INTERNAL REPORTS IN

SIMULATION, OPTIMIZATION AND CONTROL

No. SOC-42

EFFICIENT, INTERACTIVE SEMI-AUTOMATED OPTIMIZATION OF MODELS AND DESIGNS

J.W. Bandler, W.Y. Chu and J.R. Popović

May 1974

FACULTY OF ENGINEERING
McMASTER UNIVERSITY
HAMILTON, ONTARIO, CANADA

EFFICIENT, INTERACTIVE SEMI-AUTOMATED OPTIMIZATION OF MODELS AND DESIGNS

J.W. Bandler, W.Y. Chu and J.R. Popović
Group on Simulation, Optimization and Control
and Department of Electrical Engineering
McMaster University
Hamilton, Ontario, Canada

ABSTRACT

The work described in this paper is directed towards a compromise between fully automated design and modelling, which, it is felt, is still some way off, and fully interactive design, which, in contrast, is probably unnecessarily inefficient in the use of machines. At the heart of the software are efficient gradient methods of minimizing unconstrained nonlinear functions of many variables, such as the Fletcher method. In an effort to satisfy design specifications and constraints we use least pth approximation techniques devised by Bandler and Charalambous. A number of interactive optimization programs are discussed, in particular, one that optimizes certain cascaded two-port electrical circuits in the frequency domain, and ones that minimize nonlinear functions of many variables, constrained and unconstrained. The aim of the paper is to discuss the present state of the authors' programs and to indicate future directions to be explored. To this end, results of the incorporation of extrapolation into the programs as well as the forcing of symmetry are presented.

INTRODUCTION

Fully automated optimization of engineering designs and models is still some way off. This work is aimed at a compromise which attempts to fully exploit available automatic analysis and optimization techniques as well as the facility for user-interaction. The implementation of interaction with the actual constrained optimization process at convenient points so as to influence the rate of convergence, redefine the parameters or redefine the objectives or constraints on line in a convenient manner is discussed. An ultimate aim is to study poorly defined or ill-conditioned optimization problems with the aim of automating as many features as possible which are clearly understood, leaving the designer to decide on factors which influence the outcome of his design but whose effect may not easily be predicted in advance.

This work was supported by the National Research Council of Canada under Grants A7239 and E3429.

Optimization methods require a single function of many variables to be minimized possibly subject to constraints, yet, as any designer knows, a real design or modelling problem requires a trade-off between a number of often conflicting objectives. There are many ways of setting up such design problems in the standard form of minimizing a function subject to constraints [1,2]. An important conclusion is that no matter how the original problem is specified, in general the problem can always be recast as an unconstrained minimax optimization problem and its efficient solution ultimately depends on the efficiency of the unconstrained minimization method which forms the heart of the whole process.

Most of the work described in this paper has been carried out on the CDC 6400 computer, some on the PDP 11/45.

INTERACTIVE FUNCTION MINIMIZATION

It is usually desirable to have control, at least partially, over the execution of an optimization process. Judging from information obtained from the early stages of the optimization process, the user may want to redefine parameters or request some options, e.g., extrapolation techniques to speed up convergence. Implementation of user-interaction with the optimization process can best be done on a time-sharing system or on a dedicated mini-computer.

Among the many packages developed by our group, user interaction facilities were first incorporated into packages FLNLP1 [3] and FLOPT1 [4] and implemented with INTERCOM on the CDC 6400 computer. FLNLP1 is a general program for solving constrained optimization problems. The Bandler-Charalambous technique [2,5] is used to transform the constrained optimization problem into the minimization of an unconstrained objective function. Practical least pth approximation [2,6] is used to solve the resulting minimax problem together with Fletcher's gradient method [7]. FLOPT1 is a program written primarily for solving unconstrained functions using Fletcher's method. In using the interactive version of both packages, the user is instructed to supply the required data by answering questions. After he has entered all the data, he is free to modify any of the entries. In this way, he can be sure that correct data is entered before the optimization process starts.

```
YOU ARE WELCOME TO USE THE PACKAGE " F L O P T 1 ".
PLEASE SUPPLY DATA WHEN ASKED FOR.  YOU CAN ENTER YOUR DATA IN ANY
FORMAT, HOWEVER, BE REASONABLE.  PLEASE SEPARATE EACH VALUE BY A
COMMA, A BLANK OR TYPING THE RETURN KEY.  THANK YOU.

SPECIFY THE MAXIMUM NUMBER OF ITERATIONS ALLOWED.
1+      100
ENTER AN INTEGER SO THAT INTERMEDIATE OUTPUT WILL BE PRINTED AFTER
EVERY SPECIFIED NUMBER OF ITERATIONS.  ENTER 0 IF YOU DON"T WANT
ANY INTERMEDIATE OUTPUT.
2+      10
ENTER 1 IF YOU WANT TO HAVE A RECORD OF YOUR INPUT DATA, OTHERWISE
ENTER 0.
3+      1
SPECIFY A MINIMUM ESTIMATED VALUE OF THE OBJECTIVE FUNCTION.
4+      0
ENTER STARTING VALUES FOR THE VARIABLE PARAMETERS.
5+      -1.2,1
ENTER SMALL VALUES FOR TESTING CONVERGENCE.
6+      1.E-9  1.E-9

ANY MODIFICATION
#       YES
WHICH ENTRY
#       1
SPECIFY THE MAXIMUM NUMBER OF ITERATIONS ALLOWED.
1+      50

ANY MODIFICATION
#       NO

YOUR DATA IS NOW BEING PROCESSED.  IT MAY TAKE SOME TIME BEFORE
RESULTS ARE AVAILABLE.  PLEASE BE PATIENT.
```

Fig. 1. Typical instructions for entering data when using FLOPT1 [4].

Fig. 1 shows typical instructions for entering data when using FLOPT1 to minimize Rosenbrock's function [8]. After a complete optimization, the user can request the process to stop or restart with different input data.

In updated versions of FLNLP1 and FLOPT1 (called FLNLP2 [9] and FLOPT2 [10], respectively), the user may request an extrapolation technique to accelerate the rate of convergence to the final solution. The extrapolation technique, proposed by Fiacco and McCormick [11], involves fitting a polynomial through k points on $\boldsymbol{\phi}(r)$, where $\boldsymbol{\phi}$ is the n-dimensional parameter vector, when k minima have been obtained from minimizing the unconstrained objective function. The unconstrained objective function is obtained by using the SUMT [11,12] transformation on the original objective function subject to inequality or equality constraints. The unconstrained function, for the problem of minimizing $U(\boldsymbol{\phi})$ subject to $g_i(\boldsymbol{\phi}) \geq 0$, $i = 1,2, \ldots, m$ and $h_j(\boldsymbol{\phi}) = 0$, $j = 1,2, \ldots, s$, is usually of the form

$$B(\boldsymbol{\phi},r) = U(\boldsymbol{\phi}) + r \sum_{i=1}^{m} \frac{1}{g_i(\boldsymbol{\phi})} + \frac{1}{\sqrt{r}} \sum_{j=1}^{s} h_j^2(\boldsymbol{\phi}) \quad (1)$$

Suppose the B function has been uniquely minimized for $r_1 > \ldots > r_k > 0$ at $\check{\boldsymbol{\phi}}^1, \ldots, \check{\boldsymbol{\phi}}^k$. A polynomial in r that yields $\check{\boldsymbol{\phi}}^1, \ldots, \check{\boldsymbol{\phi}}^k$ is given by a set of equations of the form

$$\check{\boldsymbol{\phi}}^i = \sum_{j=0}^{k-1} \boldsymbol{a}_j (r_i)^j \qquad i = 1, \ldots, k \quad (2)$$

where $\boldsymbol{a}_j$ are n-component vectors. With $r_{i+1} = r_i/c$ $(c > 1)$, a simple iterative scheme to calculate a series of estimates based on using a given number of terms in the polynomial is possible.

If $\boldsymbol{\phi}_j^i$, $i = 1, \ldots, k$, $j = 1, \ldots, i-1$ signifies the jth order estimate of $\check{\boldsymbol{\phi}}(0)$ after i minima have been achieved, with r_1 being the initial value of r, then we have

$$\boldsymbol{\phi}_0^i = \check{\boldsymbol{\phi}}\left(\frac{r_1}{c^{i-1}}\right) \qquad i = 1, \ldots, k$$

and

$$\boldsymbol{\phi}_j^i = \frac{c^j \boldsymbol{\phi}_{j-1}^i - \boldsymbol{\phi}_{j-1}^{i-1}}{c^j - 1} \qquad \begin{array}{l} i = 2, \ldots, k \\ j = 1, \ldots, i-1 \end{array} \quad (3)$$

The "best" estimate of $\check{\phi}(0)$, namely ϕ_0, is given by

$$\check{\phi}(0) \doteq \phi_{k-1}^{k} = \phi_0 \qquad (4)$$

To estimate $\check{\phi}^{k+1}$, we have the recursive relation

$$\phi_{j-1}^{k+1} = \frac{(c^j-1)\phi_j^{k+1} + \phi_{j-1}^{k}}{c^j} \qquad (5)$$

and

$$\check{\phi}^{k+1} = \phi_0^{k+1} \qquad (6)$$

In practical least pth optimization, near minimax solutions can be obtained by using a very large value of p [6]. The process is usually accomplished by optimizing the objective function for $2 \leq p_1 < \ldots < p_k \to \infty$. Or in another form, we have the sequence of p as

$$\frac{1}{2} \geq \frac{1}{p_1} > \ldots > \frac{1}{p_k} \to 0.$$

The minimax solution will be the one when $p \to \infty$. Arranging the sequence of p values such that $p_{i+1} = cp_i$ $(c > 1)$, we find that the extrapolation formulas (3) and (5) are very effective in accelerating the rate of convergence to the minimax solution.

Parameters	$p = 10^5$ $\alpha = 10$	$p = 4, 16, 64, 256, 1024$ $\alpha = 10$ order of extrapolation = 3
ϕ_1	.00000	.00000
ϕ_2	1.00000	1.00000
ϕ_3	1.99999	2.00000
ϕ_4	- .99999	-1.00000
$U(\phi)$	-43.9998	-44.0000
Total no. of function evaluations	88	73

The parameter α is required in the minimax formulation of Bandler and Charalambous [5].

Table 1. Results for the Rosen-Susuki problem [8].

Table 1 compares results obtained by the package FLNLP2 [9] in solving the Rosen-Suzuki problem [8] using a p value of 10^5 and using a sequence of p values with extrapolation. Chen has also reported good results using extrapolation [13].

In addition, when a sequence of least pth optimizations is involved, information from the first optimization may reveal that the objective function exhibits symmetry in some of the parameters. For the special case of complete symmetry, a facility is available in packages FLNLP2 and FLOPT2 to the user such that upon request the dimensionality of the optimization process will be automatically scaled down to the minimum for the subsequent optimizations. Considerable computational effort and time may be saved.

Table 2, for example, compares the effort required for optimizing a 7-element LC lowpass filter with and without symmetry enforcement. The filter is to have an insertion loss of not more than .01 dB in the passband (ω = 0 to 1) and an insertion loss of 62.9 dB or more at ω = 2.5 in the stopband. In both cases, the starting point is the least squares optimum.

CASCADED NETWORK OPTIMIZATION

The cascaded network optimization package called CANOP2 will analyze and optimize cascaded linear, time-invariant networks in the frequency domain. The package features some of the latest and most efficient methods of computer-aided design currently available. The program is organized in such a way that future additions or deletions of performance specifications, constraints, optimization

Parameters	p = 10, 10^3, 10^6	p = 10, 10^3, 10^6 symmetry enforced
ϕ_1	.79839	.79839
ϕ_2	1.39221	1.39221
ϕ_3	1.74870	1.74870
ϕ_4	1.63304	1.63303
ϕ_5	1.74870	ϕ_3
ϕ_6	1.39221	ϕ_2
ϕ_7	.79839	ϕ_1
Total no. of function evaluations	80	58
Execution time (sec)	7.30	4.44

Table 2. Results for a 7-element LC lowpass filter.

methods and circuit elements are readily implemented. Presently, the network to be optimized is assumed to be a cascade of two-port building blocks terminated in a unit normalized, frequency-independent resistance at the source and a user-specified frequency-independent resistance at the load.

A variety of two-port lumped and distributed elements such as resistors, inductors, capacitors, lossless transmission lines, lossless short-circuited and open-circuited transmission-line stubs, series and parallel LC and RLC resonant circuits and microwave allpass C- and D-sections can be handled. Upper and lower bounds on all relevant parameters can be specified by the user. A generalized least pth objective function, or sequence of least pth objective functions, developed by Bandler and Charalambous [6] incorporating simultaneously input reflection coefficient, insertion loss, relative group delay and parameter constraints (if any) are automatically created. Constraints are treated by the objective function in essentially the same way as the performance specifications [14]. To distinguish conveniently between the various responses or constraint functions a scheme for interval translation and introduction of artificial points has been developed [14]. Both the Fletcher-Powell method of minimizing unconstrained functions of many variables [15] and the Fletcher method [7] are available to the user. The package was designed to incorporate the adjoint network method of sensitivity evaluation to produce accurate first derivatives needed by these efficient gradient minimization methods [16].

If symmetry of some parameters can be predicted, symmetry may be forced throughout the optimization. Results may be automatically presented numerically and graphically and analysis of different responses and/or different frequency ranges may be performed at the user's discretion and a new optimization may be requested. A summary of the latest features and options available is given in Table 3.

The package written in FORTRAN IV was originally developed for batch processing on a CDC 6400 computer and has been largely extended for use on INTERCOM [17]. The user may interact at many points with the program to change parameters, frequency range, types and options and to request plots. The interactive user enters his data in free format.

A test example will be presented here to illustrate the approach. Examples of input and output as well as actual execution times are given.

We consider a seven-section equal-ripple band-pass microwave filter of 3 to 1 bandwidth (ratio of upper band edge to lower band edge) consisting of two unit elements and five stubs which has been previously considered by Horton and Wenzel [18] as

Features	Type	Options	Parameters
Objective Functions	Least pth	$1 < p < \infty$	Value of p for each of a specified number of optimizations Artificial margin Difference in objective functions for termination
Performance Specifications and Parameter Constraints	Upper (+1.) Lower (-1.) Single (0.)	Reflection coefficient (1) Insertion loss (2) Group delay (3) Parameter value (0)	Normalization frequency Number of points and constraints — Number of bands or intervals For each: Specification/constraint Weighting factor Type Option Frequency (sample point) or parameter — Lower and upper frequencies (band edges) Number of subintervals
Optimization Methods	Gradient	Fletcher (1) Fletcher-Powell (2)	Option Number of iterations allowed Estimate of lower bound on objective function Test quantities for termination
Circuit Elements	Cascaded Two-port	See text	Number of elements Sequence of code numbers Parameter values Indicator for fixed or variable parameters Indicator for symmetrical variable parameters Load resistance Parameters for C- and D-sections
Graph	Frequency response	Given response Other response Any frequency range Automatic scaling Specified scaling	As many plots as desired Option Frequency (sample point) — Lower and upper frequencies (band edges)

Table 3. Summary of features, options and parameters required.

represented in Fig. 2. The terminations of the filter are unity. The filter is to have a 0.1 dB ripple in the passband, from 1.0875 to 3.2625 GHz, and an attenuation above 50 dB at frequency points 0.6 and 3.75 GHz in the stopband. All section lengths were kept fixed at normalized values of 1, and the normalized characteristic impedances are used as variables. The starting value of the variable vector (see Fig. 2) was ϕ^0 = [.63 .33 1.27 .26 1.27 .33 .63]^T. Table 4 shows some results. When symmetry is taken into account there is a considerable saving in the number of function evaluations and running time. Partial interactive output of CANOP2 for the example is shown in Fig. 3. Results are obtained by the Fletcher method for p=2 and p=10^3. Only 21 uniformly spaced sample points were used in the passband to demonstrate the work of the package, although there may be a need for a larger number of discrete points, as indicated by the 51 points that are used in plotting the results (Fig. 3). A uniform weighting of 1 was used. Fig. 3 also shows a plot of the group delay of the filter.

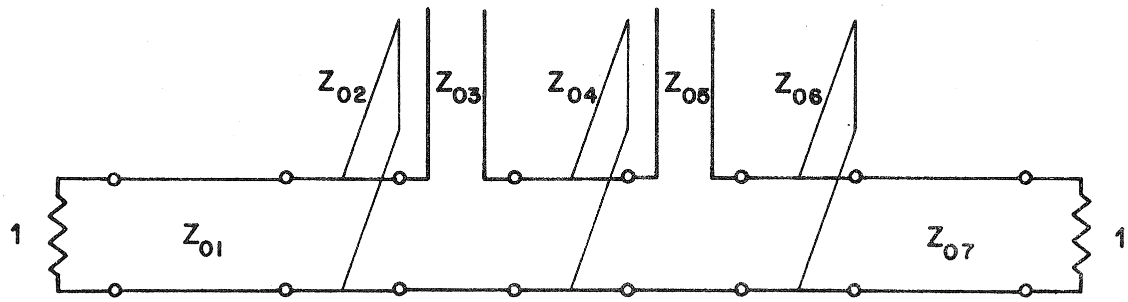

Fig. 2. Seven-section band-pass filter example.

Parameters	p = 2	p = 2 symmetry enforced	p = 1000	p = 1000 symmetry enforced
Z_{01}	.6073491	.6073491	.6064585	.6064586
Z_{02}	.3019215	.3019209	.3030585	.3030583
Z_{03}	.7192856	.7192849	.7220743	.7220734
Z_{04}	.2345477	.2345477	.2356086	.2356082
Z_{05}	.7192847	Z_{03}	.7220745	Z_{03}
Z_{06}	.3019208	Z_{02}	.3030586	Z_{02}
Z_{07}	.6073499	Z_{01}	.6064588	Z_{01}
No. of function evaluations	74	37	100	68
Execution time (sec)	25.5	15.8	34.5	29

Optimization for p = 1000 was started at the optimum for p = 2.

Table 4. Results for the microwave filter.

CONCLUSIONS

Much future work is suggested by the results presented here to obtain more practical, efficient and user-oriented design software within the scope of this paper. Increased use of a dedicated machine such as the PDP 11/45 to achieve these goals is expected. We note here that most of the packages refered to in this paper are available from the first author at nominal charge.

ACKNOWLEDGEMENTS

In alphabetical order, we would like to acknowledge C. Charalambous, J.H.K. Chen, V.K. Jha, W. Kinsner, P.C. Liu and M.R.M. Rizk.

REFERENCES

[1] J.W. Bandler, "Optimization methods for computer-aided design", IEEE Trans. Microwave Theory Tech., vol. MTT-17, Aug. 1969, pp. 533-552.

[2] C. Charalambous, "A unifed review of optimization", IEEE Trans. Microwave Theory Tech., vol. MTT-22, Mar. 1974, pp. 289-300.

[3] J.W. Bandler and W.Y. Chu, "Nonlinear programming package for constrained optimization version FLNLP1", Faculty of Engineering, McMaster University, Hamilton, Canada, Aug. 1973, Report SOC-15.

[4] J.W. Bandler and W.Y. Chu, "Function optimization package version FLOPT1", Faculty of Engineering, McMaster University, Hamilton, Canada, Aug. 1973, Report SOC-17.

[5] J.W. Bandler and C. Charalambous, "A new approach to nonlinear programming", Proc. 5th Hawaii Int. Conf. on Systems Science (Honolulu, Jan. 1972), pp. 127-129.

[6] J.W. Bandler and C. Charalambous, "Practical least pth optimization of networks", IEEE Trans. Microwave Theory Tech., vol. MTT-20, Dec. 1972, pp. 834-840.

[7] R. Fletcher, "A new approach to variable metric algorithms", Computer J. vol. 13, Aug. 1970, pp. 317-322.

[8] J. Kowalik and M.R. Osborne, Methods for Unconstrained Optimization Problems. New York: Elsevier, 1968.

[9] J.W. Bandler and W.Y. Chu, "FLNLP2 - a package for nonlinear programming with extrapolation", Faculty of Engineering, McMaster University, Hamilton, Canada, in preparation.

[10] J.W. Bandler and W.Y. Chu, "FLOPT2 - a package for function minimization with extrapolation for constrained problems", Faculty of Engineering, McMaster University, Hamilton, Canada, in preparation.

[11] A.V. Fiacco and G.P. McCormick, Nonlinear Programming: Sequential Unconstrained Minimization Techniques. New York: Wiley, 1968, pp. 188-191.

[12] A.V. Fiacco and G.P. McCormick, "Computational algorithm for the sequential unconstrained minimization technique for nonlinear programming", Management Science, vol. 10, July 1964, pp. 601-617.

[13] J.H.K. Chen, "DISOPT- a general program for continuous and discrete nonlinear programming problems", Faculty of Engineering, McMaster University, Hamilton, Canada, Mar. 1974, Report SOC-29.

[14] J.W. Bandler, J.R. Popović and V.K. Jha, "Cascaded network optimization program", IEEE Trans. Microwave Theory Tech., vol. MTT-22, Mar. 1974, pp. 300-308.

[15] R. Fletcher and M.J.D. Powell, "A rapidly convergent descent method for minimization", Computer J., vol. 6, June 1963, pp. 163-168.

[16] J.W. Bandler and R.E. Seviora, "Current trends in network optimization", IEEE Trans. Microwave Theory Tech., vol. MTT-18, Dec. 1970, pp. 1159-1170.

[17] J.W. Bandler and J.R. Popović, "CANOP2 - Interactive cascaded network optimization package", Faculty of Engineering, McMaster University, Hamilton, Canada, in preparation.

[18] M.C. Horton and R.J. Wenzel, "General theory and design of optimum quarter-wave TEM filters", IEEE Trans. Microwave Theory Tech., vol. MTT-13, May 1965, pp. 316-327.


```
DO YOU WANT TO SEE THE TABLE OF ELEMENTS AND CODE NUMBERS.          NO-
DO YOU WANT QUESTIONS FULLY WORDED TO BE PRINTED OUT.      YES
SPECIFY THE NUMBER OF ELEMENTS IN THE CIRCUIT NOT INCLUDING C- AND D-SECTIONS.
SET TO 0 IF YOU DO NOT WANT ANY.
 1)     7
SUPPLY A SEQUENCE OF    7 CODE NUMBERS OF ELEMENTS TO BE CONNECTED SEQUENTIALLY
FROM SOURCE TO LOAD.
(SEE TABLE FOR ELEMENTS AND CODE NUMBERS.)
 2)     15,14,13,14,13,14,15
IS DATA OK.       OK
SPECIFY VALUES OF  14 PARAMETERS IN THE CIRCUIT INCLUDING STARTING VALUES
FOR VARIABLES. (FOLLOW THE SUPPLIED SEQUENCE OF THE CODE NUMBERS OF ELEMENTS.)
(SEE TABLE FOR THE SEQUENCE OF PARAMETERS.)
 3)     1 .63 1 .33 1 1.27 1 .26 1 1.27 1 .33 1 .63
IS DATA OK.       YES
INDICATE WHICH OF THE  14 PARAMETERS ARE FIXED OR VARIABLE.
SET TO 0 IF FIXED AND 1 IF VARIABLE.
 4)     0 1 0 1 0 1 0 1 0 1 0 1 0 1
IS DATA OK.       YES
SPECIFY THE NUMBER OF C-SECTIONS.
SET TO 0 IF YOU DO NOT WANT ANY.
 5)     0
```

Fig. 3a. Partial instructions for entering data when using CANOP2 [17].