}$$

Using only the input source I_1^S for the first set of voltage measurements, i.e.,

$$\underset{\sim}{I}^{S1} = [I_1^{S1} \quad 0 \quad 0 \quad \ldots \quad 0]^T, \tag{33}$$

we obtain the first subsystem of equations (28) as

$$\begin{bmatrix} U_1^1 & U_2^1 & & & & & & \\ & -U_2^1 & U_3^1 & U_4^1 & & & & \\ & & & -U_4^1 & U_5^1 & U_6^1 & & \\ & & & & & \cdot\ \cdot\ \cdot\ \cdot & & \\ & & & & & & -U_{2n}^1 & U_{2n+1}^1 \end{bmatrix} \begin{bmatrix} Y_1 \\ Y_2 \\ \cdot \\ \cdot \\ Y_{2n+1} \end{bmatrix} = \begin{bmatrix} I_1^{S1} \\ 0 \\ \cdot \\ \cdot \\ \cdot \\ 0 \end{bmatrix} . \tag{34}$$

According to (25) we find m = 2, so we have to arrange for another set of measurements. Since we need exactly n more equations we can make exactly 1 short. A number of different approaches can be suggested. We will discuss three of them.

1. We use the same excitations, i.e.,

$$\underset{\sim}{I}^{S2} = \underset{\sim}{I}^{S1} \tag{35}$$

and the output port is shorted. The appropriate matrix $\underset{\sim}{\Lambda}^2$ is obtained from (32) by dropping the last row.

2. We use only the output source I^S_{2n+1} for the second set of measurements, i.e.,

$$\underset{\sim}{I}^{S2} = [0 \quad 0 \quad \dots \quad 0 \; I^{S2}_{2n+1}]^T, \tag{36}$$

and the input port is shorted. The appropriate matrix $\underset{\sim}{\Lambda}^2$ is obtained from (32) by dropping the first row.

3. We do not make any shorts, i.e., we use the same branch-node matrix

$$\underset{\sim}{\Lambda}^2 = \underset{\sim}{\Lambda}^1, \tag{37}$$

and the resulting system of equations (29) will be overdetermined. According to the previous discussion we apply the output source and the vector $\underset{\sim}{I}^{S2}$ is in the form of (36).

Note that regardless of the method chosen, for any row of $\underset{\sim}{\Lambda}^2$ we can find an <u>identical</u> row within the matrix $\underset{\sim}{\Lambda}^1$. Hence the linear independence or linear dependence of the final system (29) consists in the particular values of voltages U^1_1, U^1_2, ..., U^1_p in comparison with U^2_1, U^2_2, ..., U^2_p. Because of this the first method is likely to be ill-conditioned. It can be caused by relatively insensitive behaviour of voltages across the elements located close to the input w.r.t. a change of the output load. Using the same excitation for the two measurements we can meet the situation that the corresponding equations in both subsystems are "nearly" the same. From this point of view it is obvious that we are looking for quite a different excitation for the second set of measurements. The second and the third methods satisfy this requirement. These two methods are similar and we can discuss both simultaneously. The only difference is that the second method provides one less equation and that the values of voltages are a little different (in particular, $U_1^2 = 0$ and $U_2^2 = -U_3^2$). Therefore, we will discuss the third method and most of the following results will be applicable to the second method. Now, the second subsystem of equations (28) is similar to (34). The only difference is that superscripts "1" are replaced by superscripts "2" and the right hand side of (34) is replaced by $\underset{\sim}{I}^{S2}$ given by (36). The resulting system of equations (29), after reordering, can be expressed in the form

$$\underset{\sim}{A}\ \underset{\sim}{Y} = \underset{\sim}{B} \tag{38}$$

where

$$\underset{\sim}{B} = \begin{bmatrix} \underset{\sim}{I}^{S1} \\ \underset{\sim}{I}^{S2} \end{bmatrix} = [I_1^{S1}\ 0\ \ 0\ \ .\ .\ .\ \ 0\ I_{2n+1}^{S2}]^T \tag{39}$$

and

$$\underset{\sim}{A} = \begin{bmatrix} U_1^1 & U_2^1 & & & & & & & \\ U_1^2 & U_2^2 & & & & & & & \\ & -U_2^1 & U_3^1 & U_4^1 & & & & & \\ & -U_2^2 & U_3^2 & U_4^2 & & & & & \\ & & & -U_4^1 & U_5^1 & U_6^1 & & & \\ & & & -U_4^2 & U_5^2 & U_6^2 & & & \\ & & & & & \cdot & \cdot\ \cdot\ \cdot & \cdot & \\ & & & & & & & -U_{2n}^1 & U_{2n+1}^1 \\ & & & & & & & -U_{2n}^2 & U_{2n+1}^2 \end{bmatrix} . \qquad (40)$$

The particular sparse form of the matrix $\underset{\sim}{A}$ enables us to find explicit formulae for the solution of (38). It is more convenient to use nodal voltages $V_1^i, \ldots, V_{n+1}^i$, $i = 1, 2$, such that

$$U_{2k-1}^i = V_k^i \quad \text{and} \quad U_{2k}^i = V_k^i - V_{k+1}^i \qquad (41)$$

for $i = 1, 2$ and $k = 1, 2, \ldots$. From the last two equations we obtain

$$Y_{2n+1} = \frac{I_{2n+1}^{S2}\ (V_n^1 - V_{n+1}^1)}{\Delta_n} \qquad (42)$$

and

$$Y_{2n} = \frac{I_{2n+1}^{S2}\ V_{n+1}^1}{\Delta_n}\ , \qquad (43)$$

where

$$\Delta_n = V_n^1 V_{n+1}^2 - V_n^2 V_{n+1}^1. \tag{44}$$

Substituting Y_{2n} into the preceding two equations we can determine Y_{2n-1} and Y_{2n-2}. In this way we find the recurrent formulae

$$Y_{2k+1} = \frac{\Delta_k'}{\Delta_k} Y_{2k+2}, \tag{45}$$

$$Y_{2k} = \frac{\Delta_{k+1}}{\Delta_k} Y_{2k+2}, \tag{46}$$

where

$$\Delta_k = \det \begin{bmatrix} V_k^1 & V_{k+1}^1 \\ V_k^2 & V_{k+1}^2 \end{bmatrix} \tag{47}$$

and

$$\Delta_k' = \det \begin{bmatrix} V_k^1 - V_{k+1}^1 & V_{k+1}^1 - V_{k+2}^1 \\ V_{k+1}^2 - V_k^2 & V_{k+2}^2 - V_{k+1}^2 \end{bmatrix}. \tag{48}$$

From (46) and (43) we notice that

$$Y_{2k} \Delta_k = I_{2n+1}^{S2} \quad V_{n+1}^1 \tag{49}$$

or

$$Y_{2k} = \frac{I^{S2}_{2n+1} \quad V^{1}_{n+1}}{\Delta_k} \tag{50}$$

for $k = 1, 2, \ldots, n$.

Using (50), (45) and (42), we find Y_2, Y_3, ..., Y_{2n+1}. Finally, from the first equation of the system (38) we have

$$Y_1 = \frac{I^{S1}_1 - (V^1_1 - V^1_2)Y_2}{V^1_1} . \tag{51}$$

Alternatively, since the system is overdetermined, we obtain from the second equation

$$Y_1 = \frac{V^2_2 - V^2_1}{V^2_1} Y_2 \tag{52}$$

and both solutions should be identical. Of course, this second equation does not appear in the second method (because of shorting the input port). The above solution may be described by the term <u>backward solution</u>.

Similarly, starting from the first two equations we can derive the <u>forward solution</u> as

$$Y_1 = \frac{I^{S1}_1 (V^2_2 - V^2_1)}{\Delta_1} , \tag{53}$$

$$Y_{2k} = \frac{I_1^{S1} V_1^2}{\Delta_k}, \; k = 1, 2, \ldots, n, \tag{54}$$

$$Y_{2k+1} = \frac{\Delta_k'}{\Delta_{k+1}} Y_{2k}, \; k = 1, 2, \ldots, n-1, \tag{55}$$

and

$$Y_{2n+1} = \frac{I_{2n+1}^{S2} - (V_{n+1}^2 - V_n^2) Y_{2n}}{V_{n+1}^2}, \tag{56}$$

or

$$Y_{2n+1} = \frac{V_n^1 - V_{n+1}^1}{V_{n+1}^1} Y_{2n}. \tag{57}$$

For the second method, only the backward solution exists and Y_1 is expressed by (51). For the third method we can use the backward as well as the forward solution and the two solutions should be identical. They can be different from each other if the measurements are inaccurate. Then the question arises of how to take advantage of the fact that the system (38) is overdetermined.

The solvability of the problem depends on the determinants (47). They have to be different from zero. In other words any two successive nodal voltages for the two tests cannot be linearly dependent. Also V_{n+1}^2 and V_1^1 should have nonzero values. From a physical point of view we see that none of the nodal voltages (except V_{n+1}^1 and V_1^2) can be equal to zero. This is because if $V_k^1 = 0$ or $V_k^2 = 0$ then $V_{k+1}^1 = V_{k+2}^1 = \ldots = 0$

or $V^2_{k-1} = V^2_{k-2} = \dots = 0$, respectively. Then also $\Delta_k = \Delta_{k+1} = \dots = 0$ or $\Delta_k = \Delta_{k-1} = \dots = 0$ and the solution does not exist. This corresponds, for instance, to the situation when the frequency of excitation is the resonant frequency of a shunt element. To remedy the situation we can change the frequency of excitation and/or arrange for other measurements, i.e., use other ports of excitations. Otherwise, if all nodal voltages are different from zero the solution is likely to exist. For instance, for a resistive ladder network the voltages V^1_1, V^1_2, ... and V^2_{n+1}, V^2_n, V^2_{n-1}, ... are consecutively smaller and, as a consequence, the determinants (47) are different from zero. The only exception occurs when two successive voltages are identical, i.e., a series element is a short circuit. In this case, although we can identify $Y_{2k} = \infty$, we cannot identify Y_{2k-1} and Y_{2k+1} separately. Only the composite parallel connection of Y_{2k-1} and Y_{2k+1} can be determined. This corresponds to the assumption that the network does not contain parallel connections of elements which are to be identified.

The above example of the ladder network parameter identification gives us some guidance as to how to arrange for independent tests of measurements as well as some problems which can arise. However, these are not satisfactory enough and more general and precise methods and properties should be sought. In particular, methods for active networks are of great importance. We deal with this problem in the following section.

V. ACTIVE NETWORKS

We now consider a network which consists of passive as well as active lumped elements. Control sources are taken into account as models of active elements. We will consider only voltage controlled current sources (VCCS) which are typical for the nodal approach. It is sufficiently general for many practical cases.

The general formulation discussed at the beginning of this section can easily be extended to identify unknown control coefficients besides all other passive admittances. Of course, if the control coefficient of a VCCS is known, we can treat this source as independent since the controlling voltage is also known.

Consider a network with passive branches and s voltage controlled current sources. The VCCS elements are described by the equation

$$J_k^c = Y_k^c \, U_k^c, \tag{58}$$

for k = 1, 2, ..., s.

For our purposes we have to treat the controlled branches as different from those which contain passive elements and/or independent sources even if they are parallel. Hence, the branch-node incidence matrix for the network can be expressed as

$$\underset{\sim}{\Lambda} = [\underset{\sim}{\Lambda}_p \quad -\underset{\sim}{\Lambda}_a] \tag{59}$$

where $\underset{\sim}{\Lambda}_p$ is the (r-1) x p matrix described by (16) and $\underset{\sim}{\Lambda}_a$ is an (r-1) x s matrix constructed for all controlled branches in the same way as $\underset{\sim}{\Lambda}_p$. Now, Kirchhoff's current law can be written in the form

$$\underset{\sim}{\Lambda}_p \, \underset{\sim}{I} + \underset{\sim}{\Lambda}_a \, \underset{\sim}{J}^c = -\underset{\sim}{I}^S, \tag{60}$$

where

$$\underset{\sim}{J}^c = [J_1^c \; J_2^c \; \ldots \; J_s^c]^T. \tag{61}$$

Using the notation

$$\underset{\sim}{U} = \mathrm{diag}(U_1 \; U_2 \; \ldots \; U_p \; U_1^c \; U_2^c \; \ldots \; U_s^c) \tag{62}$$

we finally find the equation

$$(\underset{\sim}{\Lambda} \, \underset{\sim}{U})\underset{\sim}{Y} = \underset{\sim}{I}^S, \tag{63}$$

where $\underset{\sim}{\Lambda}$ is given by (59), $\underset{\sim}{I}^S$ is described by (18) and $\underset{\sim}{Y}$ is the vector of unknown parameters

$$\underset{\sim}{Y} = [Y_1 \; Y_2 \; \ldots \; Y_p \; Y_1^c \; Y_2^c \; \ldots \; Y_s^c]^T. \tag{64}$$

The system (63) contains r-1 equations with p+s unknowns. As before, in order to obtain an appropriate number of independent equations we have to arrange for other tests. The number of tests which we need is at least

$$m = \mathrm{int}\left(\frac{p+s}{r-1} \right). \tag{65}$$

The same approaches are valid as for the choice of independent measurements.

VI. EXAMPLE

As an example consider the identification of unknown parameters G_1, G_2, G_3, G_4, G_5 and G^c of a resistive active circuit shown in Fig. 8. According to (59) and (62) we have

$$\underset{\sim}{\Lambda} = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & -1 & 1 & -1 \end{bmatrix}$$

and

$$\underset{\sim}{U} = \text{diag}(U_1 \quad U_2 \quad U_3 \quad U_4 \quad U_5 \quad U_1).$$

The number of tests required is 2. First, we apply only the source I_1^S = 2A, so

$$\underset{\sim}{I}^{S1} = [2 \quad 0 \quad 0]^T$$

and measure voltages

$$\underset{\sim}{U}^1 = \text{diag}(1 \quad 1 \quad 0 \quad -1 \quad 1 \quad 1).$$

Second, applying only the source I_5^S = 8A, i.e.,

$$\underset{\sim}{I}^{S2} = [0 \quad 0 \quad 8]^T$$

we measure

$$\underset{\sim}{U}^2 = \text{diag}(1 \quad -1 \quad 2 \quad -4 \quad 6 \quad 1).$$

The two tests give us the final system of equations (29) as

$$\begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & -1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & -1 \\ 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 2 & -4 & 0 & 1 \\ 0 & 0 & 0 & 4 & 6 & -1 \end{bmatrix} \begin{bmatrix} G_1 \\ G_2 \\ G_3 \\ G_4 \\ G_5 \\ G^c \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ 0 \\ 0 \\ 0 \\ 8 \end{bmatrix},$$

whose solution is

$$\underset{\sim}{G} = [1 \quad 1 \quad 0.5 \quad 1 \quad 1 \quad 2]^T.$$

VII. CONCLUSIONS

A very basic approach to the problem of postproduction parameter identification has been discussed. Methods presented here are oriented to linear analog electrical networks. They are based mainly on voltage measurements of the network, which is excited by current source(s). The limitations for the selected element identification have been derived and formulated in Theorems 1 and 2.

For identification of all elements, a simple approach based on nodal analysis has been proposed. As a very important example we present a method for ladder networks. The method is much simpler than that of Trick and Sakla [13] and, because of a particular sparse form of the equations, we obtain explicit recurrent formulae for the solution. For arbitrary network topologies, however, there are still many open questions and unsolved problems.

It is to be noted that the presented nodal approach to the identification of all elements is also valid under limited measurements. In such a case we simply do not have all of equations in (29). The equations containing explicitly the voltages which are not available have to be dropped from (29). If necessary, we should perform more tests. Then, the identification can be done if the network is element-value-solvable.

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FIGURE CAPTIONS

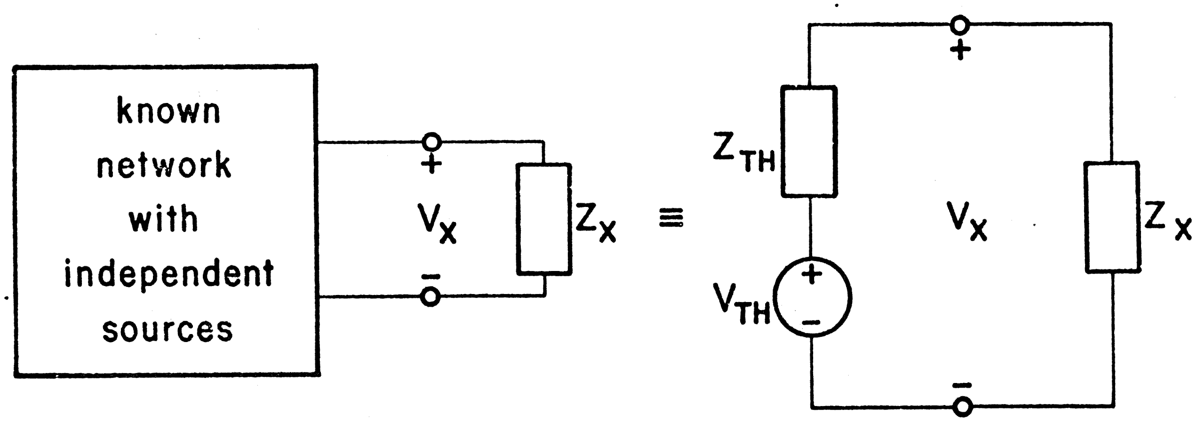
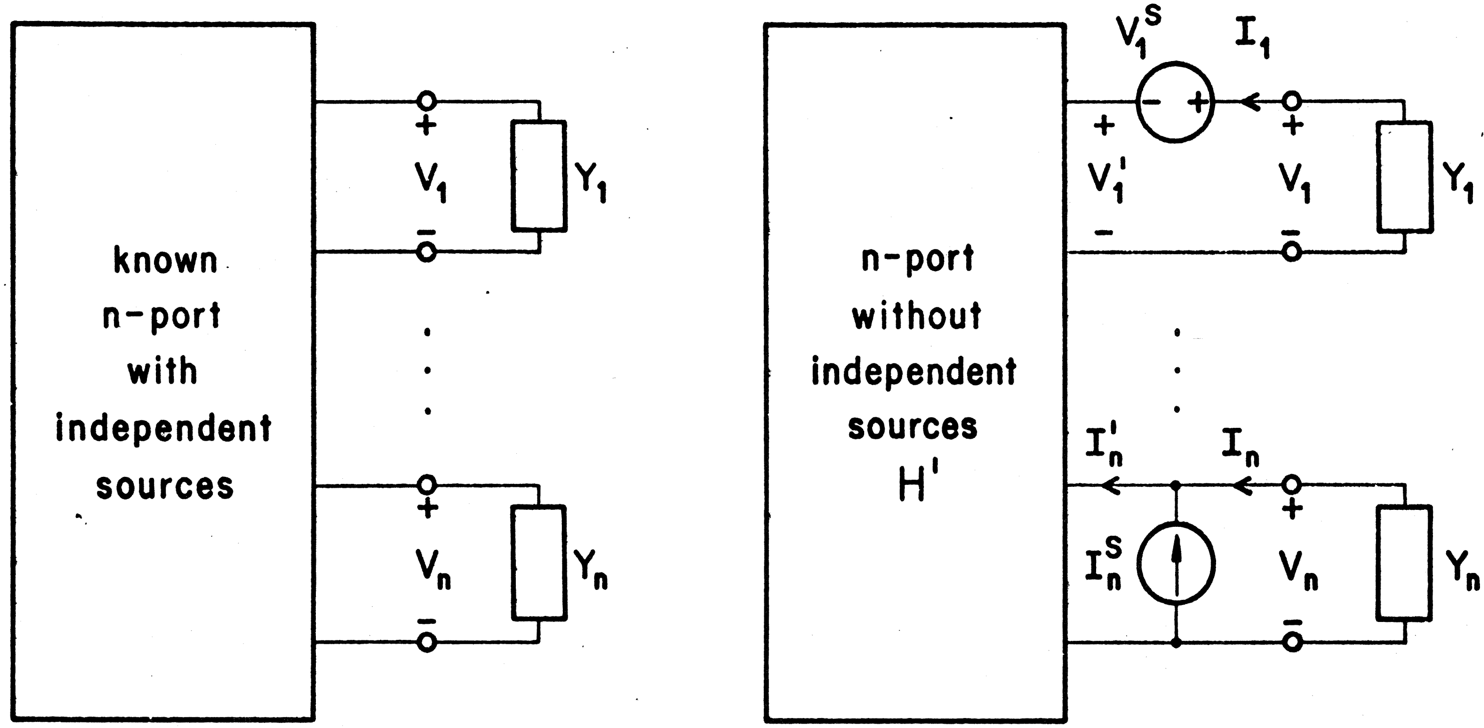


Fig. 1


known
n-port
with
independent
sources
n-port
without
independent
sources
H'
V_1
Y_1
V_n
Y_n
V_1^S
I_1
V_1'
I_n'
I_n
I_n^S

(a)

(b)

Fig. 2

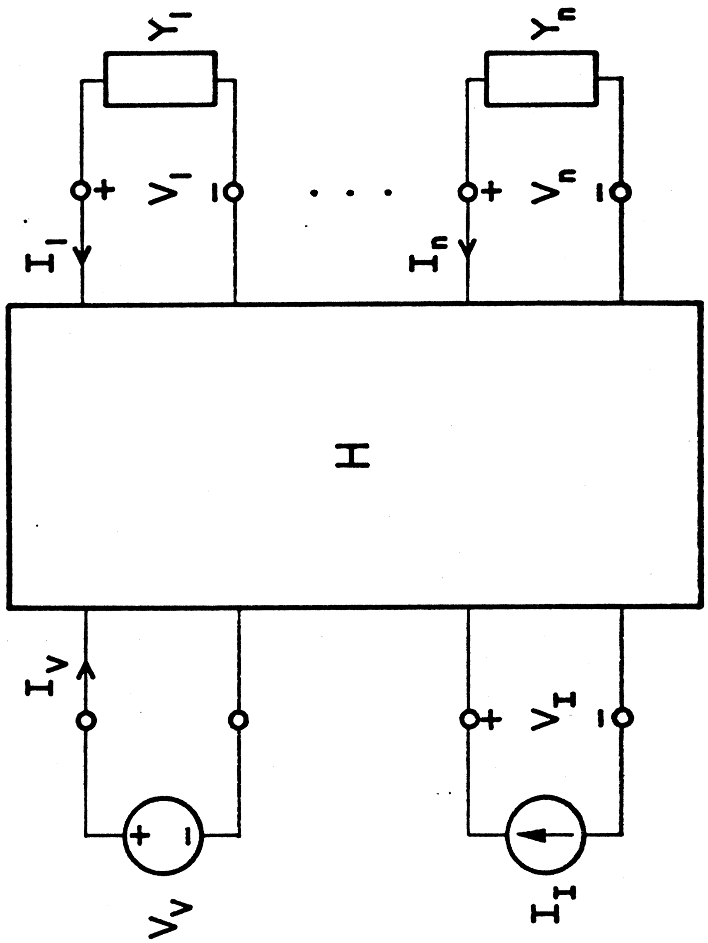


Fig. 3

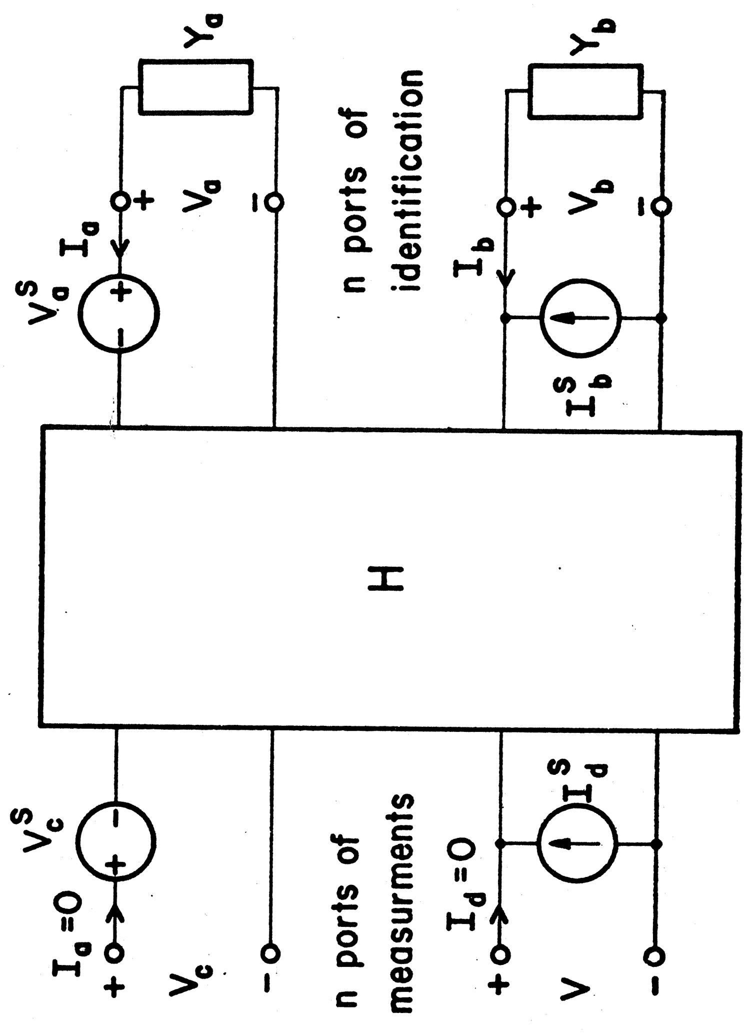


Fig. 4

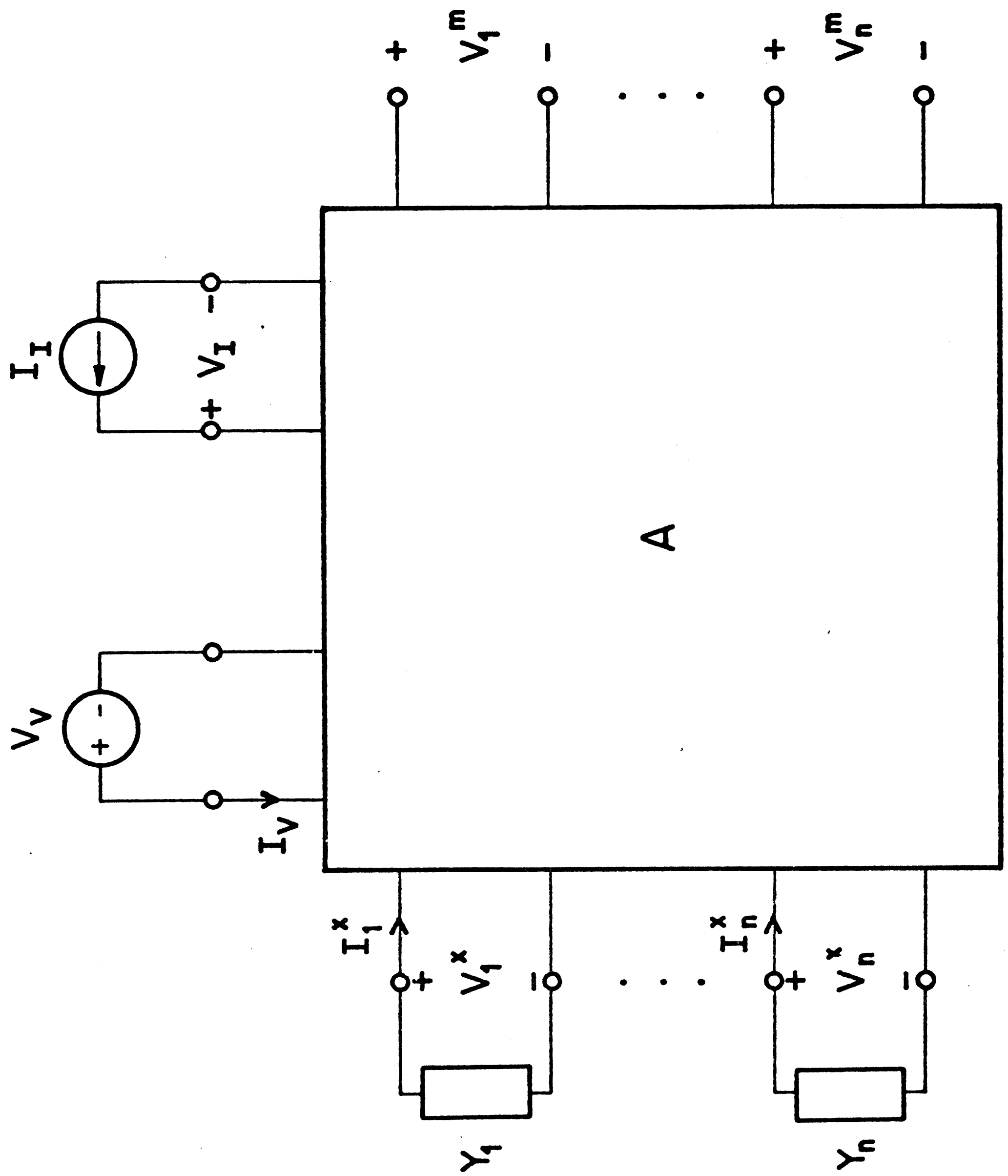


Fig. 5

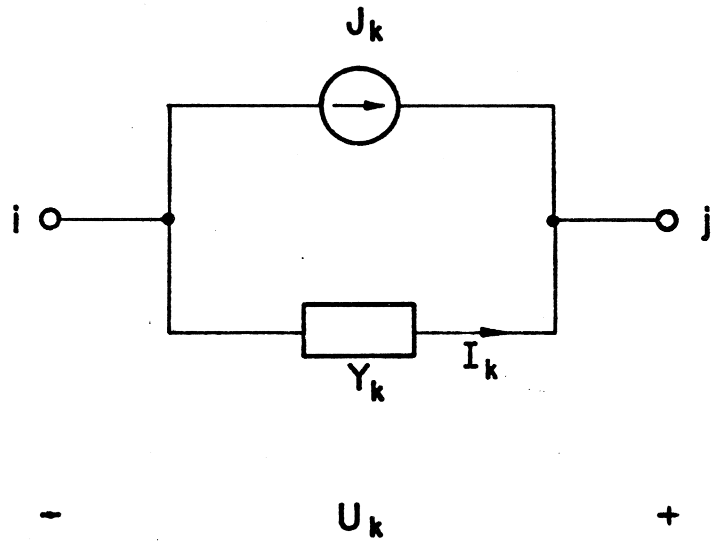


Fig. 6

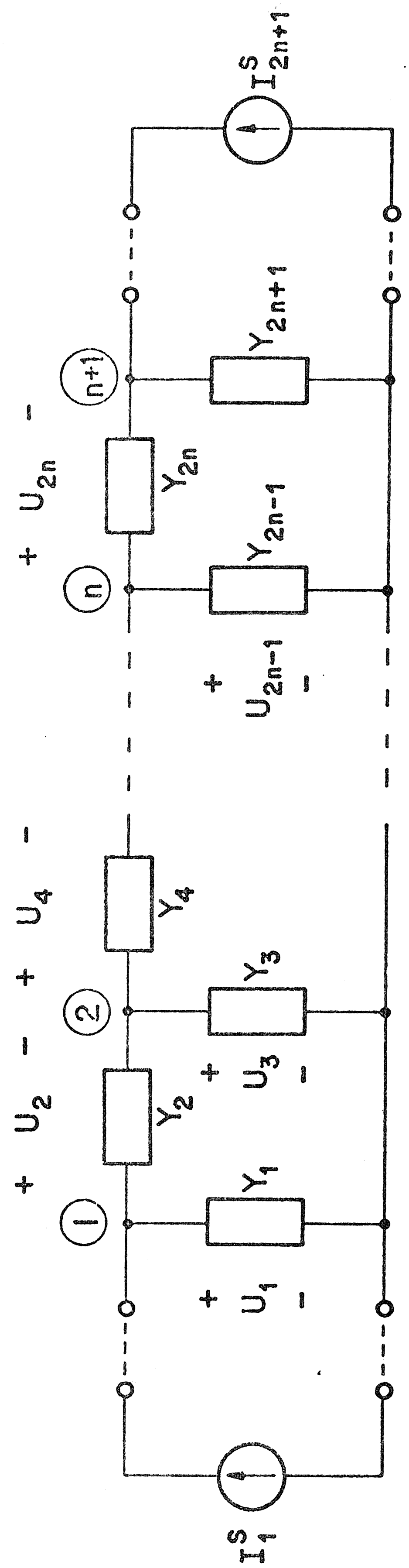


Fig. 7

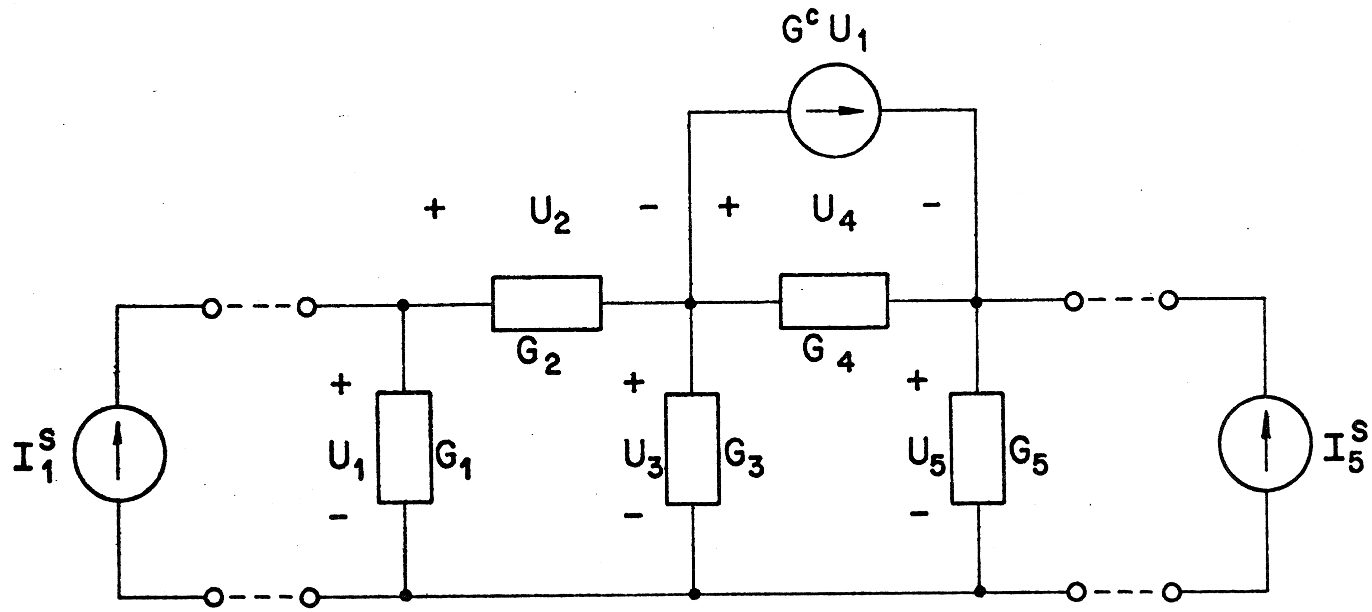


Fig. 8