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No. SOC-280

MINI5W - A FORTRAN PACKAGE FOR MINIMAX OPTIMIZATION

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(Adapted and Edited by J.W. Bandler and W.M. Zuberek)

December 1981

# FACULTY OF ENGINEERING MCMASTER UNIVERSITY HAMILTON, ONTARIO, CANADA



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#### Abstract

MINI5W is a package of subroutines for solving unconstrained, nonlinear minimax optimization problems. First derivatives of all functions w.r.t. all variables are assumed to be known. The solution is found by an iteration that uses either linear programming applied in connection with first derivatives or a Newton step applied in connection with first derivatives and approximate second derivatives. The method has been described by Hald and Madsen. The Fortran IV package and documentation have been adapted for the CDC 170/730 system.

K. Madsen is with the Institute of Datalogy, University of Copenhagen, Copenhagen, Denmark.

J.W. Bandler and W.M. Zuberek are with the Group on Simulation, Optimization and Control and the Department of Electrical and Computer Engineering, McMaster University, Hamilton, Canada L8S 4L7.

W.M. Zuberek is on leave from the Institute of Computer Science, Technical University of Warsaw, Warsaw, Poland.

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#### I. INTRODUCTION

Prepared by J.W. Bandler and W.M. Zuberek

This report gives a user-oriented description of a program package for unconstrained minimax optimization of absolute values of a set of differentiable nonlinear functions. The package has been developed in Fortran IV by Kaj Madsen at the Institute of Datalogy of the University of Copenhagen\* and has been adapted for the CDC 170/730 (System B) installation at McMaster University. Sections II, IV-VII contain the body of Madsen's description edited and arranged for use at McMaster University. Also given is the listing (Appendix) and tests of the package.

The package is available as a permanent group file in the form of a library of binary relocatable subroutines. The name of the library is LIBRMMU. The package is linked with the user's program by the appropriate call of the main subroutine of the package, namely, subroutine MINI5W. The general sequence of NOS commands to use the package can be as follows:

/GET(LIBRMMU/GR) - fetch the library LIBRMMU, /LIBRARY(LIBRMMU) - indicate the library to the loader, /FTN(...,GO) - compile, load and execute the program.

- 2 -

K. Madsen, "Documentation of the Fortran subroutine MINI5W for minimax optimization", Inst. of Datalogy, University of Copenhagen, Copenhagen, Denmark, April 1981.

The user must prepare programs which should be composed (at least) of:

- the main segment, which prepares parameters and calls the main subroutine of the package (subroutine MINI5W).
- the segment which calculates the values of the nonlinear functions and their first derivatives w.r.t. all variables at points determined by the package; the name of this subroutine can be arbitrary because it is transferred to the package as one of the parameters.

#### **II. GENERAL DESCRIPTION**

The purpose of the package is to find a local minimum of the minimax objective function

$$F(x) = \max_{\substack{1 \le j \le m}} |f_j(x)|$$
,

where  $f_j(x)$ , j = 1, ..., m, is a set of nonlinear functions of the n-dimensional vector of variables x. First-order derivatives w.r.t. x are assumed to be available.

The solution is found by an iteration process that uses either linear programming applied in connection with the first derivatives, or a Newton step applied in connection with first derivatives and approximate second derivatives. For a description, see Hald and Madsen [1].

#### III. STRUCTURE OF THE PACKAGE

Prepared by J.W. Bandler and W.M. Zuberek

A block diagram of the package is shown in Fig. 1. The subroutine MINI5W is the main subroutine, the aim of which is only to subdivide the work space (defined by user) into a set of vectors and arrays used by

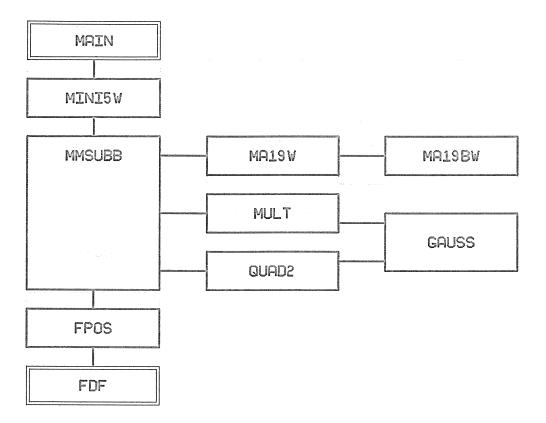


Fig. 1 Structure of the MINI5W package for unconstrained minimax optimization of absolute values of a set of nonlinear functions. the package. Minimax optimization is performed by MMSUBB, which calls FPOS (and FDF) for evaluation of the nonlinear functions and their first derivatives. Linear subproblems are solved by MA19BW and quadratic subproblems by QUAD2. MA19W is only used to simplify the call of MA19BW. MULT determines Lagrangian multipliers for linear subproblems. GAUSS is used to solve a system of linear equations.

The main program MAIN and the subroutine FDF must be supplied by the user.

#### IV. LIST OF ARGUMENTS

The main subroutine call is

#### CALL MINI5W (FDF, N, M, X, DX, EPS, MAXFUN, W, IW)

The arguments of this call statement are defined as follows.

```
FDF is the name of a subroutine written by the user. It must have the form
```

SUBROUTINE FDF(N,M,X,DF,F)

REAL X(N), DF(M,N), F(M)

and it must calculate the values of the nonlinear functions and their derivatives at the point x corresponding to X(1), X(2),...,X(N), and store these in the following way:

 $F(J) = f_{J}(x), \qquad J = 1,...,M,$ 

 $DF(J,I) = \partial f_{J} / \partial x_{T}(x), J = 1,...,M, I = 1,...,N.$ 

Note: The name of this user-supplied subroutine, which can be any name of the user's choice, must appear in an EXTERNAL statement in the calling program.

- N is an INTEGER variable and must be set to n, the number of optimization parameters. Its value must be positive, and it is not changed by the package.
- M is the INTEGER variable and must be set to m, the number of nonlinear functions defining the minimax objective function. Its value must be positive, and it is not changed by the package.
- X is a REAL array of length at least N. <u>On entry</u> it must be set by the user to an initial approximation of the solution,  $X(I) = x_{I}^{0}$ , I=1,...,N. <u>On exit</u> X contains the best solution found by the package.
- DX is a REAL variable which controls the step length of the iterative method used. It must be set by the user to an initial value corresponding to the starting vector  $\underline{x}^0$ . DX should be chosen so that in the region  $\{\underline{x} \mid \|\underline{x}-\underline{x}^0\| < DX\}$  the functions  $f_j$  can be approximated reasonably well by linear functions. If the functions are nearly linear, DX should be set to an approximate value of the distance between the starting vector and the solution, but if more curvature is present this value may be too large. In general,  $DX = 0.1^* \|\underline{x}^0\|$  is a reasonable choice. However, it is normally not severe to choose a bad initial value of DX, since DX is adjusted by the package during the iteration. The value of DX must be positive.
- EPS

is a REAL variable which must be set by the user to indicate the required accuracy of the solution. The iteration is stopped when the change  $\underline{h}^{k}$  of the approximate solution  $\underline{x}^{k}$  is smaller than EPS\* $\|\underline{x}^{k}\|$ . If EPS is chosen too small, the package will

stop when the calculation is dominated by rounding errors, and EPS will be set to 0.

- MAXFUN is an INTEGER variable. It must be set by the user, and its value gives an upper bound for the number of calls of FDF. If the number of calls required exceeds MAXFUN, the package will return and W(M+1) will be set to (MAXFUN+1).
- W is a REAL array which is used for workspace. Its length must be at least

(16\*N+4\*M+2\*M\*N+2\*N\*\*2+max{M,3\*N\*\*2+6\*N+5}+13).

<u>On entry</u> W(1) must be set to a positive value  $\delta$  which is used to determine the set of active functions at each iterate  $x^k$ . If

$$|f_{j}(x^{k}) - F(x^{k})| \leq \delta^{*}|F(x^{k})|$$

then  $f_j$  goes into the set of active functions. Normally  $\delta = W(1) = 0.01$  is an appropriate value. W(2) must be set to the number of iterations with identical set of active functions that is required before a Newton iteration is tried (see reference [1]). Normally W(2) = 3 is an appropriate value. If  $W(2) \geq MAXFUN$  then the Newton iteration is never used. <u>On exit</u> W will contain the function values at the solution, i.e.,

$$W(J) = f_{J}(x), \quad J = 1, ..., M.$$

Further, W(M+1) will give the number of calls of FDF used (see variable MAXFUN).

IW

is an INTEGER variable which must be set to the length of W.

### V. GENERAL INFORMATION

Use of COMMON:	None.
Workspace:	Provided by the user; see arguments W and IW.
Other subroutines:	MMSUBB, QUAD2, MULT, FPOS, GAUSS, MA19W, MA19BW.
Input/Output:	None.
Restrictions:	$N \ge 1$ , $M \ge 1$ , DX>0, EPS $\ge 0$ , MAXFUN>0.
Date:	March 1981.

#### VI. EXAMPLES

Example 1

Minimize

 $F(x) = max(|f_1(x)|, |f_2(x)|),$ 

where

$$f_{1}(x) = x_{1}^{2} + 2x_{2}^{2} + x_{1}x_{2},$$
  
$$f_{2}(x) = \sin(x_{1}) + \cos(x_{2}).$$

For this example,

```
N = 2
M = 2
IW = 98
```

The starting point is

$$\mathbf{x}^{0} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$
.

	PROGRAM TIRMMUCOUTPUT, TAPE3=OUTPUT)	00000010
C		00000020
C F	S. MADSEN – EXAMPLE	00000030
C		00000040
	DIMENSION X(2), W(98)	00000050
	EXTERNAL DFF	00000060
	COMMON NCOUNT	00000070
	NCOUNT=0	00000080
	N=2	00000090
	m=2	00000100
	X(1)=3.	00000110
	X(2)=1.	00000120
	DX=1.0	00000130
	EPS=1.0E-10	00000140
	MAXF=30	00000150
	IW=98	00000160
	W(1) = 0.01	00000170
	W(2) = 3.0	00000180
	WRITE(3,100) A COM 201	00000180
100	) FORMAT(" PROGRAM TIRMMU (K. MADSEN EXAMPLE) "//	00000190
100	1 13X, "X1", 12X, "X2", 15X, "F1", 12X, "F2")	00000200
	$\begin{array}{c} 1  15A,  AI  , 12A,  AZ  , 15A,  FI  , 12A,  FZ  , \\ CALL  SECOND(TM1)  \\ \end{array}$	00000210
	CALL MINISWOFF, N. M.X. DX, EPS, MAXF, W, IW)	00000220
		00000230
	CALL SECOND(TM2) CPU=TM2-TM1 LCALL=W(M+1) WRITE(3,200) CPU ) FORMAT(/" CPU TIME: ",F8.3, " SECONDS") WEITE(2,200) EPE LCALL	00000250
		00000250
		00000270
ഫെ	NETIES,2007 GLU	00000280
200	WRITE(3.300) EPS.LCALL	00000280
200	WITHER CO, COUP IN B, LUMAN.	00000290
000	) FORMAT(/" EPS VAL=",E13.6/" F.EVAL:=",I5) (%) (%) (%) (%) (%) (%) (%) (%) (%) (%	00000310
	) FORMAT(/" SOLUTION: ",2(/F20.10)//" FUNCTION VALUES: ",2(/F20.10)//	
200	STOP	00000330
	END	00000340
C		00000350
c		00000360
6	SUBROUTINE DFF(N, M, X, DF, F)	00000370
	DIMENSION $X(N)$ , $DF(M, N)$ , $F(M)$	00000380
	COMMON NCOUNT	00000390
	F(1)=X(1)**2+2*X(2)**2+X(1)*X(2)	00000390
	F(2) = SIN(X(1)) + COS(X(2))	
	$DF(1, 1) = 2 \times (1) + X(2)$	00000410
	$DF(1,2) = 4 \times X(2) + X(1)$	00000420
	DF(1,2) = 4*X(2) + X(1)	00000430 00000440
	DF(2,2) = -SIN(X(2))	
	NCOUNT=NCOUNT+1	00000450
	NGUUNI-NGUUNITI VDIWICO LOAN NCONNYCCVCIN ISI NN CECIN ISI WN	00000460
100	WRITE(3,100) NCOUNT;(X(I),I=1,N),(F(I),I=1,M) FORMAT(1X,I5,2(F13.8,1X),1X,2(1X,F13.8))	00000470
TOO	FURIALLIA, LJ, AVELJ, G, LAJ, LA, AVLA, FIG. GJJ ( )	00000480
	RETURN	00000490
	END	00000500

#### PROGRAM TIRMMUR(K. MADSEN EXAMPLE)

	X1	X2		F1	F2	
1	3.0000000	1.00000000	69 C :-	14.0000000	.68142231	<i>50</i>
2	2.31450415	0.00000000	\$ C.	5.35692946	1.73596335	$X^{(2)}$
3	2.06828201 -	1.00000000		4.20950847	1.41908752	41 -
4	1.06828201	1.29690028	. 29	3.11967189	1.14685866	( 2)
5	.06828201	97915801	$\sim r$	1.85530438	.62595058	14
6	93171799	32302946		1.37776682	.14563227	
7	-1.40213805	.67697054	53	1.93336319	20633683	500
8	94555372	.17697054	1.1	.78937382	.17356040	5 ×
9	69143806	.68801162	S.	.94908915	.13486454	10.00
10	69555372	.42697054	44	.55142171	.26941384	24
11	73863115	.17697054	11	.47749717	.31110511	S5 1
12	63924467	.42697054	김 오.	.50030280	.31363504	19 E - 1
13	64814920	.30197054	- Q (* -	.40674784	.35104032	£35
14	67685894	.17697054	λY.	.40099108	.35803401	经生产
15	64828496	.23947054	25	.37972052	.36764349	
16	64244065	.23720201	24	.37287137	.37284796	251
17	64232155	.23756544	20	.37285825	.37285786	6.1
18	64233723	.23751138	$\geq_{\rm H^{-}}$	.37285803	.37285802	27
19	64233723	.23751138	- 2 yr	.37285803	.37285803	Si V
20	64233723	.23751138	20	.37285803	.37285803	is ₹

CPU TIME: . 151 SECONDS

EPS VAL= .100000E-09 F.EVAL.= 20

SOLUTION:

-.6423372301 .2375113809

FUNCTION VALUES:

.3728580268 .3728580268 Example 2 (Prepared by J.W. Bandler and W.M. Zuberek)

This is the design of a 3-section 100-percent relative bandwidth 10:1 transmission-line transformer problem [2,3]. In this case the error functions  $f_i$  represent the modulus of the reflection coefficient sampled at the 11 normalized frequencies (w.r.t. 1 GHz)

 $\{ 0.5, 0.6, 0.7, 0.77, 0.9, 1.0, 1.1, 1.23, 1.3, 1.4, 1.5 \}.$ 

Gradient vectors with respect to section lengths and characteristic impedances are obtained using the adjoint network method.

The known quarter-wave solution is given by

$$\ell_{1} = \ell_{2} = \ell_{3} = \ell_{q},$$

$$Z_{1} = 1.63471,$$

$$Z_{2} = 3.16228,$$

$$Z_{3} = 6.11729,$$

where  $\boldsymbol{\textbf{l}}_q$  is the quarter wavelength at centre frequency, namely

$$l_{q} = 7.49481 \text{ cm} \text{ for } 1 \text{ GHz}.$$

The corresponding maximum reflection coefficient is 0.19729. The vector of optimization parameters is

$$x = \begin{bmatrix} {}^{\ell} 1^{/\ell} q \\ {}^{Z} 1 \\ {}^{\ell} 2^{/\ell} q \\ {}^{Z} 2 \\ {}^{\ell} 3^{/\ell} q \\ {}^{Z} 3 \end{bmatrix}$$

For this example,

N = 6 M = 11 IW = 506

The starting point is

	PROGRAM T2RMMU(OUTPUT, TAPE1=OUTPUT)	000010
		000020
	3-SECTION MICROWAVE TRANSMISSION-LINE TRANSFORMER	000030
	DESTRUCTION MACH IN FOCK	000040
	DIMENSION X(6), W(506)	000050
	EXTERNAL FGR	000060
	COMMON NCOUNT, RG, RL, FREQ(11) DATA X/0.8, 1.5, 1.2, 3.0, 0.8, 6.0/	000070 000080
	NCOUNT=0	000030
	RG=1.0	000100
	RL=10.0	000110
	FREQ(1)=0.5	000120
		000130
	FREQ(2)=0.6 FREQ(3)=0.7	000140
	FREQ(4) = 0.77	000150
		000160
	FREQ(6) = 1.0	000170
	FREQ(5)=0.9 FREQ(6)=1.0 FREQ(7)=1.1	000180
	FREQ(8) = 1.23	000190
	FREQ(9) = 1.3	000200
	FREQ(10) = 1.4	000210
	FREQ(11)=1.5	000220
	N=6	000230
	M= 1 1	000240
	DX=0.1	000250
	DX=0.1 EPS=1.0E-7	000260
	MAXF=150	000270
	AX=0.01	000280
	KEQS=2	000290
	W(1)=AX	000300
	W(2)=KEQS	000310
	IW=506	000320
	WRITE(1,100)	000330
100	FORMAT( " PROGRAM T2RMMU (3-SECTION (MICROWAVE TRANSFORMER) "/)	
	CALL SECOND(T1)	000350
	CALL SECOND(T1) CALL MINI5W(FGR, N, M, X, DX, EPS, MAXF, W, IW)	000360
	GALL SECONDE 121	000370
	CPU=T2-T1	000380
~~~	WRITE(1,200) CPU FORMAT("OCPU TIME: ",F8.3," SECONDS")	000390
200	FORMAT "OCPUTIME: ", F8.3, "SECONDS")	000400
	NFC=W(M+1)	000410
000	WRITE(1,300) EPS,NFC FORMAT("0EPS VAL=",E13.6/" F.EVAL.=",I5) WRITE(1,400) (X(I),I=1,N),(W(I),I=1,M)	000420
300	FURNARY VER'S VAL-", E10.07" F.EVAL-", 107	$000430 \\ 000440$
400	FORMAT( "OSOLUTION: ", 6(/F20.8)/"OFUNCTION VALUES: ", 11(/F20.8)//)	
499	STOP	000450
	END	000470
	ETATA	000480
		000-TOO

C C C

C

		000400
	SUBBOUTINE FCB(N. M. X. DF. F)	000490 000500
	SUBROUTINE FGR(N, M, X, DF, F) DIMENSION X(N), DF(M, N), F(M)	000510
	COMPLEX AI(4), V(4), A, B, C, RH, CRH, VG, TVG	000520
	DIMENSION AL(3), TH(3), Z(3)	000530
	COMMON NCOUNT, RG, RL, FREQ(11)	000540
	DATA FACT1, FACT2/7.4948125, 0.2095844728/	000550
	NS=3	000560
	NS1=NS+1	000570
	DO 40 I=1,M	000580
	Beta=Fact2*FREQ(1)	000590
	ALFA=FACT1*BETA	000600 000610
	DO 10 J=1,NS JJ=J+J	000620
	$AL(J) = FACT1 \times X(JJ-1)$	000630
	Z(J) = X(JJ)	000640
10	CONTINUE	000650
2.0	AI(NS1) = CMPLX(1.0,0.0)	000660
	V(NS1) = CMPLX(RL, 0.0)	000670
	D0 20 J=1,NS	000680
	K=NS1-J	000690
	K1=K+1	000700
	T=BETA*AL(K)	000710
	TH(K) = T	000720
	CT=COS(T)	000730
	ST=SIN(T)	000740
	A=CMPLX(CT, 0.0) B=CMPLX(0.0, Z(K) *ST) C=CMPLX(0.0, ST/Z(K)) V(K) = A*V(K1) + B*AI(K1)	000750 000760
	D - CIFLA(V, U, A(X) + SI)	000770
	G = G = G = G = G = G = G = G = G = G =	000780
	AI(K) = C*V(K1) + A*AI(K1)	000790
20	CONTINUE	000800
	VG=V(1)+AI(1)*RG	000810
	TVG=(RG+RG)/VG	000820
	RH=1.0-AI(1)*TVG	000830
	CRH=TVG*CONJG(RH)	000840
	FM=CABS(RH)	000850
	F(I)=FM	000860
	DO 30 J=1, NS	000870
	T=TH(J)	000880
	JJ=J+J J1=J+1	000890 000900
	$J_{2}=J_{2}-1$	000910
	DF(I,JJ)=REAL(CRH*(V(J)*AI(J)+V(J1)*AI(J1))/VG)/(Z(J)*FM) 20	000920
	DF(1, J2) = ALFA*REAL(CRH*(V(J)*AI(J1)-V(J1)*AI(J))/VG)/(SIN(T)*FM)	000930
30	CONTINUE	000940
	CONTINUE	000950
	FM=0.0	000960
	DO 50 I=1,M	000970
	IF(F(I).GT.FM)(FM=F(I))	000980
50	CONTINUE	000990
	IF(NCOUNT.EQ.0) WRITE(1,200)	001000
	FORMAT(//13X, "X1", 12X, "X2", 12X, "X3", 12X, "X4", 12X, "X5", 12X, "X6",	001010
	1 12X, "MAX(F)")	001020
	NCOUNT=NCOUNT+1 WRITE(1,100) NCOUNT,(X(I),I=1,N),FM	001030
100	FORMAT(1X, 15, 6(F13.8, 1X), 1X, F13.8, 2X, 11A1)	$001040 \\ 001050$
199	RETURN	001050
	END	001070

C

PROCRAM TZRMMU (3-SECTION(MICROWAVE TRANSFORMER)

	1															ŝ, i		
		j.		i K		il X			i. A	( ,	₹.× 1. tonar	n de Refe					.)0 	in and
MAX(F)	.38813233	. 25859383	.27440370	.20190611	.27854952	.23975004	.20909191	.20154102	.20260379	.19802040	.19789703	.19761384	. 19743702	, 19729193	.19729074	. 19729063	.19729063	.19729063
X6	6.00000000		5.9000000	6.0000000	5.80000000	5.9000000	5.9500000 · · · ·	5.97500000	5.95116460	5.96002514	5.97164943	6.03551229	6.11381691	6.11701215	6.11734875	6.11730887	6.11730393	. 6.11730369
X5	. 800000000	000000000000000000000000000000000000000	1.100000000	1.000000000	1.08792370	1.06102097	1.03051281	1.01525873	1.00311132	:999916669	.99983466	1.00041262	1:00025859	1.00007439	1:00000434	1.00000005	1.000000000	1.0000000
X4	3.000000000	2.90000000	3.10000000	3.000000000	3.200000000	3.10000000	3.050000000	3.02500000	3.07531822	3.07621267	3.08463835	3.12193821	3.16262159	3.16279629	3.16233863	3.16228041	3.16227774	3.16227766
CX3	1.20000000	1.10000000	.96590917	1.00117844	1.00902065	1.00627271	1.00313508	1.00156626	. 99822726	1.00008572	1.00001135	.999996044	1.00000664	1.00000073	1.00000007	1.00000000	1.00000000	1.00000000
X	1.50000000	1.51604930	0102002010	1.60944075	1.55971115	1.58268512	1,59425478	1.60003961	1.59249966	1.59399712	1.59682950	0.1.61283336	1.63349063	501.63463073	1.63471930	1.63470852	1.63470720	<pre>2 1.63470714</pre>
X1	. 800000000	. 900000000	.92577041	1.000000000	.85589970	. 900000000.	.95000000	00000226.	1.00430879	1.00088815	1.00016800	. 99955105	20672666.	. 99992489	. 99999551	266666666	1.000000000	1.00000000
	Ĩ	01	တ	ፍ	ю	9	67	63	6	10	Ĩ Ì	0 2	13	14	ю Т	16	2 F	13

.704 SECONDS CPU TIME:

. 100000E-06 18 EPS VAL= F.EVAL.=

SOLUTION:

1.00000000 1.63470714 1.00000000 3.16227766 1.00000000 6.11730369

FUNCTION VALUES: . 19729063 . 03946026 . 17197713 . 19729063 . 19729063 . 19729063 . 19729063 . 19729063 . 19729063 . 19729063 . 19729063

# $x^{0} = [0.8 \ 1.5 \ 1.2 \ 3.0 \ 0.8 \ 6.0]^{T}$ .

#### VII. REFERENCES

- J. Hald and K. Madsen, "Combined LP and quasi-Newton methods for minimax optimization", <u>Mathematical Programming</u>, vol. 20, 1981, pp. 49-62.
- [2] J.W. Bandler, T.V. Srinivasan and C. Charalambous, "Minimax optimization of networks by grazor search", <u>IEEE Trans. Microwave</u> Theory Tech., vol. MTT-20, 1972, pp. 596-604.
- [3] J.W. Bandler and D. Sinha, "FLOPT5 a program for minimax optimization using the accelerated least pth algorithm", Faculty of Engineering, McMaster University, Hamilton, Canada, Report SOC-218, 1978.

6

# APPENDIX

## LISTING OF THE MINI5W PACKAGE

Subroutine	Number of lines	Number of words	Listing from page
	(source text)	(compiled code)	
MIN15W	35	172	17
MMSUBB	257	1043	17
QUAD2	67	314	21
MULT	36	201	22
FPOS	10	75	23
GAUSS	83	317	23
MA 19W	15	103	24
MA19BW	411	1562	24

SUBROUTINE MINISW (FDF, N, M, X, RDX, EPS, MAXFUN, W, IW) 000010 000020 MINIMAX OPTIMIZATION USING QUADRATIC PROGRAMMING. OF 000030 KAJ MADSEN, NUMERISK INSTITUT, 000040 THE TECHNICAL UNIVERSITY OF DENMARK, LYNGBY, DENMARK. 000050 MARCH 1981. 000060 000070 DIMENSION X(N), W(IW) 000080 000090 IW MUST BE AT LEAST 16N+4M+2MN+2N\*\*2+MAX(M, 3N\*\*2+6N+5)+13 000100 000110 EXTERNAL FDF 000120 N1 = N+1000130 N2=N+2 000140 IIND1=5\*N1+M 000150 IWO=MAX0(N1\*(N+5)+M;(2\*N+3)\*\*2+1) 000160 IRLAM=2\*N1 000170 NFO = 1000180 NF1=NFØ+M 000190 NDFØ=NF1+M 000200 NDF1=NDF0+M\*N 000210 NX1=NDF1+M\*N 000220 NH=NX1+N 000230 NB=NH+N2 000240 NIND1=NB+N\*N 000250 NINDØ=NIND1+IIND1 000260 NWO=NINDO+M 000270 NY=NWO+IWO 000280 NBH=NY+N 000290 NRLAM=NBH+N 000300 CALL MMSUBB (FDF, N, M, X, RDX, EPS, MAXFUN, W(NF0), W(NF1), W(NDF0), W(NDF1 000310 1), W(NX1), W(NH), W(NB), W(NIND1), W(NINDØ), IIND1, W(NWO), IWO, W(NY), W(NB 000320 2H), W(NRLAM), IRLAM, N1, N2) 000330 RETURN 000340 END 000350 000360 000370 SUBROUTINE MMSUBB (FDF, N, M, X0, RDX, EPS, MAXFUN, F0, F1, DF0, DF1, X1, H, B, 000380 1 IND1, IND0, I IND1, WO, IWO, Y, BH, RLAM, IRLAM, N1, N2) 000390 EXTERNAL FDF 000400 DIMENSION X0(N), F0(M), F1(M), DF0(M,N), DF1(M,N), X1(N), H(N2), B 1(N,N), IND1(IIND1), IND0(M), W0(IWO), Y(N), BH(N), RLAM(N1) 000410 000420 LOGICAL X10K, NEWTON 000430 000440 XØ IS THE CURRENT APPROXIMATION OF THE SOLUTION. 000450 ARE THE CORRESPONDING SETS OF FUNCTION VALUES AND DERIVA-FØ.DFØ 000460 TIVES. 000470 X1IS THE CURRENT APPROXIMATION OF THE SOLUTION 000480 ARE THE CORRESPONDING SETS OF FUNCTION VALUES AND DERIVA-F1,DF1 000490 TIVES. 000500 Н IS THE TRIAL INCREMENT TO ADD TO XO. 000510 HOLDS THE INDICES CORRESPONDING TO ACTIVE CONSTRAINTS AT INDØ 000520 X0 - WHEN APPROPRIATE: (THERE IS KACTO OF THESE). 000530 IND1 AS INDO, BUT AT THE POINT X1. HOLDS AN APPROXIMATION TO THE LAGRANGE MULTIPLIERS -000540 RLAM 000550 WHEN APPROPRIATE. 000560 IS THE APPROXIMATE HESSIAN, UPDATED BY POWELL'S METHOD. B 000570 Y, BH, WO ARE WORK AREAS. 000580 X10K IS TRUE IF X1 IS ACCEPTED AS A NEW ITERATE. 000590 IS TRUE IF THE NEXT STEP OF THE ITERATION WILL BE A NEWTON 000600 NEWTON STEP: 000610 000620 EPSFL IS THE SMALLEST MACHINE NUMBER X FOR WHICH 1+X>1 000630 000640 EPSFL=16.E0\*\*(-12) 000650

C

C

C

C

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C

C

C

C C

C

G

```
IW02=SQRT(IW0+0.E0)
                                                                               000660
      NEWTON=.FALSE.
                                                                               000670
      KACT0=0
                                                                               000680
      RDXF IX=RDX
                                                                               000690
      DEL=F0(1)
                                                                               000700
      KBOUND=F0(2)
                                                                               000710
      KBOUND=MAX0(KBOUND,2)
                                                                               000720
      NTAL=0
                                                                               000730
                                                                               000740
        FIND THE LENGTH OF THE STARTING VECTOR
                                                                               000750
C
                                                                               000760
      X0MAX=0.E0
                                                                               000770
      DO 1 I=1,N
                                                                               000780
      XOMAX=AMAX1(XOMAX,ABS(XO(I)))
                                                                               000790
    1 CONTINUE
                                                                               000800
C
                                                                               000810
C
        CALCULATE FUNCTION VALUES
                                                                               000820
\mathbf{C}
                                                                               000830
      CALL FPOS (FDF, N, M, X0, DF0, F0) ×
                                                                               000840
      NTAL=NTAL+1
                                                                               000850
      FOMAX=0.E0
                                                                               000860
      DO 2 J=1,M
                                                                               000870
      IF (FO(J).GT.FOMAX) FOMAX=FO(J)
                                                                               000880
    2 CONTINUE
                                                                               000890
\mathbf{C}
                                                                               000900
C
        INITIALIZE ARRAYS
                                                                               000910
C
                                                                               000920
      DO 4 I=1,N
                                                                               000930
      DO 3 J=1,N
                                                                               000940
      B(I,J)=0.E0
                                                                               000950
    3 CONTINUE
                                                                               000960
      B(I,I)=1.E0
                                                                               000970
    4 CONTINUE
                                                                               000980
C
                                                                               000990
C
        ITERATIVE LOOP STARTS HERE
                                                                               001000
C
                                                                               001010
    5 CONTINUE
                                                                               001020
C
                                                                               001030
          FIND THE SOLUTION H OF THE LINEAR OR QUADRATIC SUBPROBLEM
\mathbb{C}
                                                                               001040
          F1PRED IS THE MINIMUM PREDICTED BY THE SUBPROBLEM
C
                                                                               001050
G
                                                                               001060
      IF (NEWTON) GO TO 7
                                                                               001070
    6 CALL MA19W (N, M, DF0; M, F0, RDX, 0. E0, H, W0, IND1, IW0, IIND1)
                                                                               001080
      F1PRED=WO(M+1)
                                                                               001090
      GO TO 8
                                                                               001100
    7 CONTINUE
                                                                               001110
      CALL QUAD2 (N, M, DF0, F0, B, H, RLAM, N1, IND1, KACT1, W0, IW02, EPSFL)
                                                                               001120
      F1PRED=H(N1)
                                                                               001130
    8 CONTINUE
                                                                               001140
      IF (F1PRED.GT.FØMAX) GO TO 38)
                                                                               001150
C
                                                                               001160
C
C
          FIND THE NORM OF H, AND FIND THE POINT X+H
                                                                               001170
                                                                               001180
      HMAX=0.E0
                                                                               001190
      DO 9 I=1,N
                                                                               001200
      HMAX=AMAX1(HMAX,ABS(H(I)))
                                                                               001210
      X1(I)=X0(I)+H(I)
                                                                               001220
    9 CONTINUE
                                                                               001230
С
                                                                               001240
G
C
           IF THE STEP LENGTH IS TOOCLARGE UNDER THE NEWTON ITERATION
                                                                               001250
          THEN USE THE LP DIRECTION
                                                                               001260
C
                                                                               001270
      IF ((HMAX.LE.RDXFIX).OR.(.NOT)NEWTON)) GO TO 10
                                                                               001280
      NEWTON=.FALSE.
                                                                               001290
      GO TO 6
                                                                               001300
```

```
С
\mathbf{C}
```

C 001310 FIND THE NEW FUNCTION VALUES  $\mathbf{C}$ 001320 C 001330 10 CALL FPOS (FDF, N, M, X1, DF1, F1) 001340 NTAL=NTAL+1 001350 F1MAX=0.E0 001360 DO 11 J=1,M 001370 F1MAX=AMAX1(F1MAX,F1(J)) 001380 **11 CONTINUE** 001390  $\mathbf{C}$ 001400  $\mathbf{C}$ TEST IF THE NEW POINT IS ACCEPTABLE 001410 С 001420 X10K=(FOMAX-F1MAX).GE.0.01\*(FOMAX-F1PRED) 001430 C 001440 FIND THE SET OF ACTIVE FUNCTIONS  $\mathbb{C}$ 001450 C 001460 KACT1=0 001470 DO 13 J=1,M 001480 SUM=F0(J) 001490 DO 12 I=1,N 001500 SUM=SUM+DF0(J,I)\*H(I) 001510 12 CONTINUE 001520 IF (ABS(SUM-F1PRED),GT.DEL\*ABS(F1PRED))%G0 T0 13 001530 KACT1=KACT1+1 001540 IND1(KAGT1)=J 001550 **13 CONTINUE** 001560 C 001570  $\mathbb{C}$ FIND THE MULTIPLIERS RLAM 001580 C 001590 IF (.NOT.NEWTON) CALL MULT (N,M, DFØ, DF1, X10K, IND1, N1, KACT1, RLAM, WO 001600 1, IWO2, EPSFL) 001610  $\mathbf{C}$ 001620  $\mathbb{C}$ FIND Y: THE DIFFERENCE IN THE LAGRANGIAN GRADIENTS 001630 C 001640 DO 14 I=1,N 001650 Y(I)=0.E0 001660 14 CONTINUE 001670 DO 16 J=1,KACT1 001680 JK=IND1(J) 001690 DO 15 I=1.N 001700 Y(I)=Y(I)+RLAM(J)\*(DF1(JK,I)-DF0(JK,I)) 001710 **15 CONTINUE** 001720 **16 CONTINUE** 001730 C 001740 ADJUST THE LOCAL BOUND RDX C 001750 C 001760 IF ((FOMAX-F1MAX).LE.0.25\*(FOMAX-F1PRED)) RDX=RDX/2. 001770 IF ((FØMAX-F1MAX).GE.0.75\*(FØMAX-F1PRED)) RDX=RDX\*2. 001780  $\mathbf{C}$ 001790 TEST FOR NEWTON ITERATION G 001800 C 001810 IF (KACT1.GT.N1) KACT0=0 001820 IF (KACTI.EQ.KACTØ) GO TO 18 001830 KACTØ=KACT1 001840 DO 17 I=1, KACT1 001850 INDØ(I)=IND1(I) 001860 **17 CONTINUE** 001870 KEQUAL=0 001880 GO TO 24 001890 **18** CONTINUE 001900 KEQUAL=KEQUAL+1 001910 DO 19 I=1,KACT1 001920 IF ((IND1(I).EQ.IND0(I)).AND.(RLAM(I).GE.0E0)) GOSTO 19 001930 INDØ(I)=IND1(I) 001940 KEQUAL=0 001950

**19 CONTINUE** 001960 IF (KEQUAL.LT. (KEOUND-1)) CO TO 24) 001970 C 001980 C FIND THE RESIDUAL-NORM OF THE SET OF NON-LINEAR EQUATIONS 001990  $\mathbf{C}$ 002000 RES=0.E0 002010 DO 21 I=1,N 002020 S=0.E0 002030 DO 20 J=1,KACT1 002040 JK=IND1(J) 002050 S=S+RLAM(J)\*DF1(JK, I) 002060 20 CONTINUE 002070 RES=AMAX1(RES,ABS(S)) 002080 **21 CONTINUE** 002090 DO 22 J=1,KACT1 002100 JK=IND1(J) 002110 RES=AMAX1(RES,F1MAX4F1(JK)) 002120 22 CONTINUE 002130 IF (KEQUAL.GE.KBOUND) GO TO 23) 002140 RES0=RES 002150 CO TO 24 23 IF (RES.LE.0.999\*RES0) GO TO 25 002160 002170 24 NEWTON=.FALSE. 002180 GO TO 26 002190 25 NEWTON=.TRUE. 002200 RES0=RES 002210 C 002220 С INTRODUCE THE NEW POINT IF IT IS ACCEPTABLE 002230 C 002240 26 IF ((.NOT.X10K).AND.(.NOT.NEWTON)) GO TO 30 002250 FOMAX=F1MAX 002260 XOMAX=0.E0 002270 DO 28 I=1,N 002280 X0(I)=X1(I) 002290 XOMAX=AMAX1(XOMAX,ABS(XO(I))) 002300 DO 27 J=1,M 002310 DFO(J, I) = DF1(J, I)002320 **27** CONTINUE 002330 **28** CONTINUE 002340DO 29 J=1,M 002350 FO(J) = F1(J)002360 **29** CONTINUE 002370 **30 CONTINUE** 002380 C 002390 C C ADJUST THE MATRIX B USING POWELL'S METHOD 002400 002410 C FIND BH AND YH 002420 C 002430 YH=0 002440 DO 32 J=1,N 002450 YH=YH+Y(J)\*H(J)002460 SUMB=0.E0 002470 DO 31 I=1,N 002480 SUMB=SUMB+B(J,I)\*H(I) 002490 **31 CONTINUE** 002500 BH(J) = SUMB002510 32 CONTINUE 002520 C 002530 C FIND T AND SEE IF THETA IS LESS THAN 1 002540 C 002550 T=0.E0 002560 DO 33 I=1.N 002570 T=T+H(I) \*BH(I) 002580 **33 CONTINUE** 002590 IF (YH.GE.0.2E0\*T) GO TO 35 002600

```
THETA=0.8E0*T/(T-YH)
                                                                                 002610
C
                                                                                 002620
         IF THETA IS TOO SMALL WE DON'T ALTER MATRIX B
\mathbf{C}
                                                                                 002630
C
                                                                                 002640
       IF (THETA.LT.0.50E0) GO TO 37)
                                                                                 002650
      S=1E0-THETA
                                                                                 002660
       YH=0.E0
                                                                                 002670
      DO 34 I=1.N
                                                                                 002680
       Y(I) = THETA*Y(I) + S*BH(I)
                                                                                 002690
      YH=YH+Y(1) *H(1)
                                                                                 002700
   34 CONTINUE
                                                                                 002710
   35 CONTINUE
                                                                                 002720
\mathbb{C}
                                                                                 002730
G
         FINALLY WE CAN CALCULATE THE NEW B
                                                                                 002740
C
                                                                                 002750
      DO 36 I=1,N
                                                                                 002760
      S1=BH(I)/T
                                                                                 002770
      S2=Y(I)/YH
                                                                                 002780
      DO 36 J=I,N
                                                                                 002790
      B(I,J)=B(I,J)-S1*BH(J)+S2*Y(J)
                                                                                 002800
   36 B(J,I) = B(I,J)
                                                                                 002810
\mathbb{C}
                                                                                 002820
   37 CONTINUE
                                                                                 002830
G
                                                                                 002840
C
        TEST THE STOPPING CRITERION
                                                                                 002850
\mathbf{C}
                                                                                 002860
      IF ((NTAL.LT. MAXFUN) AND (HMAX.GT. XOMAX*EPS)) GO TO 5
                                                                                 002870
C
                                                                                 002880
      IF (HMAX.GT.X0MAX*EPS) NTAL=NTAL+1
                                                                                 002890
      GO TO 39
                                                                                 002900
   38 EPS=0.E0
                                                                                 002910
   39 F1(1)=NTAL
                                                                                 002920
      RETURN
                                                                                 002930
      END
                                                                                 002940
C
                                                                                 002950
C
                                                                                 002960
      SUBROUTINE QUAD2 (N,M, DF0, F0, B, H, RLAM, N1, IND, KACT1, W, IW2, EPSFL)
                                                                                 002970
C
                                                                                 002980
C
                        MINIMIZE
                                    DELTA+HBH/2
                                                                                 002990
C
                        SUBJECT TO FO+DFO^H = DELTA
                                                                                 003000
                        FOR THE INDICES IN IND.
C
C
C
                                                                                 003010
           IND
                GIVES THE INDICES CORRESPONDING TO ACTIVE CONSTRAINTS.
                                                                                 003020
                THERE IS IND(N+2) OF THESE.
                                                                                 003030
Ċ
C
                   N+I.
           NI
                TS
                                                                                 003040
                MUST BE AT LEAST 2*N+2
           IW
                                                                                 003050
C
                                                                                 003060
      DIMENSION FO(M), DFO(M,N), B(N,N), H(N1), RLAM(N1), IND(N1), W(IW2
                                                                                 003070
     1, IW2)
                                                                                 003080
      NS=N+KACT1
                                                                                 003090
      NTOT=NS+1
                                                                                 003100
      NTOT1=NTOT+1
                                                                                 003110
      EPS1=EPSFL*10*NTOT
                                                                                 003120
C
                                                                                 003130
G
G
G
     SET UP THE THE LINEAR SYSTEM
                                                                                 003140
     1: THE MATRIX (IT IS SYMMETRIC)
                                                                                 003150
                                                                                 003160
      DO 1 I=1,N
                                                                                 003170
      W(I, NTOT) = 0.E0
                                                                                 003180
      W(NTOT, I) = 0. E0
                                                                                 003190
      DO 1 J=1, I
                                                                                 003200
      W(I, J) = B(I, J)
                                                                                 003210
      W(J,I) = W(I,J)
                                                                                 003220
    1 CONTINUE
                                                                                 003230
      DO 3 J=N1,NS
                                                                                 003240
      W(J, NTOT) = -1.E0
                                                                                 003250
```

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W(NTOT, J) = -1.E0003260 DO 2 I=N1,J 003270 W(I,J)=0.E0 003280 W(J,I) = 0.E0003290 2 CONTINUE 003300 DO 3 I=1,N 003310 JA=IND(J-N) 003320 W(J, I) = DFO(JA, I)003330 W(I,J) = W(J,I)003340 **3** CONTINUE 003350 W(NTOT, NTOT) = 0.E0003360 003370 2: RIGHT HAND SIDE 003380 003390 DO 4 I=1,N 003400 W(I, NTOT1) = 0.E0 003410 4 CONTINUE 003420 DO 5 J=N1,NS 003430 JA=IND(J-N) 003440 W(J, NTOT1) = -FO(JA)003450 **5** CONTINUE 003460 W(NTOT, NTOT1) =-1.E00 003470 003480 SOLVE THE LINEAR SYSTEM 003490 003500 CALL GAUSS (W, IW2, NTOT, NTOT1, EPS1) 003510 003520 STORE THE SOLUTION IN H AND RLAM 003530 003540 DO 6 I=1,N 003550 H(I) = W(I, NTOT1) 003560 6 CONTINUE 003570 H(N1) = W(NTOT, NTOT1) 003580 DO 7 J=1, KACT1 003590 RLAM(J) = W(N+J, NTOT1)003600 7 CONTINUE 003610 RETURN 003620 END 003630 003640 003650 SUBROUTINE MULT (N, M, DFØ, DF1, X10K, IND1, N1, KACT1, RLAM, W, IW2, EPSFL) 003660 003670 FIND THE MULTIPLIERS RLAM BY A LEAST SQUARES CALCULATION, SUB-003680 JECT TO THE CONSTRAINT THAT THE SUM OF THE MULTIPLIERS IS 1. 003690 003700 DIMENSION DF0(M,N), DF1(M,N), IND1(N1), RLAM(KACT1), W(IW2,IW2) 003710 LOGICAL X10K 003720 K1=KACT1+1 003730 K2=KACT1+2 003740 EPS1=EPSFL\*10\*K1 003750 DO 4 I=1, KACT1 003760 IK=IND1(I) 003770 DO 3 J=1.I 003780 JK=IND1(J) 003790 S=0.E0 003800 DO 2 L=1,N 003810 IF (X10K) GO TO 1 003820 S=S+DF0(IK,L)\*DF0(JK,L) 003830 GO TO 2 003840 1 S=S+DF1(IK,L)\*DF1(JK,L) 003850 2 CONTINUE 003860 W(I,J) = S003870 , W(J, I) = S003880 3 CONTINUE 003890 W(I, K1) = -1.E0003900

C C C

> $\mathbb{C}$ C  $\mathbb{C}$

C  $\mathbb{C}$  $\mathbf{C}$ 

C  $\mathbf{C}$ 

W(K1,I)=1.E0 003910 W(I,K2)=0.E0 003920 4 CONTINUE 003930 W(K1,K1)=0.E0 003940 W(K1, K2) = 1.E0003950 CALL GAUSS (W, IW2, K1, K2, EPS1) 003960 DO 5 I=1, KACTÍ RLAM(I)=W(I, K2) 003970 003980 5 CONTINUE 003990 RETURN 004000 END 004010 C 004020 004030 SUBROUTINE FPOS (FDF, N, M, X, DF, F) 004040 DIMENSION X(N), DF(M,N), F(M) 004050 CALL FDF (N, M, X, DF, F) 004060 DO 2 J=1,M 004070 IF (F(J).GE.0E0) GOTO 2 004080 F(J) = -F(J)004090 DO 1 I=1,N DF(J,I)=-DF(J,I) 004100 004110 1 CONTINUE 004120 2 CONTINUE 004130 RETURN 004140 END 004150 004160 004170 SUBROUTINE GAUSS (A, IA, N, M, EPS) 004180 004190 SOLUTION OF A SET OF N LINEAR EQUATIONS IN N UNKNOWNS WITH M-N RIGHT HAND SIDES. THE SET OF SOLUTIONS WILL BE STORED IN PLACE 004200  $\mathbb{C}$ 004210  $\mathbb{C}$ OF THE RIGHT HAND SIDES: IN THE LAST M-N COLUMNS OF MATRIX A. 004220 C KAJ MADSEN, NUMERISK INSTITUT, LYNGBY, AUGUST 1980. 004230 C 004240 DIMENSION ACIA, MD 004250 N1 = N+1004260 IF (N.EQ.1) GO TO 10 004270 C 004280  $\mathbb{C}$ EQUILIBRATION 004290 C 004300 DO 3 I=1,N 004310 C=ABS(A(I,1))004320 DO 1 J=2,N 004330 IF (ABS(A(I,J)).GT.C) C=ABS(A(I,J)) 004340 **1** CONTINUE 004350 DO 2 J=1,M 004360 A(I,J) = A(I,J)/C004370 2 CONTINUE 004380 **3** CONTINUE 004390 C 004400  $\mathbb{C}$ PIVOTING AND REDUCTION TO TRIANGULAR FORM 004410 C 004420 NM=N-1 004430 DO 9 K=1,NM 004440  $K_{1} = K_{1}$ 004450 IPIV=K 004460 C=ABS(A(K,K))004470 DO 4 I=K1,N 004480 IF (C.GE. ABS(ACI, K)) GO(TO 4) 004490 IPIV=I 004500 C=ABS(A(I,K)) 004510 4 CONTINUE 004520 004530  $\mathbb{C}$ TEST FOR SINGULARITY 004540 C 004550

C

 $\mathbb{C}$ 

C	IF (C.LT.EPS) GO TO 15	004560
Ğ	PIVOTING CONTINUED	$004570 \\ 004580$
C		004590
	IF (IPIV.EQ.K) GO TO 6	004600
	D0 5 J=K, M C=A(K, J)	$004610 \\ 004620$
	A(K, J) = A(IPIV, J)	004630
	A(IPIV, J) = C	004640
	5 CONTINUE 6 CONTINUE	004650
	DO 8 I=K1,N	$004660 \\ 004670$
	$C=A(I,K) \land A(K,K)$	004680
	D0 7 J=K1,M	004690
	A(I,J)=A(I,J)-C*A(K,J) 7 CONTINUE	004700
	8 CONTINUE	$004710 \\ 004720$
	9 CONTINUE	004730
G		004740
C C	END OF REDUCTION	$004750 \\ 004760$
G	10 CONTINUE	004770
C		004780
C C	TEST FOR SINGULARITY	004790
C.	IF (ABS(A(N,N)).LT.EPS) GO TOM15	$004800 \\ 004810$
C		004820
C C	BACKSUBSTITUTION	004830
G	DO 14 II=1.N	$004840 \\ 004850$
	I = N - I I + 1	004860
	$DO_{13} J=N1,M$	004870
	C=A(I,J) IF (I.EQ.N) GO TO 12	004880
	I1=I+1	004890 004900
	DO 11 K=I1,N	004910
	C=C-A(I,K)*A(K,J) 11 CONTINUE	004920
	12 A(I,J)=C/A(I,I)	$004930 \\ 004940$
	13 CONTINUE	004950
	14 CONTINUE	004960
	CO TO 16 15 EPS=-1.E0	004970
	16 RETURN	004980 004990
	END	005000
C C		005010
9	SUBROUTINE MA19W (N, M, A, IA, B, DX, EPS, X, RES, IREF, NURES, NUIREF)	005020 005030
	DIMENSION A(IA, N), B(M), X(N), RES(NURES)	005040
~	INTEGER IREF(NUIREF)	005050
C C	NURES=(N+1)*(N+5)+M	005060
Ğ	NUIREF=5*(N+1)+M	005070 005080
C		005090
	N1=N+1 N2=N+2	005100
	NURHO = MAXO(M. 3*N1) + 1	$005110 \\ 005120$
	NH= 1+NURHO	005130
	CALL MA19BW (N, M, A, IA, B, DX, EPS, X, RES(NH), N1, RES(1), IREF, NURHO, NUIR	005140
	1EF, N2) RETURN	$005150 \\ 005160$
	END	005170
G		005180
С	SUBROUTINE MA19BW (N, M, A, IA, B, DX, EPSH, X, H, N1, RHO, IREF, NURHO, NUIREF	005190
	Source of this three Burners and a start of the Burner of the Store of the Start of	005200

- 24 -

1,N2) 005210 DIMENSION A(IA, N), B(M), X(N), H(N1, N2), RHO(NURHO) 005220 REAL LAM 005230 INTEGER IREF(NUIREF) 005240 LOGICAL GAMCH 005250 IF ((DX.LT.0E0).OR.(EPSH.LT.0E0)) RETURN 005260 IF ((N.LT.1).OR.(M.LT.1)) RETURN 005270 005280 NN2=N+N2 005290 NN3=NN2+N1 005300 LREF=NN3+N1 005310 LBND=LREF+M 005320 M1=M+1 005330 SI4N=1./(4.\*N) 005340 NTAL=0 005350  $\mathbb{C}$ 005360 Ĉ FIND EQUATION 10 WHICH GOES INTO THE FIRST REFERENCE 005370  $\mathbb{C}$ 005380 C=-1. 005390 DO 1 J=1,M 005400 IREF(LREF+J)=0 005410 IF (ABS(B(J)).LT.C)(GO)TO 1 005420 C=ABS(B(J))005430 IØ=J 005440 **1** CONTINUE 005450  $\mathbb{C}$ 005460  $\mathbf{C}$ INITIALIZE REFERENCE ARRAYS 005470  $\mathbf{C}$ 005480 S=0.E0 005490 T=B(10) 005500 XM=M 005510 DO 2 I=1.N 005520 D=A(I0,I)005530 S=S+ABS(D) 005540 XM=XM+1E0 005550 IF (D.EQ.0E0) D=-1. 005560 IREF(I)=SIGN(XM,-D\*T) 005570 IREF(LBND+I) = IREF(I) 005580 2 CONTINUE 005590 XM=IØ 005600 IREF(N1)=SIGN(XM,T) 005610 IREF(LREF+I0) = IREF(N1) 005620 005630 INITIALIZE DH, DG, AND GAM 005640 005650 IF ((DX\*S).GT.C) GO)TO 3 005660 GAM=DX 005670 DG=GAM 005680 DH=C-DX\*S 005690 GO TO 4 005700 3 DG=C/S 005710 DH=0. 005720 GAM=DC 005730 005740 FIND VECTOR X 005750 005760 4 DO 5 I=1,N 005770 XM=IREF(I) 005780 X(I)=SIGN(DG, XM) 005790 **5** CONTINUE 005800 005810 FIND MATRIX H 005820 005830 S=1./(S+1.) 005840 H(N1,N1)=S 005850

C

C

C

C

C

C

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C C

		IREF(NN2+N1)=0			005860
		DO 8 I=1, N			005870
		XM = -IREF(I)			005880
		H(N1, I) = SIGN(S, XM)			
			· · ·		005890
					005900
		H(I,N1) = T			005910
		DO 6 J=1,N			005920
		XM = -IREF(J)			005930
		H(I, J) = SIGN(T, XM)			005940
	6	CONTINUE			005950
		IF (T.GT.0.) GO TO	7	1	005966
		IREF(NN2+I) = 1			005970
		H(I, N2) = 1.			005980
		GO TO 8			
	17	IREF(NN2+I)=0			00599(
			74 7 3 3 1 174 7	TTN A SA SA	006000
~	0	H(I, I) = ISIGN(1), IREP	R DD+mer,	H 2 Constant of the second sec	006010
G					006020
C		INITIALIZE SOME (	JUNSTANTS		006030
$\mathbf{C}$					006040
		RSIC=1E0-S			006050
		DGH=DG			006060
		DH1=DH			006070
		ETA=GAM			006080
		ERRX=0.E0			006090
		GAMCH=. TRUE.			
		NBIN=N			006100
		GO TO 65			006110
a		60 10 09			006120
G				8 m.	006130
G		ITERATIVE LOOP ST		J&注意:	006140
G		FIND VECTOR RHO			006150
$\mathbb{C}$					006160
	9	DGH=DG			006170
		IF (IOS.LT.O) GO TO	) 12		006180
		DO 11 I=1,N1			006190
		S=-H(I,N1)			006200
		DO 10 J=1,N			006210
		S=S-H(I,J)*A(I0,J)			006220
	10	CONTINUE			
	10	RHO(I)=S			006230
	ৰ গ	CONTINUE			006240
	l l				006250
	10	GO TO 19			006260
	12	DO 14 I=1,N1			006270
		S=-H(I,N1)			006280
		DO 13 J=1,N			006290
		S=S+H(I,J)*A(I0,J)			006300
	13	CONTINUE			006310
		RHO(1) = S			006320
	14	CONTINUE			006330
		GO TO 19			006340
C					006350
Ĉ		BOUNDS VIOLATED			006360
č		BOOMBD VIOLINELLD			
a	15	I0=M+J0			006370
	£ 9.9	DHH=DH			006380
			<b>X</b>	3	006390
		IOS=SIGN(1.E0, X(J0)	J	k .	006400
		IF (IOS.LT.O) GO TO	1116	ŧ.	006410
		DO 16 I=1,N1			006420
		RHO(I) = -H(I, J0) - H(J)	.,N1) 🗇		006430
	16	CONTINUE			006440
		GO TO 19			006450
	17	DO 18 I=1,N1			006460
		RHO(I) = H(I, J0) - H(I, I)	N1)	-3 i	006470
	18	CONTINUE			006480
C	-	and an ent the sheat of the black			006490
C		FIND FOILATION TO	WHICH PAT	VES THE REFERENCE	
<b>C</b> 4		TETAL EXCLUSE FOR FAU	WELL VILL LARAS	A 1992 A THE AND LEVEN AL	006500

FIND -H(I,N1)/RHO(I) FOR NEGATIVE VALUES OF RHO(I) C 006510 C 006520 19 LB=0 006530 RTAU=0.E0 006540 IF (IØ.GT.M) RTAU=1. 006550 DO 21 I=1,N1 006560 IF (RHO(I).GE.0.E0))GO TO 20 006570 LB=LB+1006580 IREF(N1+LB) = I006590 KK= N 1 006600 IF (IREF(NN2+1).GT.0) KK=N2 006610 RHO(N1+I) =-H(I,KK)/RHO(I) 006620 20 IF (IABS(IREF(I)).LE.M) GO TOP21 006630 RTAU=RTAU+RHO(I) 006640 21 CONTINUE 006650 C 006660 C C FIND THE COEFFICIENTS IN THE RATIONAL EXPRESSION 006670 (TT+LAM\*SS)/(RSIG+LAM\*RTAU) 006680 C 006690 DG2=DG\*2. 006700 IF (DH.GT.0E0) GO TO 22 006710 TT=DG\*RSIG 006720 SS=DG\*RTAU+DHH+DGH-DG 006730 NL=1006740 GO TO 23 006750 22 RSIG=1.-RSIG 006760 RTAU=-RTAU 006770 TT=DH\*RSIG 006780 SS=DH\*RTAU+DHH-DH+DCH-DC 006790 NL=2 006800 23 SMAX=0. 006810 L=1006820 24 LA=L 006830  $\mathbb{C}$ 006840  $\mathbf{C}$ FIND MINIMUM VALUE OF -H(I,N1)/RHO(I) 006850  $\mathbb{C}$ 006860 LO=IREF(N1+LA) 006870 LAM=RHO(N1+LØ) 006880 ILAM= IREF(NN2+LØ) 006890 IF (LA.EQ.LB) GO TO 26 006900  $T_{,=}T_{,A+1}$ 006910 LM=LA 006920 DO 25 I=L,LB 006930 IN=IREF(N1+I) 006940 S=RHO(N1+IN)006950 IS=IREF(NN2+IN) 006960 IF (((IS.EQ.ILAM).AND.(S.GE.LAM).OR.(IS.LT.ILAM)) GO TO 25) 006970 LØ=IN 006980 LAM=S 006990 ILAM=IS 007000 LM= I 007010 **25** CONTINUE 007020  $\mathbb{C}$ 007030 C REORDER 007040  $\mathbb{C}$ 007050 IREF(N1+LM) = IREF(N1+LA) 007060 IREF(N1+LA)=L0 007070  $\mathbb{C}$ 007080 C FIND MAX(-RHO(I)) 007090  $\mathbb{C}$ 007100 26 SMAX=AMAX1(SMAX,-RHO(L0)) 007110 C 007120  $\mathbb{C}$ REVISE THE COEFFICIENTS OF THE RATIONAL EXPRESSION 007130 C 007140 ML=NL 007150

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IF (IABS(IREF(L0)).GT.M. ML=NL+2
                                                                            007160
      GO TO (29,28,28,27), ML
                                                                            007170
   27 TT=TT+DG2*H(L0,N1)
                                                                            007180
      SS=SS+DG2*RHO(L0)
                                                                            007190
      GO TO 29
                                                                            007200
   28 RSIG=RSIG-2.*H(L0,N1)
                                                                            007210
      RTAU=RTAU-2.*RHO(LØ)
                                                                            007220
   29 IF ((RSIG*SS.GT.RTAU*TT).AND.(LA.LT.LB)) GO TO 24)
                                                                            007230
C
                                                                            007240
\overline{\mathbf{C}}
        TEST IF -RHO(LØ) IS TOO SMALL
                                                                            007250
\mathbf{C}
                                                                            007260
      SMAX=SMAX/4.
                                                                            007270
   30 IF (-RHO(LO).GE.SMAX) GO TO 31
                                                                            007280
      LA=LA-1
                                                                            007290
      LØ=IREF(N1+LA)
                                                                            007300
      GO TO 30
                                                                            007310
C
                                                                            007320
        UPDATE REFERENCE ARRAYS
C
                                                                            007330
C
                                                                            007340
   31 IREF(LREF+10)=ISIGN(1,10S)
                                                                            007350
      IREF(LREF+IABS(IREF(L0)))=0
                                                                            007360
      IF (IABS(IREF(LØ)).GT.M) NBINSNBIN-1
                                                                            007370
      IF (IØ.GT.M) NBIN=NBIN+1
                                                                            007380
      IREF(LO) = ISIGN(IO, IOS)
                                                                            007390
C
                                                                            007400
C
        UPDATE MATRIX H
                                                                            007410
C
                                                                            007420
      RHOLØ=RHO(LØ)
                                                                            007430
      DO 32 J=1,N1
                                                                            007440
      H(LO, J) = -H(LO, J) / RHOLO
                                                                            007450
   32 CONTINUE
                                                                            007460
      DO 34 I=1,N1
                                                                            007470
      IF (I.EQ.LO) GO TO 34
                                                                            007480
      S=RHO(I)
                                                                            007490
      DO 33 J=1,N1
                                                                           007500
      H(I,J) = S \times H(LO,J) + H(I,J)
                                                                            007510
   33 CONTINUE
                                                                            007520
   34 CONTINUE
                                                                            007530
C
                                                                           007540
C
        IF ANY SIGNS HAVE BEEN CHANGED, UPDATE H
                                                                            007550
\mathbf{C}
                                                                            007560
      RHON1=1E0
                                                                           007570
      DO 35 I=1,N1
                                                                            007580
      RHO(N1+I)=1.
                                                                            007590
   35 CONTINUE
                                                                            007600
      IF (LA.LE.1) GO TO 46
                                                                            007610
      K=LA-1
                                                                            007620
      DO 36 I=1.N1
                                                                            007630
      RHO(NN2+1)=0.E0
                                                                            007640
   36 CONTINUE
                                                                            007650
      DO 37 I=1,K
                                                                            007660
      RHO(IREF(N1+I)+N1)=-1.
                                                                           007670
   37 CONTINUE
                                                                            007680
      DO 41 I=1,N1
                                                                           007690
      IF (RHO(N1+I).GT.0.) GO TO 39)
                                                                           007700
      DO 38 J=1,N1
                                                                           007710
      RHO(NN2+J)=RHO(NN2+J)-H(I,J)
                                                                           007720
   38 CONTINUE
                                                                           007730
      GO TO 41
                                                                           007740
  39 DO 40 J=1,N1
                                                                           007750
     RHO(NN2+J)=RHO(NN2+J)+H(I,J)
                                                                           007760
  40 CONTINUE
                                                                           007770
  41 CONTINUE
                                                                           007780
      RHON1=RHO(NN2+N1)
                                                                           007790
     DO 42 J=1,N1
                                                                           007800
```

C C	43	H(J,N1)=H(J,N1)∕RHON1 CONTINUE D0 44 I=1,N S=RHO(NN2+I) D0 43 J=1,N1 H(J,I)=-S*H(J,N1)+H(J,I) CONTINUE CONTINUE CONTINUE CHANGE SIGNS IN SOME ROWS OF MATRIX H D0 45 L=1,K I=IREF(N1+L) J=-IREF(I) IREF(I)=J IREF(I)=J IREF(LREF+IABS(J))=J D0 45 J=1,N1	007810 007820 007830 007850 007850 007850 007850 007890 007900 007910 007920 007930 007930 007940 007950 007950
C	45	H(I,J) = -H(I,J)	007980
C C		UPDATE THE LAST COLUMN OF HO IN CASE OF DEGENERACIES	007990 008000
C	10		008010
	40	IF (IREF(NN2+L0).EQ.0) GO TO 52 H(L0,N2)=-H(L0,N2)/(RH0(L0)*RH0N1)	008020 008030
		DO 51 I=1.N1	008040
		IF ((IREF(NN2+I).EQ.0).OR.(I.EQ.L0)) GOOTO 51	008050
	47	IF (IREF(NN2+I)-IREF(NN2+L0)) 47,49,48 H(I.N2)=H(I.N2)/RHON1	008060 008070
		GO TO 51	008080
	48	H(I, N2) = ABS(H(L0, N2) * RHO(I))	008090
		IREF(NN2+1)=IREF(NN2+L0) GO TO 51	008100
	49	C=H(1,N2)/RHON1+H(LØ,N2)*RHO(1)	008110 008120
		IF ((LA.GT.1).AND.(RHO(N1+1).LT.0E0)) C=-C	008120
		IF (C.GT.0) GO TO 50	008140
		H(I, N2) = 1.E0 IREF(NN2+I) = IREF(NN2+I) + 1	008150
		$\frac{1}{60} \text{ TO } 51$	008160 008170
	50	H(I, N2) = C	008180
	51	CONTINUE	008190
C		GO TO 54	008200
C		TEST FOR DECENERACIES	$008210 \\ 008220$
C			008230
	52	DO 53 I=1,N1	008240
		IREF(NN2+1)=0 IF (H(1,N1).GT.0E0) GO TO 53	008250
		IREF(NN2+1)=1	$008260 \\ 008270$
		H(I,N1)=0.E0	008280
	50	H(I, N2) = 1E0	008290
C	03	CONTINUE	008300
Ğ		UPDATE GAM	$008310 \\ 008320$
C			008330
	54	GAMCH=.FALSE. IF ((NBIN.EQ.0).OR.((GAMM.LT.2.*GAMD.AND.(GAMM.LT.DX))) GO TO 55) IF (GAM.LT.GAMMD GAMCH=.TRUE.	008340 008350 008360
		GAM= GAMM	008370
C			008380
C C		UPDATE DH AND DG	008390
4	55	S=0.	$008400 \\ 008410$
	00	RSIG=0.E0	008420
		DO 57 I=1,N1	008430
		K=IABS(IREF(I))	008440
		IF (K.GT.M) GO TO 56	008450

RHO(I)=B(K)\*ISIGN(1, IREF(I)) 008460 S=S+H(I,N1) \*RHO(I) 008470 GO TO 57 008480 56 RSIG=RSIG+H(I,N1) 008490 **57 CONTINUE** 008500 IF (RSIG.NE.0E0) GONTO 58 008510 DH=ABS(S) 008520 DG=GAM 008530 GO TO 59 008540 58 DG=AMIN1(GAM, ABS(S)/RSIG) 008550 DH=0. 008560 IF (DG.EQ.GAM) DH=ABS(S-DG\*RSIG)/(1E0-RSIG) 008570  $\mathbb{C}$ 008580 C CALCULATE PARAMETER VALUES FOR 008590 C 008600 59 DGH=0.E0 008610 ERRX=0.E0 008620 DO 61 I=1,N1 008630 IF (IABS(IREF(I)).GT.M) GO TO 60 008640 RHO(I) = DH-RHO(I) 008650 GO TO 61 008660 60 RHO(I)=DG 008670 61 CONTINUE 008680 DO 64 I=1,N 008690 S=H(1, I)\*RHO(1) 008700 DO 62 J=2,N1 008710  $S=S+H(J, I) \times RHO(J)$ 008720 62 CONTINUE 008730 IF (IREF(LEND+I).EQ.0) GO TO 63 008740 T=S 008750 XM=IREF(LBND+I) 008760 S=SIGN(DG.XM) 008770 ERRX=AMAX1(ERRX,ABS(S-T)) 008780 63 X(I)=S 008790 IF (ABS(S).LE.DGH) GO TO 64 008800 DGH=ABS(S) 008810 J0=1 008820 64 CONTINUE 008830 NTAL=NTAL+1 008840 C 008850 C CALCULATE GAMM 008860  $\mathbb{C}$ 008870 65 GAMM=AMIN1(AMAX1(5.\*DG,GAM),DX) 008880 C 008890 FIND EQUATION 10 WHICH GOES NINTO THE REFERENCE AND  $\mathbf{C}$ 008900  $\mathbb{C}$ 008910 DHH=0.E0 008920 T=DH 008930 T1=DH 008940 DO 68 I=1,M 008950 S=B(I)008960 DO 66 J=1,N 008970 S=S+A(I,J)\*X(J) 008980 66 CONTINUÉ 008990 RHO(I)=S009000 IF (IREF(LREF+I).NELO) GO TO 67 009010 IF (ABS(S).LE.DHH) GO TO 68 009020 DHH=ABS(S) 009030 I0S=SIGN(1.E0,S) 009040 TO = T009050 GO TO 68 009060 67 T=AMAX1(T,ABS(S)) T1=AMIN1(T1,ABS(S)) 009070 009080 68 CONTINUE 009090 C 009100

C	CALCULATE DH1 AND ETA	009110	
G		009120	
	ETA=AMAX1(GAMM,SIGN(ETA+ERRX,DH1-T))	009130	
	IF (.NOT.GAMCH) T=AMAX1(T,DH1+(T-T1))	009140	
	DH1=AMAX1(T, DH+EPSH)	009150	
C		009160	
C	TEST IF CONSTRAINTS ARE VIOLATED	009170	
C		009180	
	IF (DGH.GT.ETA) GO TO 15	009190	
C		009200	
C	TEST OF CONVERGENCE CRITERION	009210	
Ĉ		009220	
-	IF (DHH, CT, DH1) GO TO 9	009230	
	IF ((GAM, LT, DZ), AND, (DHH+DH, GT, (DH1-DH) *2.), AND, (NBIN, GT, 0)) GO TO	009240	
	1 54	009250	
	BHO(M1) = BH	009260	
	$\operatorname{IREF}(N2) = \operatorname{NBIN}$		
		009270	
	IREF(N+3)=NTAL	009280	
	RETURN	009290	
	END	009300	

#### SOC-280

MINI5W - A FORTRAN PACKAGE FOR MINIMAX OPTIMIZATION

K. Madsen (Adapted and Edited by J.W. Bandler and W.M. Zuberek)

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Revised:

Key Words: Minimax optimization, nonlinear optimization, computeraided design, optimization program

Abstract: MINI5W is a package of subroutines for solving unconstrained, nonlinear minimax optimization problems. First derivatives of all functions w.r.t. all variables are assumed to be known. The solution is found by an iteration that uses either linear programming applied in connection with first derivatives or a Newton step applied in connection with first derivatives and approximate second derivatives. The method has been described by Hald and Madsen. The Fortran IV package and documentation have been adapted for the CDC 170/730 system.

Description: Contains Fortran listing, user's manual. Source deck or magnetic tape available for \$100.00. The listing contains 930 lines, of which 261 are comments.

Related Work: SOC-218, SOC-281.

Price: \$ 30.00.