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PRACTICAL DESIGN CENTERING, TOLERANCING AND TUNING

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Abstract This paper presents the results of a numerical investigation of simultaneous optimal design centering, tolerancing and tuning of circuits. The general worst-case optimal tolerance-tuning problem is briefly reviewed. Practical implementation requires a reasonable and relevant number of parameters and constraints to be identified to make the problem tractable. Two circuits, a simple LC lowpass filter and a realistic highpass filter, are studied under a variety of different problem situations to illustrate both the benefits to be derived from our approach and the difficulties encountered in its implementation.

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I INTRODUCTION

This paper presents the results of a numerical investigation of simultaneous optimal design centering, tolerancing and tuning of circuits. The optimal worst-case tolerance problem has received much attention in the literature [1-4] and benefits in terms of increased tolerances by permitting the nominal point to move have been established [2,4]. This work brings in the tuning of one or more circuit components basically in order to further increase tolerances on all the components.

Theoretical background to our work has already been presented [5]. This paper, therefore, briefly reviews the essential ideas in the general worst-case optimal tolerance-tuning problem before turning to practical implementation. We have to minimize an objective function representing the cost of the circuit. There are, in general, an infinite number of variables and an infinite number of constraints even for a small circuit. To make the problem tractable we need a sufficient but reasonable number of variables and constraints to be identified. The approach of selecting these is not yet automated, due to its complexity, and (except in very small problems) usually requires a few preliminary runs to determine relevant parameters and active constraints.

Two circuits, a simple LC lowpass filter and a realistic highpass filter, are studied under a variety of different problem situations to illustrate both the benefits to be derived from our approach and the difficulties encountered in its implementation.

II GENERAL CONSIDERATIONS

The problem we are considering may, in general, be stated as [5]

$$\text{minimize } C(\underset{\sim}{\phi}^{o}, \underset{\sim}{\varepsilon}, \underset{\sim}{t})$$

where

$$\phi^o \triangleq \begin{bmatrix} \phi_1^o \\ \phi_2^o \\ \cdot \\ \cdot \\ \cdot \\ \phi_k^o \end{bmatrix}, \quad \epsilon \triangleq \begin{bmatrix} \epsilon_1 \\ \epsilon_2 \\ \cdot \\ \cdot \\ \cdot \\ \epsilon_k \end{bmatrix}, \quad t \triangleq \begin{bmatrix} t_1 \\ t_2 \\ \cdot \\ \cdot \\ \cdot \\ t_k \end{bmatrix}. \tag{1}$$

k is the number of designable parameters, ϕ^o is the nominal point, ϵ is the tolerance vector and t the tuning vector. It is required, furthermore, that

$$\phi \in R_c \tag{2}$$

where R_c is the constraint region given by

$$R_c \triangleq \{\phi | g(\phi) \geq 0\} \tag{3}$$

and where

$$\left.\begin{aligned} \phi_i &= \phi_i^o + \epsilon_i\mu_i + t_i\rho_i \\ \phi_i^o&, \epsilon_i, t_i \geq 0 \end{aligned}\right\} i = 1,2,\ldots,k \tag{4}$$

for all specified values of μ_i and some allowable values of ρ_i. In this work we consider

$$\mu_i, \rho_i \in [-1, 1], i = 1,2,\ldots,k \tag{5}$$

Intuitively, we require that for each outcome $\{\phi^o, \epsilon, \mu\}$ of a design $\{\phi^o, \epsilon, t\}$ there must be a ρ such that $\phi \in R_c$, where

$$\mu \triangleq \begin{bmatrix} \mu_1 \\ \mu_2 \\ \cdot \\ \cdot \\ \cdot \\ \mu_k \end{bmatrix}, \quad \rho \triangleq \begin{bmatrix} \rho_1 \\ \rho_2 \\ \cdot \\ \cdot \\ \cdot \\ \rho_k \end{bmatrix}. \tag{6}$$

We let the tolerance region R_ε be given by [5]

$$R_\varepsilon \triangleq \{\phi \mid \phi_i^o - \varepsilon_i \leq \phi_i \leq \phi_i^o + \varepsilon_i, \ i = 1,2,\ldots,k\} \tag{7}$$

and the tuning region $R_t(\mu)$ be given by

$$R_t(\mu) \triangleq \{\phi \mid \phi_i^o + \varepsilon_i \mu_i - t_i \leq \phi_i \leq \phi_i^o + \varepsilon_i \mu_i + t_i, \ i = 1,2,\ldots,k \} \ . \tag{8}$$

Other essential concepts for this work are

$$\varepsilon_i' \triangleq \varepsilon_i - t_i \ \text{for} \ I_\varepsilon \triangleq \{i \mid \varepsilon_i > t_i, \ i \in \{1,2,\ldots,k\}\} \tag{9}$$

$$t_i' \triangleq t_i - \varepsilon_i \ \text{for} \ I_t \triangleq \{i \mid t_i \geq \varepsilon_i, \ i \in \{1,2,\ldots,k\}\} \tag{10}$$

called the effective tolerance and effective tuning, respectively, and

$$P \triangleq \begin{bmatrix} p_1 & & & \\ & p_2 & & \\ & & \ddots & \\ & & & p_k \end{bmatrix}, \qquad p_i = \begin{cases} 0 & \text{for} \ i \in I_t \\ 1 & \text{for} \ i \in I_\varepsilon \end{cases} \tag{11}$$

In using effective tuning or tolerancing we may replace (4) by

$$\phi_i = \phi_i^o + \begin{cases} \varepsilon_i' \mu_i, \ \varepsilon_i' \geq 0 & \text{for} \ i \in I_\varepsilon \\ t_i' \rho_i', \ t_i' \geq 0 & \text{for} \ i \in I_t \end{cases} \tag{12}$$

$$\phi_i^o \geq 0, \ i = 1,2,\ldots,k$$

where

$$\mu_i, \ \rho_i' \in [-1, 1] \ . \tag{13}$$

Instead of considering $g(\phi) \geq 0$ as in (3) we might then take constraints of the form

$$g(P\phi + \sum_{i \in I_t} (\phi_i^o + t_i' \rho_i') e_i) \geq 0 \tag{14}$$

where $\underset{\sim}{e}_i$ is the ith unit vector, and $\underset{\sim}{\phi}$ describes an outcome or an effective outcome, whichever is appropriate. This leads to a special case for which results have been obtained.

III IMPLEMENTATION

The constraints associated with response specifications are of the form

$$g = w(S - F) \geq 0 \tag{15}$$

with appropriate subscripts, where F is the circuit response function of $\underset{\sim}{\phi}$ and ψ, which is an independent parameter denoting frequency or any number to identify a particular function. S is a specification and w a weighting factor. Both are functions of ψ. In our present work

$$w_i = \begin{cases} +1 \text{ if } S_i \text{ is an upper specification} \\ -1 \text{ if } S_i \text{ is a lower specification .} \end{cases} \tag{16}$$

Data for a specific problem is contained in a vector $\underset{\sim}{a}$, which has the form

$$\underset{\sim}{a} = \begin{bmatrix} i \\ \underset{\sim}{\mu} \\ \psi \\ S \\ w \end{bmatrix} \Rightarrow \{\underset{\sim}{\phi}^i, \psi\} \Rightarrow g \text{ ,} \tag{17}$$

where i is an integer indexing a distinct outcome to be considered in the subspace spanned by the effectively toleranced components.

If vertices of the tolerance region are considered, then we employ the numbering scheme [2,4]

$$\underset{\sim}{\mu}^1 \triangleq \begin{bmatrix} -1 \\ -1 \\ \cdot \\ \cdot \\ \cdot \\ -1 \end{bmatrix}, \quad \underset{\sim}{\mu}^2 \triangleq \begin{bmatrix} +1 \\ -1 \\ \cdot \\ \cdot \\ \cdot \\ -1 \end{bmatrix}, \quad \underset{\sim}{\mu}^3 \triangleq \begin{bmatrix} -1 \\ +1 \\ \cdot \\ \cdot \\ \cdot \\ -1 \end{bmatrix}, \ldots, \underset{\sim}{\mu}^{2^k} \triangleq \begin{bmatrix} +1 \\ +1 \\ \cdot \\ \cdot \\ \cdot \\ +1 \end{bmatrix} \qquad (18a)$$

or, more formally, the rth vertex corresponds to

$$r = 1 + \sum_{j=1}^{k} \left(\frac{\mu_j(r) + 1}{2} \right) 2^{j-1}, \quad \mu_j(r) \in \{-1, 1\}. \qquad (18b)$$

We assume, unless otherwise specified, that vertices provide active constraints. The validity or otherwise of this has been discussed elsewhere [5].

The number of variables is designated n and the number of constraints m.

IV LOWPASS FILTER

The 3-element LC lowpass filter (Fig. 1) to be discussed has already been considered in the context of optimal centering and tolerancing [2,4]. Table I specifies the passband and stopband requirements. The sample points used in the optimization procedure are $\omega_1 = 0.45$, $\omega_2 = 0.5$, $\omega_3 = 0.55$ and $\omega_4 = 1.0$ rad/s for the passband and $\omega_5 = 2.5$ rad/s in the stopband.

The optimization program used is based on recent work in least pth approximation and nonlinear programming by Charalambous [6] and incorporates the quasi-Newton method of unconstrained optimization developed by Fletcher [7] and Gill and Murray [8].

Example 1: No Tuning ($\underset{\sim}{t} = \underset{\sim}{0}$)

For each frequency point $2^k = 8$ vertices for the tolerance region R_ε given by (7) can be obtained. The active vertices correspond to $\underset{\sim}{\mu}^6$ at $\omega = \omega_1, \omega_2, \omega_3$; $\underset{\sim}{\mu}^8$ at $\omega = \omega_4$; and $\underset{\sim}{\mu}^1$ at $\omega = \omega_5$, where

$$\underset{\sim}{\phi}^o = \begin{bmatrix} L_1^o \\ C^o \\ L_2^o \end{bmatrix}, \qquad \underset{\sim}{\varepsilon} = \begin{bmatrix} \varepsilon_{L_1} \\ \varepsilon_C \\ \varepsilon_{L_2} \end{bmatrix}.$$

For this problem, therefore, from (17)

$$\underset{\sim}{a}^1 = \begin{bmatrix} 1 \\ +1 \\ -1 \\ +1 \\ 0.45 \\ 1.5 \\ 1 \end{bmatrix} \Rightarrow g_1, \quad \underset{\sim}{a}^2 = \begin{bmatrix} 1 \\ +1 \\ -1 \\ +1 \\ 0.5 \\ 1.5 \\ 1 \end{bmatrix} \Rightarrow g_2, \quad \underset{\sim}{a}^3 = \begin{bmatrix} 1 \\ +1 \\ -1 \\ +1 \\ 0.55 \\ 1.5 \\ 1 \end{bmatrix} \Rightarrow g_3, \quad \underset{\sim}{a}^4 = \begin{bmatrix} 2 \\ +1 \\ +1 \\ +1 \\ 1.0 \\ 1.5 \\ 1 \end{bmatrix} \Rightarrow g_4, \quad \underset{\sim}{a}^5 = \begin{bmatrix} 3 \\ -1 \\ -1 \\ -1 \\ 2.5 \\ 2.5 \\ -1 \end{bmatrix} \Rightarrow g_5 . \tag{19}$$

We note also that

$$\underset{\sim}{\phi}^1 = \underset{\sim}{\phi}(\underset{\sim}{\mu}^6) = \begin{bmatrix} \phi_1^o + \varepsilon_1 \\ \phi_2^o - \varepsilon_2 \\ \phi_3^o + \varepsilon_3 \end{bmatrix}, \quad \underset{\sim}{\phi}^2 = \underset{\sim}{\phi}(\underset{\sim}{\mu}^8) = \begin{bmatrix} \phi_1^o + \varepsilon_1 \\ \phi_2^o + \varepsilon_2 \\ \phi_3^o + \varepsilon_3 \end{bmatrix}, \quad \underset{\sim}{\phi}^3 = \underset{\sim}{\phi}(\underset{\sim}{\mu}^1) = \begin{bmatrix} \phi_1^o - \varepsilon_1 \\ \phi_2^o - \varepsilon_2 \\ \phi_3^o - \varepsilon_3 \end{bmatrix}. \tag{20}$$

The results for this problem are shown in Table II for the cost function

$$C(\underset{\sim}{\phi}^o, \underset{\sim}{\varepsilon}) = \frac{L_1^o}{\varepsilon_{L_1}} + \frac{C^o}{\varepsilon_C} + \frac{L_2^o}{\varepsilon_{L_2}} . \tag{21}$$

<u>Example 2: Effective Tuning for One Component</u>

(a) L_1 tuned, C and L_2 toleranced.

We consider an objective function based on the relative tolerances of C and L_2 in the form

$$C(x_2,x_3,x_5,x_6) = \frac{x_2}{x_5^2} + \frac{x_3}{x_6^2} \tag{22}$$

where, assuming $t_2 = t_3 = 0$,

$$L_1^o = \phi_1^o = x_1$$

$$C^o = \phi_2^o = x_2$$

$$L_2^o = \phi_3^o = x_3$$

$$t_{L_1} = t_1' = x_4^2$$

$$\varepsilon_C = \varepsilon_2 = x_5^2$$

$$\varepsilon_{L_2} = \varepsilon_3 = x_6^2$$

the last three transformations chosen to avoid changes of sign. The functions $g_1, g_2, \ldots, g_5$ are chosen as in (19) except that from (11), (14) and (20)

$$\boldsymbol{\phi}^1 = \begin{bmatrix} x_1 + x_4^2 x_7 \\ x_2 - x_5^2 \\ x_3 + x_6^2 \end{bmatrix} = \boldsymbol{P}\boldsymbol{\phi}(\boldsymbol{\mu}^6) + (\phi_1^o + t_1'\rho_1^1)\boldsymbol{e}_1$$

$$\boldsymbol{\phi}^2 = \begin{bmatrix} x_1 + x_4^2 x_8 \\ x_2 + x_5^2 \\ x_3 + x_6^2 \end{bmatrix} = \boldsymbol{P}\boldsymbol{\phi}(\boldsymbol{\mu}^8) + (\phi_1^o + t_1'\rho_1^2)\boldsymbol{e}_1$$

$$\boldsymbol{\phi}^3 = \begin{bmatrix} x_1 + x_4^2 x_9 \\ x_2 - x_5^2 \\ x_3 - x_6^2 \end{bmatrix} = \boldsymbol{P}\boldsymbol{\phi}(\boldsymbol{\mu}^1) + (\phi_1^o + t_1'\rho_1^3)\boldsymbol{e}_1$$

where

$$\underset{\sim}{R} = \begin{bmatrix} 0 & & \\ & 1 & \\ & & 1 \end{bmatrix}$$

and where

$$\rho_1^1 = x_7,\ \rho_1^2 = x_8,\ \rho_1^3 = x_9.$$

Additional constraints are given by

$$\left.\begin{aligned} g_{5+2i-1} &= 1+x_{6+i} \\ g_{5+2i} &= 1-x_{6+i} \end{aligned}\right\}\ i = 1,2,3$$

$$g_{12} = t_r - x_4^2/x_1.$$

The last constraint g_{12} is designed to limit the tuning range to t_r. Table III shows results for three values of t_r. The same results are obtained replacing the term $x_1+x_4^2x_i$, by $x_1(1+t_r x_i)$, i= 7,8,9, allowing g_{12} to be removed, and reducing the number of variables by one, since g_{12} is active.

(b) C tuned, L_1 and L_2 toleranced.

We consider an objective function based on the relative tolerances of L_1 and L_2 in the form

$$C(x_1,x_3,x_4,x_6) = \frac{x_1}{x_4^2} + \frac{x_3}{x_6^2} \qquad (23)$$

where x_1, x_2, x_3 and x_6 are as before but where,

$$\varepsilon_{L_1} = \varepsilon_1 = x_4^2$$

$$t_C = t_2' = x_5^2$$

with $t_1 = 0$. In this case

$$\underset{\sim}{\phi}^1 = \begin{bmatrix} x_1 + x_4^2 \\ x_2 + x_5^2 x_7 \\ x_3 + x_6^2 \end{bmatrix} = \underset{\sim}{P}\underset{\sim}{\phi}(\underset{\sim}{\mu}^6) + (\phi_2^o + t_2' \rho_2^1)\underset{\sim}{e}_2 = \underset{\sim}{P}\underset{\sim}{\phi}(\underset{\sim}{\mu}^8) + (\phi_2^o + t_2' \rho_2^1)\underset{\sim}{e}_2$$

$$\underset{\sim}{\phi}^2 = \begin{bmatrix} x_1 - x_4^2 \\ x_2 + x_5^2 x_8 \\ x_3 - x_6^2 \end{bmatrix} = \underset{\sim}{P}\underset{\sim}{\phi}(\underset{\sim}{\mu}^1) + (\phi_2^o + t_2' \rho_2^2)\underset{\sim}{e}_2$$

where

$$\underset{\sim}{P} = \begin{bmatrix} 1 & & \\ & 0 & \\ & & 1 \end{bmatrix}$$

and where

$$\rho_2^1 = x_7, \; \rho_2^2 = x_8 .$$

Additional constraints are given by

$$\left.\begin{aligned} g_{5+2i-1} &= 1 + x_{6+i} \\ g_{5+2i} &= 1 - x_{6+i} \end{aligned}\right\} \quad i = 1,2$$

$$g_{10} = t_r - x_5^2/x_2 .$$

Table IV shows results for three values of t_r. The same results are obtained replacing the term $x_2 + x_5^2 x_i$ by $x_2(1+t_r x_i)$, $i = 7,8$, removing g_{10} and reducing the number of variables by one. We note that larger tolerances are obtained than before for corresponding tuning ranges.

Example 3: Tolerancing and Tuning for One Component

We consider C to be both toleranced and tuned and minimize

$$C(x_1,x_2,\ldots,x_6) = \frac{x_1}{x_4^2} + \frac{x_2}{x_5^2} + \frac{x_3}{x_6^2} \tag{24}$$

where x_1, x_2, and x_3 are as before but where

$$\varepsilon_{L_1} = \varepsilon_1 = x_4^2$$
$$\varepsilon_C = \varepsilon_2 = x_5^2$$
$$\varepsilon_{L_2} = \varepsilon_3 = x_6^2$$

and $t_1 = t_2 = 0$. Here,

$$\phi^1 = \begin{bmatrix} x_1 + x_4^2 \\ x_2 - x_5^2 + t_r x_2 x_7 \\ x_3 + x_6^2 \end{bmatrix} = \phi(\mu^6) + t_2 \rho_2^1 e_2$$

$$\phi^2 = \begin{bmatrix} x_1 + x_4^2 \\ x_2 + x_5^2 + t_r x_2 x_8 \\ x_3 + x_6^2 \end{bmatrix} = \phi(\mu^8) + t_2 \rho_2^2 e_2$$

$$\phi^3 = \begin{bmatrix} x_1 - x_4^2 \\ x_2 - x_5^2 + t_r x_2 x_9 \\ x_3 - x_6^2 \end{bmatrix} = \phi(\mu^1) + t_2 \rho_2^3 e_2$$

with

$$t_2 = t_r C^o$$

and

$$\rho_2^1 = x_7, \ \rho_2^2 = x_8, \ \rho_2^3 = x_9 \quad .$$

Constraints g_6 to g_{11} are as in Example 2(a).

The results are shown in Table V where we note that for 5% and 10% tuning we have an effective tolerance problem, whereas for 20% tuning we have an effective tuning problem. Rerunning the same problem with $t_r = 0.05$ and $x_7 = 1$, $x_8 = -1$, $x_9 = 1$, which imply effective tolerances the same solution as for the 5% tuning range is obtained.

Example 4:Optimal Tuning

In this example we include the tuning range in the objective function.

(a) Tolerancing and tuning for one component.

We take a similar formulation to Example 3 except that

$$C(x_1,x_2,\ldots,x_7) = \frac{x_1}{x_4^2} + \frac{x_2}{x_5^2} + \frac{x_3}{x_6^2} + c\frac{x_7^2}{x_2} \qquad (25)$$

where c is a weighting factor and the term $t_r x_2$ is replaced by x_7^2, x_i by x_{i+1}, $i = 7,8,9$. This implies that $t_2 = x_7^2$. The constraints remain the same except for g_6 to g_{11} with x_i updated by x_{i+1}.

Table VI shows results for different values of c. Note that a threshold value of c seems to occur somewhere between 10 and 20. Below that threshold, the solution in terms of an effective tuning and tolerance problem is unaffected. Note also the transition for c = 50 from effective tuning to effective tolerancing. When c is very large we obtain the tolerance solution of Example 1.

(b) Tolerancing and tuning for 3 components.

The objective function considered is of the form

$$C(\phi^o, \varepsilon, t) = \sum_{i=1}^{3} \left(\frac{\phi_i^o}{\varepsilon_i} + c \frac{t_i}{\phi_i^o} \right) . \tag{26}$$

We consider one additional distinct vertex such that ϕ^1, ϕ^2, and ϕ^3 are as in (20), and $\phi^4 = \phi(\mu^3)$ in order to bound the solution during optimization.

We omit details of the constraints, and summarize the final results in Table VII for different c. The results are the same as in Table VI, but the computational effort has substantially increased. This formulation, however, has verified that ϕ_2 should be effectively tuned for c less than 50, and the other parameters effectively toleranced. The values of ρ^1, ρ^2, ρ^3 and ρ^4 confirm these observations.

V HIGHPASS FILTER

This problem was suggested by Pinel and Roberts [9,10]. The circuit diagram is shown in Fig. 2 and the basic specifications for the design are listed in Table VIII. The insertion loss relative to the loss at 990 Hz is to be constrained as indicated with resistances R_5 and R_7 related to L_5^o and L_7^o with constant Q. The terminations are fixed, the designable parameters being C_1, C_2, C_3, C_4, L_5, C_6 and L_7.

The objective function throughout was taken as

$$\sum_{i=1}^{7} \frac{\phi_i^o}{\varepsilon_i} \tag{27}$$

where

$$\underset{\sim}{\phi}^o = \begin{bmatrix} C_1^o \\ C_2^o \\ C_3^o \\ C_4^o \\ L_5^o \\ C_6^o \\ L_7^o \end{bmatrix}, \qquad \underset{\sim}{\varepsilon} = \begin{bmatrix} \varepsilon_{C_1} \\ \varepsilon_{C_2} \\ \varepsilon_{C_3} \\ \varepsilon_{C_4} \\ \varepsilon_{L_5} \\ \varepsilon_{C_6} \\ \varepsilon_{L_7} \end{bmatrix}.$$

The optimization package used here is DISOPT [11], which has been previously employed in worst-case tolerance problems [4]. The same quasi-Newton unconstrained minimization procedure as for the work described in the previous section is incorporated into DISOPT. In most cases the extrapolation feature [12] was chosen to accelerate convergence to the constrained optimimum.

Verification of the designs to be described was carried out using all 2^7 vertices plus the nominal point at 170, 360, 440, 630-680 and 680-1800 Hz. 42 logarithmically spaced points were taken for the latter interval, and 8 for the former interval.

Problem 1: No Tuning ($\underset{\sim}{t} = \underset{\sim}{0}$)

Table IX summarizes the particular frequencies, specifications and the particular vertex number (r of (18b)) employed to obtain the final tolerances listed in Table X. The total number of variables and constraints are indicated in Table IX. Table X also lists the shifts in nominal parameter values with respect to those of an uncentered design [9,10].

Problem 2: 3% Tuning for L_5

Results corresponding to the ones for Problem 1 are tabulated in Tables IX and X. Note that all the tolerances have increased over the results of Problem 1. Fig. 3 shows the nominal response as well as the worst upper and lower outcomes based on all 2^7 vertices.

A more detailed verification of the results was made. 60 logarithmically spaced points were taken from the critical region 630-680 Hz as well as 40 from 600-630 Hz. All the vertices were checked plus the nominal point, followed by 4000 Monte Carlo simulations uniformly distributed in the effective tolerance region. No violations were detected, and the upper and lower limits of response given by the vertices bounded the results from the Monte Carlo analysis except at 638.2 Hz, where the lowest relative loss obtained from the vertices was -0.0243 dB, whereas the Monte Carlo analysis yielded -0.0246 dB.

As a further check on the optimality of these results, L_5 was allowed to be both toleranced and tuned as distinct from being effectively toleranced from the point of view of optimization. The same vertices, an additional 25 ρ variables and 50 additional constraints on the ρ variables were used without any significant improvement in the results. The values of the ρ variables confirmed the assumption that L_5 should be effectively toleranced for 3% tuning.

Problem 3: 3% Tuning for L_5 and L_7

As indicated by Table X a further improvement in all tolerances has been obtained.

Problem 4: 3% Tuning for L_7

The results for this problem are, as shown by Table X, slightly worse than those for Problem 2. A slight violation of the specification at 658 Hz was detected. We conclude that if only one inductor is to be tuned, L_5 should be chosen.

VI CONCLUSIONS

As expected, the inclusion of tunable elements can increase the tolerances on the components. The results of the problems we have studied seem to justify the reduction of the general tolerance-tuning problem into one containing effectively toleranced and effectively tuned components, where appropriate. If the separation of the components is not decided in advance, the general problem as in Example 4(b) with the cost function reflecting both tolerances and tuning ranges is appropriate, since an optimization program requires an explicit number of variables and constraints in advance.

A cost function tending to maximize tolerances and minimizing tuning has been implemented successfully in this context. Zero tuning ranges were indicated when the cost became too high. For the highpass filter the 3% tuning range on the inductors was considered free, thus tuning did not enter into the objective function. A reduced problem involving effective tolerances was found adequate.

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TABLE I

SPECIFICATIONS FOR THE LC LOWPASS FILTER

Frequency Range (rad/s)	Sample Points (rad/s)	Insertion Loss Specification (dB)	Type	Weight w
0 - 1	0.45, 0.50, 0.55, 1.0	1.5	upper (passband)	+1
2.5	2.5	25	lower (stopband)	-1

TABLE II

LC LOWPASS FILTER (EXAMPLE 1)

Parameters	Toleranced Solution	Minimax Solution
L_1^o	1.9990	1.6280
C^o	0.9056	1.0897
L_2^o	1.9990	1.6280
$100\ \varepsilon_1/L_1^o$	9.89 %	-
$100\ \varepsilon_2/C^o$	7.60 %	-
$100\ \varepsilon_3/L_2^o$	9.89 %	-
$n = 6$	$m = 5$	-

TABLE III

L_1 TUNED, C AND L_2 TOLERANCED (EXAMPLE 2(a))

Parameters	$t_r = 0.2$	$t_r = 0.1$	$t_r = 0.05$
L_1^o	2.0932	2.2442	2.1953
C^o	0.9360	0.9059	0.9062
L_2^o	1.7718	1.7569	1.7920
$100\ t_1'/L_1^o$	20.00 %	10.00 %	5.00 %
$100\ \varepsilon_2/C^o$	15.99 %	14.23 %	12.60 %
$100\ \varepsilon_3/L_2^o$	21.62 %	18.41 %	16.23 %
ρ_1^1		-1.0000	
ρ_1^2		-1.0000	
ρ_1^3		1.0000	

n = 9 m = 12

TABLE IV

C TUNED, L_1 AND L_2 TOLERANCED (EXAMPLE 2(b))

Parameters	$t_r = 0.2$	$t_r = 0.1$	$t_r = 0.05$
L_1^o	1.8664	1.9536	2.0002
C^o	1.1336	1.0077	0.9546
L_2^o	1.8664	1.9536	2.0002
$100\ \varepsilon_1/L_1^o$	27.54 %	21.84 %	19.00 %
$100\ t_2'/C^o$	20.00 %	10.00 %	5.00 %
$100\ \varepsilon_3/L_2^o$	27.54 %	21.84 %	19.00 %
ρ_2^1		-1.0000	
ρ_2^2		1.0000	
	n = 8	m = 10	

TABLE V

TOLERANCING AND TUNING FOR C
L_1 AND L_2 TOLERANCED (EXAMPLE 3)

Parameters	$t_r = 0.2$	$t_r = 0.1$	$t_r = 0.05$
L_1^o	2.0178	2.0380	2.0209
C^o	0.9366	0.9061	0.9040
L_2^o	2.0178	2.0380	2.0209
$100\ \varepsilon_1/L_1^o$	17.96 %	14.81 %	12.41 %
$100\ \varepsilon_2/C^o$	16.83 %	11.66 %	9.64 %
$100\ \varepsilon_3/L_2^o$	17.96 %	14.81 %	12.41 %
$100\ t_2/C^o$	20.00 %	10.00 %	5.00 %
ρ_2^1		1.0000	
ρ_2^2		-1.0000	
ρ_3^3		1.0000	
$100/C^o x$	$t_2' = 3.17$ %	$\varepsilon_2' = 1.66$ %	$\varepsilon_2' = 4.64$ %
	n = 9	m = 11	

TABLE VI

OPTIMAL TUNING (EXAMPLE 4(a))

Parameters	c = 1	c = 10	c = 20	c = 50	c = 100	c = 1000
L_1^o	1.8440	1.8440	1.9221	2.0492	2.0227	1.9990
C^o	1.1730	1.1730	1.0486	0.9069	0.9043	0.9056
L_2^o	1.8440	1.8440	1.9221	2.0492	2.0227	1.9990
100 ε_1/L_1^o	29.08 %	29.08 %	23.84 %	16.15 %	12.69 %	9.89 %
100 ε_2/C^o	100.00 %	31.62 %	22.36 %	14.14 %	10.00 %	7.60 %
100 ε_3/L_2^o	29.08 %	29.08 %	23.84 %	16.15 %	12.69 %	9.89 %
100 t_2/C^o	122.69 %	54.31 %	35.88 %	14.14 %	5.71 %	0.00 %
ρ_2^1			1.0000			
ρ_2^2			-1.0000			
ρ_2^3			1.0000			
$100/C^o x$	$t_2' = 22.69$ %	t_2'=22.69%	t_2'=13.52%	t_2'=0.00%	ε_2'=4.29%	ε_2'=7.60%

n = 10 m = 11

TABLE VII

OPTIMAL TUNING (EXAMPLE 4(b))

Parameters	c = 10	c = 20	c = 50
$L_1^o = L_2^o$	1.8440	1.9221	2.0492
C^o	1.1730	1.0486	0.9069
$100\ \varepsilon_1/L_1^o = 100\ \varepsilon_3/L_2^o$	31.62 %	23.84 %	16.15 %
$100\ \varepsilon_2/C^o$	31.62 %	22.36 %	14.14 %
$100\ t_1/L_1^o = 100\ t_3/L_2^o$	2.54 %	0.00 %	0.00 %
$100\ t_2/C^o$	54.31 %	35.89 %	14.14 %
ρ_1^1	-1.0000	-0.7165	0.9743
ρ_2^1	0.1645	0.2466	1.0000
ρ_3^1	-1.0000	-0.9992	-0.9846
ρ_1^2	-1.0000	-1.0000	-0.8813
ρ_2^2	-1.0000	-1.0000	-1.0000
ρ_3^2	-1.0000	-1.0000	-0.9876
ρ_1^3	1.0000	0.9887	0.9933
ρ_2^3	1.0000	1.0000	1.0000
ρ_3^3	1.0000	0.9989	0.9029
ρ_1^4	1.0000	0.8433	-0.6051
ρ_2^4	-0.1645	-0.1468	0.6434
ρ_3^4	1.0000	0.8944	0.6441
$100\ \varepsilon_1'/L_1^o = 100\ \varepsilon_3'/L_2^o$	29.08 %	23.84 %	14.14 %
$100\ t_2'/C^o$	22.69 %	13.53 %	0.00 %

n = 21 m = 36

TABLE VIII

SPECIFICATIONS FOR THE HIGHPASS FILTER

Frequency Range (Hz)	Basic Sample Points (Hz)	Relative Insertion Loss (dB)	Weight w
170	170	45.	-1
360	360	49.	-1
440	440	42.	-1
630 - 680	630	4.	+1
680 - 1800	680 710 725 740	1.75	+1
630 - 1800	630 650 680 860 910 930 1050	-0.05	-1

Reference Frequency: 990 Hz

R_5, R_7 related to L_5^o and L_7^o through $Q = \frac{2\pi 990 L_5^o}{R_5} = \frac{2\pi\ 990 L_7^o}{R_7} = 1456$

TABLE IX

DATA FOR CONSTRAINTS

Frequency (Hz)	S (dB)	w	Vertex Number			
			Problem 1 No Tuning	Problem 2 L_5 Tuned	Problem 3 L_5 and L_7 Tuned	Problem 4 L_7 Tuned
170	45	-1	8	8	8	8
360	49	-1	48	48	48	48
440	42	-1	128	128	128	128
630	4	+1	1	1	1	1
630	-0.05	-1	60,100,104, 108,120,126	58,60,100, 104,108,120 126	60,108,120	60,87,95 100,104,108, 120,126
637	-0.05	-1	-	-	-	87
640	-0.05	-1	-	58	108	52,58,60
643	-0.05	-1	-	-	-	85,93,117
650	-0.05	-1	nominal,12, 50,58,102	nominal,12, 34,42,50,58 102,106,126	nominal,12,34, 42,44,58,106, 126	nominal,12, 36,42,50,58, 85,93,94, 102,106,126
658	-0.05	-1	-	-	42	58,69,85
665	-0.05	-1	-	-	34,42	34,58
670	-0.05	-1	-	-	-	2
680	1.75	+1	123	123	123	123
680	-0.05	-1	2,6	2,6	2,6	2,6
710	1.75	+1	43,83	43,83	43,83,123	43,83
725	1.75	+1	43,83	43,83	43,83	43,83
730	1.75	+1	-	-	43,83	43
740	1.75	+1	43,83	43,83	43,83	43,83
860	-0.05	-1	118,126	118,126	118,126	118,126
910	-0.05	-1	118,126	118,126	118,126	118,126
930	-0.05	-1	118,126	118,126	118,126	118,126

1040	-0.05	-1	-	-	-	3
1050	-0.05	-1	3	3	3	3
Number of Response Constraints			31	37	37	55
Total Number of Constraints m			45	51	51	69
Number of Variables n			14	14	14	14

TABLE X

RESULTS FOR HIGHPASS FILTER

Parameters		Problem 1 No Tuning	Problem 2 L_5 Tuned	Problem 3 L_5 and L_7 Tuned	Problem 4 L_7 Tuned
C_1	tolerance (%)	5.71	6.77	7.90	6.63
	nom. shift(%)	+18.1	+17.8	+18.3	+17.6
C_2	tolerance (%)	4.33	4.97	5.32	4.77
	nom. shift(%)	+16.2	+15.2	+14.4	+15.3
C_3	tolerance (%)	4.72	5.81	7.23	5.83
	nom. shift(%)	+16.6	+18.0	+18.8	+17.8
C_4	tolerance (%)	4.54	5.03	5.15	4.78
	nom. shift(%)	-3.8	-2.2	-1.2	-3.1
L_5	tolerance (%)	3.29	3.95	4.44	3.82
	nom. shift(%)	-3.0	-3.0	-4.3	-4.1
C_6	tolerance (%)	6.32	7.05	7.27	6.66
	nom. shift(%)	-7.3	-5.1	-3.6	-6.0
L_7	tolerance (%)	3.64	4.34	5.04	4.32
	nom. shift(%)	-6.4	-7.9	-7.9	-6.3
Cost		157	135	121	138*

*Violation of specifications. Relative Loss = -0.052 dB at 658 Hz

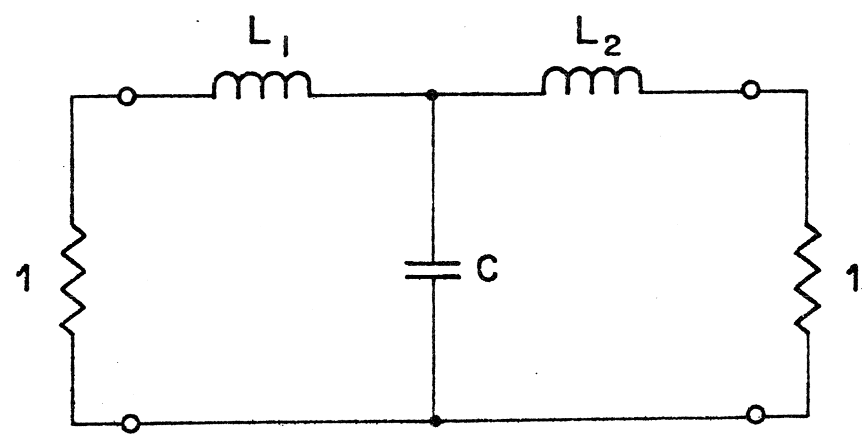

Fig. 1 The LC lowpass filter

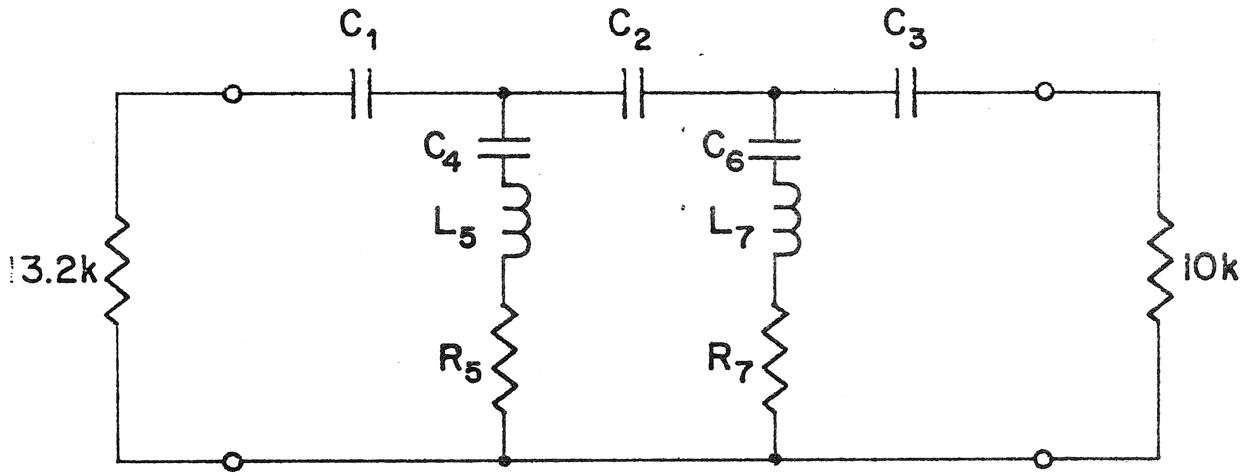

Fig. 2 The highpass filter

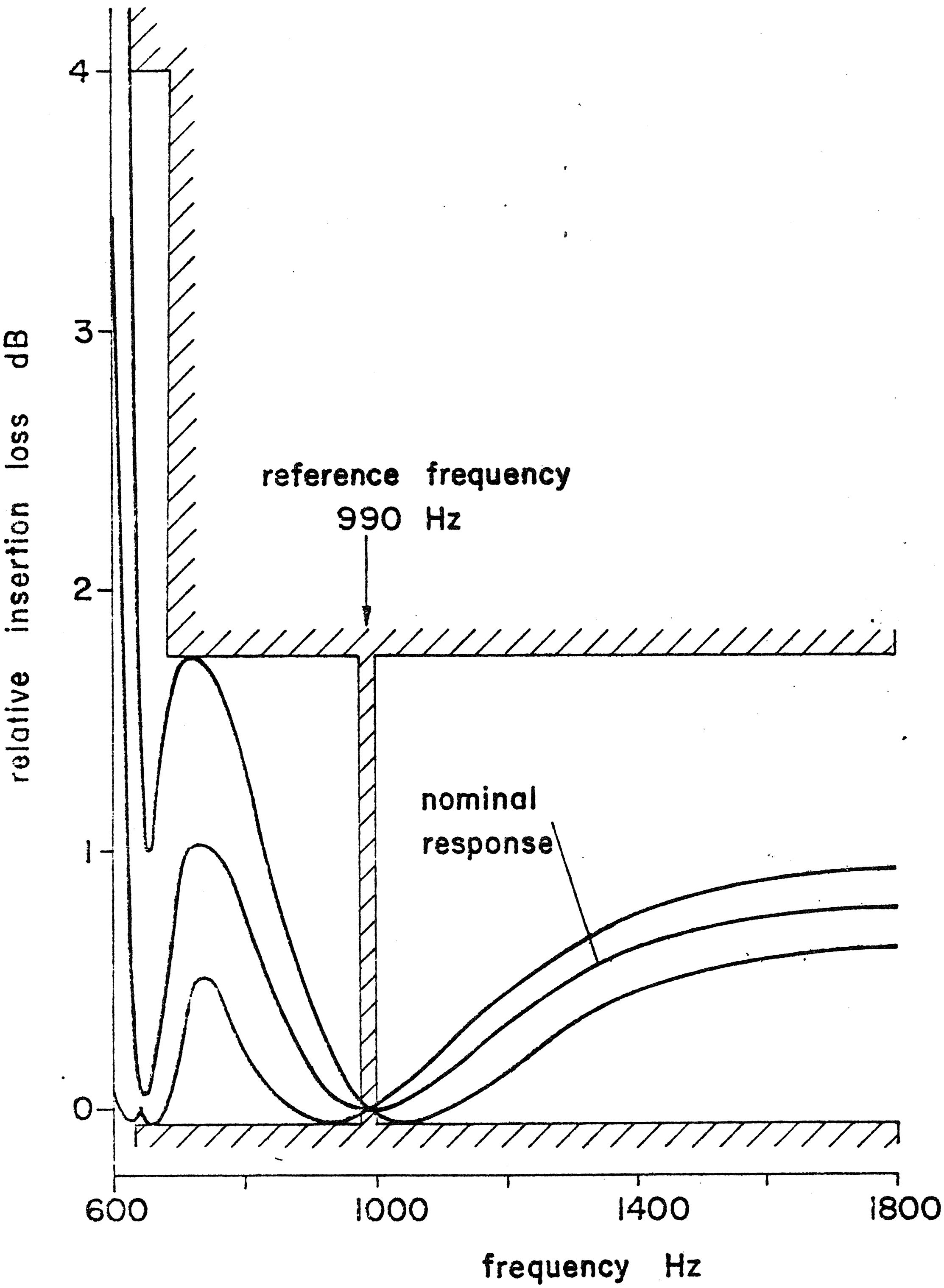

Fig. 3a Passband details of the optimized highpass filter (Problem 2)

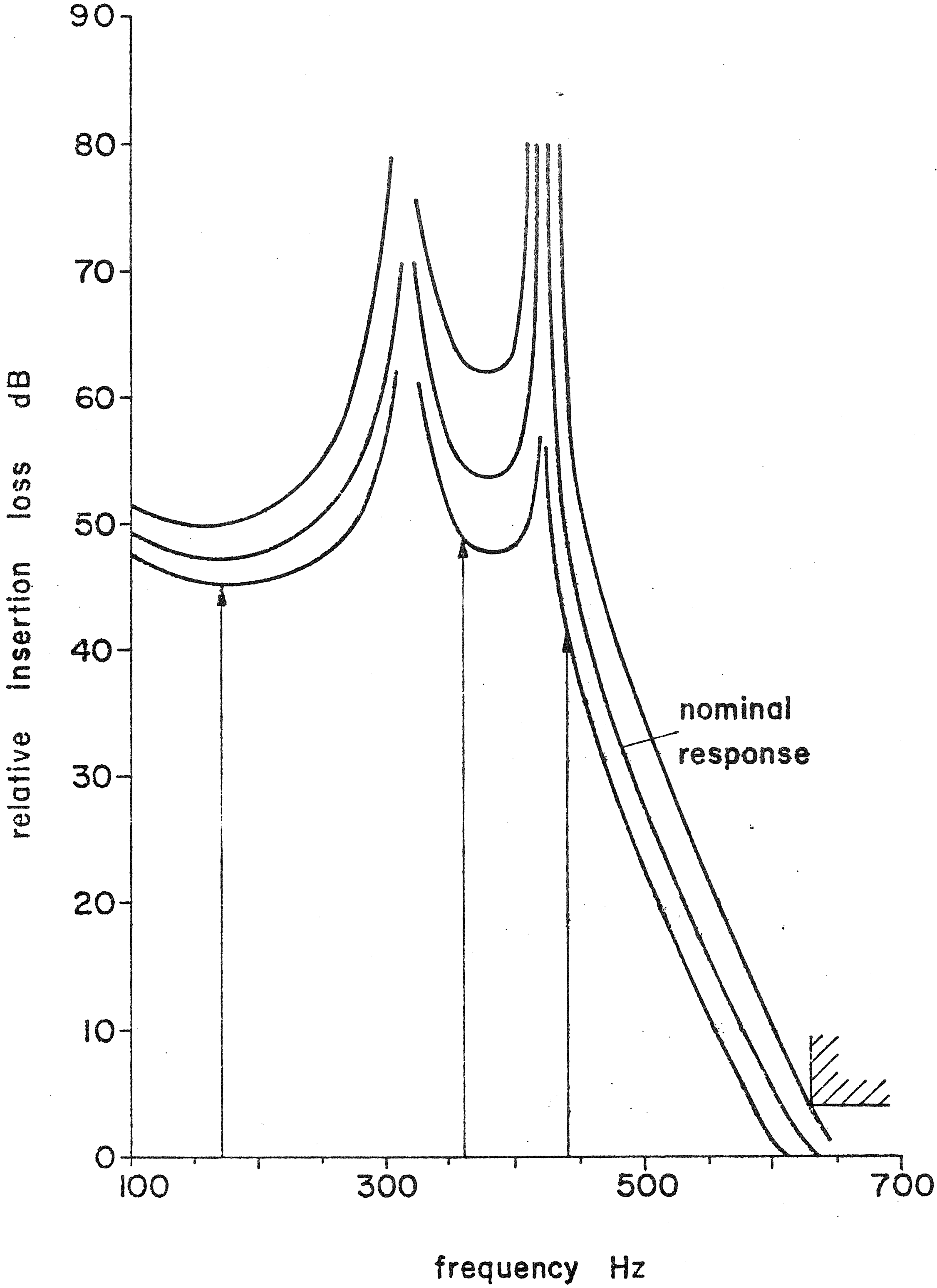

Fig. 3b Stopband details of the optimized highpass filter (Problem 2)