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THE TOLERANCE-TUNING PROBLEM: A NONLINEAR PROGRAMMING APPROACH

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A NONLINEAR PROGRAMMING APPROACH

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Abstract A theory of optimal worst case design embodying centering, tolerancing and tuning is presented. Some simplified problems and special cases are discussed. Projections and slack variables are used to explain some of the concepts. The worst case tolerance assignment and design centering problem falls out as a special case. Possible practical implementation of the ideas in circuit design is suggested.

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I INTRODUCTION

Much attention has been drawn recently to the component tolerance assignment and the design centering problem [1] - [4]. An approach whereby the nominal point as well as the tolerances are simultaneously optimized to meet minimum cost requirement in the fabrication of the design has been proposed [1], [2], [4]. On the other hand, component tuning, a very important related subject, is usually dealt with separately on a case-by-case study, or as an alignment procedure after the product is built [5].

A theory of optimal worst-case design embodying all the centering, tolerancing and tuning problems in a unified formulation at the design stage is presented here. Our approach incorporates the nominal circuit parameter values, the corresponding tolerances and tuning variables simultaneously into an optimization procedure designed to obtain the best values for all of them in an effort to reduce cost, or make an otherwise impractically toleranced design more attractive. Intuitively, our aim is to produce the best nominal point to permit the largest tolerances and the smallest tuning ranges (preferably zero) such that we can guarantee in advance that, in the worst case, the design will meet all the constraints and specifications.

Although our work seems to have more general implications we envisage that circuit design will be the most common application.

II THE GENERAL FORMULATION

A design consists of design data of the nominal point $\phi^o \triangleq [\phi_1^o \ \phi_2^o \ \dots \ \phi_k^o]^T$, the tolerances $\epsilon \triangleq [\epsilon_1 \ \epsilon_2 \ \dots \ \epsilon_k]^T$ and the tuning vector $t \triangleq [t_1 \ t_2 \dots t_k]^T$, where k is the number, for example, of network parameters. Note that not all the components of ϕ^o, ϵ and t need to be variables. Let $I_\phi \triangleq \{1,2,\ldots,k\}$ be the index set for these parameters. The problem is formulated as the nonlinear programming problem:

$$\text{minimize } C(\phi, \epsilon, t)$$

subject to

$$\phi \in R_c \tag{1}$$

where

$$\phi = \phi^o + E\mu + T\rho \tag{2}$$

and constraints on ϕ^o, ϵ, $t \geq 0$, for all $\mu \in R_\mu$ and some $\rho \in R_\rho$, where

$$E \triangleq \begin{bmatrix} \epsilon_1 & & & & \\ & \epsilon_2 & & & \\ & & \cdot & & \\ & & & \cdot & \\ & & & & \cdot \\ & & & & & \epsilon_k \end{bmatrix}, \qquad T \triangleq \begin{bmatrix} t_1 & & & & \\ & t_2 & & & \\ & & \cdot & & \\ & & & \cdot & \\ & & & & \cdot \\ & & & & & t_k \end{bmatrix}.$$

R_μ and R_ρ are sets of real number multipliers that will determine the outcome and the amount of tuning required to bring the point into R_c. R_μ and R_ρ are determined from realistic situations of the tolerance spread and tuning range. For example,

$$R_\mu = \{\mu \mid -1 \leq \mu_i \leq -a_i,\ a_i \leq \mu_i \leq 1,\ i \in I_\phi\} \tag{3}$$

where $0 \leq a_i \leq 1$, $i \in I_\phi$. The most commonly used continuous range is obtained by setting a_i to zero. A commercial stock will probably have the better toleranced components taken out, thus $0 < a_i \leq 1$. If $a_i = 1$, R_μ is identical to the set of vertices of the tolerance region. Some of the common examples for R_ρ will be

$$R_\rho = \{\underset{\sim}{\rho} | -1 \leq \rho_i \leq 1,\ i \in I_\phi\} \tag{4}$$

or, in the case of one-way tuning or irreversible trimming, $R_\rho = \{\underset{\sim}{\rho} | 0 \leq \rho_i \leq 1, i \in I_\phi\}$, or $R_\rho = \{\underset{\sim}{\rho} | -1 \leq \rho_i \leq 0, i \in I_\phi\}$. Unless otherwise indicated, we will consider $-1 \leq \mu_i \leq 1$, $-1 \leq \rho_i \leq 1$, $i \in I_\phi$.

<u>The Tuning Region</u>

The constraint region R_c is given by $R_c \triangleq \{\underset{\sim}{\phi} | \underset{\sim}{g}(\underset{\sim}{\phi}) \geq \underset{\sim}{0}\}$ and the tolerance region $R_\varepsilon \triangleq \{\underset{\sim}{\phi} | \underset{\sim}{\phi}^o - \underset{\sim}{\varepsilon} \leq \underset{\sim}{\phi} \leq \underset{\sim}{\phi}^o + \underset{\sim}{\varepsilon}\}$. Thus, a tuning region is given by

$$R_t(\underset{\sim}{\mu}) \triangleq \{\underset{\sim}{\phi} | \underset{\sim}{\phi}^o + \underset{\sim}{E}\underset{\sim}{\mu} - \underset{\sim}{t} \leq \underset{\sim}{\phi} \leq \underset{\sim}{\phi}^o + \underset{\sim}{E}\underset{\sim}{\mu} + \underset{\sim}{t}\}. \tag{5}$$

It is required that

$$R_t(\underset{\sim}{\mu}) \cap R_c \neq \emptyset , \tag{6}$$

where $\emptyset$ is the empty set. See Fig. 1 for an illustration of these concepts.

<u>Tunable Constraint Region</u>

R_c is defined by a set of specifications and constraints. For tunable constraint regions the problem is subject to

$$\underset{\sim}{\phi} \in R_c(\underset{\sim}{\psi}) \tag{7}$$

where $\underset{\sim}{\psi}$ represents other independent variables, and

$$\underset{\sim}{\phi} = \underset{\sim}{\phi}^o + \underset{\sim}{E}\underset{\sim}{\mu} + \underset{\sim}{T}\underset{\sim}{\rho}(\underset{\sim}{\psi}) . \tag{8}$$

Fig. 2 depicts three different regions of R_c. Overlapping is, of course, allowable.

III REDUCED PROBLEM 1

Assumptions and Definitions

Consider the separation of the components into effectively tuned and effectively toleranced parameters. Let

$$I_\varepsilon \triangleq \{i \mid \varepsilon_i > t_i,\ i \in I_\phi\} \tag{9}$$

$$I_t \triangleq \{i \mid t_i \geq \varepsilon_i,\ i \in I_\phi\} \tag{10}$$

$$\varepsilon_i' \triangleq \varepsilon_i - t_i\ ,\ i \in I_\varepsilon \tag{11}$$

and

$$t_i' \triangleq t_i - \varepsilon_i,\ i \in I_t. \tag{12}$$

It is obvious that I_t and I_ε are disjoint and $I_t \cup I_\varepsilon = I_\phi$. Now, we may consider the problem subject to

$$\underset{\sim}{\phi} \in R_c$$

where

$$\phi_i = \phi_i^o + \begin{cases} \varepsilon_i' \mu_i & \text{for } i \in I_\varepsilon \\ t_i' \rho_i' & \text{for } i \in I_t \end{cases} \tag{13}$$

for all $-1 \leq \mu_i \leq 1$, $i \in I_\varepsilon$ and for some $-1 \leq \rho_i' \leq 1$, $i \in I_t$.

Theorem 1

A feasible solution to the reduced problem 1 is a feasible solution to the original problem.

Proof

Given $\underset{\sim}{\phi}^o$, $\underset{\sim}{\varepsilon}$, $\underset{\sim}{t}$ we will show that

(1) $\varepsilon_i \mu_i + t_i \rho_i = \varepsilon_i' \mu_i$, $i \in I_\varepsilon$

(2) $\varepsilon_i \mu_i + t_i \rho_i = t_i' \rho_i'$, $i \in I_t$

under the restrictions on μ_i, ρ_i and $\rho_i^{'}$.

(1) Since ρ_i can be freely chosen from $-1 \leq \rho_i \leq 1$, we can let $\rho_i = -\mu_i$ giving

$$(\varepsilon_i - t_i)\mu_i = \varepsilon_i^{'}\, \mu_i.$$

(2) For any $-1 \leq \rho_i^{'} \leq 1$ and all $-1 \leq \mu_i \leq 1$, we can choose

$$-1 \leq \rho_i = \frac{(t_i - \varepsilon_i)\rho_i^{'} - \varepsilon_i \mu_i}{t_i} \leq 1.$$

Thus, any point with components represented by (13) of the reduced problem can be represented by (2) of the original problem.

Theorem 2

A feasible solution to the original problem implies a feasible solution to the reduced problem if R_c is one-dimensionally convex.

Proof

(1) We note for $i \in I_\varepsilon$ that

$$\phi_i^o - \varepsilon_i + t_i\rho_i(-1) \leq \phi_i^o - \varepsilon_i + t_i \leq \phi_i^o + (\varepsilon_i - t_i)\mu_i \leq \phi_i^o + \varepsilon_i - t_i \leq \phi_i^o + \varepsilon_i + t_i\rho_i(1)$$

where $\rho_i(-1)$ corresponds to $\mu_i = -1$ and $\rho_i(1)$ corresponds to $\mu_i = 1$.

If R_c is one-dimensionally convex [1]

$$\begin{bmatrix} \vdots \\ \phi_i^o - \varepsilon_i + t_i\ \rho_i(-1) \\ \vdots \end{bmatrix}, \quad \begin{bmatrix} \vdots \\ \phi_i^o + \varepsilon_i + t_i\ \rho_i(1) \\ \vdots \end{bmatrix} \in R_c \qquad (14)$$

implies that

$$\begin{bmatrix} \vdots \\ \phi_i^o + (\varepsilon_i - t_i)\mu_i \\ \vdots \end{bmatrix} \in R_c$$

where we consider changes in the ith component only and impose the required restrictions on μ_i and ρ_i.

(2) On the other hand, for $i \varepsilon I_t$, given feasible $\rho_i(-1)$ and $\rho_i(1)$ such that $\phi_i^o - \varepsilon_i + t_i \rho_i(-1) \leq \phi_i^o + \varepsilon_i + t_i \rho_i(1)$ there exists a feasible ρ_i' such that $\phi_i^o - \varepsilon_i + t_i \rho_i(-1) \leq \phi_i^o + (t_i - \varepsilon_i) \rho_i' \leq \phi_i^o + \varepsilon_i + t_i \rho_i(1)$. This is true for $t_i = \varepsilon_i$ and can be seen for $t_i > \varepsilon_i$ by rewriting this inequality as

$$\frac{-\varepsilon_i + t_i \rho_i(-1)}{t_i - \varepsilon_i} \leq \rho_i' \leq \frac{\varepsilon_i + t_i \rho_i(1)}{t_i - \varepsilon_i} .$$

Hence, if R_c is one-dimensionally convex, assumption (14) implies that

$$\begin{bmatrix} \vdots \\ \phi_i^o + (t_i - \varepsilon_i) \rho_i' \\ \vdots \end{bmatrix} \varepsilon R_c .$$

Thus, a feasible solution to the original problem can be transformed to a feasible solution of the reduced problem.

<u>A Geometric Interpretation</u>

Let

$$R_{\varepsilon t} \triangleq \{\phi | \phi_i^o - \varepsilon_i' \leq \phi_i \leq \phi_i^o + \varepsilon_i', i \varepsilon I_\varepsilon\} \tag{15}$$

and

$$R_{t\varepsilon} \triangleq \{\phi | \phi_i^o - t_i' \leq \phi_i \leq \phi_i^o + t_i', i \varepsilon I_t\}. \tag{16}$$

Consider the following regions

$$R_{\varepsilon tp} \triangleq \{\phi_p | \phi_p = P\phi, \phi \varepsilon R_{\varepsilon t}\} , \tag{17}$$

$$R_{ct\varepsilon} \triangleq R_c \cap R_{t\varepsilon}$$

and

$$R_{ct\varepsilon p} \triangleq \{\underset{\sim}{\phi}_p | \underset{\sim}{\phi}_p = \underset{\sim}{P}\underset{\sim}{\phi}, \underset{\sim}{\phi} \in R_{ct\varepsilon}\} \tag{18}$$

where

$$\underset{\sim}{P} \triangleq \begin{bmatrix} p_1 & & & & \\ & p_2 & & & \\ & & \cdot & & \\ & & & \cdot & \\ & & & & \cdot \\ & & & & & p_k \end{bmatrix}$$

and

$$p_i = \begin{cases} 0 \text{ for } & i \in I_t \\ 1 \text{ for } & i \in I_\varepsilon \end{cases} .$$

The constraints of the problem are now interpreted by the requirement that

$$R_{\varepsilon tp} \subseteq R_{ct\varepsilon p} \quad . \tag{19}$$

Fig. 3 depicts the definition of the different regions. Any point whose components are given by (13) is in the intersection of $R_{\varepsilon t}$ and $R_{t\varepsilon}$, may be tuned into $R_{ct\varepsilon}$ by changing the value of ρ_i', $i \in I_t$ if the projection of $R_{ct\varepsilon}$ onto the subspace spanned by the components subject to tolerances includes the projection of that point.

The reduced problem may be stated as: solve a pure tolerance problem (i.e., no tuning) in the subspace spanned by the toleranced variables with $R_{\varepsilon tp}$ as the tolerance region and $R_{ct\varepsilon p}$ as the constraint region.

<u>Special Cases</u>

(1) $I_\varepsilon = \emptyset$, the pure tuning problem.

In this case, $R_{\varepsilon t}$ is the entire space and $\underset{\sim}{P}$ is a zero matrix. The problem has a solution if

$$R_{ct\varepsilon} \neq \emptyset. \tag{20}$$

(2) $I_t = \emptyset$, the pure tolerance problem.

In this case, $R_{t\varepsilon}$ is the entire space and $\underset{\sim}{P}$ is a unit matrix. This problem has been treated previously. It is required that

$$R_{\varepsilon t} \subseteq R_c. \tag{21}$$

Fig. 4 illustrates a case where $R_{\varepsilon tp} \not\subseteq R_{ct\varepsilon p}$. An outcome at $\underset{\sim}{\phi}^o$ cannot be tuned to R_c within the effective tuning range. However, there exists a solution to the original formulation by tuning both components. R_c is not one-dimensionally convex in this case.

<u>Extension for Tunable Constraint Region</u>

Three types of components can be identified when the constraint region is considered to be tunable. They are:

(a) Toleranced components

(b) Components tuned by the manufacturer

(c) Components tunable by the customer

In this case,

$$\underset{\sim}{\phi} \in R_c(\underset{\sim}{\psi})$$

where

$$\phi_i = \phi_i^o + \begin{cases} \varepsilon_i' \mu_i & \text{for } i \in I_\varepsilon \\ t_i' \rho_i' & \text{for } i \in I_{tm} \\ t_i' \rho_i'(\underset{\sim}{\psi}) & \text{for } i \in I_{tc} \end{cases} \tag{22}$$

where I_{tm} identifies components (b) and I_{tc} identifies components (c).

Setting the value $\underset{\sim}{\psi}$ to a particular value will control the setting of the value of ρ_i', $i \in I_{tc}$ such that $\underset{\sim}{\phi}$ will be in that particular constraint region $R_c(\underset{\sim}{\psi})$.

IV REDUCED PROBLEM 2

We consider $-1 \leq \mu_i \leq 1$ replaced by $\mu_i \in \{-1,1\}$, $i \in I_\varepsilon$.

Theorem 3 A feasible solution to reduced problem 2 implies a feasible solution to reduced problem 1 if $R_{ct\varepsilon p}$ is one-dimensionally convex.

This is a pure tolerance problem in the subspace. The proof follows [1].

Implementation for Frequency Domain Problem

Let $\underset{\sim}{\phi}_p^r$ be a vertex of $R_{\varepsilon tp}$. The constraints can be expressed explicitly as

$$\underset{\sim}{g}(\underset{\sim}{\phi}) = \underset{\sim}{g}(\underset{\sim}{\phi}_p^r + \sum_{i\varepsilon I_t} (\phi_i^o + t_i' \rho_i^r)\underset{\sim}{e}_i) \geq \underset{\sim}{0} \tag{23}$$

where $\underset{\sim}{e}_i$ is the ith unit vector. The slack variables ρ_i^r are also to be constrained by $-1 \leq \rho_i^r \leq 1$. Strategies to eliminate inactive vertices [4] will not be discussed here, but must be considered in any practical implementation in order to make the problem tractable.

The optimization parameters $\underset{\sim}{x}$ may now be identified as a n-dimensional vector consisting of the variable nominal values, tolerances, tuning variables and all the appropriate slack variables $\underset{\sim}{\rho}$. A total of m constraint functions may be formed. in general,

$$n = k_o + k_\varepsilon + k_t(1 + n_v) \tag{24}$$

and

$$m = \left(\sum_{i=1}^{n_\omega} n_v(i) \right) + 2k_t n_v + \ldots \tag{25}$$

where k_o, k_ε and k_t are the numbers of nominal parameter values, toleranced and tuned parameters, respectively; $n_v \leq 2^{k_\varepsilon}$ is the number of distinct vertices chosen; n_ω is the number of frequency points considered; $n_v(i)$ is the number of vertices chosen at the ith frequency point and $2k_t n_v$ is the number of slack variable bounds.

Any conventional nonlinear programming algorithm may be used to minimize $C(\underset{\sim}{x})$ subject to $g_i(\underset{\sim}{x}) \geq 0$, $i = 1,2,\ldots,m$.

V EXAMPLE

Consider the constraints

$$\phi_2 - \phi_1 - 2 \geq 0 \tag{26}$$

$$-\phi_2^2 + 16\phi_1 \geq 0. \tag{27}$$

Tolerance Example

Minimize $\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2}$

subject to

$$g_1 = \varepsilon_1 \geq 0,\ g_2 = \varepsilon_2 \geq 0,\ g_3 = \phi_1^o \geq 0,\ g_4 = \phi_2^o \geq 0$$

$$g_5(i) = (\phi_2^o + \varepsilon_2\mu_2(i)) - (\phi_1^o + \varepsilon_1\mu_1(i)) - 2 \geq 0\ , i = 1,2,\ldots \tag{28}$$

$$g_6(i) = -(\phi_2^o + \varepsilon_2\mu_2(i))^2 + 16(\phi_i^o + \varepsilon_1\mu_1(i)) \geq 0\ ,\ i = 1,2,\ldots \tag{29}$$

where $-1 \leq \mu_1(i) \leq 1$ and $-1 \leq \mu_2(i) \leq 1$.

Necessary conditions for optimality require that

$$\begin{bmatrix} -\frac{1}{\varepsilon_1^2} \\ -\frac{1}{\varepsilon_2^2} \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{bmatrix} + \sum_i u_5(i) \begin{bmatrix} -\mu_1(i) \\ \mu_2(i) \\ -1 \\ 1 \end{bmatrix} + \sum_i u_6(i) \begin{bmatrix} 16\mu_1(i) \\ -2\mu_2(i)\ (\phi_2^o + \varepsilon_2\mu_2(i)) \\ 16 \\ -2(\phi_2^o + \varepsilon_2\mu_2(i)) \end{bmatrix} \tag{30}$$

$$u_1 g_1 = \ldots = u_4 g_4 = u_5(i)\ g_5(i) = u_6(i)\ g_6(i) = 0, i = 1,2,\ldots \quad (31)$$

$$u_1,\ \ldots,\ u_4,\ u_5(i),\ u_6(i) \geq 0\ ,\ i = 1,2,\ldots \quad (32)$$

Assume that ε_1, ε_2, ϕ_1^o and ϕ_2^o are not zero, therefore, set u_1, u_2, u_3 and u_4 to zero. Minimize $g_5(i)$ of (28) and $g_6(i)$ of (29) w.r.t. $\underset{\sim}{\mu}(i)$. This leads, respectively, to

$$(\phi_2^o - \varepsilon_2) - (\phi_1^o + \varepsilon_1) - 2 \geq 0$$

using $\underset{\sim}{\mu}(i) = [1 \quad -1]^T$ and

$$-(\phi_2^o + \varepsilon_2)^2 + 16(\phi_1^o - \varepsilon_1) \geq 0$$

using $\underset{\sim}{\mu}(i) = [-1 \quad 1]^T$. The optimality conditions (30) - (32) are correspondingly reduced yielding the solution

$$\varepsilon_1 = 0.5,\ \varepsilon_2 = 0.5,\ \phi_1^o = 3.5,\ \phi_2^o = 7.5,$$

which have been verified by numerical optimization.

Tuning Example

Suppose ϕ_1 has a 10% tuning range and ϕ_2 is toleranced. Consider the problem of minimizing

$$\frac{1}{\varepsilon_2},$$

subject to

$$g_1 = t_1' \geq 0,\ g_2 = \varepsilon_2' \geq 0,\ g_3 = \phi_1^o \geq 0,\ g_4 = \phi_2^o \geq 0$$

$$g_5 = .1 - \frac{t_1'}{\phi_1^o} \geq 0 \tag{33}$$

$$g_6(i) = (\phi_2^o + \varepsilon_2'\mu_2(i)) - (\phi_1^o + t_1'\rho_1'(i)) - 2 \geq 0\,,\; i = 1,2,\ldots \tag{34}$$

$$g_7(i) = -(\phi_2^o + \varepsilon_2'\mu_2(i))^2 + 16(\phi_1^o + t_1'\rho_1'(i)) \geq 0\,,\; i = 1,2,\ldots \tag{35}$$

$$g_8(i) = 1 - \rho_1'(i) \geq 0\,,\quad i = 1,2,\ldots \tag{36}$$

$$g_9(i) = 1 + \rho_1'(i) \geq 0\,,\quad i = 1,2,\ldots \tag{37}$$

The variables are t_1', ε_2', ϕ_1^o, ϕ_2^o and $\rho_1'(i)$, and $-1 \leq \mu_2(i) \leq 1$

Optimality requires in this case

$$\begin{bmatrix} 0 \\ -\frac{1}{\varepsilon_2'^2} \\ 0 \\ 0 \\ \underset{\sim}{0} \end{bmatrix} = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \\ \underset{\sim}{\ell} \end{bmatrix} + u_5 \begin{bmatrix} -\frac{1}{\phi_1^o} \\ 0 \\ \frac{t_1'}{\phi_1^{o2}} \\ 0 \\ \underset{\sim}{\ell} \end{bmatrix} + \sum_i u_6(i) \begin{bmatrix} -\rho_1'(i) \\ \mu_2(i) \\ -1 \\ 1 \\ -t_1'\underset{\sim}{e}_i \end{bmatrix} + \sum_i u_7(i) \begin{bmatrix} 16\rho_1'(i) \\ -2(\phi_2^o+\varepsilon_2'\mu_2(i))\mu_2(i) \\ 16 \\ -2(\phi_2^o+\varepsilon_2'\mu_2(i)) \\ 16t_1'\underset{\sim}{e}_i \end{bmatrix}$$

$$+ \sum_i u_8(i) \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ -\underset{\sim}{e}_i \end{bmatrix} + \sum_i u_9(i) \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \underset{\sim}{e}_i \end{bmatrix} \tag{38}$$

$$u_1 g_1 = \ldots = u_5 g_5 = u_6(i)\ g_6(i) = \ldots = u_9(i)\ g_9(i) = 0, i = 1,2,\ldots \qquad (39)$$

$$u_1,\ldots,u_5,\ u_6(i),\ldots,u_9(i) \geq 0\ ,\ i = 1,2,\ldots \qquad (40)$$

where $\underset{\sim}{e}_i$ is the ith unit vector. Minimize $g_6(i)$ of (34) and $g_7(i)$ of (35) w.r.t. $\mu_2(i)$. We use $\mu_2(i) = -1$ in (34) and $\mu_2(i) = 1$ in (35) for this purpose. The corresponding $\rho_1'(i) = -1$ and $\rho_1'(i) = 1$, respectively, are obtained by maximizing $g_6(i)$ and $g_7(i)$ w.r.t. $\rho_1'(i)$. This yields the solution $t_1' = 0.5432$, $\varepsilon_2' = 1.444$, $\phi_1^o = 5.4321$, $\phi_2^o = 8.3333$.

VI CONCLUSIONS

A theory of optimal worst case design embodying centering, tolerancing and tuning has been presented. The concept of a tunable constraint region that allows variable specifications as set by the customer has also been incorporated. This may find application, for example, in tunable filters. Components can be separated into effectively tuned and effectively toleranced parameters to simplify the solution, but possibly at the expense of optimality. The purely toleranced and purely tuned problems become special cases. Further simplification has been discussed in the light of one-dimensional convexity.

It may be added that as far as the authors are aware, this seems to be the most general formulation to date dealing with the centering, tolerancing and tuning problems at the design stage. Tuning uncertainties can also be taken care of in the formulation by associating tolerances with the tunable elements.

By its very nature the problem is a large one, even for designs with a relatively small number of parameters. Practical implementation will depend heavily on one's ability to select a sufficiently small number of relevant vertices or critical points and constraints likely to be active, as well as meaningful variables.

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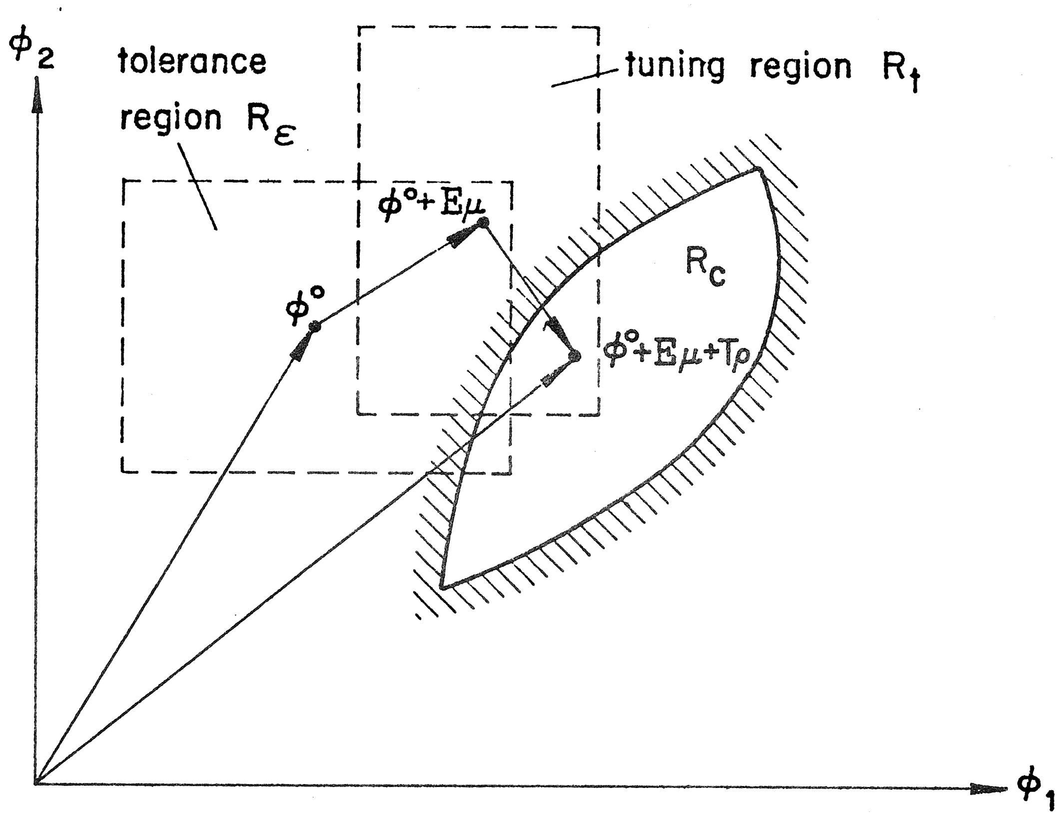

Fig. 1 An illustration of the regions R_ε, R_t, and R_c.

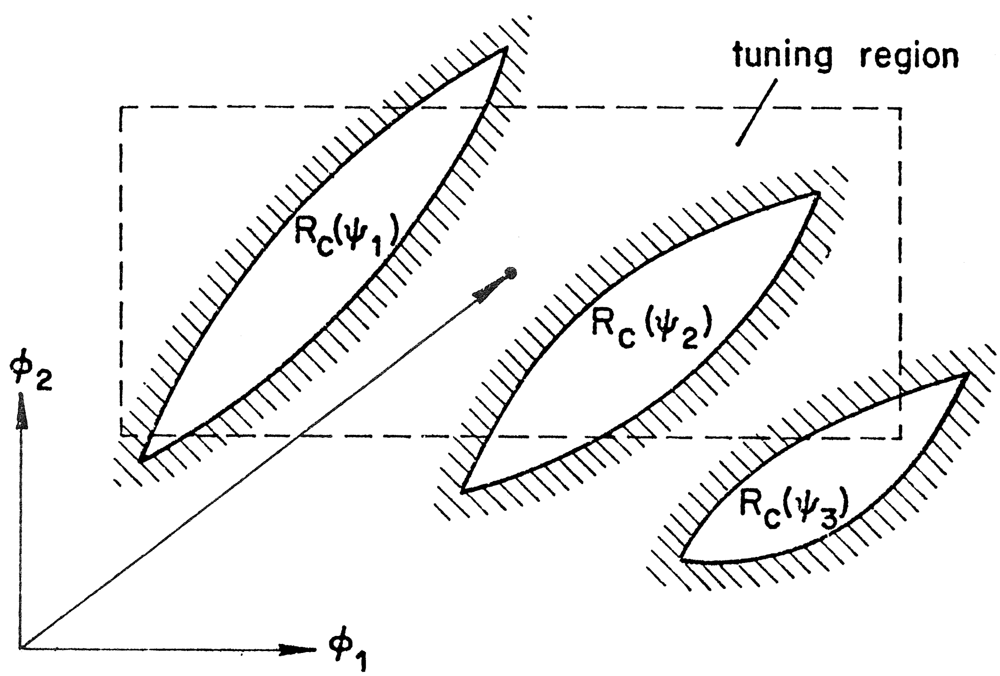

Fig. 2 An example of three different settings of the tunable constraint regions.

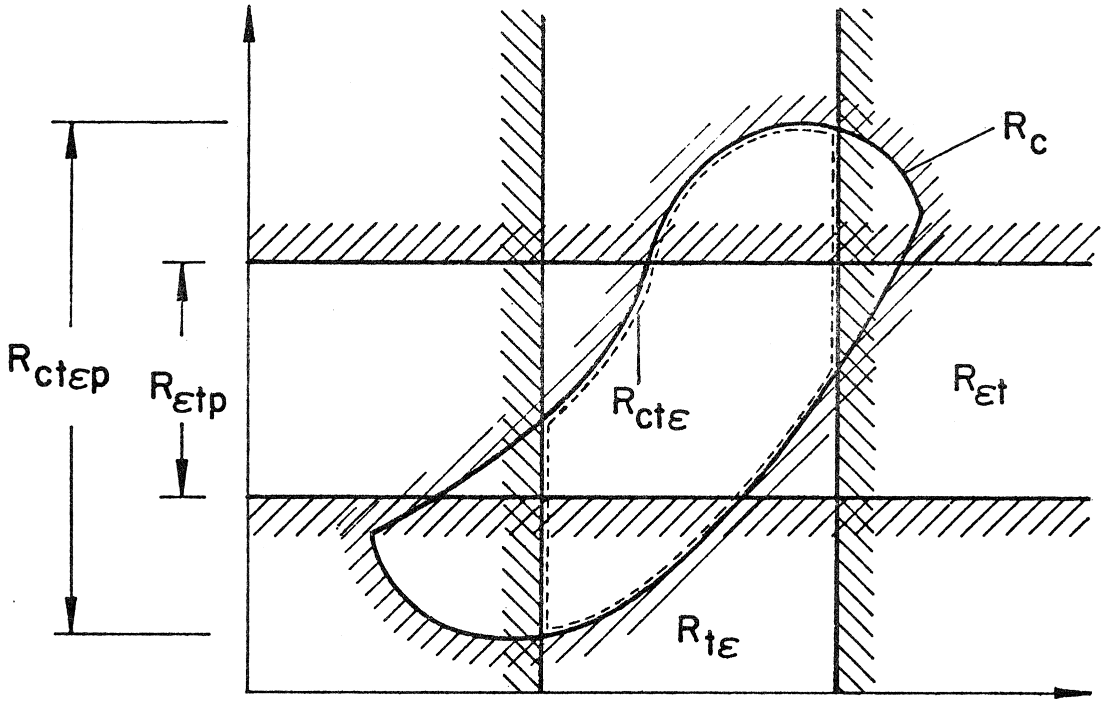

Fig. 3 A geometric interpretation of the reduced problem 1.

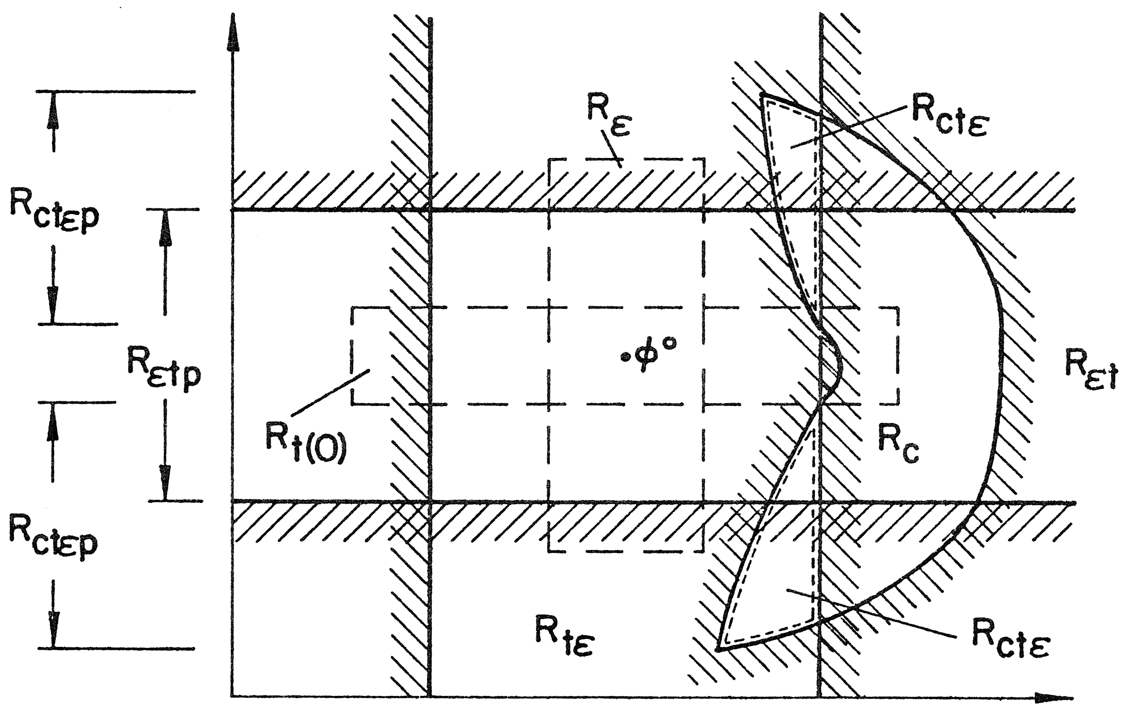

Fig. 4 An example of $R_{\varepsilon tp} \subsetneq R_{ct\varepsilon p}$.