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A HIERARCHICAL DECOMPOSITION APPROACH FOR NETWORK ANALYSIS

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Abstract

A novel approach for analyzing large electrical networks is presented in which the network is decomposed into subnetworks in a hierarchical manner by removing few interconnections. These subnetworks are solved separately. The results are then interconnected at a number of computing levels. The solution of the original network is thereby obtained.

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I. INTRODUCTION

Recently, much effort has been devoted to decompostion methods for the analysis of large networks. The idea of decomposition or tearing was originated by Kron [1,2], in which a part of a given network is torn away so that the remaining subnetworks can be analyzed independently. The solutions of the separate subnetworks are then combined and modified to take the part torn away into consideration and thus the solution of the original network is obtained at two levels [3]. Happ [2,4] has expanded the theory and applications along the same lines. Chua and Chen [5], and Wu [6] have generalized the concept of tearing. Happ [7] has generalized the two-level computation into a multilevel computation process. However, the calculation at the levels except for the first can not be carried out in parallel and thus this method may not be suitable for analyzing large scale networks.

In this paper, a method is presented to solve a large scale network by decomposing it in a hierarchical manner. The network is decomposed into subnetworks and blocks by removing few interconnections and applying arbitrary current sources at the terminals created by removal of interconnections. As the decomposition imposes a hierarchical structure on the computations, the calculation at each level can be done in parallel.

II. NOTATION

N_k a subnetwork of the original network, which is denoted N_1.

N_k' a subnetwork made up of equivalent multipoles of divisions of subnetwork N_k.

T_{ij} a set of interconnection nodes common to subnetworks N_i and N_j.

$\underset{\sim}{Y}_k$ the nodal admittance matrix of subnetwork N_k.

$\underset{\sim}{Z}_k$ $\underset{\sim}{Y}_k^{-1}$

$\underset{\sim}{V}_{k\ell}$ a column vector of voltages on the external terminals of subnetwork N_k incident to subnetwork N_ℓ.

$\underset{\sim}{I}_{k\ell}$ a column vector of arbitrary impressed currents applied at the external terminals of subnetwork N_k which were connected to $T_{k\ell}$.

$\underset{\sim}{V}_k^I$, $\underset{\sim}{I}_k^I$ column vectors of voltages and currents, respectively, on the internal nodes of subnetwrok N_k.

$\underset{\sim}{V}_k^O$, $\underset{\sim}{I}_k^O$ column vectors of voltages and currents, respectively, on external nodes of subnetwork N_k.

$\underset{\sim}{V}_k$ nodal voltage vector of subnetwork N_k.

$\underset{\sim}{I}_k$ current excitation vector of subnetwork N_k.

$\underset{\sim}{V}_{km}^I$, $\underset{\sim}{I}_{km}^I$ column vectors of voltages and currents, respectively, on the nodes which are external to subnetwork $N_k^{'}$ and internal to subnetwork $N_m^{'}$, where $N_k^{'} \in Q^{L'}$ and $N_m^{'} \in Q^{L-1'}$.

$\underset{\sim}{V}_{km}^O$, $\underset{\sim}{I}_{km}^O$ column vectors of voltages and currents, respectively, on the nodes which are external to subnetwork $N_k^{'}$ and external to subnetwork $N_m^{'}$, where $N_k^{'} \in Q^{L'}$ and $N_m^{'} \in Q^{L-1'}$.

$\underset{\sim}{E}_{k\ell} \equiv \underset{\sim}{E}^m$ $\underset{\sim}{V}_{\ell k} - \underset{\sim}{V}_{k\ell}$ if $T_{\ell k}$ exists and N_k, $N_\ell \subset N_m$.

Q^L a set of subnetworks at decomposition level L.

$Q^{L'}$ a set of subnetworks made up of multipoles at decomposition level L.

III. NETWORK DECOMPOSITION

Consider a large network N_1. Let us decompose N_1 into subnetworks N_2, N_3, ..., N_i connected by a small number of interconnections. Each of them can be still too large for direct analysis so we decompose N_2, ..., N_i into smaller subnetworks and continue this process until we reach sufficiently small subnetworks. The last ones, which are not further divided, we call blocks. This decomposition procedure gives us a hierarchical structure of subnetworks as illustrated in Fig. 1. Subnetworks N_i and N_k are connected by T_{ik} interconnection nodes (see Fig. 1(a)). The network is decomposed, such that no mutual coupling is present between blocks. For simplicity, it is assumed that each block contains a common ground node. When some blocks do not contain a common ground, the analysis can be performed in the same way after slight modification of these blocks. Modification of ungrounded blocks is discussed in Section VI.

IV. ANALYSIS OF BLOCKS

Let us assume that the circuit N_1 is linear. For simplicity, assume that every block can be described by nodal equations. In order to decompose network N_1 into subnetworks, we apply the arbitrary current sources to all the interconnection nodes as shown in Fig. 2 and compute voltages on them. The network with added current sources will be equivalent to the orignal one when voltages on these sources will be zero. We obtain the conditions on node to datum voltages as

$$\underset{\sim}{V}_{jk} = \underset{\sim}{V}_{kj}, \quad \forall\, T_{jk}. \tag{1}$$

Every block is now separated from the rest of the network by the set of added current sources which can be treated as external excitations. We solve them separately and obtain

$$\underset{\sim}{V}_k = [\underset{\sim}{Y}_k]^{-1}\, \underset{\sim}{I}_k \tag{2}$$

for all blocks, where $\underset{\sim}{Y}_k$ is the nodal admittance matrix of the kth block. Equation (2) can be written as

$$\begin{bmatrix} \underset{\sim}{V}_k^I \\ \underset{\sim}{V}_k^O \end{bmatrix} = \begin{bmatrix} \underset{\sim}{Z}_k^{II} & \underset{\sim}{Z}_k^{IO} \\ \underset{\sim}{Z}_k^{OI} & \underset{\sim}{Z}_k^{OO} \end{bmatrix} \begin{bmatrix} \underset{\sim}{I}_k^I \\ \underset{\sim}{I}_k^O \end{bmatrix} . \tag{3}$$

If we assume current excitations $\underset{\sim}{I}_k^O$ for all blocks of added sources arbitrarily and solve (3), then the outside voltages $\underset{\sim}{V}_k^O$ may not satisfy the condition (1). Therefore, our aim is to correct $\underset{\sim}{I}_k^O$ by an amount $\Delta\underset{\sim}{I}_k^O$ to satisfy condition (1). From (3)

$$\begin{bmatrix} \underset{\sim}{V}_k^{I'} \\ \underset{\sim}{V}_k^{O'} \end{bmatrix} = \begin{bmatrix} \underset{\sim}{V}_k^I \\ \underset{\sim}{V}_k^O \end{bmatrix} + \begin{bmatrix} \underset{\sim}{Z}_k^{IO} \\ \underset{\sim}{Z}_k^{OO} \end{bmatrix} \Delta\underset{\sim}{I}_k^O, \tag{4}$$

where changes in outside voltages

$$\Delta\underset{\sim}{V}_k^O = \underset{\sim}{Z}_k^{OO} \; \Delta\underset{\sim}{I}_k^O \tag{5}$$

should satisfy the conditions

$$\Delta\underset{\sim}{V}_{ki} - \Delta\underset{\sim}{V}_{ik} = \underset{\sim}{V}_{ik} - \underset{\sim}{V}_{ki}, \quad k \neq i. \tag{6}$$

Let us denote

$$\underset{\sim}{E}_{ki} = \underset{\sim}{V}_{ik} - \underset{\sim}{V}_{ki}, \quad k \neq i. \tag{7}$$

To find the corrections $\Delta\underset{\sim}{I}_k^O$ for all blocks we can put correction voltage sources (7) in place of added current sources $\underset{\sim}{I}_{ki}$ and calculate $\Delta\underset{\sim}{I}_k^O$ for blocks removing all other internal energy sources. The problem of network analysis has been reduced to determining $\Delta\underset{\sim}{I}_k^O$ flowing through interconnections. Blocks can now be represented by multipoles for which the matrix description is known (from (5)). For a large scale network, however, even this reduced network description, which contains multipoles and correction voltage sources $\underset{\sim}{E}_{ki}$, may still be too large for direct analysis. This is the reason for developing a hierarchical decomposition approach.

V. ANALYSIS OF SUBNETWORKS

In this section we will discuss the hierarchical analysis of the network which is decomposed into subnetworks and blocks in a manner in which, at each level of decomposition, each subnetwork is decomposed into two smaller subnetworks only. This type of decomposition is general enough to represent any type of network partitioning into blocks, and can be used in an effective way for computer programming.

Now we will discuss the way of connecting two subnetworks described by equations of the form (5). Consider multipoles $N_k^{'}$ and $N_\ell^{'}$, elements of $Q^{L'}$, being linked by correction voltage sources $\underset{\sim}{E}^m$. The only difference between the original subnetworks N_k, N_ℓ and subnetworks $N_k^{'}$, $N_\ell^{'}$ is that the latter do not contain independent sources inside them. Equations of subnetwork $N_m^{'} \in Q^{L-1'}$ which consists of $N_k^{'}$ and $N_\ell^{'}$ are obtained in the following way (see Fig. 3).

1. Present (5) for $N_k^{'}$ and $N_\ell^{'}$ in the form

$$\begin{bmatrix} \Delta \underset{\sim}{V}_{tm}^{I} \\ \Delta \underset{\sim}{V}_{tm}^{O} \end{bmatrix} = \begin{bmatrix} \underset{\sim}{Z}_{tm}^{II} & \underset{\sim}{Z}_{tm}^{IO} \\ \underset{\sim}{Z}_{tm}^{OI} & \underset{\sim}{Z}_{tm}^{OO} \end{bmatrix} \begin{bmatrix} \Delta \underset{\sim}{I}_{tm}^{I} \\ \Delta \underset{\sim}{I}_{tm}^{O} \end{bmatrix} , \begin{matrix} t = k, \ell \\ N_k, N_\ell \subset N_m \end{matrix} . \tag{8}$$

2. The nodal equations of subnetwork $N_m^{'}$ can be written as

$$\begin{bmatrix} \underset{\sim}{Z}_{km}^{II} & \underset{\sim}{Z}_{km}^{IO} & 0 & 0 \\ \underset{\sim}{Z}_{km}^{OI} & \underset{\sim}{Z}_{km}^{OO} & 0 & 0 \\ 0 & 0 & \underset{\sim}{Z}_{\ell m}^{II} & \underset{\sim}{Z}_{\ell m}^{IO} \\ 0 & 0 & \underset{\sim}{Z}_{\ell m}^{OI} & \underset{\sim}{Z}_{\ell m}^{OO} \end{bmatrix} \begin{bmatrix} \Delta \underset{\sim}{I}_{km}^{I} \\ \Delta \underset{\sim}{I}_{km}^{O} \\ \Delta \underset{\sim}{I}_{\ell m}^{I} \\ \Delta \underset{\sim}{I}_{\ell m}^{O} \end{bmatrix} = \begin{bmatrix} \Delta \underset{\sim}{V}_{km}^{I} \\ \Delta \underset{\sim}{V}_{km}^{O} \\ \Delta \underset{\sim}{V}_{\ell m}^{I} \\ \Delta \underset{\sim}{V}_{\ell m}^{O} \end{bmatrix} , \tag{9}$$

where

$$\Delta \underset{\sim}{I}_{km}^{I} = -\Delta \underset{\sim}{I}_{\ell m}^{I}, \tag{10}$$

$$\Delta \underset{\sim}{V}^{I}_{km} - \Delta \underset{\sim}{V}^{I}_{\ell m} = \underset{\sim}{E}^{m}. \tag{11}$$

3. With the help of (10) and (11), (9) is reduced to

$$\begin{bmatrix} \underset{\sim}{Z}^{II}_{km} + \underset{\sim}{Z}^{II}_{\ell m} & \underset{\sim}{Z}^{IO}_{km} & -\underset{\sim}{Z}^{IO}_{\ell m} \\ \underset{\sim}{Z}^{OI}_{km} & \underset{\sim}{Z}^{OO}_{km} & \underset{\sim}{0} \\ -\underset{\sim}{Z}^{OI}_{\ell m} & \underset{\sim}{0} & \underset{\sim}{Z}^{OO}_{\ell m} \end{bmatrix} \begin{bmatrix} \Delta \underset{\sim}{I}^{I}_{km} \\ \Delta \underset{\sim}{I}^{O}_{km} \\ \Delta \underset{\sim}{I}^{O}_{\ell m} \end{bmatrix} = \begin{bmatrix} \underset{\sim}{E}^{m} \\ \Delta \underset{\sim}{V}^{O}_{km} \\ \Delta \underset{\sim}{V}^{O}_{\ell m} \end{bmatrix} . \tag{12}$$

4. Equation (12) can be written as

$$\begin{bmatrix} \underset{\sim}{Z}^{II'}_{m} & \underset{\sim}{Z}^{IO'}_{m} \\ \underset{\sim}{Z}^{OI'}_{m} & \underset{\sim}{Z}^{OO'}_{m} \end{bmatrix} \begin{bmatrix} \Delta \underset{\sim}{I}^{I}_{m} \\ \Delta \underset{\sim}{I}^{O}_{m} \end{bmatrix} = \begin{bmatrix} \underset{\sim}{E}^{m} \\ \Delta \underset{\sim}{V}^{O}_{m} \end{bmatrix} , \tag{13}$$

where

$$\Delta \underset{\sim}{I}^{I}_{m} = \Delta \underset{\sim}{I}^{I}_{km} , \tag{14}$$

$$\Delta \underset{\sim}{I}^{O}_{m} = \begin{bmatrix} \Delta \underset{\sim}{I}^{O}_{km} \\ \Delta \underset{\sim}{I}^{O}_{\ell m} \end{bmatrix} , \tag{15}$$

$$\Delta \underset{\sim}{V}^{O}_{m} = \begin{bmatrix} \Delta \underset{\sim}{V}^{O}_{km} \\ \Delta \underset{\sim}{V}^{O}_{\ell m} \end{bmatrix} . \tag{16}$$

From (13) we have

$$\Delta \underset{\sim}{I}^{I}_{m} = [\underset{\sim}{Z}^{II'}_{m}]^{-1} [\underset{\sim}{E}^{m} - \underset{\sim}{Z}^{IO'}_{m} \Delta \underset{\sim}{I}^{O}_{m}], \tag{17}$$

$$\Delta \underset{\sim}{V}^{O}_{m} = \underset{\sim}{Z}^{OI'}_{m} [\underset{\sim}{Z}^{II'}_{m}]^{-1} \underset{\sim}{E}^{m} + [\underset{\sim}{Z}^{OO'}_{m} - \underset{\sim}{Z}^{OI'}_{m} [\underset{\sim}{Z}^{II'}_{m}]^{-1} \underset{\sim}{Z}^{IO'}_{m}] \Delta \underset{\sim}{I}^{O}_{m}. \tag{18}$$

Now adjust $\underset{\sim}{V}^{O}_{m}$ to

$$\underset{\sim}{V}^{O'}_{m} = \underset{\sim}{V}^{O}_{m} + \underset{\sim}{Z}^{OI'}_{m} [\underset{\sim}{Z}^{II'}_{m}]^{-1} \underset{\sim}{E}^{m}. \tag{19}$$

Then (18) can be written as

$$\Delta \underset{\sim}{V}_m^{O'} = \Delta \underset{\sim}{V}_m^{O} - \underset{\sim}{Z}_m^{OI'} [\underset{\sim}{Z}_m^{II'}]^{-1} \underset{\sim}{E}^m$$

$$= [\underset{\sim}{Z}_m^{OO'} - \underset{\sim}{Z}_m^{OI'} [\underset{\sim}{Z}_m^{II'}]^{-1} \underset{\sim}{Z}_m^{IO'}] \Delta \underset{\sim}{I}_m^{O} \tag{20}$$

or

$$\Delta \underset{\sim}{V}_m^{O'} = \underset{\sim}{Z}_m^{OO} \Delta \underset{\sim}{I}_m^{O}. \tag{21}$$

Again, to compute $\Delta \underset{\sim}{I}_m^{O}$ we replace subnetworks from level L by multipoles described by (21) and join them by correction voltage sources and put L = L-1. If L > 1, the form of the subnetwork is similar to N_m' and equations (8)-(21) describing this subnetwork can be written and we can go for the next lower level. If L = 1, we obtain a subnetwork without outside current excitations and can determine $\Delta \underset{\sim}{I}_1^{I}$ from (17), which is reduced to

$$\Delta \underset{\sim}{I}_1^{I} = [\underset{\sim}{Z}_1^{II'}]^{-1} \underset{\sim}{E}^1, \tag{22}$$

where $\Delta \underset{\sim}{I}_1^{I}$ describes the change in current excitations at the 2nd level. Using (5)-(17), we return to the highest level determining all the corrections in the arbitrary current sources and then the various node voltages of the original network are calculated from (4). Note that in the procedure described above only small interconnection matrices are to be inverted. The computations can be carried out in series parallel as shown in Fig. 1(b). In the above analysis, subnetwork N_m' consists of only two multipoles (N_k' and N_ℓ').

The method can be extended to the case when on each level of decomposition the subnetwork is partitioned into more than two smaller subnetworks. However, the authors are convinced that such an extension may change the computational efficiency slightly, with serious complications to the algorithm and larger memory demands.

VI. UNGROUNDED BLOCKS

In the case of ungrounded blocks, we cannot assume an independent description nor can we apply independent current excitations. At least three different approaches are possible in such a case. In the first we calculate voltages in the ungrounded block with respect to one of its interconnection nodes and the outside current source incident to this node equals the sum of all other added current sources with the opposite sign. This approach, however, requires a more complicated algorithm of network analysis and more computer memory.

The second approach is illustrated with the help of equivalent networks in Fig. 4. In this approach, ungrounded blocks (or all blocks of the original network) are separated by applying the arbitrary voltage sources in place of current sources and forcing the conditions (instead of condition (1)) that current drawn from arbitrary voltage sources will be zero, i.e.,

$$\underset{\sim}{I}_{ij} + \underset{\sim}{I}_{ji} = \underset{\sim}{0}. \tag{23}$$

This approach is simple and does not require extra computational effort. This concept is obtained from the node tearing approach [8].

The third approach is also simple and we describe it shortly. Fig. 5 illustrates the way of proving this approach on the basis of equivalent networks.

Finally, we can add current source I'_{ij} and admittances of value y and -y as shown in Fig. 5 and obtain an equivalent network in which blocks N_i and N_j are separated by current source I'_{ij} and both blocks are grounded as shown in Fig. 6. It is clear that only one such source I'_{ij} with admittances y and -y is to be added for every ungrounded block.

The third method can be used directly in the method of hierarchical analysis we have described.

VII. ALGORITHM

Let us consider a linear network decomposed into blocks with a known structure of hierarchical decomposition. We write the steps of the algorithm as follows.

Step 1 Assume $\underset{\sim}{I}_k^O$ and solve the nodal equations for all blocks, i.e.,

$$\begin{bmatrix} \underset{\sim}{V}_k^I \\ \underset{\sim}{V}_k^O \end{bmatrix} = \begin{bmatrix} \underset{\sim}{Z}_k^{II} & \underset{\sim}{Z}_k^{IO} \\ \underset{\sim}{Z}_k^{OI} & \underset{\sim}{Z}_k^{OO} \end{bmatrix} \begin{bmatrix} \underset{\sim}{I}_k^I \\ \underset{\sim}{I}_k^O \end{bmatrix}, \ \forall N_k \in Q^L.$$

Step 2 Set $L \leftarrow L-1$ and calculate $\underset{\sim}{E}^m$, $\forall N_m' \in Q^{L'}$ from (11).

Step 3 If $L = 1$, go to Step 7.

Step 4 Obtain matrices $\underset{\sim}{Z}_m' \triangleq \begin{bmatrix} \underset{\sim}{Z}_m^{II'} & \underset{\sim}{Z}_m^{IO'} \\ \underset{\sim}{Z}_m^{OI'} & \underset{\sim}{Z}_m^{OO'} \end{bmatrix}$, $\forall N_m \in Q^{L'}$ with the help of (8) and (12).

Step 5 Adjust voltages $\underset{\sim}{V}_m^O \leftarrow \underset{\sim}{V}_m^O + \underset{\sim}{Z}_m^{OI'}[\underset{\sim}{Z}_m^{II'}]^{-1} \underset{\sim}{E}^m$, $\forall N_m' \in Q^{L'}$ from (19).

Step 6 Calculate matrices $\underset{\sim}{Z}_m^{OO} \triangleq [\underset{\sim}{Z}_m^{OO'} - \underset{\sim}{Z}_m^{OI'} [\underset{\sim}{Z}_m^{II'}]^{-1} \underset{\sim}{Z}_m^{IO'}]$, $\forall N_m' \in Q^{L'}$ and go to Step 2.

Step 7 Set $\underset{\sim}{Z}_1^{II} \leftarrow \underset{\sim}{Z}_2^{OO} + \underset{\sim}{Z}_3^{OO}$, calculate $\Delta\underset{\sim}{I}_1^I$ from (22). Using (14) and (10) we have $\Delta\underset{\sim}{I}_{21}^I = \Delta\underset{\sim}{I}_1^I$ and $\Delta\underset{\sim}{I}_{31}^I = -\Delta\underset{\sim}{I}_{21}^I$; using (5) and (8) we have $\Delta\underset{\sim}{I}_2^O = \Delta\underset{\sim}{I}_{21}^I$ and $\Delta\underset{\sim}{I}_3^O = \Delta\underset{\sim}{I}_{31}^I$.

Step 8 Set $L \leftarrow L+1$, calculate $\Delta \underset{\sim}{I}_m^I$ from (17), $\forall\, N_m' \in Q^{L'}$.

Step 9 Use (14) and (15) to determine $\Delta \underset{\sim}{I}_{km}^I$ and $\Delta \underset{\sim}{I}_{km}^O$, $\forall\, N_k' \in Q^{L+1'}$.

Step 10 Determine $\Delta \underset{\sim}{I}_k^O$ with the help of (5) and (8) $\forall\, N_k' \in Q^{L+1'}$.

Step 11 If L+1 is not the highest level then go to Step 8.

Step 12 Calculate $\underset{\sim}{I}_k^O \leftarrow \underset{\sim}{I}_k^O + \Delta \underset{\sim}{I}_k^O$ for all blocks and use (3) to find all nodal voltages.

VIII. EXAMPLE

A simple example of an analog linear network is considered to illustrate the algorithm. Consider the network N_1 in Fig. 7, which is decomposed into two subnetworks N_2 and N_3 as shown in Fig. 8(a). Let us assume that subnetworks N_2 and N_3 are not small enough. Subnetworks N_2 and N_3 are, therefore, further decomposed into smaller subnetworks N_4, N_5 and N_6, N_7 as shown in Fig. 8(b). Further decomposition of N_4, N_5, N_6, N_7 is not needed and we call them blocks.

Analysis of Blocks

Impedances of various elements of the network N_1 are in ohms.

$$Z_1 = 1, \quad Z_2 = 1, \quad Z_3 = 1, \quad Z_4 = 0.5,$$
$$Z_5 = 0.5, \quad Z_6 = 1, \quad Z_7 = 0.5, \quad Z_8 = 1,$$
$$Z_9 = 1, \quad Z_{10} = 0.5, \quad Z_{11} = 0.5, \quad Z_{12} = 0.5$$

and internal current excitations are $I_4^I \equiv I_1 = 10$ A, $I_7^I = I_2 = 10$ A.

The solutions of the nodal equations for blocks N_4, N_5, N_6 and N_7 have the form

$$\begin{bmatrix} V_4^I(1) \\ V_4^I(2) \\ V_{45} \end{bmatrix} = \begin{bmatrix} 1.6 & 1 & 1.4 \\ 1 & 1 & 1 \\ 1.4 & 1 & 1.6 \end{bmatrix} \begin{bmatrix} I_4^I \\ 0 \\ I_{45} \end{bmatrix}, \tag{24}$$

$$\begin{bmatrix} V_{54} \\ V_{56} \\ V_{57} \end{bmatrix} = \begin{bmatrix} 1.5 & 1 & 1 \\ 1 & 1.5 & 1 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} I_{54} \\ I_{56} \\ I_{57} \end{bmatrix}, \tag{25}$$

$$\begin{bmatrix} V_{65} \\ V_{67} \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} I_{65} \\ I_{67} \end{bmatrix}, \tag{26}$$

$$\begin{bmatrix} V_7^I \\ V_{75} \\ V_{76} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0.5 \\ 1 & 1.5 & 0.5 \\ 0.5 & 0.5 & 0.5 \end{bmatrix} \begin{bmatrix} I_7^I \\ I_{75} \\ I_{76} \end{bmatrix}. \tag{27}$$

Let us assume arbitrary current excitations in the decomposed network

$$I_{45} = -I_{54} = 5\text{A}, \qquad I_{56} = -I_{65} = 5\text{A},$$
$$I_{57} = -I_{75} = 5\text{A}, \qquad I_{67} = -I_{76} = 5\text{A}.$$

With the given excitations we solve (24)-(27) to obtain

$$V_{45} = 22\text{V}, \quad V_{54} = 2.5\text{V}, \quad V_{56} = 7.5\text{ V}, \quad V_{57} = 5.0\text{ V},$$
$$V_{65} = 0\text{ V}, \quad V_{67} = 5.0\text{V}, \quad V_{75} = 0\text{ V}, \quad V_{76} = 0\text{ V}.$$

Analysis of Subnetworks

Now we combine blocks N_4, N_5 and N_6, N_7 to obtain corrections $\Delta \underset{\sim}{I}_k^0$ for subnetworks N_2 and N_3. Using equations (11) we calculate correction voltages

$$E_2 = \Delta V_{42}^I - \Delta V_{52}^I = V_{52}^I - V_{42}^I = V_{54} - V_{45} = 2.5 - 22 = -19.5 \text{ V},$$

$$E_3 = \Delta V_{63}^I - \Delta V_{73}^I = V_{73}^I - V_{63}^I = V_{76} - V_{67} = 0 - 5 = -5 \text{ V}.$$

From (8) and (12) we have

$$\underset{\sim}{Z}_2' = \left[\begin{array}{c:cc} 3.1 & -1 & -1 \\ \hdashline -1 & 1.5 & 1 \\ -1 & 1 & 1 \end{array}\right], \quad \underset{\sim}{Z}_3' = \left[\begin{array}{c:cc} 2.5 & 1 & -0.5 \\ \hdashline 1 & 1 & 0 \\ -0.5 & 0 & 1.5 \end{array}\right]$$

From (19) adjusted voltages $\underset{\sim}{V}_2^0$ and $\underset{\sim}{V}_3^0$ are

$$\underset{\sim}{V}_2^0 = \begin{bmatrix} V_{56} \\ V_{57} \end{bmatrix} = \begin{bmatrix} 7.5 \\ 5.0 \end{bmatrix} + \begin{bmatrix} 19.5/3.1 \\ 19.5/3.1 \end{bmatrix} = \begin{bmatrix} 13.7903 \\ 11.2903 \end{bmatrix},$$

$$\underset{\sim}{V}_3^0 = \begin{bmatrix} V_{65} \\ V_{75} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} + \begin{bmatrix} -5/2.5 \\ 2.5/2.5 \end{bmatrix} = \begin{bmatrix} -2.0 \\ 1.0 \end{bmatrix}.$$

From (20) and (21) we have

$$\underset{\sim}{Z}_2^{00} = \begin{bmatrix} 1.5 & 1 \\ 1 & 1 \end{bmatrix} - \begin{bmatrix} -1 \\ -1 \end{bmatrix} [3.1]^{-1} [-1 \quad -1] = \begin{bmatrix} 1.1774 & 0.6774 \\ 0.6774 & 0.6774 \end{bmatrix},$$

$$\underset{\sim}{Z}_3^{00} = \begin{bmatrix} 1 & 0 \\ 0 & 1.5 \end{bmatrix} - \begin{bmatrix} 1 \\ -0.5 \end{bmatrix} [2.5]^{-1} [1 \quad -0.5] = \begin{bmatrix} 0.6 & 0.2 \\ 0.2 & 1.4 \end{bmatrix}.$$

Now to combine the subnetworks N_2 and N_3, using (11), we obtain the correction voltage vector

$$\underset{\sim}{E}^1 = \Delta\underset{\sim}{V}_{23}^I - \Delta\underset{\sim}{V}_{32}^I = [\underset{\sim}{V}_{32}^I - \underset{\sim}{V}_{23}^I] = [\underset{\sim}{V}_3^0 - \underset{\sim}{V}_2^0] = \begin{bmatrix} -15.7903 \\ -10.2903 \end{bmatrix}$$

and from (8) and (12) we have

$$\underset{\sim}{Z}_1^{II'} = \underset{\sim}{Z}_2^{00} + \underset{\sim}{Z}_3^{00} = \begin{bmatrix} 1.7774 & 0.8774 \\ 0.8774 & 2.0774 \end{bmatrix}$$

then

$$\Delta \underset{\sim}{I}_1^I = [\underset{\sim}{Z}_1^{II'}]^{-1} \underset{\sim}{E}^1 = \begin{bmatrix} -8.1346 \\ -1.5183 \end{bmatrix} .$$

From (10) and (14) we have

$$\Delta \underset{\sim}{I}_{21}^I = -\Delta \underset{\sim}{I}_{31}^I = \Delta \underset{\sim}{I}_1^I$$

then

$$\Delta \underset{\sim}{I}_2^0 = \Delta \underset{\sim}{I}_{21}^I \text{ and } \Delta \underset{\sim}{I}_3^0 = \Delta \underset{\sim}{I}_{31}^I .$$

Now we calculate $\Delta \underset{\sim}{I}_2^I$ and $\Delta \underset{\sim}{I}_3^I$ from (17).

$$\Delta I_2^I = [1/3.1][-19.5 - [-1 \quad -1] \begin{bmatrix} -8.1346 \\ -1.5183 \end{bmatrix}] = -9.4042$$

and

$$\Delta I_3^I = -4.9502.$$

Using (14) and (15) we have

$$\Delta I_{42}^I = \Delta I_2^I = -9.4042, \ \Delta I_{63}^I = \Delta I_3^I = -4.9502$$

$$\Delta \underset{\sim}{I}_{52}^0 = \Delta \underset{\sim}{I}_2^0 = \begin{bmatrix} -8.1346 \\ -1.5183 \end{bmatrix} , \begin{bmatrix} \Delta I_{63}^0 \\ \Delta I_{73}^0 \end{bmatrix} = \Delta \underset{\sim}{I}_3^0 = \begin{bmatrix} 8.1346 \\ 1.5183 \end{bmatrix}$$

With the help of (5) and (8) we have

$$\Delta I_4^0 = \Delta I_{42}^I = -9.4042, \quad \Delta \underset{\sim}{I}_5^0 = \begin{bmatrix} \Delta I_{52}^I \\ \Delta \underset{\sim}{I}_{52}^0 \end{bmatrix} = \begin{bmatrix} 9.4042 \\ -8.1346 \\ -1.5183 \end{bmatrix}$$

$$\Delta \underset{\sim}{I}_6^0 = \begin{bmatrix} \Delta I_{63}^I \\ \Delta I_{63}^0 \end{bmatrix} = \begin{bmatrix} -4.9502 \\ 8.1346 \end{bmatrix}, \quad \Delta \underset{\sim}{I}_7^0 = \begin{bmatrix} \Delta I_{73}^I \\ \Delta I_{73}^0 \end{bmatrix} = \begin{bmatrix} 4.9502 \\ 1.5183 \end{bmatrix}.$$

At the last step of the algorithm, we correct $\underset{\sim}{I}_k^0 = \underset{\sim}{I}_k^0 + \Delta \underset{\sim}{I}_k$, $k = 4, 5, 6, 7$ and obtain

$$I_{45} = -4.4042, \quad I_{56} = -3.1346, \quad I_{57} = 3.4817, \quad I_{67} = 0.0498.$$

Now, putting these values of I_{45}, I_{56}, I_{57} and I_{67} into (24), (25), (26) and (27), the node voltages obtained are

$$V_1 = 9.8341V, \quad V_2 = 5.5958V, \quad V_3 = 6.9533V, \quad V_4 = 4.7513V,$$
$$V_5 = 3.1840V, \quad V_6 = 3.2342V, \quad V_7 = 6.4934V.$$

IX. CONCLUSIONS

A hierarchical decomposition approach for simulating a large network has been presented. The network is decomposed, by removing some interconnections, into quite small subnetworks, then the network analysis is realized in two stages. First, analysis of blocks is performed and after this subnetworks are combined in a hierarchical manner joining two subnetworks at any time. Thus, combining the solution of the subnetworks can be performed in a series-parallel way. The analysis of very large networks is possible, therefore, in a short time. We have described the method for linear networks which can easily be extended to the case of nonlinear networks. For the analysis of nonlinear electronic circuits the modified Newton method [9] can be used, which is comparatively efficient. It is recommended that for analysis of nonlinear circuits, at each iteration of Newton's or the modified Newton method, an incremental model of the network should be used (instead of the companion model [10]) and solved for increments in network variables. As the network is to be solved for increments in variables, round off errors will be less and also less computation time will be required.

There is no efficient algorithm available for optimally decomposing large networks. In this approach, however, it is possible to use an efficient algorithm which gives suboptimal decomposiiton of large networks because only the number of external nodes of the subnetworks is important. Sparsity techniques at blocks or the subnetwork level can be used in implementing the algorithm.

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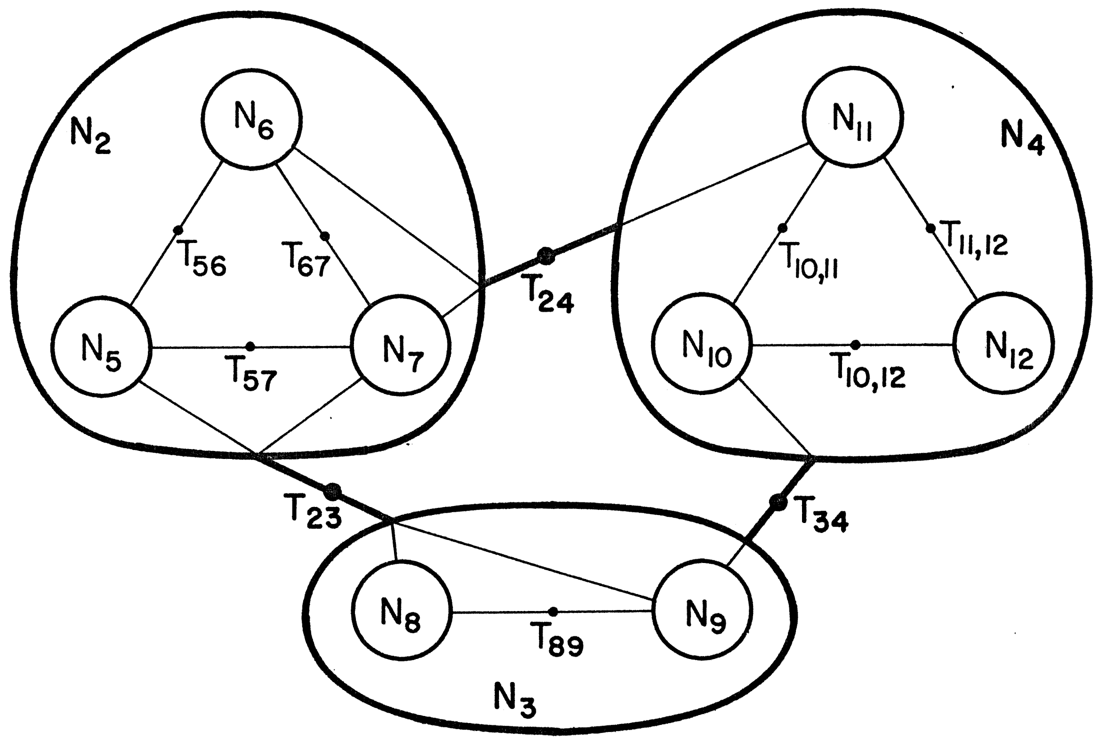

Fig. 1(a) Decomposed network with interconnections and blocks.

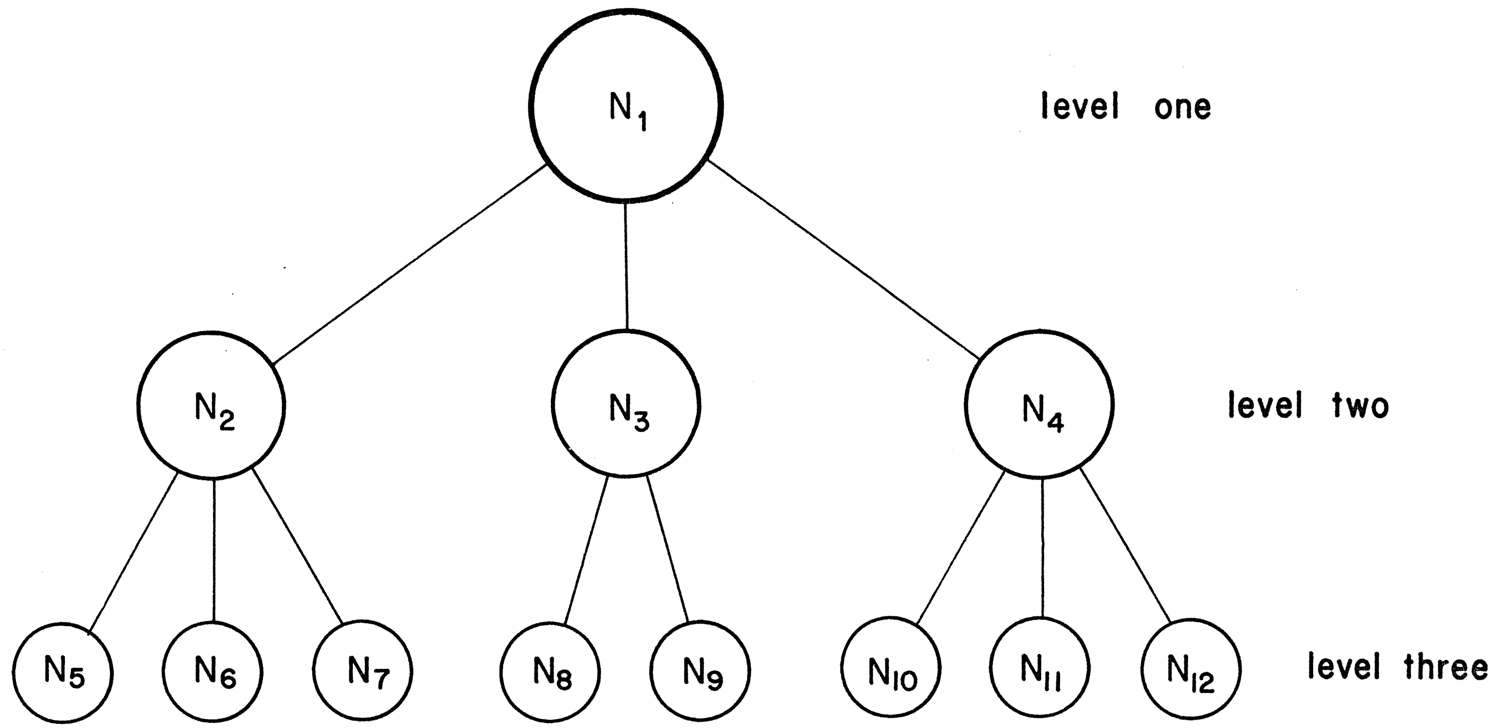

Fig. 1(b) Hierarchical structure of subnetworks obtained by decomposition.

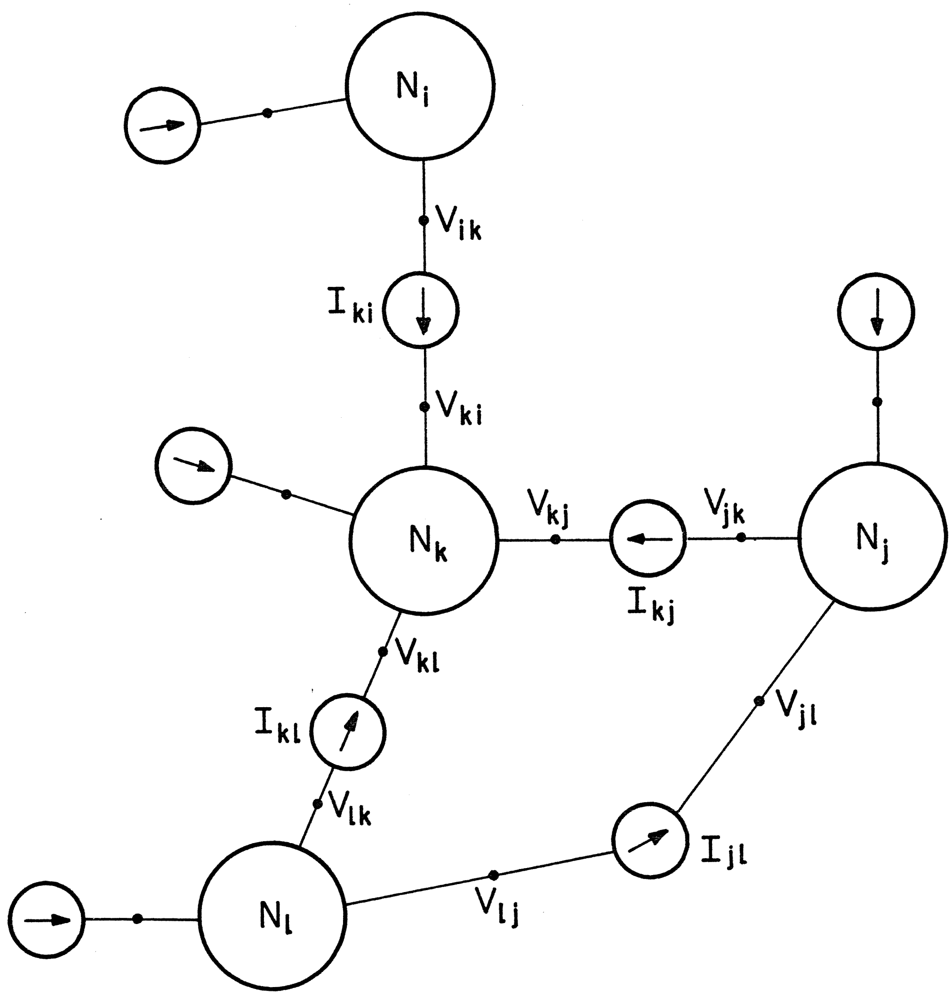

Fig. 2 Network blocks with added current sources.

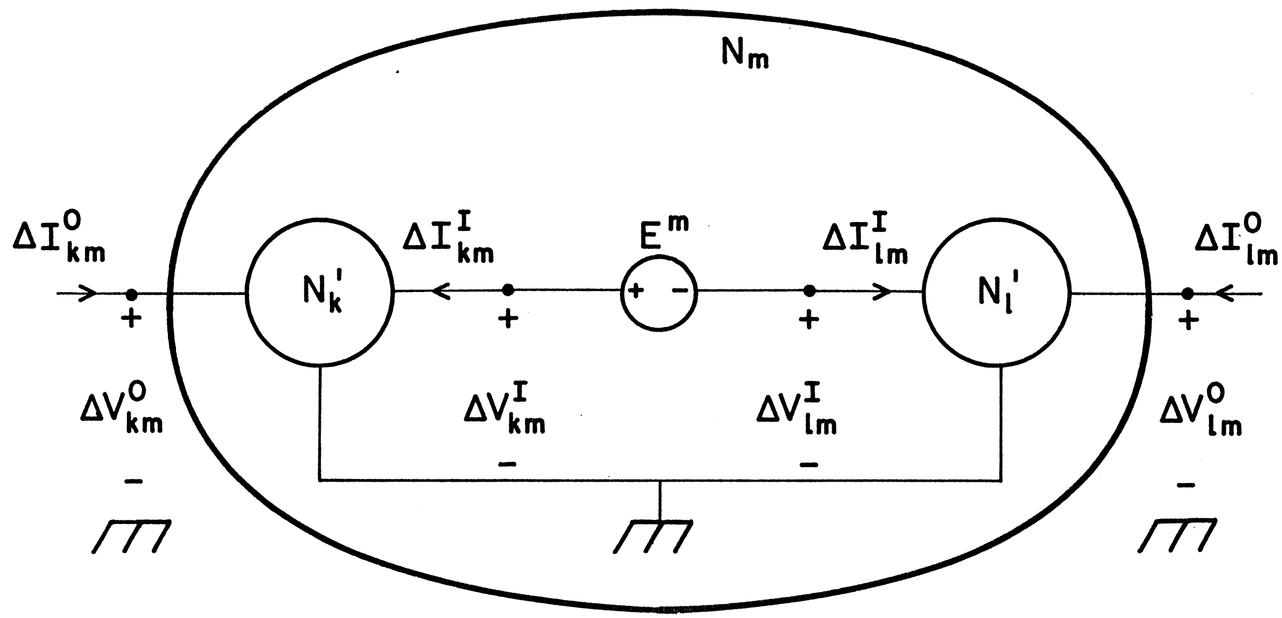

Fig. 3 Interconnection of two multipoles.

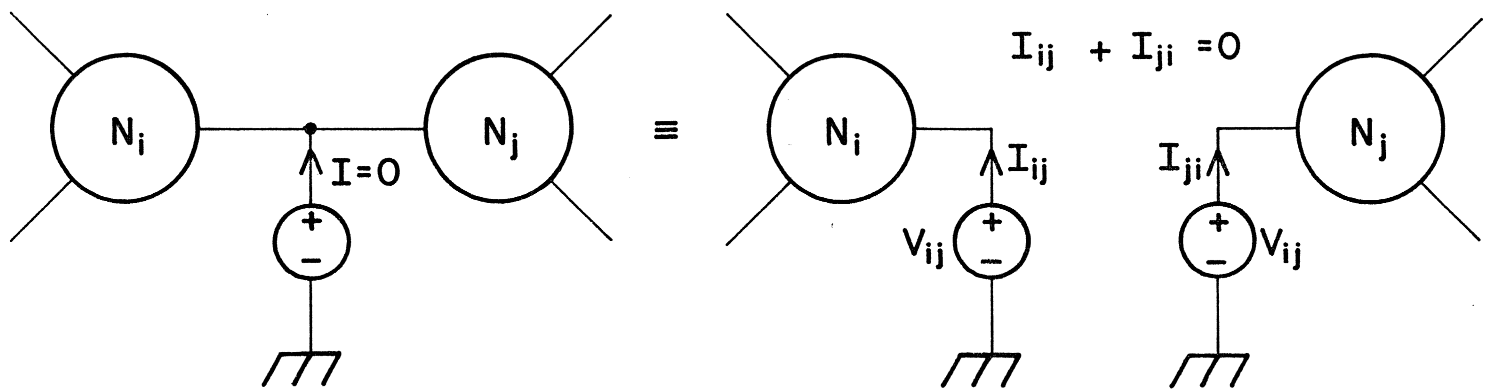

Fig. 4 Equivalent networks for the second approach.

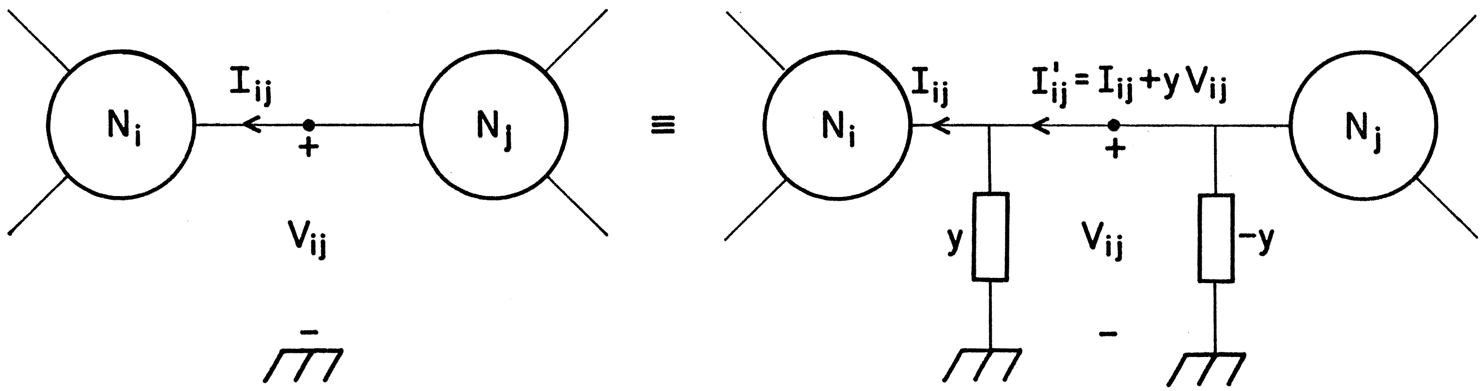

Fig. 5 Equivalent networks for the third approach.

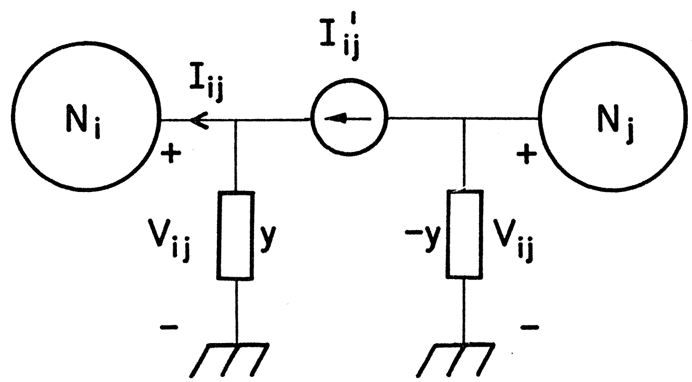

Fig. 6 Separation of two ungrounded blocks.

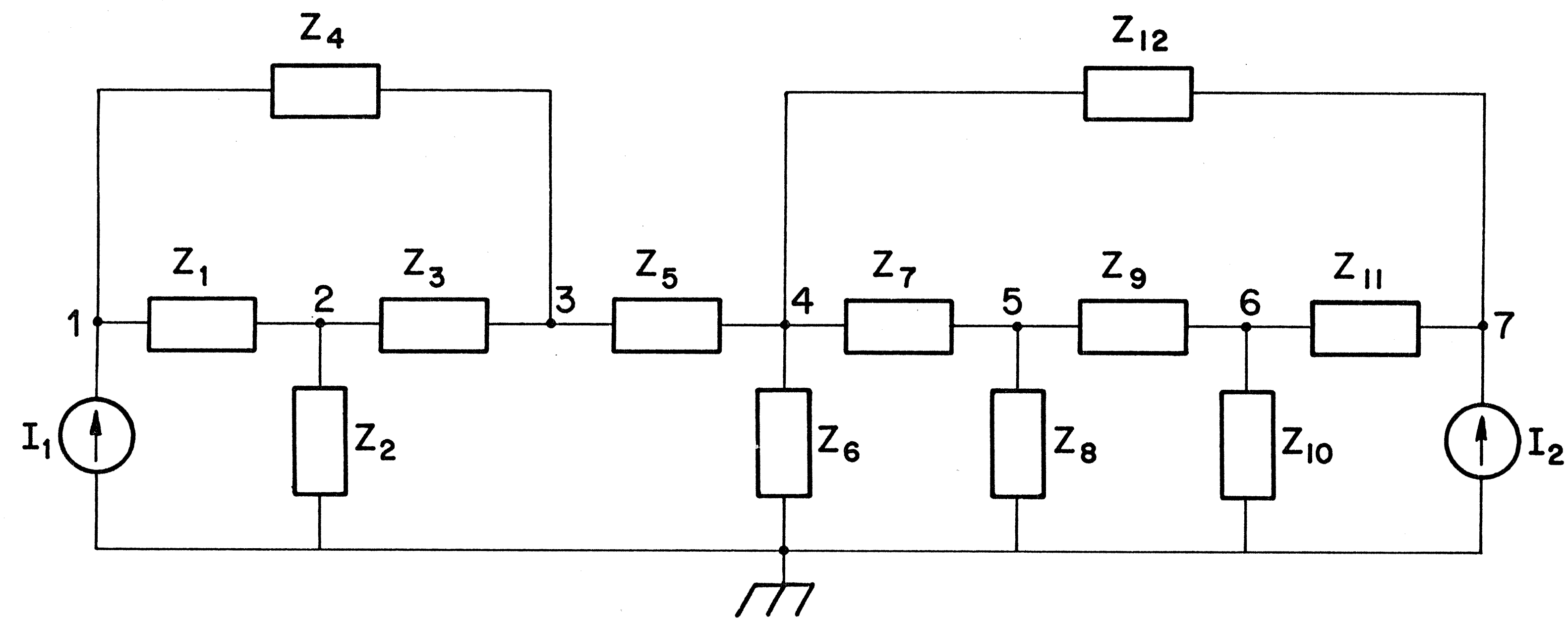

Fig. 7 A linear network example constituting N_1.

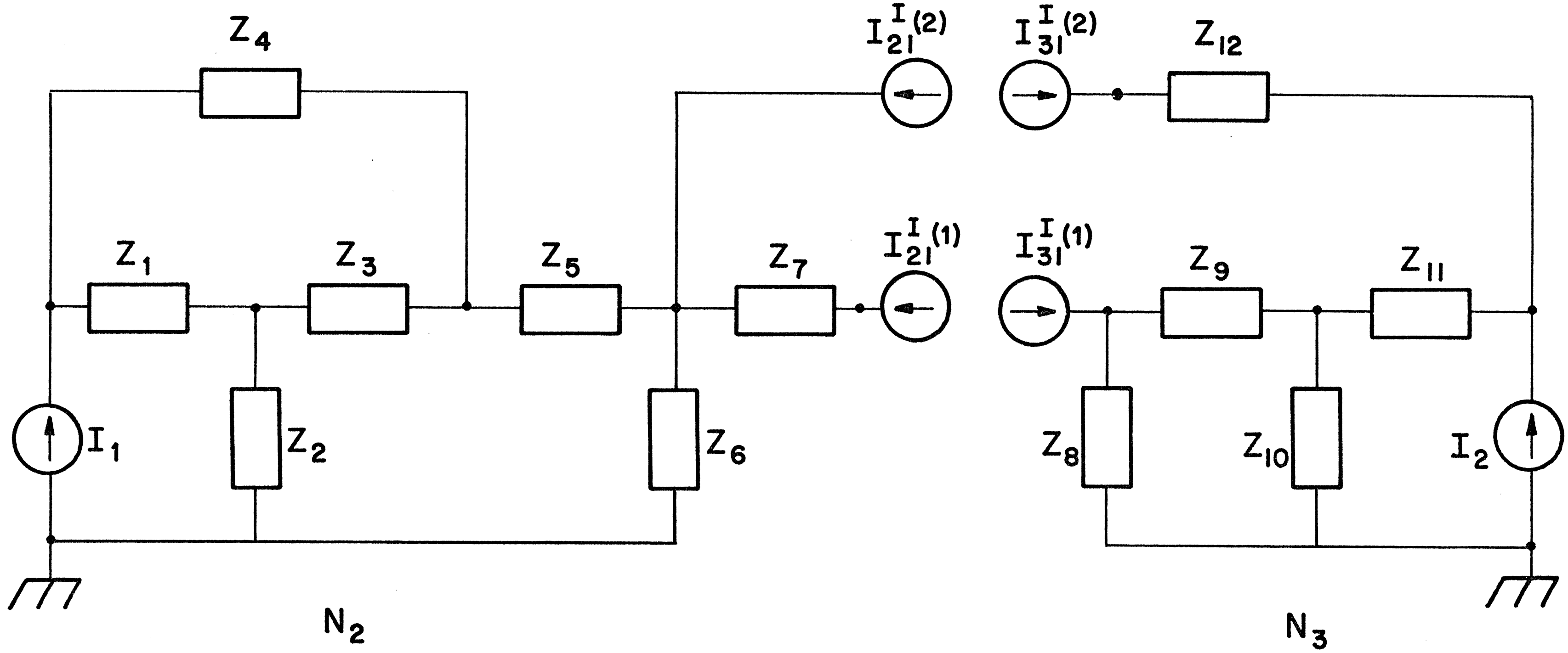

Fig. 8(a) Subnetworks N_2 and N_3.

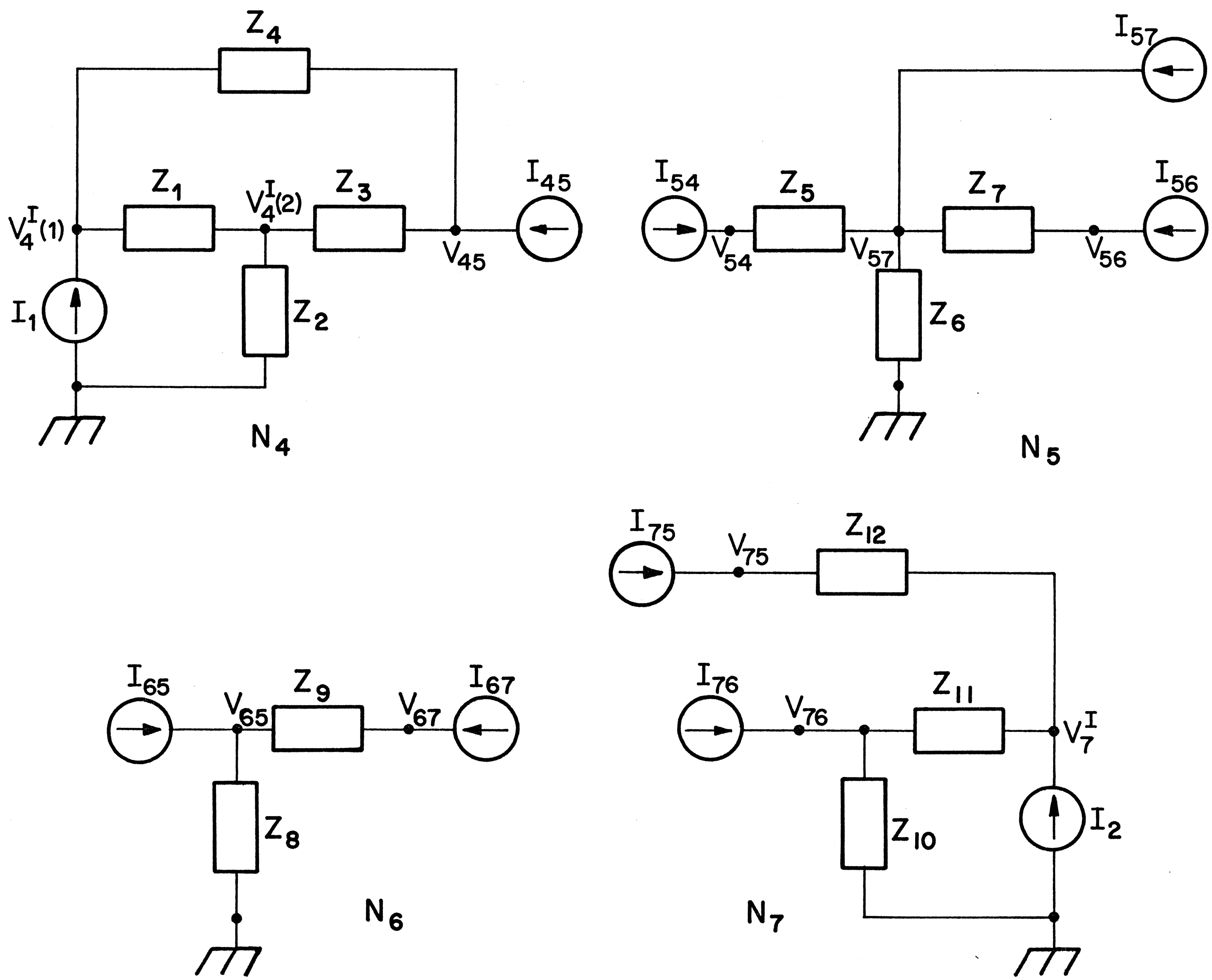

Fig. 8(b) Blocks N_4,N_5,N_6 and N_7.

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A HIERARCHICAL DECOMPOSITION APPROACH FOR NETWORK ANALYSIS

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Key Words: Network decomposition, hierarchical analysis of linear networks, multilevel algorithms, subnetworks and multipoles

Abstract: A novel approach for analyzing large electrical networks is presented in which the network is decomposed into subnetworks in a hierarchical manner by removing few interconnections. These subnetworks are solved separately. The results are then interconnected at a number of computing levels. The solution of the original network is thereby obtained.

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