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A UNIFIED APPROACH TO POWER SYSTEM SENSITIVITY ANALYSIS AND PLANNING

PART II: SPECIAL CLASS OF ADJOINT SYSTEMS

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Abstract

A unified approach to power system sensitivity analysis and planning has been presented in a companion paper, where a family of adjoint systems based on the exact a.c. power flow model was described. Here, we consider a class of this family in which the extended Tellegen sum is a real quantity. An important practical case is discussed in which the adjoining complex coefficients are set to particular values, which result in an adjoint system of a special structure. The adjoint matrix of coefficients is shown to be of the same size and sparsity as the Jacobian matrix of the original power network. The required sensitivity expressions are derived and tabulated for direct use in sensitivity analysis and gradient evaluation and are common to all relevant power system studies. Numerical examples are presented based on a 6-bus sample power system.

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I. INTRODUCTION

In power system studies such as optimal power flow, contingency analysis and planning [1] the sensitivities of the system states and the derivatives of objective functions and constraints w.r.t. the control variables are required to be calculated in an efficient way. The adjoint network approaches to these problems, which are based on the exact a.c. power flow model [2,3] appear to be superior to the Lagrange multiplier method [4] since they afford a great deal of dimensionality reduction and scheme compactness. A unified approach to power system sensitivity analysis and planning exploiting the adjoint network concept has been presented in Part I of this paper [5]. A family of adjoint systems based on the exact a.c. power flow model has been fully described. In this part, namely Part II, we consider a class of this family in which the extended Tellegen sum is a real quantity.

We discuss in detail an important practical case in which the adjoining complex coefficients are set to particular values which result in an adjoint system of a special structure. The transmission line admittances of both the original and the adjoint systems turn out to be identical. The required sensitivity expressions for this special case are derived and tabulated for direct use in sensitivity analysis and gradient evaluation. These sensitivity expressions are common to all relevant power system studies.

Numerical examples are presented for a 6-bus sample power system. The sensitivities and gradient vectors of several types of functions are considered for the purpose of illustration and comparison.

II. REAL EXTENDED TELLEGEN SUM

Equation (7) of Part I describes the general form of the extended Tellegen sum. To obtain a real extended Tellegen sum, it is sufficient for the adjoining complex coefficients to satisfy the conditions

$$\bar{\alpha} = \alpha^*, \ \bar{\beta} = \beta^*, \ \bar{\xi} = \xi^*, \ \bar{\nu} = \nu^* \tag{1a}$$

and

$$\bar{\Gamma}_k = \Gamma_k^* \text{ for all } k. \tag{1b}$$

Hence, the extended Tellegen sum is written as

$$\sum_b [\alpha \hat{I}_b V_b + \alpha^* \hat{I}_b^* V_b^* - \beta \hat{V}_b I_b - \beta^* \hat{V}_b^* I_b^* + \xi \hat{I}_b V_b^* + \xi^* \hat{I}_b^* V_b$$
$$- \nu \hat{V}_b I_b^* - \nu^* \hat{V}_b^* I_b + \sum_k \Gamma_k \lambda_{bk} C_b^k + \sum_k \Gamma_k^* \lambda_{bk} C_b^{k*}] = 0. \tag{2}$$

Expressing (2) in terms of first-order changes in V and I, the resulting perturbed sum is also a real quantity and equations (39) of Part I are consistent for all values of α, β, ξ, ν and Γ_k.

Equation (2) describes a class of adjoint networks corresponding to different values of the arbitrary complex coefficients α, β, ξ, ν and Γ_k.

III. AN IMPORTANT SPECIAL CASE

We consider the special case

$$\alpha = \beta = 1 + j0, \tag{3a}$$

$$\xi = \nu = 0 + j0, \tag{3b}$$

and

$$\Gamma_k = 0 + j0 \text{ for all } k, \tag{3c}$$

which, as we shall see, provides a special structure of the adjoint network. The transmission admittances of both original and adjoint systems are identical. The required gradients in power system sensitivity analysis and planning studies are supplied via one adjoint network analysis.

The matrices $\underset{\sim}{\Lambda}_i^b$, $\bar{\underset{\sim}{\Lambda}}_i$, $\underset{\sim}{\Lambda}_v^b$ and $\bar{\underset{\sim}{\Lambda}}_v$ of (33) of Part I are given by

$$\underset{\sim}{\Lambda}_i^b = \underset{\sim}{\Lambda}_i = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \tag{4a}$$

$$\bar{\underset{\sim}{\Lambda}}_i = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \tag{4b}$$

$$\underset{\sim}{\Lambda}_v^b = \underset{\sim}{\Lambda}_v = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \tag{4c}$$

and

$$\bar{\underset{\sim}{\Lambda}}_v = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}, \tag{4d}$$

hence, the matrices $\bar{\theta}_{bi}$, $\bar{\theta}_{bv}$ and vector θ_b of (39) of Part I are simply

$$\bar{\theta}_{bi} = M^b_{11}, \tag{5a}$$

$$\bar{\theta}_{bv} = M^b_{12} \tag{5b}$$

and

$$\theta_b = \hat{\eta}_{bx}. \tag{5c}$$

Tables I and II show the corresponding matrices $\bar{\theta}_{bi}$ and $\bar{\theta}_{bv}$ and vector θ_b for different power system elements considering the sets of element variables $\bar{z}_b$ and $\tilde{z}_b$, respectively.

IV. POWER SYSTEM ADJOINT ELEMENTS

Using the results outlined in Table I, the equations defining the adjoint elements for the set of element variables $\bar{z}_\ell$ are, for a load

$$\hat{I}_\ell = -(S_\ell/V^2_\ell)\hat{V}^*_\ell + [\,|V_\ell|\,\frac{\partial f}{\partial |V_\ell|} - j\,\frac{\partial f}{\partial \delta_\ell}\,]/(2V_\ell), \tag{6}$$

for a generator

$$V_g\,\hat{I}_g - V^*_g\,\hat{I}^*_g = (S^*_g/V^*_g)\,\hat{V}_g - (S_g/V_g)\,\hat{V}^*_g - j\,\frac{\partial f}{\partial \delta_g} \tag{7a}$$

and

$$V_g\,\hat{V}_g - V^*_g\,\hat{V}^*_g = -\,j\,V_g\,V^*_g\,\frac{\partial f}{\partial Q_g}\,, \tag{7b}$$

where each of (7a) and (7b) represents only one condition, for the slack generator

$$\hat{V}_n = -V_n^* \left(\frac{\partial f}{\partial P_n} + j \frac{\partial f}{\partial Q_n} \right)/2, \tag{8}$$

and for a transmission element

$$\hat{I}_t = Y_t \hat{V}_t + Y_t \left[\frac{\partial f}{\partial \mathrm{Re}\{I_t\}} - j \frac{\partial f}{\partial \mathrm{Im}\{I_t\}} \right]/2. \tag{9}$$

Similarly, using the results outlined in Table II, the equations defining the adjoint elements using the set of element variables $\tilde{z}_\ell$ are, for a load

$$\hat{I}_\ell = - (S_\ell/V_\ell^2) \hat{V}_\ell^* + \frac{\partial f}{\partial V_\ell}, \tag{10}$$

for a generator

$$V_g \hat{I}_g - V_g^* \hat{I}_g^* = -j\, 2\, Q_g \hat{V}_g^*/V_g + V_g \frac{\partial f}{\partial V_g}, \tag{11a}$$

and

$$V_g \hat{V}_g - V_g^* \hat{V}_g^* = -V_g \frac{\partial f}{\partial I_g}, \tag{11b}$$

where each of (11a) and (11b) represents only one condition, for the slack generator

$$\hat{V}_n = - \frac{\partial f}{\partial I_n} \tag{12}$$

and for a transmission element

$$\hat{I}_t = Y_t \hat{V}_t + Y_t \frac{\partial f}{\partial I_t}. \tag{13}$$

Note incidentally that, for transmission elements, equations (9) and (13) have the form

$$\hat{I}_t = Y_t \hat{V}_t + \hat{I}_t^S. \tag{14}$$

Comparing (53) of Part I and (14), we get

$$\bar{Y}_{t1} = \tilde{Y}_{t1} = Y_{t1} \tag{15a}$$

and

$$\bar{Y}_{t2} = \tilde{Y}_{t2} = Y_{t2}, \tag{15b}$$

where

$$Y_t = Y_{t1} + jY_{t2}, \tag{15c}$$

hence, the line admittances of both original and adjoint systems are the same, and (76) of Part I has the form

$$\underset{\sim}{Y}_T \hat{\underset{\sim}{V}}_M = -(\hat{\underset{\sim}{I}}_M + \hat{\underset{\sim}{J}}_M), \tag{16}$$

where $\underset{\sim}{Y}_T$ is the symmetrical bus admittance matrix of the original system. Moreover, since the matrices $\underset{\sim}{\Phi}_{ij}$, $\underset{\sim}{\Psi}_{ij}$; i,j = 1, 2 of (74) of Part I are diagonal matrices, the adjoint matrix of coefficients is of the same sparsity as the Jacobian matrix of the original network.

V. THE ADJOINT EQUATIONS

We write equation (16) in the form

$$\begin{pmatrix} \underset{\sim}{Y}_{LL} & \underset{\sim}{Y}_{LG} & \underset{\sim}{Y}_{LN} \\ \underset{\sim}{Y}_{GL} & \underset{\sim}{Y}_{GG} & \underset{\sim}{Y}_{GN} \\ \underset{\sim}{Y}_{NL} & \underset{\sim}{Y}_{NG} & Y_{nn} \end{pmatrix} \begin{pmatrix} \hat{\underset{\sim}{V}}_L \\ \hat{\underset{\sim}{V}}_G \\ \hat{V}_n \end{pmatrix} = - \begin{pmatrix} \hat{\underset{\sim}{I}}_L + \hat{\underset{\sim}{J}}_L \\ \hat{\underset{\sim}{I}}_G + \hat{\underset{\sim}{J}}_G \\ \hat{I}_n + \hat{J}_n \end{pmatrix}, \tag{17}$$

where the nxn bus admittance matrix has been partitioned into blocks associated with the sets of load, generator and slack buses of appropriate dimension, and $\hat{\underset{\sim}{V}}_M$, $\hat{\underset{\sim}{I}}_M$ and $\hat{\underset{\sim}{J}}_M$ have been partitioned correspondingly.

We also write the diagonal matrices $\underset{\sim}{\Phi}_{ij}$ and $\underset{\sim}{\Psi}_{ij}$; i, j = 1, 2 and the vector $\hat{\underset{\sim}{W}}_M^S$ of (74) of Part I in the corresponding partitioned forms

$$\underset{\sim}{\Phi}_{ij} = \begin{pmatrix} \underset{\sim}{\Phi}_{ij}^L & \underset{\sim}{0} & \underset{\sim}{0} \\ \underset{\sim}{0} & \underset{\sim}{\Phi}_{ij}^G & \underset{\sim}{0} \\ \underset{\sim}{0} & \underset{\sim}{0} & \phi_{ij}^n \end{pmatrix} , \tag{18a}$$

$$\underset{\sim}{\Psi}_{ij} = \begin{pmatrix} \underset{\sim}{\Psi}_{ij}^L & \underset{\sim}{0} & \underset{\sim}{0} \\ \underset{\sim}{0} & \underset{\sim}{\Psi}_{ij}^G & \underset{\sim}{0} \\ \underset{\sim}{0} & \underset{\sim}{0} & \psi_{ij}^n \end{pmatrix} , \tag{18b}$$

where $\underset{\sim}{\Phi}_{ij}^L$, $\underset{\sim}{\Phi}_{ij}^G$, $\underset{\sim}{\Psi}_{ij}^L$ and $\underset{\sim}{\Psi}_{ij}^G$ are diagonal submatrices, and

$$\hat{\underset{\sim}{W}}_M^S = \begin{pmatrix} \hat{\underset{\sim}{W}}_L^S \\ \hat{\underset{\sim}{W}}_G^S \\ \hat{\underset{\sim}{W}}_n^S \end{pmatrix} . \tag{18c}$$

Using the forms (17) and (18) it is straight forward to show that the adjoint system of equations (74) of Part I can be written in the form

$$
\left[\begin{array}{c|c|c|c}
\underset{\sim}{\Phi}{}^{L}_{11}\underset{\sim}{G}_{LL}+\underset{\sim}{\Phi}{}^{L}_{12}\underset{\sim}{B}_{LL}+\underset{\sim}{\Psi}{}^{L}_{11} & \underset{\sim}{\Phi}{}^{L}_{11}\underset{\sim}{G}_{LG}+\underset{\sim}{\Phi}{}^{L}_{12}\underset{\sim}{B}_{LG} & -\underset{\sim}{\Phi}{}^{L}_{11}\underset{\sim}{B}_{LL}+\underset{\sim}{\Phi}{}^{L}_{12}\underset{\sim}{G}_{LL}+\underset{\sim}{\Psi}{}^{L}_{12} & -\underset{\sim}{\Phi}{}^{L}_{11}\underset{\sim}{B}_{LG}+\underset{\sim}{\Phi}{}^{L}_{12}\underset{\sim}{G}_{LG} \\
\hline
\underset{\sim}{\Phi}{}^{G}_{11}\underset{\sim}{G}_{GL}+\underset{\sim}{\Phi}{}^{G}_{12}\underset{\sim}{B}_{GL} & \underset{\sim}{\Phi}{}^{G}_{11}\underset{\sim}{G}_{GG}+\underset{\sim}{\Phi}{}^{G}_{12}\underset{\sim}{B}_{GG}+\underset{\sim}{\Psi}{}^{G}_{11} & -\underset{\sim}{\Phi}{}^{G}_{11}\underset{\sim}{B}_{GL}+\underset{\sim}{\Phi}{}^{G}_{12}\underset{\sim}{G}_{GL} & -\underset{\sim}{\Phi}{}^{G}_{11}\underset{\sim}{B}_{GG}+\underset{\sim}{\Phi}{}^{G}_{12}\underset{\sim}{G}_{GG}+\underset{\sim}{\Psi}{}^{G}_{12} \\
\hline
\underset{\sim}{\Phi}{}^{L}_{21}\underset{\sim}{G}_{LL}+\underset{\sim}{\Phi}{}^{L}_{22}\underset{\sim}{B}_{LL}+\underset{\sim}{\Psi}{}^{L}_{21} & \underset{\sim}{\Phi}{}^{L}_{21}\underset{\sim}{G}_{LG}+\underset{\sim}{\Phi}{}^{L}_{22}\underset{\sim}{B}_{LG} & -\underset{\sim}{\Phi}{}^{L}_{21}\underset{\sim}{B}_{LL}+\underset{\sim}{\Phi}{}^{L}_{22}\underset{\sim}{G}_{LL}+\underset{\sim}{\Psi}{}^{L}_{22} & -\underset{\sim}{\Phi}{}^{L}_{21}\underset{\sim}{B}_{LG}+\underset{\sim}{\Phi}{}^{L}_{22}\underset{\sim}{G}_{LG} \\
\hline
\underset{\sim}{\Phi}{}^{G}_{21}\underset{\sim}{G}_{GL}\;\;\underset{\sim}{\Phi}{}^{G}_{22}\underset{\sim}{B}_{GL} & \underset{\sim}{\Phi}{}^{G}_{21}\underset{\sim}{G}_{GG}+\underset{\sim}{\Phi}{}^{G}_{22}\underset{\sim}{B}_{GG}+\underset{\sim}{\Psi}{}^{G}_{21} & -\underset{\sim}{\Phi}{}^{G}_{21}\underset{\sim}{B}_{GL}+\underset{\sim}{\Phi}{}^{G}_{22}\underset{\sim}{G}_{GL} & -\underset{\sim}{\Phi}{}^{G}_{21}\underset{\sim}{B}_{GG}+\underset{\sim}{\Phi}{}^{G}_{22}\underset{\sim}{G}_{GG}+\underset{\sim}{\Psi}{}^{G}_{22}
\end{array}\right]
$$

$$
\cdot\left[\begin{array}{c}
\hat{\underset{\sim}{V}}_{L1} \\ \hline
\hat{\underset{\sim}{V}}_{G1} \\ \hline
\hat{\underset{\sim}{V}}_{L2} \\ \hline
\hat{\underset{\sim}{V}}_{G2}
\end{array}\right]
= -
\left[\begin{array}{c}
\underset{\sim}{\Phi}{}^{L}_{11}\hat{\underset{\sim}{J}}_{L1}+\underset{\sim}{\Phi}{}^{L}_{12}\hat{\underset{\sim}{J}}_{L2}+\hat{\underset{\sim}{W}}{}^{S}_{L1} \\
(\underset{\sim}{\Phi}{}^{L}_{11}\underset{\sim}{G}_{LN}+\underset{\sim}{\Phi}{}^{L}_{12}\underset{\sim}{B}_{LN})\hat{V}_{n1}-(\underset{\sim}{\Phi}{}^{L}_{11}\underset{\sim}{B}_{LG}-\underset{\sim}{\Phi}{}^{L}_{12}\underset{\sim}{G}_{LG})\hat{V}_{n2} \\ \hline
\underset{\sim}{\Phi}{}^{G}_{11}\hat{\underset{\sim}{J}}_{G1}+\underset{\sim}{\Phi}{}^{G}_{12}\hat{\underset{\sim}{J}}_{G2}+\hat{\underset{\sim}{W}}{}^{S}_{G1} \\
(\underset{\sim}{\Phi}{}^{G}_{11}\underset{\sim}{G}_{GN}+\underset{\sim}{\Phi}{}^{G}_{12}\underset{\sim}{B}_{GN})\hat{V}_{n1}-(\underset{\sim}{\Phi}{}^{G}_{11}\underset{\sim}{B}_{GN}-\underset{\sim}{\Phi}{}^{G}_{12}\underset{\sim}{G}_{GN})\hat{V}_{n2} \\ \hline
\underset{\sim}{\Phi}{}^{L}_{21}\hat{\underset{\sim}{J}}_{L1}+\underset{\sim}{\Phi}{}^{L}_{22}\hat{\underset{\sim}{J}}_{L2}+\hat{\underset{\sim}{W}}{}^{S}_{L2} \\
(\underset{\sim}{\Phi}{}^{L}_{21}\underset{\sim}{G}_{LN}+\underset{\sim}{\Phi}{}^{L}_{22}\underset{\sim}{B}_{LN})\hat{V}_{n1}-(\underset{\sim}{\Phi}{}^{L}_{21}\underset{\sim}{B}_{LN}-\underset{\sim}{\Phi}{}^{L}_{22}\underset{\sim}{G}_{LN})\hat{V}_{n2} \\ \hline
\underset{\sim}{\Phi}{}^{G}_{21}\hat{\underset{\sim}{J}}_{G1}+\underset{\sim}{\Phi}{}^{G}_{22}\hat{\underset{\sim}{J}}_{G2}+\hat{\underset{\sim}{W}}{}^{S}_{G2} \\
(\underset{\sim}{\Phi}{}^{G}_{21}\underset{\sim}{G}_{GN}+\underset{\sim}{\Phi}{}^{G}_{22}\underset{\sim}{B}_{GN})\hat{V}_{n1}-(\underset{\sim}{\Phi}{}^{G}_{21}\underset{\sim}{B}_{GN}-\underset{\sim}{\Phi}{}^{G}_{22}\underset{\sim}{G}_{GN})\hat{V}_{n2}
\end{array}\right]. \qquad (19)
$$

where

$$\underset{\sim}{Y}_{IJ} = \underset{\sim}{G}_{IJ} + j\,\underset{\sim}{B}_{IJ}, \qquad (20a)$$

$$\hat{\underset{\sim}{V}}_{K} = \hat{\underset{\sim}{V}}_{K1} + j\,\hat{\underset{\sim}{V}}_{K2}, \qquad (20b)$$

$$\hat{\underset{\sim}{J}}_{K} = \hat{\underset{\sim}{J}}_{K1} + j\,\hat{\underset{\sim}{J}}_{K2}, \qquad (20c)$$

$$\hat{W}_{K}^{S} = \hat{W}_{K1}^{S} + j\,\hat{W}_{K2}^{S} \tag{20d}$$

and I, J and K can be G, L, N or n.

Note that the form (19) is general for any set of element variables. The adjoint matrix of coefficients has dimension 2(n-1) x 2(n-1) where the slack bus equations have been omitted.

Tables III and IV show the parameters of the adjoint system (19) for the sets of element variables $\bar{z}_b$ and $\tilde{z}_b$, respectively. For simplicity, only general elements of the diagonal matrices and of the vectors are shown.

The structure of the adjoint system (19) for any of the two cases is simplified to

$$\begin{bmatrix} G_{LL} + \Psi_{11}^{L} & G_{LG} & -B_{LL} + \Psi_{12}^{L} & -B_{LG} \\ \hline \Phi_{11}^{G} G_{GL} + \Phi_{12}^{G} B_{GL} & \Phi_{11}^{G} G_{GG} + \Phi_{12}^{G} B_{GG} + \Psi_{11}^{G} & -\Phi_{11}^{G} B_{GL} + \Phi_{12}^{G} G_{GL} & -\Phi_{11}^{G} B_{GG} + \Phi_{12}^{G} G_{GG} + \Psi_{12}^{G} \\ \hline B_{LL} + \Psi_{21}^{L} & B_{LG} & G_{LL} + \Psi_{22}^{L} & G_{LG} \\ \hline 0 & \Psi_{21}^{G} & 0 & \Psi_{22}^{G} \end{bmatrix}$$

$$\begin{bmatrix} \hat{V}_{L1} \\ \hline \hat{V}_{G1} \\ \hline \hat{V}_{L2} \\ \hline \hat{V}_{G2} \end{bmatrix} = - \begin{bmatrix} \hat{J}_{L1} + \hat{W}_{L1}^{S} + G_{LN}\,\hat{V}_{n1} - B_{LG}\,\hat{V}_{n2} \\ \hline \Phi_{11}^{G}[\hat{J}_{G1} + G_{GN}\hat{V}_{n1} - B_{GN}\hat{V}_{n2}] + \Phi_{12}^{G}[\hat{J}_{G2} + B_{GN}\hat{V}_{n1} + G_{GN}\hat{V}_{n2}] + \hat{W}_{G1}^{S} \\ \hline \hat{J}_{L2} + \hat{W}_{L2}^{S} + B_{LN}\,\hat{V}_{n1} + G_{LN}\,\hat{V}_{n2} \\ \hline \hat{W}_{G2}^{S} \end{bmatrix} \tag{21}$$

VI. GRADIENT CALCULATIONS

The derivatives of the function f w.r.t. the control variables are calculated from

$$\frac{df}{d\underset{\sim}{u}_b} = \frac{\partial f}{\partial \underset{\sim}{u}_b} - \hat{\underset{\sim}{\eta}}_{bu} \tag{22}$$

or

$$\frac{df}{\partial \zeta_{bi}} = [(\frac{\partial f}{\partial \underset{\sim}{u}_b})^T - \hat{\underset{\sim}{\eta}}_{bu}^T] \frac{\partial \underset{\sim}{u}_b}{\partial \zeta_{bi}} , \tag{23}$$

derived in Part I. The vector $\hat{\underset{\sim}{\eta}}_{bu}$ is obtained from (32) of Part I, namely

$$\hat{\underset{\sim}{\eta}}_{bu} = \underset{\sim}{M}_{21}^{b} \hat{\underset{\sim}{f}}_{bi} + \underset{\sim}{M}_{22}^{b} \hat{\underset{\sim}{f}}_{bv}, \tag{24}$$

where the vector $\hat{\underset{\sim}{f}}_b$ is given, for the considered case, by

$$\hat{\underset{\sim}{f}}_b = \begin{bmatrix} \hat{\underset{\sim}{f}}_{bi} \\ --- \\ \hat{\underset{\sim}{f}}_{bv} \end{bmatrix} = \begin{bmatrix} \hat{I}_b \\ \hat{I}_b^* \\ --- \\ -\hat{V}_b \\ -\hat{V}_b^* \end{bmatrix} , \tag{25}$$

and the matrices $\underset{\sim}{M}_{21}^{b}$ and $\underset{\sim}{M}_{22}^{b}$ are given in Tables III and IV of Part I.

Tables V and VI show the vector $\hat{\underset{\sim}{\eta}}_{bu}$ for different power system elements in the special considered case using the sets of element variables $\bar{\underset{\sim}{z}}_b$ and $\tilde{\underset{\sim}{z}}_b$, respectively.

VII. EXAMPLES

In this section, we present some numerical results to illustrate the use of the derived formulas. A 6-bus sample power system [3,6] shown in Fig. 1 is considered.

Required data for the problem is shown in Tables VII and VIII. Powers injected into buses are shown. The corresponding a.c. load flow solution is shown in Table IX. Tables X and XI show the coefficient matrices of the adjoint systems corresponding to element variables $\bar{z}_b$ and $\tilde{z}_b$, respectively. These matrices are common to all the sensitivity calculations.

So as not to be restricted to any particular application, we consider the following examples where we consider, without loss of generality, the sensitivities of some system states and a function representing the total transmission losses in the system. The control variables associated with the transmission elements are taken as the line conductances G_t and susceptances B_t.

The results presented have been checked by small perturbations about the base point.

Example 1

In this example, we consider the states associated with the load bus number 3. Element variables $\bar{z}_b$ are used. Table XII shows the RHS vector of the adjoint equations for both states and the adjoint voltages resulting from the solution of (21). Table XIII shows the derivatives calculated by our approach.

Example 2

Now, we consider the states associated with the generator bus number 5. Element variables $\bar{\underset{\sim}{z}}_b$ are used. The RHS vector of the adjoint equations for both states and the adjoint voltages resulting from the solution of (21) are shown in Table XIV. Table XVa shows the derivatives calculated by our approach.

Using element variables $\tilde{\underset{\sim}{z}}_b$, the derivatives w.r.t. S_ℓ, for example, are shown in Table XVb. Note that from (3) of Part I

$$\frac{\partial f}{\partial S_\ell^*} = \left(\frac{\partial f}{\partial S_\ell} \right)^*.$$

Example 3

In this example, we consider the function

$$f = \sum_t |I_t|^2 R_t,$$

which represents the total transmission losses in the power network.

Table XVI shows the RHS vector of the adjoint equations for this function and the adjoint voltages resulting from the solution of (21). Table XVII shows the derivatives calculated by our approach. Element variables $\bar{\underset{\sim}{z}}_b$ are used.

Example 4

In this example, we investigate line removals by considering functions of the form

$$f = |I_t|^2.$$

The control variables associated with generator and load buses are maintained at their base-case values.

Table XVIII shows some results of different contingencies.

VIII. CONCLUSIONS

We have considered a class of adjoint systems in which the extended Tellegen sum (introduced in Part I) is a real quantity. A detailed discussion of an important case in which the selection of the adjoining complex coefficients lead to an adjoint system of a special structure has been presented. The adjoint matrix of coefficients is of the same size and sparsity as the Jacobian matrix of the load flow equations of the original power network.

Sensitivity expressions have been presented and adjoint elements have been defined permitting a wide variety of problems to be handled. A number of relevant problems have been numerically explored for a 6-bus sample power system. The application of the formulas, which have been derived for two important sets of variables, has been illustrated.

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TABLE Ia

ELEMENTS OF $\bar{\underset{\sim}{\theta}}_{bi}$, $\bar{\underset{\sim}{\theta}}_{bv}$ AND $\underset{\sim}{\theta}_b$ USING ELEMENT VARIABLES $\bar{\underset{\sim}{z}}_b$

	Load Elements	Generator Elements
$\bar{\underset{\sim}{\theta}}_{bi}$	$\begin{bmatrix} V_\ell/\lvert V_\ell\rvert & V_\ell^*/\lvert V_\ell\rvert \\ jV_\ell & -jV_\ell^* \end{bmatrix}$	$\begin{bmatrix} jV_g & -jV_g^* \\ 0 & 0 \end{bmatrix}$
$\bar{\underset{\sim}{\theta}}_{bv}$	$\begin{bmatrix} -S_\ell^*/(V_\ell^*\lvert V_\ell\rvert) & -S_\ell/(V_\ell\lvert V_\ell\rvert) \\ jS_\ell^*/V_\ell^* & -jS_\ell/V_\ell \end{bmatrix}$	$\begin{bmatrix} jS_g^*/V_g^* & -jS_g/V_g \\ -j/V_g^* & j/V_g \end{bmatrix}$
$\underset{\sim}{\theta}_b$	$\begin{bmatrix} \frac{\partial f}{\partial\lvert V_\ell\rvert} \\ \frac{\partial f}{\partial\delta_\ell} \end{bmatrix}$	$\begin{bmatrix} \frac{\partial f}{\partial\delta_g} \\ \frac{\partial f}{\partial Q_g} \end{bmatrix}$

TABLE Ib

ELEMENTS OF $\bar{\underset{\sim}{\theta}}_{bi}$, $\bar{\underset{\sim}{\theta}}_{bv}$ AND $\underset{\sim}{\theta}_{b}$ USING ELEMENT VARIABLES $\bar{\underset{\sim}{z}}_{b}$

	Slack Generator	Transmission Elements
$\bar{\underset{\sim}{\theta}}_{bi}$	$\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 1/Y_t & 1/Y_t^* \\ j/Y_t & -j/Y_t^* \end{pmatrix}$
$\bar{\underset{\sim}{\theta}}_{bv}$	$\begin{pmatrix} 1/V_n^* & 1/V_n \\ -j/V_n^* & j/V_n \end{pmatrix}$	$\begin{pmatrix} 1 & 1 \\ j & -j \end{pmatrix}$
$\underset{\sim}{\theta}_{b}$	$\begin{pmatrix} \dfrac{\partial f}{\partial P_n} \\ \dfrac{\partial f}{\partial Q_n} \end{pmatrix}$	$\begin{pmatrix} \dfrac{\partial f}{\partial Re\{I_t\}} \\ \dfrac{\partial f}{\partial Im\{I_t\}} \end{pmatrix}$

TABLE II

ELEMENTS OF $\underset{\sim}{\bar{\theta}}_{bi}$, $\underset{\sim}{\bar{\theta}}_{bv}$ and $\underset{\sim}{\theta}_{b}$ USING ELEMENT VARIABLES $\underset{\sim}{\tilde{z}}_{b}$

	Load Elements	Generator Elements	Slack Generator	Transmission Elements
$\underset{\sim}{\bar{\theta}}_{bi}$	$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 1 & -V_g^*/V_g \\ 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 1/Y_t & 0 \\ 0 & 1/Y_t^* \end{pmatrix}$
$\underset{\sim}{\bar{\theta}}_{bv}$	$\begin{pmatrix} 0 & -S_\ell/V_\ell^2 \\ -S_\ell^*/V_\ell^{*2} & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & -j2Q_g/V_g^2 \\ 1 & -V_g^*/V_g \end{pmatrix}$	$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$
$\underset{\sim}{\theta}_{b}$	$\begin{pmatrix} \frac{\partial f}{\partial V_\ell} \\ \frac{\partial f}{\partial V_\ell^*} \end{pmatrix}$	$\begin{pmatrix} \frac{\partial f}{\partial V_g} \\ \frac{\partial f}{\partial I_g} \end{pmatrix}$	$\begin{pmatrix} \frac{\partial f}{\partial I_n} \\ \frac{\partial f}{\partial I_n^*} \end{pmatrix}$	$\begin{pmatrix} \frac{\partial f}{\partial I_t} \\ \frac{\partial f}{\partial I_t^*} \end{pmatrix}$

TABLE III

PARAMETERS OF ADJOINT SYSTEM USING ELEMENT VARIABLES $\bar{\underset{\sim}{z}}_b$

Load Elements

$$\phi_{11}^{\ell} = 1 \qquad \psi_{11}^{\ell} = \mathrm{Re}\{-S_\ell/V_\ell^2\}$$

$$\hat{W}_{\ell 1}^{S} = \mathrm{Re}\{(|V_\ell| \frac{\partial f}{\partial |V_\ell|} - j \frac{\partial f}{\partial \delta_\ell})/(2V_\ell)\}$$

$$\phi_{12}^{\ell} = 0 \qquad \psi_{12}^{\ell} = \mathrm{Im}\{-S_\ell/V_\ell^2\}$$

$$\phi_{21}^{\ell} = 0 \qquad \psi_{21}^{\ell} = \mathrm{Im}\{-S_\ell/V_\ell^2\}$$

$$\hat{W}_{\ell 2}^{S} = \mathrm{Im}\{(|V_\ell| \frac{\partial f}{\partial |V_\ell|} - j \frac{\partial f}{\partial \delta_\ell})/(2V_\ell)\}$$

$$\phi_{22}^{\ell} = 1 \qquad \psi_{22}^{\ell} = \mathrm{Re}\{S_\ell/V_\ell^2\}$$

Generator Elements

$$\phi_{11}^{g} = 2\mathrm{Im}\{V_g\} \qquad \psi_{11}^{g} = -2\mathrm{Im}\{S_g/V_g\}$$

$$\hat{W}_{g1}^{S} = -\frac{\partial f}{\partial \delta_g}$$

$$\phi_{12}^{g} = 2\mathrm{Re}\{V_g\} \qquad \psi_{12}^{g} = 2\mathrm{Re}\{S_g/V_g\}$$

$$\phi_{21}^{g} = 0 \qquad \psi_{21}^{g} = 2\mathrm{Im}\{V_g\}$$

$$\hat{W}_{g2}^{S} = |V_g|^2 \frac{\partial f}{\partial Q_g}$$

$$\phi_{22}^{g} = 0 \qquad \psi_{22}^{g} = 2\mathrm{Re}\{V_g\}$$

Slack Generator

$$\hat{V}_n = -V_n^* \left(\frac{\partial f}{\partial P_n} + j \frac{\partial f}{\partial Q_n} \right)/2$$

Transmission Elements

$$\hat{I}_t^S = Y_t \left(\frac{\partial f}{\partial \mathrm{Re}\{I_t\}} - j \frac{\partial f}{\partial \mathrm{Im}\{I_t\}} \right)/2, \text{ hence } \hat{J}_{m1} = \sum_t a_{mt} \mathrm{Re}\{\hat{I}_t^S\} \text{ and}$$

$$\hat{J}_{m2} = \sum_t a_{mt} \mathrm{Im}\{\hat{I}_t^S\}; \; m = \ell \text{ or } g.$$

The a_{mt} are elements of the reduced incidence matrix (68) of Part I.

TABLE IV

PARAMETERS OF ADJOINT SYSTEM USING ELEMENT VARIABLES $\tilde{z}_b$

Load Elements

$\phi^{\ell}_{11} = 1$ $\psi^{\ell}_{11} = \mathrm{Re}\{-S_{\ell}/V^2_{\ell}\}$

$\phi^{\ell}_{12} = 0$ $\psi^{\ell}_{12} = \mathrm{Im}\{-S_{\ell}/V^2_{\ell}\}$

$\hat{W}^S_{\ell 1} = \mathrm{Re}\{ \frac{\partial f}{\partial V_{\ell}} \}$

$\phi^{\ell}_{21} = 0$ $\psi^{\ell}_{21} = \mathrm{Im}\{-S_{\ell}/V^2_{\ell}\}$

$\phi^{\ell}_{22} = 1$ $\psi^{\ell}_{22} = \mathrm{Re}\{S_{\ell}/V^2_{\ell}\}$

$\hat{W}^S_{\ell 2} = \mathrm{Im}\{ \frac{\partial f}{\partial V_{\ell}} \}$

Generator Elements

$\phi^{g}_{11} = 2\mathrm{Im}\{V_g\}$ $\psi^{g}_{11} = \mathrm{Im}\{-j2Q_g/V_g\}$

$\phi^{g}_{12} = 2\mathrm{Re}\{V_g\}$ $\psi^{g}_{12} = \mathrm{Re}\{j2Q_g/V_g\}$

$\hat{W}^S_{g1} = \mathrm{Im}\{V_g \frac{\partial f}{\partial V_g} \}$

$\phi^{g}_{21} = 0$ $\psi^{g}_{21} = 2\mathrm{Im}\{V_g\}$

$\phi^{g}_{22} = 0$ $\psi^{g}_{22} = 2\mathrm{Re}\{V_g\}$

$\hat{W}^S_{g2} = \mathrm{Im}\{V_g \frac{\partial f}{\partial I_g} \}$

Slack Generator

$$\hat{V}_n = -\frac{\partial f}{\partial I_n}$$

Transmission Elements

$\hat{I}^S_t = Y_t \frac{\partial f}{\partial I_t}$, hence $\hat{J}_{m1} = \sum_t a_{mt} \mathrm{Re}\{\hat{I}^S_t\}$ and $\hat{J}_{m2} = \sum_t a_{mt} \mathrm{Im}\{\hat{I}^S_t\}$; $m = \ell$ or g.

The a_{mt} are elements of the reduced incidence matrix (68) of Part I.

TABLE Va

THE VECTOR $\hat{\underset{\sim}{\eta}}_{bu}$ USING ELEMENT VARIABLES $\bar{\underset{\sim}{z}}_b$

	Load Elements	Generator Elements
$\hat{\underset{\sim}{\eta}}_{bu}$	$\begin{pmatrix} -\hat{V}_\ell/V_\ell^* - \hat{V}_\ell^*/V_\ell \\ j\hat{V}_\ell/V_\ell^* - j\hat{V}_\ell^*/V_\ell \end{pmatrix}$	$\begin{pmatrix} [V_g\hat{I}_g+V_g^*\hat{I}_g^*+S_g^*\hat{V}_g/V_g^*+S_g\hat{V}_g^*/V_g]/\lvert V_g\rvert \\ -\hat{V}_g/V_g^*-\hat{V}_g^*/V_g \end{pmatrix}$

TABLE Vb

THE VECTOR $\hat{\underset{\sim}{\eta}}_{bu}$ USING ELEMENT VARIABLES $\bar{\underset{\sim}{z}}_b$

	Slack Generator	Transmission Elements
$\hat{\underset{\sim}{\eta}}_{bu}$	$\begin{pmatrix} [V_n\hat{I}_n+V_n^*\hat{I}_n^*+S_n^*\hat{V}_n/V_n^*+S_n\hat{V}_n^*/V_n]\lvert V_n\rvert \\ j[V_n\hat{I}_n-V_n^*\hat{I}_n^*-S_n^*\hat{V}_n/V_n^*+S_n\hat{V}_n^*/V_n] \end{pmatrix}$	$\begin{pmatrix} -V_t\hat{I}_t/Y_t-V_t^*\hat{I}_t^*/Y_t^* \\ j[-V_t\hat{I}_t/Y_t+V_t^*\hat{I}_t^*/Y_t^*] \end{pmatrix}$

TABLE VI

THE VECTOR $\hat{\underset{\sim}{n}}_{bu}$ USING ELEMENT VARIABLES $\tilde{\underset{\sim}{z}}_b$

	Load Elements	Generator Elements	Slack Generator	Transmission Elements
$\hat{\underset{\sim}{n}}_{bu}$	$\begin{pmatrix} -\hat{V}_\ell^*/V_\ell \\ -\hat{V}_\ell/V_\ell^* \end{pmatrix}$	$\begin{pmatrix} \hat{I}_g^*/V_g + S_g^*\hat{V}_g^*/V_g^*V_g^2 \\ -\hat{V}_g^*/V_g \end{pmatrix}$	$\begin{pmatrix} \hat{I}_n \\ \hat{I}_n^* \end{pmatrix}$	$\begin{pmatrix} V_t\ \hat{I}_t/Y_t \\ V_t^*\ \hat{I}_t^*/Y_t^* \end{pmatrix}$

TABLE VII

BUS DATA

Bus Index, i	Bus Type	P_i (pu)	Q_i (pu)	$\lvert V_i \rvert \angle \delta_i$ (pu)
1	load	-2.40	0	- ∠-
2	load	-2.40	0	- ∠-
3	load	-1.60	-0.40	- ∠-
4	generator	-0.30	-	1.02 ∠-
5	generator	1.25	-	1.04 ∠-
6	slack	-	-	1.04 ∠0

TABLE VIII

LINE DATA

Branch Index, t	Terminal Buses	Resistance R_t (pu)	Reactance X_t (pu)	Number of Lines
7	1,4	0.05	0.20	1
8	1,5	0.025	0.10	2
9	2,3	0.10	0.40	1
10	2,4	0.10	0.40	1
11	2,5	0.05	0.20	1
12	2,6	0.01875	0.075	4
13	3,4	0.15	0.60	1
14	3,6	0.0375	0.15	2

TABLE IX

LOAD FLOW SOLUTION

Load Buses

$$V_1 = 0.9787 \angle -0.6602$$

$$V_2 = 0.9633 \angle -0.2978$$

$$V_3 = 0.9032 \angle -0.3036$$

Generator Buses

$Q_4 = 0.7866;$ $\delta_4 = -0.5566$

$Q_5 = 0.9780;$ $\delta_5 = -0.4740$

Slack Bus

$P_6 = 6.1298;$ $Q_6 = 1.3546$

TABLE X

ADJOINT MATRIX OF COEFFICIENTS USING ELEMENT VARIABLES $\bar{\underset{\sim}{z}}_b$

2.9085	0	0	- 1.1765	- 2.3529	11.6900	0	0	-4.7059	-9.4118
0	3.3490	- 0.5882	- 0.5882	- 1.1765	0	20.5097	-2.3529	-2.3529	-4.7059
0	- 0.5882	1.2179	- 0.3922	0	0	- 2.3529	8.6744	-1.5686	0
9.4189	4.7095	3.1396	-16.2693	0	3.0338	1.5169	1.0113	-4.2477	0
19.6518	9.8259	0	0	-26.7071	4.5821	2.2910	0	0	-8.1534
-16.5453	0	0	4.7059	9.4118	4.1503	0	0	-1.1765	-2.3529
0	-23.4119	2.3529	2.3529	4.7059	0	7.6314	-0.5882	-0.5882	-1.1765
0	2.3529	-11.7178	1.5686	0	0	- 0.5882	3.8802	-0.3922	0
0	0	0	- 1.0777	0	0	0	0	1.7321	0
0	0	0	0	- 0.9495	0	0	0	0	1.8506

TABLE XI

ADJOINT MATRIX OF COEFFICIENTS USING ELEMENT VARIABLES $\tilde{z}_b$

2.9085	0	0	- 1.1765	- 2.3529	11.6900	0	0	-4.7059	-9.4118
0	3.3490	- 0.5882	- 0.5882	- 1.1765	0	20.5097	-2.3529	-2.3529	-4.7059
0	- 0.5882	1.2179	- 0.3922	0	0	- 2.3529	8.6744	-1.5686	0
9.4189	4.7095	3.1396	-15.9585	0	3.0338	1.5169	1.0113	-4.7471	0
19.6519	9.8259	0	0	-27.8045	4.5821	2.2910	0	0	-6.0146
-16.5453	0	0	4.7059	9.4118	4.1503	0	0	-1.1765	-2.3529
0	-23.4119	2.3529	2.3529	4.7059	0	7.6314	-0.5882	-0.5882	-1.1765
0	2.3529	-11.7178	1.5686	0	0	- 0.5882	3.8802	-0.3922	0
0	0	0	- 1.0777	0	0	0	0	1.7321	0
0	0	0	0	- 0.9495	0	0	0	0	1.8506

TABLE XII

RHS AND SOLUTION VECTORS OF THE ADJOINT NETWORKS FOR THE STATES OF BUS 3

Element No.	$f = \lvert V_3 \rvert$		$f = \delta_3$	
	RHS Vector	Solution Vector	RHS Vector	Solution Vector
1	0	-0.1016×10^{-1}	0	-0.2232×10^{-1}
2	0	-0.5260×10^{-2}	0	-0.1421×10^{-1}
3	-0.4771	-0.8741×10^{-2}	-0.1655	-0.5699×10^{-1}
4	0	-0.1301×10^{-1}	0	-0.2867×10^{-1}
5	0	-0.1003×10^{-1}	0	-0.2200×10^{-1}
6	0	-0.8205×10^{-2}	0	-0.1803×10^{-1}
7	0	-0.9182×10^{-2}	0	-0.8189×10^{-2}
8	-0.1495	-0.5868×10^{-1}	0.5283	-0.1878×10^{-1}
9	0	-0.8095×10^{-2}	0	-0.1784×10^{-1}
10	0	-0.5148×10^{-2}	0	-0.1129×10^{-1}

TABLE XIII

RESULTS OF EXAMPLE 1

Line Quantities

Line	Derivatives w.r.t. G_t		Derivatives w.r.t. B_t	
	$f = \|V_3\|$	$f = \delta_3$	$f = \|V_3\|$	$f = \delta_3$
1,4	-0.5441×10^{-3}	-0.1205×10^{-2}	0.3294×10^{-3}	0.7433×10^{-3}
1,5	-0.7289×10^{-3}	-0.1595×10^{-2}	-0.9625×10^{-3}	-0.2133×10^{-2}
2,3	0.1664×10^{-2}	0.5312×10^{-2}	-0.5748×10^{-2}	-0.1659×10^{-3}
2,4	0.1407×10^{-2}	-0.3359×10^{-2}	-0.3853×10^{-2}	-0.8465×10^{-2}
2,5	0.1507×10^{-2}	-0.1260×10^{-2}	-0.1870×10^{-2}	-0.2965×10^{-2}
2,6	0.2439	0.2466	-0.5931	-0.5979
3,4	0.2716×10^{-1}	0.7436×10^{-3}	-0.2716×10^{-2}	0.1522×10^{-1}
3,6	0.3419	0.3806	-0.5872	-0.5990

Load Bus Quantities

Bus	Derivatives w.r.t. P_ℓ		Derivatives w.r.t. Q_ℓ	
	$f = \|V_3\|$	$f = \delta_3$	$f = \|V_3\|$	$f = \delta_3$
1	0.2668×10^{-1}	0.5862×10^{-1}	0.5125×10^{-3}	0.1132×10^{-2}
2	0.1603×10^{-1}	0.3320×10^{-1}	0.1502×10^{-1}	0.7596×10^{-2}
3	0.5731×10^{-1}	0.1329	0.1182	0.1969×10^{-2}

Generator Bus Quantities

Bus	Derivatives w.r.t. $\|V_g\|$		Derivatives w.r.t. P_g	
	$f = \|V_3\|$	$f = \delta_3$	$f = \|V_3\|$	$f = \delta_3$
4	0.1948	-0.8082×10^{-2}	0.3005×10^{-1}	0.6620×10^{-1}
5	0.7978×10^{-1}	0.5671×10^{-1}	0.2169×10^{-1}	0.4755×10^{-1}

TABLE XIV

RHS AND SOLUTION VECTORS OF THE ADJOINT NETWORKS
FOR THE STATES OF BUS 5

Element No.	$f = \delta_5$		$f = Q_5$	
	RHS Vector	Solution Vector	RHS Vector	Solution Vector
1	0	-0.9467×10^{-1}	0	0.6004×10^{-1}
2	0	-0.3649×10^{-1}	0	0.2754×10^{-1}
3	0	-0.2049×10^{-1}	0	0.3282×10^{-1}
4	0	-0.7454×10^{-1}	0	0.1354
5	1.0	-0.1171	0	-0.2398×10^{-1}
6	0	-0.7455×10^{-1}	0	0.4884
7	0	-0.2466×10^{-1}	0	0.1466
8	0	-0.1804×10^{-1}	0	0.5839×10^{-1}
9	0	-0.4638×10^{-1}	0	0.8427×10^{-1}
10	0	-0.6008×10^{-1}	1.0816	0.5721

TABLE XVa

RESULTS OF EXAMPLE 2

Line Quantities

Line	Derivatives w.r.t. G_t		Derivatives w.r.t. B_t	
	$f = \delta_5$	$f = Q_5$	$f = \delta_5$	$f = Q_5$
1,4	0.2892×10^{-3}	0.6361×10^{-1}	-0.7712×10^{-2}	0.6595×10^{-1}
1,5	-0.1046×10^{-1}	-0.4660×10^{-1}	0.1216×10^{-2}	-0.4421×10^{-2}
2,3	-0.2054×10^{-2}	0.1612×10^{-2}	0.3751×10^{-3}	-0.1053×10^{-1}
2,4	-0.6958×10^{-2}	-0.4376×10^{-1}	-0.2188×10^{-1}	0.4846×10^{-1}
2,5	-0.1433×10^{-1}	0.1630	-0.3065×10^{-1}	-0.2359×10^{-1}
2,6	0.2426	0.3241	-0.6144	-0.5374
3,4	-0.1569×10^{-1}	0.1477×10^{-1}	-0.2883×10^{-1}	0.5497×10^{-1}
3,6	0.3680	0.3903	-0.5790	-0.5230

Load Bus Quantities

Bus	Derivatives w.r.t. P_ℓ		Derivatives w.r.t. Q_ℓ	
	$f = \delta_5$	$f = Q_5$	$f = \delta_5$	$f = Q_5$
1	0.2462	-0.7091	0.1696×10^{-2}	-0.7132
2	0.8745×10^{-1}	-0.1440	0.2672×10^{-1}	-0.2742
3	0.5524×10^{-1}	-0.1080	0.2456×10^{-1}	-0.1017

Generator Bus Quantities

Bus	Derivatives w.r.t. $\lvert V_g \rvert$		Derivatives w.r.t. P_g	
	$f = \delta_5$	$f = Q_5$	$f = \delta_5$	$f = Q_5$
4	0.1736	-4.5187	0.1721	-0.3128
5	-0.8889×10^{-1}	7.5809	0.2531	-0.4612

TABLE XVb

RESULTS OF EXAMPLE 2

Bus	Derivatives w.r.t. S_ℓ	
	$f = \delta_5$	$f = Q_5$
1	$0.1231 - j0.8481\times10^{-3}$	$-0.3545 + j0.3566$
2	$0.4372\times10^{-1} - j0.1336\times10^{-1}$	$-0.7199\times10^{-1} + j0.1371$
3	$0.2762\times10^{-1} - j0.1228\times10^{-1}$	$-0.5400\times10^{-1} + j0.5083\times10^{-1}$

TABLE XVI

RHS AND SOLUTION VECTORS OF THE ADJOINT NETWORK OF EXAMPLE 3

Element No.	RHS Vector	Solution Vector
1	0.4678	0.1692
2	0.3121	0.0852
3	0.3157	0.0828
4	-0.2337	0.1627
5	0.4732	0.1447
6	-0.3673	0.1440
7	-0.5174	0.0534
8	-0.3106	0.0707
9	0	0.1013
10	0	0.0743

TABLE XVII

RESULTS OF EXAMPLE 3

Line Quantities

Line	Derivatives w.r.t. G_t	Derivatives w.r.t. B_t
1,4	0.1646×10^{-1}	0.8741×10^{-2}
1,5	0.4900×10^{-1}	0.2737×10^{-1}
2,3	0.3490×10^{-2}	0.2102×10^{-2}
2,4	0.8466×10^{-1}	0.4496×10^{-1}
2,5	0.4547×10^{-1}	0.2268×10^{-1}
2,6	0.3518	-0.5270
3,4	0.8940×10^{-1}	0.4276×10^{-1}
3,6	0.4838	-0.4917

Load Bus Quantities

Bus	Derivatives w.r.t. P_ℓ	Derivatives w.r.t. Q_ℓ
1	-0.4535	-0.2039×10^{-1}
2	-0.2017	-0.5410×10^{-1}
3	-0.2217	-0.9465×10^{-1}

Generator Bus Quantities

Bus	Derivatives w.r.t. V_g	Derivatives w.r.t. P_g
4	-0.3736×10^{-1}	-0.3758
5	-0.1840	-0.3128

TABLE XVIII

CONTINGENCY RESULTS OF EXAMPLE 4

Function Line Index	Removed Line Index	Calculated Function Change	Exact Function Change
1,4	2,4	-0.200	-0.224
2,3	1,5*	0.002	0.005
2,3	2,3	-0.029	-0.021
2,4	2,4	-0.470	-0.404

* Only one line of branch 1,5 is removed.

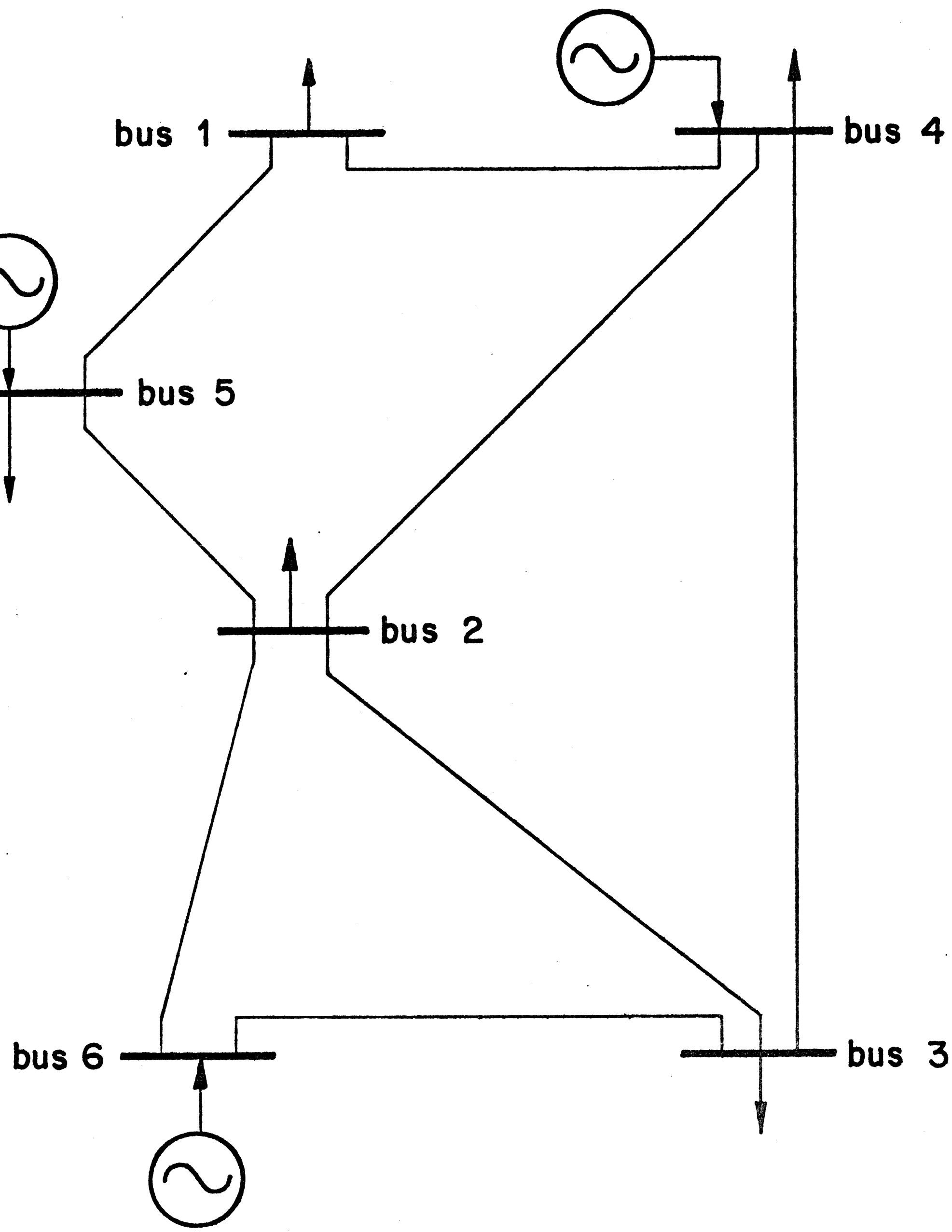

Fig. 1 6-bus sample power system

SOC-238

A UNIFIED APPROACH TO POWER SYSTEM SENSITIVITY ANALYSIS AND PLANNING PART II: SPECIAL CLASS OF ADJOINT SYSTEMS

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Key Words: Power system analysis, adjoint networks, Tellegen's theorem, sensitivity analysis, optimal load flows, contingency analysis

Abstract: A unified approach to power system sensitivity analysis and planning has been presented in a companion paper, where a family of adjoint systems based on the exact a.c. power flow model was described. Here, we consider a class of this family in which the extended Tellegen sum is a real quantity. An important practical case is discussed in which the adjoining complex coefficients are set to particular values, which result in an adjoint system of a special structure. The adjoint matrix of coefficients is shown to be of the same size and sparsity as the Jacobian matrix of the original power network. The required sensitivity expressions are derived and tabulated for direct use in sensitivity analysis and gradient evaluation and are common to all relevant power system studies. Numerical examples are presented based on a 6-bus sample power system.

Description:

Related Work: SOC-234, SOC-237.

Price: $ 6.00.