OPTIMAL CENTERING, TOLERANCING AND TUNING OF AN LC FILTER USING RECENT OPTIMIZATION PACKAGES

J.W. Bandler, S. Daijavad and W. Kellermann

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Abstract

A simple LC low-pass filter is studied to illustrate the theory of optimal worst-case design embodying centering, tolerancing and tuning presented by Bandler, Liu and Tromp. Three computer program packages (MFNC, MMLC and MINOS), which implement recent nonlinear optimization techniques, are used and the results are compared. A basic discussion of centering, tolerancing and tuning and the formulation of such electrical circuit design problems in terms of mathematical programming problems are presented. The documentation is developed for the CDC 170/730 system with the NOS 1.4 level 552 operating system and the Fortran Extended (FTN) version 4.8 compiler. Details and the full Fortran listing are made available for problems considered by Bandler, Kellermann and Zuberek.

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I. INTRODUCTION

In the nominal design problem, we are interested in finding a single point in the feasible region. This solution is impractical from the manufacturing point of view. Since there are other points that can also meet the specifications, the designer can assign tolerances on component values so as to minimize production cost. Postproduction tuning of some elements is quite common in electrical circuit fabrication. This has the effect of increasing tolerances to reduce cost or to make unrealistically toleranced solutions more attractive.

The optimal design of an LC low-pass filter embodying centering, tolerancing and tuning developed by Bandler, Liu and Tromp [1-2] is considered. Assuming that there are tolerances associated with component values and tuning is possible, we seek nominal component values and tolerancing and tuning parameter values that satisfy certain specifications on insertion loss at different frequencies, and minimize a realistically defined cost function at the same time.

The three optimization packages, which have also been studied by Bandler, Kellermann and Zuberek [3], employ different techniques. MFNC [4] is a package for minimization of a nonlinear objective function subject to nonlinear constraints. It is an extension and modification of a set of subroutines from the Harwell Subroutine Library [5]. The method implemented was presented by Han [6] and Powell [7]. First derivatives of all functions with respect to all variables are assumed to be available. The solution is found by an iteration that minimizes a quadratic approximation of the objective function subject to linearized constraints. The MMLC package [8] solves linearly constrained minimax optimization problems and is based on the method described by Hald and Madsen [9]. The solution is found by an iteration that uses either linear programming applied in connection with first-order derivatives or a quasi-Newton method applied in connection with first-order and approximate second-order derivatives.

The MINOS/AUGMENTED system [10] is a general purpose programming system to solve large-scale optimization problems involving sparse linear and nonlinear constraints.

MINOS employs a projected augmented Lagrangian algorithm to solve problems with nonlinear constraints presented by Murtagh and Saunders [11].

The present report reviews the basic theory of optimal worst-case design embodying centering, tolerancing and tuning. It uses a typical electrical circuit design example, namely an LC low-pass filter design, to illustrate the formulation of practical engineering design problems for which the objective function and constraints are flexible. The structure of the programs for the simulation of the circuit using all three packages, complete results for different test cases, discussion and comparison of results are also presented.

II. THE TOLERANCING AND TUNING PROBLEM [1-2]

A design consists of design data of nominal point $\boldsymbol{\phi}^0$, the tolerance vector $\boldsymbol{\varepsilon}$ and the tuning vector $\mathbf{t}$ where, for k parameters,

$$\boldsymbol{\phi}^0 \triangleq \begin{bmatrix} \phi_1^0 \\ \phi_2^0 \\ \cdot \\ \cdot \\ \cdot \\ \phi_k^0 \end{bmatrix}, \quad \boldsymbol{\varepsilon} \triangleq \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \cdot \\ \cdot \\ \cdot \\ \varepsilon_k \end{bmatrix} \quad \text{and } \mathbf{t} \triangleq \begin{bmatrix} t_1 \\ t_2 \\ \cdot \\ \cdot \\ \cdot \\ t_k \end{bmatrix}. \tag{1}$$

An *outcome* $\{\boldsymbol{\phi}^0, \boldsymbol{\varepsilon}, \boldsymbol{\mu}\}$ of a *design* $\{\boldsymbol{\phi}^0, \boldsymbol{\varepsilon}, \mathbf{t}\}$ implies a point

$$\boldsymbol{\phi} = \boldsymbol{\phi}^0 + \mathbf{E}\boldsymbol{\mu}, \tag{2}$$

where

$$\mathbf{E} \triangleq \begin{bmatrix} \varepsilon_1 & & & & \\ & \varepsilon_2 & & & \\ & & \cdot & & \\ & & & \cdot & \\ & & & & \cdot \\ & & & & & \varepsilon_k \end{bmatrix} \tag{3}$$

and $\boldsymbol{\mu} \in R_\mu$. R_μ is a set of multipliers determined from realistic situations of the tolerance spread. For example,

$$R_\mu \triangleq \{\boldsymbol{\mu} \mid -1 \leq \mu_i \leq 1, i \in I_\phi\}, \tag{4}$$

where

$$I_\phi \triangleq \{1, 2, ..., k\}. \tag{5}$$

The *tolerance region* R_ε is defined as

$$R_\varepsilon \triangleq \{\boldsymbol{\phi} \mid \boldsymbol{\phi}^0 - \boldsymbol{\varepsilon} \leq \boldsymbol{\phi} \leq \boldsymbol{\phi}^0 + \boldsymbol{\varepsilon}\}. \tag{6}$$

The *tuning region* $R_t(\boldsymbol{\mu})$ is defined as the set of points

$$\boldsymbol{\phi} = \boldsymbol{\phi}^0 + \mathbf{E}\,\boldsymbol{\mu} + \mathbf{T}\,\boldsymbol{\rho} \tag{7}$$

for all $\boldsymbol{\rho} \in R_\rho$, where

$$\mathbf{T} \triangleq \begin{bmatrix} t_1 & & & & \\ & t_2 & & & \\ & & \cdot & & \\ & & & \cdot & \\ & & & & \cdot \\ & & & & t_k \end{bmatrix}, \tag{8}$$

and

$$R_\rho \triangleq \{\boldsymbol{\rho} \mid -1 \leq \rho_i \leq 1, i \in I_\phi\}. \tag{9}$$

The *constraint region* is given by

$$R_c \triangleq \{\boldsymbol{\phi} \mid g_i(\boldsymbol{\phi}) \geq 0, i \in I_c\} \tag{10}$$

where

$$I_c \triangleq \{1, 2, ..., m_c\}, \tag{11}$$

and m_c is the total number of constraints. The definitions are illustrated in Fig. 1 for a two-dimensional example.

The problem is formulated as the nonlinear programming problem:

$$\text{minimize } C(\boldsymbol{\phi}^0, \boldsymbol{\varepsilon}, \mathbf{t}) \text{ subject to } \boldsymbol{\phi} \in R_c,$$

where $\boldsymbol{\phi}$ is defined in (7) and constraints $\boldsymbol{\phi}^0, \boldsymbol{\varepsilon}, \mathbf{t} \geq 0$, for all $\boldsymbol{\mu} \in R_\mu$ and some $\boldsymbol{\rho} \in R_\rho$.

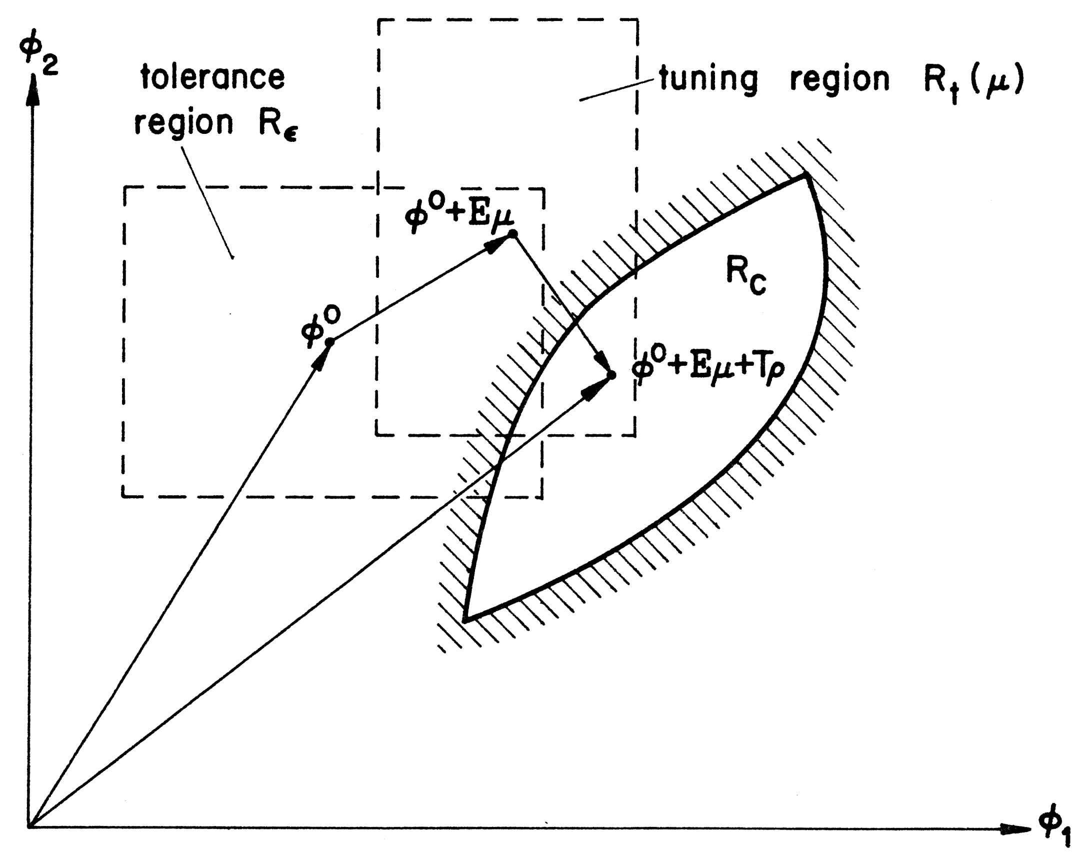

Fig. 1 Illustration of concepts in design centering, tolerancing and tuning

$C(\boldsymbol{\phi}^0, \boldsymbol{\varepsilon}, \mathbf{t})$ is a function which represents the component cost, for example,

$$C = \sum_{i=1}^{k} \left(c_i \frac{\phi_i^0}{\varepsilon_i} + c_i' \frac{t_i}{\phi_i^0} \right), \tag{12}$$

where c_i and c_i' are constants.

The Vertices Elimination Method

The tolerance region R_ε is a convex polytope of k dimensions with sides of length 2 ε_i, $i \in I_\phi$, and centered at $\boldsymbol{\phi}^0$. The extreme points of R_ε, namely the vertices, are

$$R_v \triangleq \{\boldsymbol{\phi} \mid \phi_i = \phi_i^0 + \varepsilon_i \mu_i \,,\ \mu_i \in \{-1,1\}, i \in I_\phi\}. \tag{13}$$

Each vertex can be indexed by $\boldsymbol{\phi}^i$, $i \in I_v$, where

$$I_v \triangleq \{1, 2, ..., 2^k\} \tag{14}$$

It is proved, that if the regions of points in the parameter space for which designs are both feasible and acceptable satisfies a certain condition (less restrictive than convexity), then no more than 2^k points, the vertices of the tolerance regions need to be considered during optimization [12].

Further effort can be made to identify or to predict the most critical vertices that are likely to give rise to active constraints. If g_i is monotonic in the same direction with respect to ϕ_j throughout R_c, the minimum of g_i is on the hyperplane $\phi_j = \phi_j^0 - \varepsilon_j \, \mathrm{sgn}(\partial g_i/\partial \phi_j)$. Hence, only the vertices which lie on that hyperplane need to be constrained. This implies that an investigation of $\mathrm{sgn}(\partial g_i/\partial \phi_j)$ in a locality of the point $\boldsymbol{\phi}^0$, (assuming monotonicity), will select the worst vertex for g_i.

III. LC LOW-PASS FILTER

The LC low-pass filter shown in Fig. 2 is considered. Table I summarizes the specifications. Now, consider the network in Fig. 3 as the original network. The nodal equations for this network are

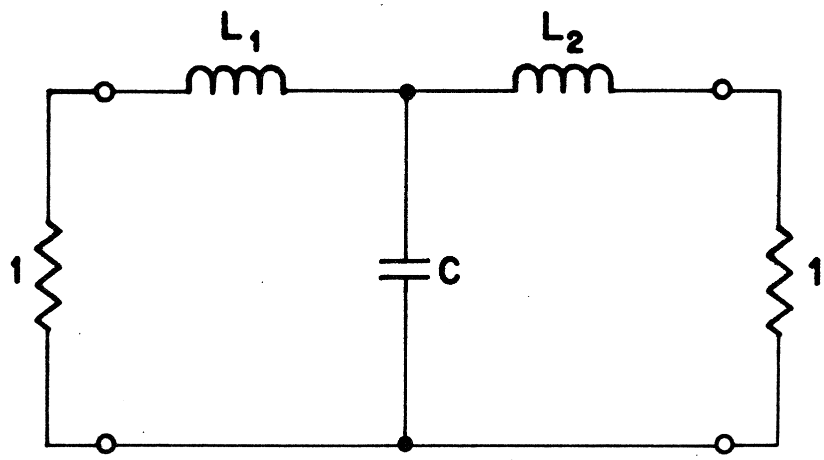

Fig. 2 The LC low-pass filter

TABLE I

SPECIFICATIONS FOR LC LOW-PASS FILTER

Frequency Range (rad/s)	Sample Points (rad/s)	Insertion Loss Specification (dB)	Type	Weight w
0 - 1	0.45, 0.50, 0.55, 1.0	1.5	upper (passband)	+1
2.5	2.5	25	lower (stopband)	−1

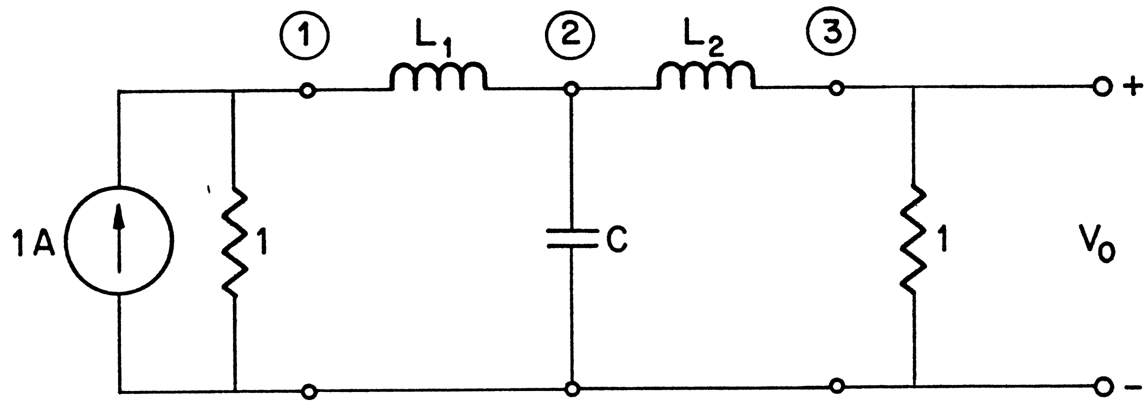

Fig. 3 Original LC filter with excitation

$$\begin{bmatrix} 1+1/(j\omega L_1) & -1/(j\omega L_1) & 0 \\ -1/(j\omega L_1) & 1/(j\omega L_1)+1/(j\omega L_2)+j\omega C & -1/(j\omega L_2) \\ 0 & -1/(j\omega L_2) & 1+1/(j\omega L_2) \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \qquad (15)$$

or

$$\mathbf{Y}\,\mathbf{V} = \mathbf{B} \ . \qquad (16)$$

These equations can be solved for **V** using Crout's algorithm for LU factorization.

The insertion loss between the source and the load is defined as

$$\text{insertion loss} \triangleq \frac{\text{power to the load when the circuit is removed}}{\text{power to the load when the circuit is in place}} \ . \qquad (17)$$

When the circuit is removed we have, for source and load resistances R_s and R_L,

$$P_{L0} = |V_I\,/\,(R_S + R_L)|^2\, R_L\,, \qquad (18)$$

where V_I is the source voltage, and with circuit in place, we get

$$P_0 = |V_0\,/\,R_L|^2\, R_L\,, \qquad (19)$$

where V_0 is the voltage across R_L. Substituting (18) and (19) into (17) gives

$$\text{insertion loss (in dB)} = 10\,\log_{10}\left[\frac{|V_I\,/\,(R_S + R_L)|^2 R_L}{|V_0\,/\,R_L|^2 R_L}\right]$$

$$= 20\,\log_{10}\frac{R_L}{R_S + R_L}\left|\frac{V_I}{V_0}\right| . \qquad (20)$$

For $R_S = R_L = 1\Omega$ and $V_I = 1$ volt (corresponding to a 1 ampere excitation), (20) becomes

$$\text{insertion loss (in dB)} = 20\,\log_{10}(1/2) + 20\,\log_{10}(1/|V_0|) = -6.0206 - 8.6859\,\ln|V_0|\,, \qquad (21)$$

where $20\,\log_{10} e = 8.6859$.

To calculate the derivatives of insertion loss with respect to L_1, C and L_2, we use adjoint network analysis. The equations for the adjoint network, shown in Fig. 4, can be summarized as

$$\mathbf{Y}^T\,\hat{\mathbf{V}} = \hat{\mathbf{B}}\ , \qquad (22)$$

where $\mathbf{Y}^T = \mathbf{Y}$ because of symmetry and $\hat{\mathbf{B}} = [0\ \ 0\ \ -1]^T$.

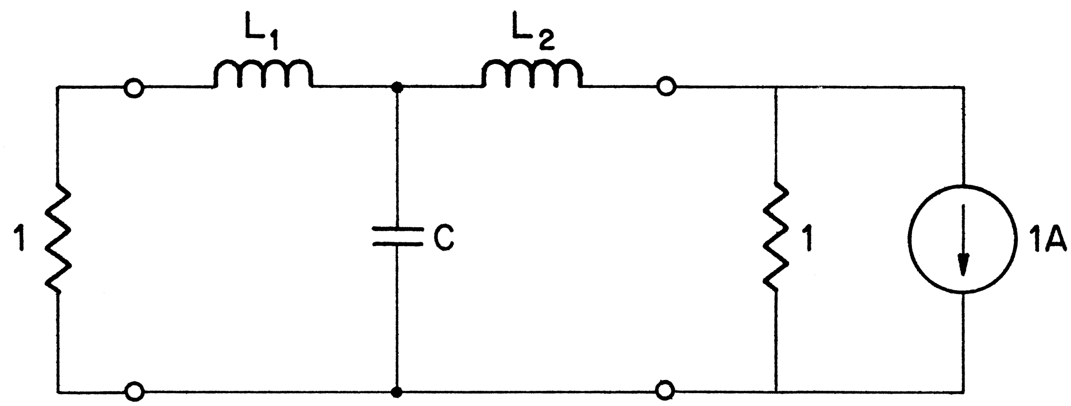

Fig. 4 Adjoint LC filter with excitation

Again, Crout's algorithm is used to find $\hat{\mathbf{V}} = [\hat{V}_1 \ \hat{V}_2 \ \hat{V}_3]^T$. It is important to note that only forward and backward substitutions are necessarily repeated for the adjoint network.

The sensitivities are

$$\frac{\partial V_0}{\partial L_1} = -j\omega I_{L_1} \hat{I}_{L_1} = -j\omega \frac{(V_1 - V_2)}{j\omega L_1} \frac{(\hat{V}_1 - \hat{V}_2)}{j\omega L_1} \tag{23}$$

$$\frac{\partial V_0}{\partial L_2} = -j\omega I_{L_2} \hat{I}_{L_2} = -j\omega \frac{(V_2 - V_3)}{j\omega L_2} \frac{(\hat{V}_2 - \hat{V}_3)}{j\omega L_2} \tag{24}$$

$$\frac{\partial V_0}{\partial C} = j\omega V_2 \hat{V}_2 .$$

Denoting insertion loss by L, (21) gives

$$\frac{\partial L}{\partial \boldsymbol{\phi}} = -8.6859 \frac{1}{|V_0|} \frac{\partial}{\partial \boldsymbol{\phi}} |V_0| , \tag{25}$$

where

$$\boldsymbol{\phi} = [L_1 \ C \ L_2]^T . \tag{26}$$

Also,

$$\frac{\partial}{\partial \boldsymbol{\phi}} |V_0| = \frac{1}{|V_0|} \operatorname{Re}\left[V_0^* \frac{\partial}{\partial \boldsymbol{\phi}} V_0 \right] . \tag{27}$$

(27) is proved as

$$\frac{1}{|V_0|} \operatorname{Re}\left[V_0^* \frac{\partial}{\partial \boldsymbol{\phi}} V_0 \right] = \frac{1}{2|V_0|} \left[V_0^* \frac{\partial}{\partial \boldsymbol{\phi}} V_0 + V_0 \frac{\partial}{\partial \boldsymbol{\phi}} V_0^* \right]$$

$$= \frac{1}{2|V_0|} \frac{\partial}{\partial \boldsymbol{\phi}} (V_0 V_0^*)$$

$$= \frac{1}{2|V_0|} \frac{\partial}{\partial \boldsymbol{\phi}} |V_0|^2$$

$$= \frac{1}{2|V_0|} \left(2|V_0| \frac{\partial}{\partial \boldsymbol{\phi}} |V_0|\right)$$

$$= \frac{\partial}{\partial \boldsymbol{\phi}} |V_0| .$$

Having calculated L and $\partial L/\partial \boldsymbol{\phi}$, we can formulate the performance specifications g_i, $i=1, 2, ..., m_c$, as

$$g_i = w(\omega_i)\,[S(\omega_i) - L(\boldsymbol{\phi}, \omega_i)], \tag{28}$$

where w denotes the weighting factor and S represents specification. Moreover,

$$\frac{\partial g_i}{\partial \boldsymbol{\phi}} = -(w)_i \frac{\partial}{\partial \boldsymbol{\phi}} (L)_i , \tag{29}$$

where

$$(w)_i \triangleq w(\omega_i), \text{ and } (L)_i \triangleq L(\boldsymbol{\phi}, \omega_i) .$$

An investigation of $\mathrm{sgn}(\partial g_i/\partial \boldsymbol{\phi})$ at a feasible point determines the worst vertex for constraint g_i, using

$$\boldsymbol{\mu}_i = -\mathrm{sgn}(\partial g_i/\partial \boldsymbol{\phi}) . \tag{30}$$

The critical vertices are found to be

vertex 6 or $\boldsymbol{\mu} = [1 \;\; -1 \;\; 1]^T$ at $\omega = 0.45, 0.50, 0.55$

vertex 8 or $\boldsymbol{\mu} = [1 \;\; 1 \;\; 1]^T$ at $\omega = 1.0$

vertex 1 or $\boldsymbol{\mu} = [-1 \;\; -1 \;\; -1]^T$ at $\omega = 2.5$.

One additional vertex, namely vertex 3, is considered for each frequency in order to bound the solution during optimization. Hence, there is a total of 10 nonlinear constraints. These are five constraints due to the five sample frequencies and their corresponding critical vertices, and five constraints due to five sample frequencies and vertex 3.

Assuming that L_1, C and L_2 all have tolerances associated with them and tuning is possible for all three, a total of 21 variables are readily identified. Table II shows these variables.

The cost function is taken as

$$C = \sum_{i=1}^{3} \left[\frac{\phi_i^0}{\varepsilon_i} + c\, \frac{t_i}{\phi_i^0} \right] , \tag{31}$$

TABLE II

21 MAJOR VARIABLES OF THE LC FILTER PROBLEM

Name	Variable	Name	Variable
x_1	L_1^0	x_{12}	$\rho_3(6)$
x_2	C^0	x_{13}	$\rho_1(8)$
x_3	L_2^0	x_{14}	$\rho_2(8)$
x_4	ε_{L_1}	x_{15}	$\rho_3(8)$
x_5	ε_C	x_{16}	$\rho_1(1)$
x_6	ε_{L_2}	x_{17}	$\rho_2(1)$
x_7	t_{L_1}	x_{18}	$\rho_3(1)$
x_8	t_C	x_{19}	$\rho_1(3)$
x_9	t_{L_2}	x_{20}	$\rho_2(3)$
x_{10}	$\rho_1(6)$	x_{21}	$\rho_3(3)$
x_{11}	$\rho_2(6)$		

where

$$\mathbf{\Phi}^0 = [L_1^0\ C^0\ L_2^0]^T, \tag{32}$$

and c is a constant.

Using the 21 variables, we construct $\mathbf{\Phi}$ for the four critical vertices. Table III summarizes the results for the construction of $\mathbf{\Phi}$.

The nonlinear constraints g_i, $i = 1, 2, \ldots, 10$, are functions of $\mathbf{\Phi}$ and are calculated in terms of the variables x_j, $j = 1, 2, \ldots, 21$, using Table III. Also, $\partial g_i/\partial x_j$ can be obtained using chain rule of differentiation as

$$\frac{\partial g_i}{\partial x_j} = \frac{\partial g_i}{\partial L_1}\frac{\partial L_1}{\partial x_j} + \frac{\partial g_i}{\partial C}\frac{\partial C}{\partial x_j} + \frac{\partial g_i}{\partial L_2}\frac{\partial L_2}{\partial x_j}. \tag{33}$$

Constraints g_i are numbered so that

$i = 1, 2, 3$ corresponds to vertex 6,

$i = 4$ corresponds to vertex 8,

$i = 5$ corresponds to vertex 1,

and $i = 6, \ldots, 10$ corresponds to vertex 3.

There are 33 linear constraints resulting from (9) and the fact that $\mathbf{\Phi}, \boldsymbol{\varepsilon} > \mathbf{0}$, and $\mathbf{t} \geq \mathbf{0}$. Numbering these constraints from 11 to 43 gives

$g_i, i=11, \ldots, 16$ correspond to $\mathbf{\Phi}, \boldsymbol{\varepsilon} > \mathbf{0}$,

$g_i, i=17, 18, 19$ correspond to $\mathbf{t} \geq \mathbf{0}$,

$g_i, i=20, \ldots, 31$ correspond to $\boldsymbol{\rho} \geq -\mathbf{1}$,

and $g_i, i=32, \ldots, 43$ correspond to $\boldsymbol{\rho} \leq \mathbf{1}$.

IV. THE PROGRAMS AND THE ACTUAL OUTPUTS

1. Program LCMFN

The program uses the package MFNC. The objective function and 43 constraints are defined in Section III. Apart from the main subroutine for minimization with general constraints (subroutine MFNC1A), and the associated main program and subroutine FCD,

TABLE III

CONSTRUCTION OF ϕ USING 21 MAJOR VARIABLES

vertex	μ	$L_1 \equiv \phi_1$	$C \equiv \phi_2$	$L_2 \equiv \phi_3$
6	$[\ 1\ \ -1\ \ \ 1]^T$	$L_1^0 + \varepsilon_{L_1} + t_{L_1}\rho_1(6)$	$C^0 - \varepsilon_C + t_C\,\rho_2(6)$	$L_2^0 + \varepsilon_{L_2} + t_{L_2}\rho_3(6)$
8	$[\ 1\ \ \ 1\ \ \ 1]^T$	$L_1^0 + \varepsilon_{L_1} + t_{L_1}\rho_1(8)$	$C^0 + \varepsilon_C + t_C\,\rho_2(8)$	$L_2^0 + \varepsilon_{L_2} + t_{L_2}\rho_3(8)$
1	$[-1\ \ -1\ \ -1]^T$	$L_1^0 - \varepsilon_{L_1} + t_{L_1}\rho_1(1)$	$C^0 - \varepsilon_C + t_C\,\rho_2(1)$	$L_2^0 - \varepsilon_{L_2} + t_{L_2}\rho_3(1)$
3	$[-1\ \ \ 1\ \ -1]^T$	$L_1^0 - \varepsilon_{L_1} + t_{L_1}\rho_1(3)$	$C^0 + \varepsilon_C + t_C\,\rho_2(3)$	$L_2^0 - \varepsilon_{L_2} + t_{L_2}\rho_3(3)$

there are three subroutines specifically written for the LC filter problem. These are FIDFI, SIMGRD, and SOLLU. These subroutines serve the following purposes.

Subroutine FCD calculates the values of the objective function F, its gradient G, the constraint functions $f_i(\mathbf{x})$, and their derivatives $\partial f_i(\mathbf{x})/\partial x_j$ at the point $\mathbf{x}$ corresponding to x(1), x(2), ..., x(N). Subroutine FIDFI uses the information about critical vertices to evaluate $\boldsymbol{\phi}$ from the 21 variables and finds $\partial\boldsymbol{\phi}/\partial x_j$. Subroutine SIMGRD simulates the LC network and its adjoint to evaluate insertion loss and its derivatives with respect to $\boldsymbol{\phi}$ at different frequencies. Therefore, it provides the necessary information for calculating 10 nonlinear constraints and their gradients. Subroutine SOLLU uses Crout's algorithm for LU factorization with forward and backward substitution to solve $\mathbf{Y}\ \mathbf{V} = \mathbf{B}$ for $\mathbf{V}$, given $\mathbf{Y}$ and $\mathbf{B}$. A block diagram illustrating the interaction of the subroutines is shown in Fig. 5.

The main subroutine (MFNC1A) is available as a group file LIBRMFN under the charge RJWBAND. The sequence of NOS commands to use the package may be as follows:

/GET,LIBRMFN/GR.

/LIBRARY,LIBRMFN.

/FTN,...,GO.

Fortran listing for LCMFN starts on page 59. The following three actual outputs show the starting points and the solutions for three values of constant c in the objective of (31), using LCMFN. The outputs correspond to $c = 50$, $c = 20$, and $c = 10$, respectively.

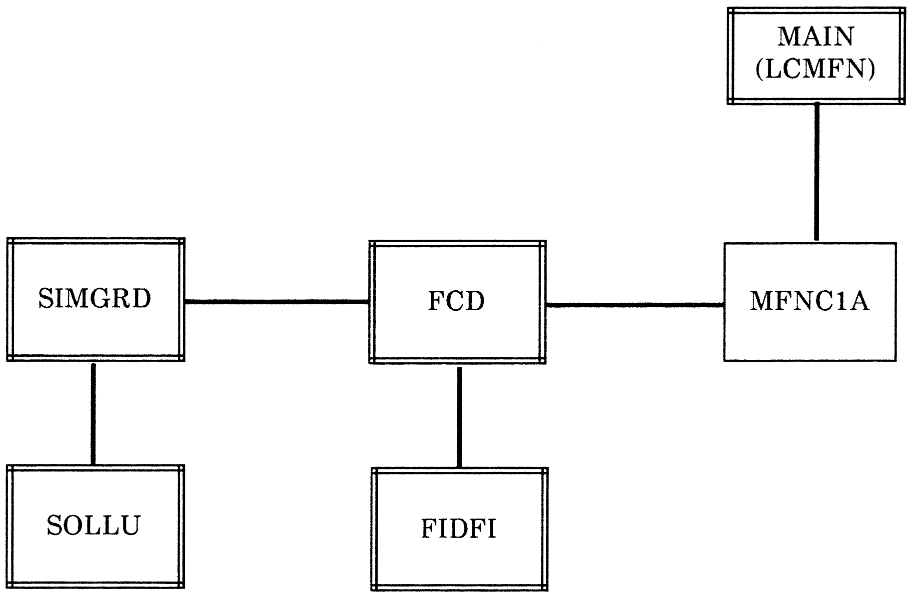

Fig. 5. Structure of the main program and the auxiliary subroutines for LCMFN.

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MINIMIZATION WITH NONLINEAR CONSTRAINTS (MFNC PACKAGE) (V:82.05)

LC FILTER PROBLEM

INPUT DATA

NUMBER OF VARIABLES (N) 21

NUMBER OF EQUALITY CONSTRAINTS (LEQ) 0

TOTAL NUMBER OF CONSTRAINTS (L) 43

ACCURACY (EPS) 1.000E-06

MAX NUMBER OF FUNCTION EVALUATIONS (MAXF) 100

WORKING SPACE (IW) 5000

PRINTOUT CONTROL (IPR) 0

STARTING POINT. OBJECTIVE FUNCTION : 3.500000000000E+01

	VARIABLES	GRADIENT
1	2.000000000000E+00	3.7500000000E+00
2	1.000000000000E+00	-2.8421709430E-14
3	2.000000000000E+00	3.7500000000E+00
4	2.000000000000E-01	-5.0000000000E+01
5	2.000000000000E-01	-2.5000000000E+01
6	2.000000000000E-01	-5.0000000000E+01
7	1.000000000000E-01	2.5000000000E+01
8	1.000000000000E-01	5.0000000000E+01
9	1.000000000000E-01	2.5000000000E+01
10	1.000000000000E+00	0.
11	1.000000000000E+00	0.
12	-1.000000000000E+00	0.
13	-1.000000000000E+00	0.
14	-1.000000000000E+00	0.
15	-1.000000000000E+00	0.
16	1.000000000000E+00	0.
17	1.000000000000E+00	0.
18	1.000000000000E+00	0.
19	-1.000000000000E+00	0.
20	1.000000000000E+00	0.
21	1.000000000000E+00	0.

	CONSTRAINTS
1	2.038374977360E-01
2	1.502893538221E-01
3	1.588580687292E-01
4	-9.711370323340E-01
5	1.761468818365E+00
6	1.058177013315E+00
7	1.091004927772E+00
8	1.158746004374E+00
9	-1.329505413865E+00
10	4.530084956314E+00
11	1.999000000000E+00
12	9.999000000000E-01
13	1.999000000000E+00
14	1.999000000000E-01
15	1.999000000000E-01
16	1.999000000000E-01
17	1.000000000000E-01
18	1.000000000000E-01
19	1.000000000000E-01
20	2.000000000000E+00
21	2.000000000000E+00
22	0.
23	0.
24	0.
25	0.
26	2.000000000000E+00
27	2.000000000000E+00
28	2.000000000000E+00
29	0.
30	2.000000000000E+00
31	2.000000000000E+00
32	0.
33	0.
34	2.000000000000E+00
35	2.000000000000E+00
36	2.000000000000E+00

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MINIMIZATION WITH NONLINEAR CONSTRAINTS (MFNC PACKAGE) (V:82.05)

LC FILTER PROBLEM

37	2.000000000000E+00
38	0.
39	0.
40	0.
41	2.000000000000E+00
42	0.
43	0.

SOLUTION

OBJECTIVE FUNCTION : 2.639283669757E+01

	VARIABLES	GRADIENT
1	2.066958840409E+00	2.6866251766E+00
2	9.075814424827E-01	-1.2609351546E+00
3	2.066955719104E+00	2.6866182747E+00
4	3.722141848172E-01	-1.4919215565E+01
5	1.283510802962E-01	-5.5091747771E+01
6	3.722151410242E-01	-1.4919116382E+01
7	5.293386570103E-14	2.4190128522E+01
8	1.491244651853E-01	5.5091474615E+01
9	9.205686290409E-16	2.4190165052E+01
10	3.882314692252E-01	0.
11	1.000000000000E+00	0.
12	-1.000000000000E+00	0.
13	-9.915777429239E-01	0.
14	-1.000000000000E+00	0.
15	-1.000000000000E+00	0.
16	9.813804117187E-01	0.
17	1.000000000000E+00	0.
18	9.740095827064E-01	0.
19	-9.728818965723E-01	0.
20	6.356249743061E-01	0.
21	1.000000000000E+00	0.

	CONSTRAINTS
1	2.049800747719E-02
2	1.233502189280E-09
3	6.095733825890E-02
4	3.203183496225E-09
5	-1.012585926219E-08
6	9.700513129670E-01
7	9.659931855141E-01
8	9.954229795973E-01
9	5.502316198332E-01
10	2.070853923816E+00
11	2.066858840409E+00
12	9.074814424827E-01
13	2.066855719104E+00
14	3.721141848172E-01
15	1.282510802962E-01
16	3.721151410242E-01
17	5.293386570103E-14
18	1.491244651853E-01
19	9.205686290409E-16
20	1.388231469225E+00
21	2.000000000000E+00
22	7.105427357601E-15
23	8.422257076106E-03
24	2.131628207280E-14
25	0.
26	1.981380411719E+00
27	2.000000000000E+00
28	1.974009582706E+00
29	2.711810342768E-02
30	1.635624974306E+00
31	2.000000000000E+00
32	6.117685307748E-01
33	2.842170943040E-14
34	2.000000000000E+00
35	1.991577742924E+00
36	2.000000000000E+00
37	2.000000000000E+00
38	1.861958828128E-02
39	7.105427357601E-15
40	2.599041729356E-02
41	1.972881896572E+00
42	3.643750256939E-01

```
DATE : 83/07/11.                   TIME : 11.18.11.                  PAGE :  3
MINIMIZATION WITH NONLINEAR CONSTRAINTS (MFNC PACKAGE)              (V:82.05)

LC FILTER PROBLEM

                                                       43    1.421085471520E-14

  TYPE OF SOLUTION (IFLAG) . . . . . . . . . . . . . . . . . . . . . . . .   0
  NUMBER OF FUNCTION EVALUATIONS  . . . . . . . . . . . . . . . . . . . . .  21
  NUMBER OF QUADRATIC ITERATIONS  . . . . . . . . . . . . . . . . . . . . .  13
  EXECUTION TIME (IN SECONDS) . . . . . . . . . . . . . . . . . . . . . 29.891
```

DATE : 83/07/11. TIME : 11.26.36. PAGE : 1
MINIMIZATION WITH NONLINEAR CONSTRAINTS (MFNC PACKAGE) (V:82.05)

LC FILTER PROBLEM

INPUT DATA

NUMBER OF VARIABLES (N) 21

NUMBER OF EQUALITY CONSTRAINTS (LEQ). 0

TOTAL NUMBER OF CONSTRAINTS (L) 43

ACCURACY (EPS) 1.000E-06

MAX NUMBER OF FUNCTION EVALUATIONS (MAXF) 100

WORKING SPACE (IW) 5000

PRINTOUT CONTROL (IPR) 0

STARTING POINT. OBJECTIVE FUNCTION : 2.900000000000E+01

	VARIABLES	GRADIENT		CONSTRAINTS
1	2.000000000000E+00	4.5000000000E+00	1	2.038374977360E-01
2	1.000000000000E+00	3.0000000000E+00	2	1.502893538221E-01
3	2.000000000000E+00	4.5000000000E+00	3	1.588580687292E-01
4	2.000000000000E-01	-5.0000000000E+01	4	-9.711370323340E-01
5	2.000000000000E-01	-2.5000000000E+01	5	1.761468818365E+00
6	2.000000000000E-01	-5.0000000000E+01	6	1.058177013315E+00
7	1.000000000000E-01	1.0000000000E+01	7	1.091004927772E+00
8	1.000000000000E-01	2.0000000000E+01	8	1.158746004374E+00
9	1.000000000000E-01	1.0000000000E+01	9	-1.329505413865E+00
10	1.000000000000E+00	0.	10	4.530084956314E+00
11	1.000000000000E+00	0.	11	1.999900000000E+00
12	-1.000000000000E+00	0.	12	9.999000000000E-01
13	-1.000000000000E+00	0.	13	1.999900000000E+00
14	-1.000000000000E+00	0.	14	1.999000000000E-01
15	-1.000000000000E+00	0.	15	1.999000000000E-01
16	1.000000000000E+00	0.	16	1.999000000000E-01
17	1.000000000000E+00	0.	17	1.000000000000E-01
18	1.000000000000E+00	0.	18	1.000000000000E-01
19	-1.000000000000E+00	0.	19	1.000000000000E-01
20	1.000000000000E+00	0.	20	2.000000000000E+00
21	1.000000000000E+00	0.	21	2.000000000000E+00
			22	0.
			23	0.
			24	0.
			25	0.
			26	2.000000000000E+00
			27	2.000000000000E+00
			28	2.000000000000E+00
			29	0.
			30	2.000000000000E+00
			31	2.000000000000E+00
			32	0.
			33	0.
			34	2.000000000000E+00
			35	2.000000000000E+00
			36	2.000000000000E+00

DATE : 83/07/11. TIME : 11.26.36. PAGE : 2
MINIMIZATION WITH NONLINEAR CONSTRAINTS (MFNC PACKAGE) (V:82.05)

LC FILTER PROBLEM

37	2.000000000000E+00
38	0.
39	0.
40	0.
41	2.000000000000E+00
42	0.
43	0.

SOLUTION

OBJECTIVE FUNCTION : 1.909262271365E+01

	VARIABLES	GRADIENT
1	2.180105558285E+00	1.6366048288E+00
2	9.196994178133E-01	-3.2754062054E+00
3	2.180111889675E+00	1.6366086394E+00
4	6.110210494223E-01	-5.8393590326E+00
5	2.056510328401E-01	-2.1746237236E+01
6	6.110196267464E-01	-5.8394031835E+00
7	-4.885142493048E-14	9.1738677166E+00
8	3.441756793447E-01	2.1746235360E+01
9	2.273747537106E-14	9.1738410743E+00
10	7.236786744035E-01	0.
11	1.000000000000E+00	0.
12	-1.000000000000E+00	0.
13	-9.952202271024E-01	0.
14	-1.000000000000E+00	0.
15	-9.916366748673E-01	0.
16	9.951065855990E-01	0.
17	9.999999999999E-01	0.
18	9.422755666244E-01	0.
19	-8.489709470598E-01	0.
20	4.025430061518E-01	0.
21	7.025830421173E-01	0.

	CONSTRAINTS
1	7.691483006056E-10
2	1.038556855426E-01
3	3.216125529296E-01
4	4.147580057179E-09
5	-4.133767106396E-09
6	1.172350938347E+00
7	1.185419740199E+00
8	1.223132971563E+00
9	9.228411169545E-02
10	1.844712169127E+00
11	2.180005558285E+00
12	9.195994178133E-01
13	2.180011889675E+00
14	6.109210494223E-01
15	2.055510328401E-01
16	6.109196267464E-01
17	-4.885142493048E-14
18	3.441756793447E-01
19	2.273747537106E-14
20	1.723678674403E+00
21	2.000000000000E+00
22	2.131628207280E-14
23	4.779772897614E-03
24	2.842170943040E-14
25	8.363325132727E-03
26	1.995106585599E+00
27	2.000000000000E+00
28	1.942275566624E+00
29	1.510290529402E-01
30	1.402543006152E+00
31	1.702583042117E+00
32	2.763213255965E-01
33	-7.815970093361E-14
34	2.000000000000E+00
35	1.995220227102E+00
36	2.000000000000E+00
37	1.991636674867E+00
38	4.893414400975E-03
39	6.394884621841E-14
40	5.772443337555E-02
41	1.848970947060E+00
42	5.974569938482E-01

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DATE : 83/07/11.                          TIME : 11.26.36.                        PAGE :  3
MINIMIZATION WITH NONLINEAR CONSTRAINTS (MFNC PACKAGE)                             (V:82.05)

LC FILTER PROBLEM

                                                                     43    2.974169578827E-01

  TYPE OF SOLUTION (IFLAG) . . . . . . . . . . . . . . . . . . . . . . . . . .     0

  NUMBER OF FUNCTION EVALUATIONS . . . . . . . . . . . . . . . . . . . . . . .    22

  NUMBER OF QUADRATIC ITERATIONS . . . . . . . . . . . . . . . . . . . . . . .    16

  EXECUTION TIME (IN SECONDS) . . . . . . . . . . . . . . . . . . . . . . . . 38.716
```

DATE : 83/07/11. TIME : 11.34.14. PAGE : 1
MINIMIZATION WITH NONLINEAR CONSTRAINTS (MFNC PACKAGE) (V:82.05)

LC FILTER PROBLEM

INPUT DATA

NUMBER OF VARIABLES (N) 21

NUMBER OF EQUALITY CONSTRAINTS (LEQ). 0

TOTAL NUMBER OF CONSTRAINTS (L) 43

ACCURACY (EPS) 1.000E-06

MAX NUMBER OF FUNCTION EVALUATIONS (MAXF) 100

WORKING SPACE (IW) 5000

PRINTOUT CONTROL (IPR) 0

STARTING POINT. OBJECTIVE FUNCTION : 2.700000000000E+01

	VARIABLES	GRADIENT		CONSTRAINTS
1	2.000000000000E+00	4.7500000000E+00	1	2.038374977360E-01
2	1.000000000000E+00	4.0000000000E+00	2	1.502893538221E-01
3	2.000000000000E+00	4.7500000000E+00	3	1.588580687292E-01
4	2.000000000000E-01	-5.0000000000E+01	4	-9.711370323340E-01
5	2.000000000000E-01	-2.5000000000E+01	5	1.761468818365E+00
6	2.000000000000E-01	-5.0000000000E+01	6	1.058177013315E+00
7	1.000000000000E-01	5.0000000000E+00	7	1.091004927772E+00
8	1.000000000000E-01	1.0000000000E+01	8	1.158746004374E+00
9	1.000000000000E-01	5.0000000000E+00	9	-1.329505413865E+00
10	1.000000000000E+00	0.	10	4.530084956314E+00
11	1.000000000000E+00	0.	11	1.999900000000E+00
12	-1.000000000000E+00	0.	12	9.999000000000E-01
13	-1.000000000000E+00	0.	13	1.999900000000E+00
14	-1.000000000000E+00	0.	14	1.999000000000E-01
15	-1.000000000000E+00	0.	15	1.999000000000E-01
16	1.000000000000E+00	0.	16	1.999000000000E-01
17	1.000000000000E+00	0.	17	1.000000000000E-01
18	1.000000000000E+00	0.	18	1.000000000000E-01
19	-1.000000000000E+00	0.	19	1.000000000000E-01
20	1.000000000000E+00	0.	20	2.000000000000E+00
21	1.000000000000E+00	0.	21	2.000000000000E+00
			22	0.
			23	0.
			24	0.
			25	0.
			26	2.000000000000E+00
			27	2.000000000000E+00
			28	2.000000000000E+00
			29	0.
			30	2.000000000000E+00
			31	2.000000000000E+00
			32	0.
			33	0.
			34	2.000000000000E+00
			35	2.000000000000E+00
			36	2.000000000000E+00

LC FILTER PROBLEM

37	2.000000000000E+00
38	0.
39	0.
40	0.
41	2.000000000000E+00
42	0.
43	0.

SOLUTION

OBJECTIVE FUNCTION : 1.431338745674E+01

	VARIABLES	GRADIENT
1	2.479622102908E+00	9.8424003521E-01
2	9.083115192866E-01	-3.4215814589E+00
3	2.479680278008E+00	9.8419167396E-01
4	1.016012318360E+00	-2.4020804687E+00
5	2.872357635403E-01	-1.1009251592E+01
6	1.016062243213E+00	-2.4019007694E+00
7	-1.857376377676E-14	4.0328725850E+00
8	5.695215538341E-01	1.1009438709E+01
9	-1.285407367241E-15	4.0327779709E+00
10	5.658882145085E-01	0.
11	1.000000000000E+00	0.
12	-9.464920996361E-01	0.
13	-9.999408945276E-01	0.
14	-1.000000000000E+00	0.
15	-1.000000000000E+00	0.
16	9.993818546536E-01	0.
17	1.000000000000E+00	0.
18	9.977469236730E-01	0.
19	-4.132427835367E-01	0.
20	-8.691458957342E-03	0.
21	7.965945025993E-01	0.

	CONSTRAINTS
1	-1.035025718465E-07
2	3.854924509296E-01
3	8.999418109616E-01
4	9.823679647525E-09
5	-3.716081664606E-09
6	1.184419260254E+00
7	1.183937013816E+00
8	1.204372563283E+00
9	8.345856019515E-01
10	-3.400941750442E-09
11	2.479522102908E+00
12	9.082115192866E-01
13	2.479580278008E+00
14	1.015912318360E+00
15	2.871357635403E-01
16	1.015962243213E+00
17	-1.857376377676E-14
18	5.695215538341E-01
19	-1.285407367241E-15
20	1.565888214509E+00
21	2.000000000000E+00
22	5.350790036388E-02
23	5.910547238130E-05
24	4.263256414561E-14
25	-2.842170943040E-14
26	1.999381854654E+00
27	2.000000000000E+00
28	1.997746923673E+00
29	5.867572164633E-01
30	9.913085410427E-01
31	1.796594502599E+00
32	4.341117854915E-01
33	0.
34	1.946492099636E+00
35	1.999940894528E+00
36	2.000000000000E+00
37	2.000000000000E+00
38	6.181453464364E-04
39	4.973799150321E-14
40	2.253076327030E-03
41	1.413242783537E+00
42	1.008691458957E+00

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DATE : 83/07/11.                    TIME : 11.34.14.                    PAGE :  3
MINIMIZATION WITH NONLINEAR CONSTRAINTS (MFNC PACKAGE)                  (V:82.05)

LC FILTER PROBLEM

                                                      43   2.034054974007E-01

  TYPE OF SOLUTION (IFLAG) . . . . . . . . . . . . . . . . . . . . . . . . .      0
  NUMBER OF FUNCTION EVALUATIONS . . . . . . . . . . . . . . . . . . . . . .     25
  NUMBER OF QUADRATIC ITERATIONS . . . . . . . . . . . . . . . . . . . . . .     19
  EXECUTION TIME (IN SECONDS) . . . . . . . . . . . . . . . . . . . . . . . 46.920
```

2. Program LCMML

A nonlinear programming problem, i.e.,

$$\text{minimize } U(\boldsymbol{\phi}) \tag{34}$$

subject to

$$g_i(\boldsymbol{\phi}) \geq 0 \quad i=1, 2, ..., m \tag{35}$$

can be formulated as a minimax problem in the following way [13]:

$$\text{minimize } V(\boldsymbol{\phi},\boldsymbol{\alpha}) = \max\,[U(\boldsymbol{\phi}), U(\boldsymbol{\phi}) - \alpha_i\, g_i(\boldsymbol{\phi})]\,, \tag{36}$$

where

$$\boldsymbol{\alpha} \triangleq [\alpha_1\, \alpha_2 \ldots \alpha_m]^T \tag{37}$$

and

$$\alpha_i > 0, \quad i=1, 2, ..., m\,. \tag{38}$$

In particular, for the LC filter problem, we use the objective function defined previously and 10 nonlinear constraints to form 11 functions for a minimax problem. The constant value of $\alpha = 10$ is used throughout. The other 33 constraints can be formulated in the main program in matrix form since they are linear.

The program uses subroutine MMLC1A. A main program and subroutine FCD provide the information for MMLC1A. Subroutines FIDFI, SIMGRD, and SOLLU are exactly the same subroutines used for MFNC package. Subroutine FCD is different from the one for MFNC, and calculates the values of 11 residual functions $f_i(\mathbf{x})$ and their derivatives $\partial f_i(\mathbf{x})/\partial x_j$ at the point $\mathbf{x}$.

The block diagram for the interaction of subroutines is similar to Fig. 5 with MMLC1A replacing MFNC1A. Subroutine MMLC1A is available as a group file LIBRMML under the charge RJWBAND. The sequence of NOS commands to use the package may be as follows:

/GET,LIBRMML/GR.

/LIBRARY,LIBRMML.

/FTN,...,GO.

The Fortran listing for LCMML starts on page 63. The following three actual outputs show the starting points and the solutions for three values of constant c, using LCMML. The outputs correspond to $c = 50$, $c = 20$, and $c = 10$, respectively.

DATE : 83/07/14. TIME : 13.50.01. PAGE : 1
LINEARLY CONSTRAINED MINIMAX OPTIMIZATION (MMLC PACKAGE) (V:82.04)

LC FILTER PROBLEM

INPUT DATA

NUMBER OF VARIABLES (N) 21

NUMBER OF FUNCTIONS (M) 11

TOTAL NUMBER OF LINEAR CONSTRAINTS (L) 33

NUMBER OF EQUALITY CONSTRAINTS (LEQ) 0

STEP LENGTH (DX) 1.000E-01

ACCURACY (EPS) 1.000E-06

MAX NUMBER OF FUNCTION EVALUATIONS (MAXF) 100

NUMBER OF SUCCESSIVE ITERATIONS (KEQS) 2

WORKING SPACE (IW) 5000

PRINTOUT CONTROL (IPR) 0

STARTING POINT :

	VARIABLES		FUNCTION VALUES
1	2.000000000000E+00	1	2.036103081094E+01
2	1.000000000000E+00	2	2.075342909785E+01
3	2.000000000000E+00	3	2.038365388158E+01
4	4.000000000000E-01	4	4.854468761818E+01
5	2.000000000000E-01	5	3.323140798737E+01
6	4.000000000000E-01	6	1.044000454821E+01
7	1.000000000000E-02	7	1.056729521494E+01
8	1.000000000000E-01	8	1.040560001253E+01
9	1.000000000000E-02	9	1.094995176769E+01
10	-1.000000000000E+00	10	1.182165687945E+01
11	1.000000000000E+00	11	2.050000000000E+01
12	-1.000000000000E+00		
13	-1.000000000000E+00		
14	-1.000000000000E+00		
15	-1.000000000000E+00		
16	1.000000000000E+00		
17	1.000000000000E+00		
18	1.000000000000E+00		
19	1.000000000000E+00		
20	-1.000000000000E+00		
21	1.000000000000E+00		

SOLUTION

	VARIABLES		FUNCTION VALUES
1	2.066957689956E+00	1	2.618786002738E+01
2	9.075814509833E-01	2	2.639283670328E+01

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LINEARLY CONSTRAINED MINIMAX OPTIMIZATION (MMLC PACKAGE) (V:82.04)

LC FILTER PROBLEM

3	2.066957634343E+00	3	2.578325878957E+01
4	3.722155598539E-01	4	2.639283670318E+01
5	1.283514248823E-01	5	2.639283670350E+01
6	3.722154408380E-01	6	1.883260260320E+01
7	0.	7	1.935698051800E+01
8	1.491252263368E-01	8	1.956450588789E+01
9	0.	9	1.184782756038E+01
10	-9.998564100021E-01	10	2.639283670179E+01
11	1.000000000000E+00	11	2.639283670336E+01
12	-9.998564100021E-01		
13	-9.992439474718E-01		
14	-1.000000000000E+00		
15	-9.992439474718E-01		
16	9.984195142677E-01		
17	1.000000000000E+00		
18	9.984195142677E-01		
19	9.998167127124E-01		
20	-7.213911828030E-01		
21	9.998167127124E-01		

TYPE OF SOLUTION (IFALL) 1

NUMBER OF FUNCTION EVALUATIONS 42

NUMBER OF SHIFTS TO STAGE-2 3

EXECUTION TIME (IN SECONDS) 11.232

DATE : 83/07/14. TIME : 11.57.48. PAGE : 1
LINEARLY CONSTRAINED MINIMAX OPTIMIZATION (MMLC PACKAGE) (V:82.04)

LC FILTER PROBLEM

INPUT DATA

NUMBER OF VARIABLES (N) 21

NUMBER OF FUNCTIONS (M) 11

TOTAL NUMBER OF LINEAR CONSTRAINTS (L) 33

NUMBER OF EQUALITY CONSTRAINTS (LEQ) 0

STEP LENGTH (DX) 1.000E-01

ACCURACY (EPS) 1.000E-06

MAX NUMBER OF FUNCTION EVALUATIONS (MAXF) 100

NUMBER OF SUCCESSIVE ITERATIONS (KEQS) 2

WORKING SPACE (IW) 5000

PRINTOUT CONTROL (IPR) 0

STARTING POINT :

	VARIABLES		FUNCTION VALUES
1	2.000000000000E+00	1	1.603857319932E+01
2	1.000000000000E+00	2	1.525290820799E+01
3	2.000000000000E+00	3	1.353122666402E+01
4	6.000000000000E-01	4	2.626172263935E+01
5	2.000000000000E-01	5	2.866116025756E+01
6	6.000000000000E-01	6	9.064315526223E+00
7	5.000000000000E-02	7	9.574559048255E+00
8	3.000000000000E-01	8	9.891977620785E+00
9	5.000000000000E-02	9	3.792538734043E+00
10	-1.000000000000E+00	10	5.051744307275E+01
11	1.000000000000E+00	11	1.866666666667E+01
12	-1.000000000000E+00		
13	-1.000000000000E+00		
14	-1.000000000000E+00		
15	-1.000000000000E+00		
16	1.000000000000E+00		
17	1.000000000000E+00		
18	1.000000000000E+00		
19	1.000000000000E+00		
20	-1.000000000000E+00		
21	1.000000000000E+00		

SOLUTION

	VARIABLES		FUNCTION VALUES
1	2.180106625293E+00	1	1.909262271222E+01
2	9.196992928945E-01	2	1.805408801912E+01

DATE : 83/07/14. TIME : 11.57.48. PAGE : 2
LINEARLY CONSTRAINED MINIMAX OPTIMIZATION (MMLC PACKAGE) (V:82.04)

LC FILTER PROBLEM

3	2.180106625291E+00	3	1.587654610112E+01
4	6.110166665287E-01	4	1.909262271222E+01
5	2.056510139774E-01	5	1.909262271222E+01
6	6.110166665377E-01	6	9.091839454992E+00
7	0.	7	9.306510951834E+00
8	3.441739856866E-01	8	9.254343811983E+00
9	0.	9	6.991186832635E+00
10	-9.998636156382E-01	10	1.909262271222E+01
11	1.000000000000E+00	11	1.909262271222E+01
12	-9.998636156381E-01		
13	-9.997371971152E-01		
14	-1.000000000000E+00		
15	-9.997371971152E-01		
16	9.997374236533E-01		
17	1.000000000000E+00		
18	9.997374236533E-01		
19	9.998391183443E-01		
20	-1.950410114067E-01		
21	9.998391183443E-01		

TYPE OF SOLUTION (IFALL) 1

NUMBER OF FUNCTION EVALUATIONS 33

NUMBER OF SHIFTS TO STAGE-2 2

EXECUTION TIME (IN SECONDS) 11.466

DATE : 83/07/11. TIME : 11.45.50. PAGE : 1
LINEARLY CONSTRAINED MINIMAX OPTIMIZATION (MMLC PACKAGE) (V:82.04)

LC FILTER PROBLEM

INPUT DATA

NUMBER OF VARIABLES (N) 21

NUMBER OF FUNCTIONS (M) 11

TOTAL NUMBER OF LINEAR CONSTRAINTS (L) 33

NUMBER OF EQUALITY CONSTRAINTS (LEQ) 0

STEP LENGTH (DX) 1.000E-01

ACCURACY (EPS) 1.000E-06

MAX NUMBER OF FUNCTION EVALUATIONS (MAXF) 100

NUMBER OF SUCCESSIVE ITERATIONS (KEQS) 2

WORKING SPACE (IW) 5000

PRINTOUT CONTROL (IPR) 0

STARTING POINT :

	VARIABLES		FUNCTION VALUES
1	2.000000000000E+00	1	1.203857319932E+01
2	1.000000000000E+00	2	1.125290820799E+01
3	2.000000000000E+00	3	9.531226664023E+00
4	6.000000000000E-01	4	2.226172263936E+01
5	4.000000000000E-01	5	2.466116025756E+01
6	6.000000000000E-01	6	5.064315526223E+00
7	5.000000000000E-02	7	5.574559048255E+00
8	5.000000000000E-01	8	5.891977620785E+00
9	5.000000000000E-02	9	-2.074612659565E-01
10	-1.000000000000E+00	10	4.651744307275E+01
11	1.000000000000E+00	11	1.466666666667E+01
12	-1.000000000000E+00		
13	-1.000000000000E+00		
14	-1.000000000000E+00		
15	-1.000000000000E+00		
16	1.000000000000E+00		
17	1.000000000000E+00		
18	1.000000000000E+00		
19	1.000000000000E+00		
20	-1.000000000000E+00		
21	1.000000000000E+00		

SOLUTION

	VARIABLES		FUNCTION VALUES
1	2.479685241591E+00	1	1.431338749494E+01
2	9.083082799714E-01	2	1.045816495277E+01

```
DATE : 83/07/11.                    TIME : 11.45.50.                  PAGE :  2
LINEARLY CONSTRAINED MINIMAX OPTIMIZATION (MMLC PACKAGE)              (V:82.04)

LC FILTER PROBLEM

                 3    2.479685237789E+00          3    5.313417478946E+00
                 4    1.016076147414E+00          4    1.431338749494E+01
                 5    2.872322983423E-01          5    1.431338749494E+01
                 6    1.016076158355E+00          6    2.461982993635E+00
                 7    0.                          7    2.465437320096E+00
                 8    5.695279591497E-01          8    2.259839988292E+00
                 9    0.                          9    6.015334366507E+00
                10   -9.998706394004E-01         10    1.422592445029E+01
                11    1.000000000000E+00         11    1.431338749494E+01
                12   -9.998706394002E-01
                13   -9.996280752160E-01
                14   -1.000000000000E+00
                15   -9.996280752160E-01
                16    9.993627042182E-01
                17    1.000000000000E+00
                18    9.999982481925E-01
                19    9.996962932009E-01
                20   -6.916176057441E-03
                21    9.996962932009E-01

  TYPE OF SOLUTION (IFALL) . . . . . . . . . . . . . . . . . . . . . . . . .      1

  NUMBER OF FUNCTION EVALUATIONS . . . . . . . . . . . . . . . . . . . . . .     38

  NUMBER OF SHIFTS TO STAGE-2 . . . . . . . . . . . . . . . . . . . . . . . .      3

  EXECUTION TIME (IN SECONDS) . . . . . . . . . . . . . . . . . . . . . . . . 12.239
```

3. Program LCMNS and Associated Text Files

This program uses the package MINOS. Again, we have the nonlinear programming problem defined in Section III with 10 nonlinear and 33 linear constraints. All linear constraints are upper or lower bounds on 21 variables and are treated in a special way using data files.

The program consists of:

(a) the main segment that defines input, output and working files, and calls the main subroutine (MINOS1);

(b) subroutine CALCFG which evaluates the objective function F and its gradient at point **x**;

(c) subroutine CALCON which evaluates nonlinear constraints f_i and $\partial f_i/\partial x_j$, $i=1, \ldots, 10$, $j = 1, \ldots, 21$;

(d) subroutines FIDFI, SIMGRD, and SOLLU are the same as in the previous programs.

In addition, a data file provides the appropriate SPECS and MPS data. SPECS data contain the keywords and values to define various optimization parameters. The MPS data contain bounds for variables and the definition of the starting point. The MPS data are defined in the format accepted by the IBM Mathematical Programming Systems MPS/360, MPSX and MPSX/370.

Since MINOS can be used in batch jobs only, a job description for the SUBMIT command may be written on a separate file, e.g., JCLMNS, and the NOS command /SUBMIT,JLCMNS,Q=... is sufficient to run the program. The listing for JLCMNS is shown after the listings for LCMNS and the corresponding data file.

A block diagram for the interaction of subroutines is shown in Fig. 6.

The Fortran listing for LCMNS starts on page 67. It is followed by SPECS and MPS data, and JLCMNS. The following output represents the actual output corresponding to $c = 50$ for MINOS. Due to the size of output, the actual outputs for $c = 20$ and $c = 10$ are not shown, however, Tables V and VI contain MINOS results for these two cases.

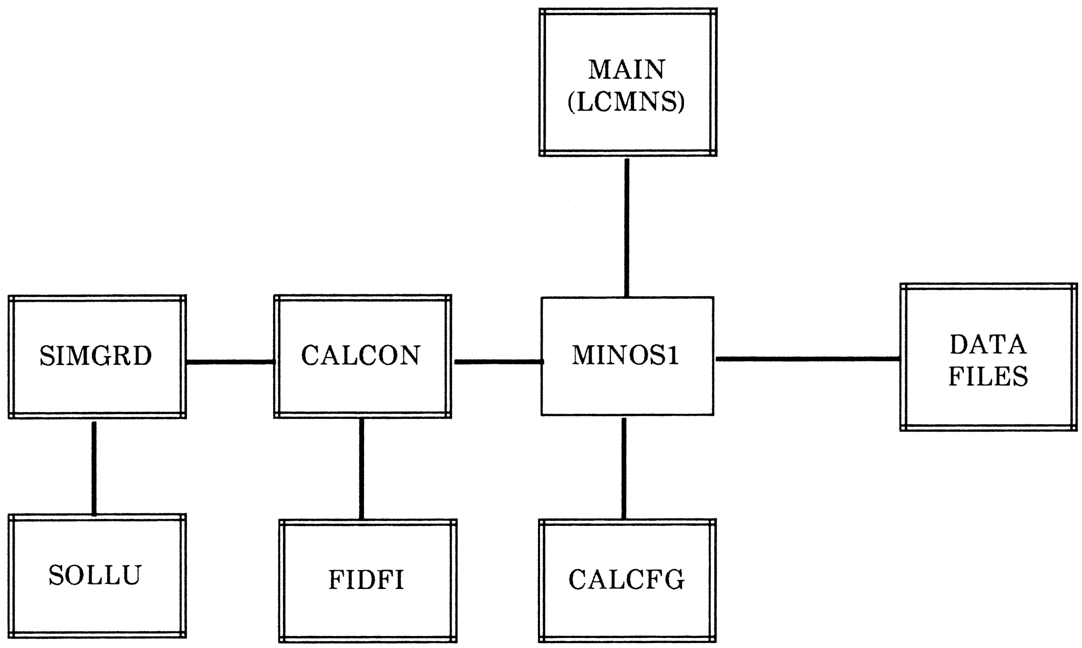

Fig. 6. Structure of the main program, auxiliary subroutines and data files for LCMNS.

```
    M I N O S     ---     VERSION  4.0     MAR  1981
    = = = = =
```

SPECS FILE

```
BEGIN  LC FILTER PROBLEM
*
*    SPECS AND MPS DATA FOR LC FILTER PROBLEM
*
   MINIMIZE
   ROWS                                        100
   COLUMNS                                     100
   NONLINEAR VARIABLES                          21
   NONLINEAR CONSTRAINTS                        10
   JACOBIAN                                  DENSE
   SUPERBASICS LIMIT                            21
   HESSIAN DIMENSION                            21
   MAJOR ITERATIONS                             20
   MINOR ITERATIONS                             25
   ITERATIONS LIMIT                            300
   LOWER BOUNDS                               -1.0
   UPPER BOUNDS                              100.0
   PRINT LEVEL                                  10
END
```

```
MPS FILE
--------
     1     NAME          LC FILTER PROBLEM
     2     ROWS
    14     COLUMNS
    36     RHS
    37     BOUNDS
    92     ENDATA


NAMES SELECTED
--------------
OBJECTIVE      OBJECT   (MIN)       1
RHS            RHS                  0
RANGES                              0
BOUNDS                             33


MATRIX STATISTICS
-----------------
              TOTAL     NORMAL      FREE      FIXED    BOUNDED
ROWS            11        10          1         0          0
COLUMNS         21         0          0         0         21

NO. OF MATRIX ELEMENTS      210    DENSITY     86.777
NO. OF REJECTED COEFFS        0     AIJTOL 1.00000E-10
BIGGEST AND SMALLEST COEFFS   0.           1.00000E+20  (EXCLUDING OBJ AND RHS)

LENGTH OF ROW-NAME HASH TABLE          211
COLLISIONS DURING TABLE LOOKUP           0

NO. OF JACOBIAN ENTRIES SPECIFIED        0
NO. OF LAGRANGE MULTIPLIERS SPECIFIED    0

NO. OF  INITIAL BOUNDS PROCESSED        21
NO. OF SUPERBASICS SPECIFIED            21

PARTITION SIZE FOR PARTIAL PRICING      21

CORE REQUIRED TO SOLVE THE PROBLEM    1771
REMAINING WORKSPACE                   3229
```

```
PARAMETERS
----------
MPS INPUT DATA.
ROW LIMIT.............        100     LIST LIMIT.............          0     LOWER BOUND DEFAULT.....  -1.00E+00
COLUMN LIMIT..........        100     ERROR MESSAGE LIMIT....         10     UPPER BOUND DEFAULT.....   1.00E+02
ELEMENTS LIMIT (COEFFS)       500     PHANTOM ELEMENTS.......          0     AIJ TOLERANCE...........   1.00E-10

FILES.
MPS FILE (INPUT FILE)..         0     OLD BASIS FILE (MAP)...          0     (CARD READER)...........          1
SOLUTION FILE.........          0     NEW BASIS FILE (MAP)...          0     (PRINTER)...............          6
INSERT FILE...........          0     BACKUP BASIS FILE......          0     (SCRATCH FILE)..........          9
PUNCH FILE............          0     LOAD FILE..............          0     DUMP FILE...............          0

FREQUENCIES.
LOG ITERATIONS........          1     CHECK ROW ERROR........         30     CYCLE LIMIT.............          1
SAVE NEW BASIS MAP....        100     FACTORIZE (INVERT).....         60     CYCLE TOLERANCE.........  0.

LP PARAMETERS.
ITERATIONS LIMIT......        300     FEASIBILITY TOLERANCE..  1.00E-05     PARTIAL PRICE FACTOR....          1
CRASH OPTION..........          1     DJ TOLERANCE...........  1.00E-06     MULTIPLE PRICE..........          0
WEIGHT ON OBJECTIVE....  0.           PIVOT TOLERANCE........  8.43E-08

NONLINEAR PROBLEMS.
NONLINEAR CONSTRAINTS..        10     SUPERBASICS LIMIT......         21     DERIVATIVE LEVEL........          3
NONLINEAR JACOBIAN VARS        21     HESSIAN DIMENSION......         21     VERIFY LEVEL............          0
NONLINEAR OBJECTIV VARS        21     LINESEARCH TOLERANCE...     .01000     DIFFERENCE INTERVAL.....  1.69E-07
PROBLEM NUMBER........          0     REDUCED-GRADIENT TOL...     .20000     CONJUGATE-GRADNT METHOD            1

AUGMENTED LAGRANGIAN.
JACOBIAN..............      DENSE     MAJOR ITERATIONS LIMIT.         20     RADIUS OF CONVERGENCE..   1.00E-02
LAGRANGIAN............        YES     MINOR ITERATIONS LIMIT.         25     ROW TOLERANCE...........  1.00E-05
PENALTY PARAMETER.....   1.00E+01     COMPLETION.............       FULL

MISCELLANEOUS.
LU ROW TOLERANCE......   1.00E-04     PRINT LEVEL..(JFLXI)...         10     IMBED...................        YES
LU COL TOLERANCE......     .10000     DEBUG LEVEL............          0     PRINT SPIKES............         NO
LU MOD TOLERANCE......     .90000     UNBOUNDED OBJECTV VALUE  1.00E+20     UNBOUNDED STEP SIZE.....   1.00E+10

NUMBER OF WORDS OF CORE AVAILABLE FOR WORKSPACE    5000
```

```
ITERATIONS
----------

CRASH OPTION   1
FREE ROWS      1      FREE COLS      0      PASS2 (E ROWS)      0      PASS3      0      REMAINDER     10

NONLINEAR VARIABLES
-------------------
  2.0000000E+00   1.0000000E+00   2.0000000E+00   2.0000000E-01   2.0000000E-01
  2.0000000E-01   1.0000000E-01   1.0000000E-01   1.0000000E+01   1.0000000E+00
  1.0000000E+00  -1.0000000E+00  -1.0000000E+00  -1.0000000E+00  -1.0000000E+00
  1.0000000E+00   1.0000000E+00   1.0000000E+00  -1.0000000E+00   1.0000000E+00
  1.0000000E+00

FACTORIZE    1     DEMAND      0     ITERATION     0     INFEAS      1     OBJECTV   0.

ITN     0 --  INFEASIBLE.  NUM =     2    SUM =    2.300642446E+00

END OF PHASE 1 LP ITERATIONS.    STILL INFEASIBLE AT NON-VERTEX POINT.

  ITN PH PP NOPT  DJ/RG   +SBS -SBS  -BS    STEP     PIVOT    NSPK    L      U NINF  SINF/OBJECTIVE    NFG   NSB  R1M H-COND
    1  4  1    0  2.1E+01    0    1   31  3.3E-03 -2.8E+00    0      1      1    2  2.30064245E+00      1    20  1 0 0.
    2  4  1    0  1.0E+00    0   14    0  3.8E-03 -1.0E+00    1      1     10    4  6.53631371E-03      1    19  0 0 0.

ITN     2 -- FEASIBLE SOLUTION.  OBJECTIVE =    3.835843604E+01

CHOLESKY FACTOR OF HESSIAN RESET TO  I.
    3  4  1    0  3.9E+01    0    2   26  4.1E-04 -2.4E+00    1      1     10    0  3.34463381E+01      3    18  2 2 1.6E+02
    4  4  1    0  3.9E+01    0   16    0  0.       -2.2E-02    2      2     18    0  3.34463381E+01      3    17  0 0 1.6E+02
    5  4  1    0  3.8E+01    0   20    0  8.1E-04 -3.5E+00    2      2     18    0  3.26764366E+01      4    16  0 0 1.6E+02
    6  4  1    0  3.8E+01    0   21    0  8.7E-06 -1.1E+00    2      2     18    0  3.26680381E+01      5    15  0 0 1.6E+02
    7  4  1    0  9.2E+01    0    4   27  1.7E-03  1.0E+01    2      2     18    0  3.10487480E+01      6    14  1 0 1.7E+03
    8  4  1    0  9.1E+01    0   17    0  3.1E-04 -7.0E-01    3      4     25    0  3.08226479E+01      7    13  0 0 1.7E+03
    9  4  1    0  9.0E+01    0   18    0  1.5E-04 -2.0E-01    3      4     25    0  3.07180160E+01      8    12  0 0 1.7E+03
   10  4  1    0  8.5E+01    0   15    0  8.7E-04  7.7E-01    3      4     25    0  3.01654767E+01      9    11  0 0 1.7E+03
   11  4  1    0  8.4E+01    0    7    0  2.6E-04  2.0E+01    3      4     25    0  3.00257821E+01     10    10  0 0 1.7E+03
   12  4  1    0  8.4E+01    0   10    0  0.       -1.3E-02    3      4     25    0  3.00257821E+01     10     9  0 0 1.7E+03
   13  4  1    0  8.3E+01    0   13    0  5.6E-04  5.8E-01    3      4     25    0  2.99844615E+01     11     8  0 0 1.7E+03
   14  4  1    0  1.4E+02    0    9    0  8.5E-03  6.7E+00    3      4     25    0  2.95439654E+01     12     7  3 3 5.2E+03
   15  4  1    0  1.2E+02    0    0    0  2.9E-02  0.          3      4     25    0  2.94448011E+01     17     7  4 4 5.5E+03
   16  4  1    0  5.9E+01    0    0    0  3.5E-02  0.          3      4     25    0  2.84468779E+01     21     7  4 4 4.0E+03
   17  4  1    0  1.9E+01    0    0    0  6.0E-02  0.          3      4     25    0  2.79187929E+01     24     7  4 4 1.5E+04
   18  4  1    0  1.9E+01    0   11    0  2.3E-02 -1.4E-02    3      4     25    0  2.79048628E+01     25     6  0 0 1.5E+04
   19  4  1    0  1.3E+01    0   19    0  3.3E-01  6.4E-01    3      4     25    0  2.78187310E+01     26     5  3 3 1.5E+04
```

20	4	1	0	7.7E-01	0	0	0	2.1E+00	0.	3.	4	25	0	2.77889074E+01	29	5	4	4	1.2E+04
21	4	1	2	3.6E-01	-31	0	0	3.0E+00	0.	3.	4	25	0	2.77875084E+01	31	5	4	4	1.2E+04
22	4	1	2	1.6E-01	0	0	0	2.0E+00	0.	3.	4	25	0	2.77874516E+01	33	5	4	4	2.7E+04
23	3	1	2	4.4E-01	31	0	0	1.3E+01	0.	3.	4	25	0	2.77646473E+01	37	6	4	4	3.3E+04
24	4	1	2	5.1E+00	0	0	0	1.7E+00	0.	3.	4	25	0	2.77444603E+01	39	6	4	4	3.6E+04
25	4	1	2	9.8E-01	0	0	0	2.1E+00	0.	3.	4	25	0	2.77368931E+01	41	6	4	4	5.0E+04

END OF MAJOR ITN 1 - MINOR ITNS TERMINATED AT 25 - TOTAL ITNS = 25

START OF MAJOR ITN 2 -- PENALTY PARAMETER = 1.00E+01

NONLINEAR VARIABLES

1.9165943E+00	9.0769450E-01	2.1179749E+00	3.0041703E-01	1.3984145E-01
3.2265980E-01	0.	1.4175045E-01	0.	1.0000000E+00
1.0000000E+00	-1.8571922E-01	-1.0000000E+00	-1.0000000E+00	-1.0000000E+00
1.0000000E+00	1.0000000E+00	1.0000000E+00	-1.0000000E+00	1.0000000E+00
1.0000000E+00				

ROW ERROR AFTER RELINEARIZATION = 9.6744E-03
RELATIVE CHANGE IN MULTIPLIERS = 8.1624E-01

FACTORIZE 2 DEMAND 0 ITERATION 25 INFEAS 0 OBJECTV 2.773689313E+01

ITN 25 -- INFEASIBLE. NUM = 3 SUM = 6.555021203E-01

ITN	PH	PP	NOPT	DJ/RG	+SBS	-SBS	-BS	STEP	PIVOT	NSPK	L	U	NINF	SINF/OBJECTIVE	NFG	NSB	RIM	H-COND
26	4	1	1	1.5E+01	0	5	32	2.4E-03	2.9E+01	2	15	3	3	6.55502120E-01	42	5	1 0	0.
27	4	1	1	0.	0	31	1	2.0E-03	8.1E-01	3	15	10	1	7.62315259E-02	42	4	1 0	0.

ITN 27 -- FEASIBLE SOLUTION. OBJECTIVE = 8.701753936E+01

CHOLESKY FACTOR OF HESSIAN RESET TO I.

ITN	PH	PP	NOPT	DJ/RG	+SBS	-SBS	-BS	STEP	PIVOT	NSPK	L	U	NINF	SINF/OBJECTIVE	NFG	NSB	RIM	H-COND
28	4	1	1	3.4E+01	0	0	0	1.3E-04	0.	3	16	17	0	8.69018143E+01	46	4	4 4	2.2E+02
29	4	1	1	1.5E+01	0	0	0	6.6E-04	0.	3	16	17	0	8.60711636E+01	48	4	4 4	7.1E+02
30	4	1	1	1.2E+01	0	0	0	7.9E-03	0.	3	16	17	0	8.52130955E+01	53	4	4 4	3.5E+03
31	4	1	1	5.5E-01	0	0	0	8.9E-01	0.	3	16	17	0	8.52085504E+01	55	4	4 4	3.5E+03
32	4	1	1	1.0E-02	0	0	0	9.4E-01	0.	3	16	17	0	8.52084928E+01	57	4	4 4	3.6E+03
33	3	1	6	-3.8E+02	7	8	31	1.4E-04	-7.6E+00	3	16	17	0	7.58081922E+01	59	4	2 2	4.7E+03
34	4	1	6	6.4E+01	0	0	0	1.0E+00	0.	3	16	21	0	7.18723127E+01	60	4	4 4	3.9E+03
35	4	1	6	1.1E+01	0	0	0	2.0E+00	0.	3	16	21	0	7.14292730E+01	62	4	4 4	2.3E+03
36	3	1	8	-1.5E+02	1	0	0	3.2E-03	0.	3	16	21	0	5.96381881E+01	65	5	4 4	2.3E+03
37	4	1	8	1.2E+02	0	3	23	4.0E+00	-1.0E+00	3	16	21	0	4.66834972E+01	67	4	2 2	4.1E+03
38	4	1	8	3.5E+01	0	0	0	4.7E+00	0.	4	18	27	0	4.14103915E+01	70	4	4 4	3.9E+03
39	4	1	8	1.1E+01	0	0	0	1.4E+01	0.	4	18	27	0	3.94839702E+01	74	4	4 4	3.5E+02
40	4	1	8	3.3E+00	0	0	0	6.3E+00	0.	4	18	27	0	3.87466526E+01	77	4	4 4	3.3E+02
41	4	1	8	8.8E-01	0	0	0	1.6E+00	0.	4	18	27	0	3.85888808E+01	79	4	4 4	2.7E+02
42	3	1	5	-2.2E+01	19	0	0	4.6E-01	0.	4	18	27	0	3.52201977E+01	82	5	4 4	3.2E+02
43	4	1	5	3.3E+00	0	0	0	1.5E+00	0.	4	18	27	0	3.44867108E+01	85	5	4 4	3.1E+02
44	4	1	5	2.8E+00	0	0	0	9.7E-01	0.	4	18	27	0	3.43719350E+01	87	5	4 4	3.7E+02
45	3	1	4	6.3E+00	31	0	0	8.6E-01	0.	4	18	27	0	3.10598757E+01	91	6	4 4	3.8E+02
46	4	1	4	5.5E+00	0	31	24	1.6E+00	9.3E-02	4	18	27	0	3.00886474E+01	93	5	2 2	2.9E+02
47	4	1	4	1.4E+00	0	0	0	1.0E+00	0.	5	21	32	0	2.99972387E+01	95	5	4 4	3.0E+02
48	4	1	4	6.8E-01	0	0	0	1.2E+00	0.	5	21	32	0	2.99766358E+01	97	5	4 4	3.0E+02
49	3	1	4	8.3E+00	23	0	0	4.8E-04	0.	5	21	32	0	2.99600032E+01	102	6	4 4	1.2E+03
50	4	1	4	2.7E+00	0	0	0	2.2E+00	0.	5	21	32	0	2.99090234E+01	104	6	4 4	8.7E+02

```
END OF MAJOR ITN    2  -  MINOR ITNS TERMINATED AT  25  -  TOTAL ITNS =    50
```

START OF MAJOR ITN 3 - PENALTY PARAMETER = 1.00E+01

NONLINEAR VARIABLES

```
 2.0095727E+00   9.4290263E-01   1.9225077E+00   4.2200353E-01   1.1899103E-01
 3.6455904E-01   2.1609977E-01   1.0911508E-01   0.              1.0000000E+00
 1.0000000E+00  -1.8571922E-01  -1.0000000E+00  -1.0000000E+00  -1.0000000E+00
 1.0000000E+00   1.0000000E+00   1.0000000E+00  -8.5848405E-01   1.0000000E+00
 1.0000000E+00
```

ROW ERROR AFTER RELINEARIZATION = 3.1371E-02
RELATIVE CHANGE IN MULTIPLIERS = 5.3419E-01

FACTORIZE 3 DEMAND 0 ITERATION 50 INFEAS 0 OBJECTV 2.990902342E+01

ITN 50 -- FEASIBLE SOLUTION. OBJECTIVE = 2.955453265E+01

ITN	PH	PP	NOPT	DJ/RG	+SBS	-SBS	-BS	STEP	PIVOT	NSPK	L	U	NINF	SINF/OBJECTIVE	NFG	NSB	R1M	H-COND
51	4	1	1	1.4E+01	0	0	0	1.6E+00	0.	4	28	5	0	2.93934228E+01	109	6	4 4	7.8E+02
52	4	1	1	1.3E+01	0	19	0	1.6E-01	1.5E-01	4	28	5	0	2.93533855E+01	110	5	0 0	1.2E+03
53	4	1	1	5.7E-01	0	0	0	1.3E+00	0.	4	28	5	0	2.93017608E+01	112	5	4 4	9.3E+02
54	3	1	3	1.2E+00	32	23	0	4.2E+00	-4.5E-03	4	28	5	0	2.81436387E+01	115	5	3 3	1.8E+03
55	4	1	3	1.1E+01	0	7	0	2.4E-01	2.0E-01	4	28	5	0	2.75574800E+01	116	4	3 3	3.5E+02
56	4	1	3	2.8E+00	0	0	0	1.0E+00	0.	4	28	5	0	2.69494363E+01	117	4	4 4	3.1E+02
57	4	1	3	2.6E-01	0	0	0	1.9E+00	0.	4	28	5	0	2.69180512E+01	119	4	4 4	2.2E+02
58	4	1	3	5.1E-02	0	0	0	2.0E+00	0.	4	28	5	0	2.69161023E+01	121	4	4 4	2.1E+02
59	3	1	4	1.9E+01	23	0	0	3.4E-04	0.	4	28	5	0	2.68493237E+01	126	5	4 4	1.7E+03
60	4	1	4	5.6E-01	0	0	0	3.4E+00	0.	4	28	5	0	2.66704469E+01	129	5	4 4	7.8E+02
61	3	1	4	-2.0E+00	19	0	0	1.0E+00	0.	4	28	5	0	2.64734977E+01	131	6	4 4	7.8E+02
62	4	1	4	5.7E-01	0	0	0	1.0E+00	0.	4	28	5	0	2.64657968E+01	132	6	4 4	7.8E+02
63	4	1	4	5.4E-02	0	0	0	1.0E+00	0.	4	28	5	0	2.64655414E+01	134	6	4 4	7.8E+02
64	3	1	3	1.2E+00	16	0	0	1.0E+00	0.	4	28	5	0	2.63901373E+01	136	7	4 4	7.8E+02
65	4	1	3	4.3E-01	0	0	0	9.6E-01	0.	4	28	5	0	2.63887305E+01	138	7	4 4	8.4E+02
66	4	1	3	7.7E-03	0	0	0	1.1E+00	0.	4	28	5	0	2.63886867E+01	140	7	4 4	8.4E+02
67	3	1	2	7.3E-01	20	0	0	4.3E-01	0.	4	28	5	0	2.62583997E+01	144	8	4 4	8.9E+02
68	4	1	2	1.8E-01	0	0	0	9.9E-01	0.	4	28	5	0	2.62395975E+01	146	8	4 4	9.7E+02
69	4	1	2	6.4E-01	0	0	0	7.7E-01	0.	4	28	5	0	2.62382213E+01	148	8	4 4	1.0E+03
70	4	1	2	2.7E-02	0	0	0	1.4E+00	0.	4	28	5	0	2.62380520E+01	150	8	4 4	9.8E+02

RG TOLS REDUCED. TOLRG = 2.386E-06

ITN	PH	PP	NOPT	DJ/RG	+SBS	-SBS	-BS	STEP	PIVOT	NSPK	L	U	NINF	SINF/OBJECTIVE	NFG	NSB	R1M	H-COND
71	4	1	0	2.2E-02	0	0	0	1.4E+00	0.	4	28	5	0	2.62380302E+01	152	8	4 4	9.6E+02
72	4	1	0	3.6E-03	0	0	0	1.2E+00	0.	4	28	5	0	2.62380290E+01	154	8	4 4	9.5E+02
73	4	1	0	1.2E-04	0	0	0	1.0E+00	0.	4	28	5	0	2.62380290E+01	156	8	4 4	9.6E+02
74	4	1	0	1.9E-05	0	0	0	1.0E+00	0.	4	28	5	0	2.62380290E+01	158	8	4 4	9.3E+02
75	4	1	0	7.0E-07	0	0	0	1.0E+00	0.	4	28	5	0	2.62380290E+01	159	8	4 4	9.3E+02

BIGGEST DJ = 0. NORM RG = 7.028E-07 NORM PI = 2.384E+00

END OF MAJOR ITN 3 - OPTIMAL SOLN AT MINOR ITN 25 - TOTAL ITNS = 75

```
START OF MAJOR ITN   4  -  PENALTY PARAMETER =     1.00E+01

NONLINEAR VARIABLES
-------------------
  2.0621229E+00    9.0243168E-01    2.0914325E+00    3.7149688E-01    1.2710412E-01
  3.7156179E-01    0.               1.5016245E-01    0.               1.0000000E+00
  1.0000000E+00   -1.8571922E-01   -1.0000000E+00   -1.0000000E+00   -1.0000000E+00
  8.7081714E-01    1.0000000E+00    1.0000000E+00   -9.6981462E-01    6.1992494E-01
  1.0000000E+00

ROW ERROR AFTER RELINEARIZATION =    1.8530E-02
RELATIVE CHANGE IN MULTIPLIERS  =    5.1107E-01

FACTORIZE    4    DEMAND     0    ITERATION    75    INFEAS     0    OBJECTV   2.623802903E+01

ITN   75 -- FEASIBLE SOLUTION.  OBJECTIVE =   2.653443096E+01

 ITN PH PP NOPT  DJ/RG  +SBS -SBS  -BS   STEP    PIVOT    NSPK    L    U NINF  SINF/OBJECTIVE   NFG  NSB  R1M H-COND
  76  4  1   0  8.4E-01   0    0    0  6.7E-01  0.          4    28    5   0  2.63941186E+01   163    8  4 4 9.5E+02
  77  4  1   0  1.6E-01   0    0    0  1.4E+00  0.          4    28    5   0  2.63929277E+01   165    8  4 4 9.8E+02
  78  4  1   0  9.3E-03   0    0    0  1.0E+00  0.          4    28    5   0  2.63928891E+01   167    8  4 4 8.9E+02
  79  4  1   0  5.3E-03   0    0    0  1.0E+00  0.          4    28    5   0  2.63928884E+01   168    8  4 4 9.4E+02
  80  4  1   0  1.1E-02   0    0    0  3.7E+00  0.          4    28    5   0  2.63928883E+01   170    8  4 4 2.5E+03
  81  4  1   0  6.5E-03   0    0    0  6.1E+01  0.          4    28    5   0  2.63928843E+01   174    8  4 4 9.6E+04
  82  4  1   0  1.2E-04   0    0    0  1.0E+00  0.          4    28    5   0  2.63928843E+01   175    8  4 4 9.5E+04

RG TOLS REDUCED.   TOLRG =    2.399E-06

  83  4  1   0  1.0E-05   0    0    0  1.0E+00  0.          4    28    5   0  2.63928843E+01   178    8  4 4 8.7E+04

OUTPUT FROM LINEAR SEARCH (GETPTC)
 IENTRY, ITEST, MINMZE, RELTOL        2        3         T    2.00471858532E-01
 BOUNDS, STRICT UPPER BND, TOL      0.                        4.86989174138E-01    4.67905187717E+03    4.86989174138E-01
 XBEST, FBEST, GBEST, CBEST         0.                        2.63928842650E+01   -3.92571114949E-13   -1.98070406286E+28
 XW, FW, GW, CW                     4.86989174138E-01         2.63928842650E+01   -2.01085403032E-13    4.54766468675E-13
 YU, FU, GU, CU                     4.86989174138E-01         2.63928842650E+01   -2.01085403032E-13    4.54766468675E-13

XXX  NO FUNCTION DECREASE.   NORM RG =   1.026E-05   NORM PI =   2.399E+00   SUPERBASICS    8   LAST SB =   20   -5.683E-06
NFAIL =   1
NFAIL =   2

BIGGEST DJ =   0.            NORM RG =   1.026E-05   NORM PI =   2.399E+00

END OF MAJOR ITN   4  -  OPTIMAL SOLN AT MINOR ITN   8  -  TOTAL ITNS =    83
```

```
START OF MAJOR ITN   5  -  PENALTY PARAMETER =     1.00E+01

NONLINEAR VARIABLES
-------------------
  2.0663775E+00   9.0755599E-01   2.0673749E+00   3.7209604E-01   1.2834769E-01
  3.7211168E-01   0.              1.4903582E-01   0.              1.0000000E+00
  1.0000000E+00  -1.8571922E-01  -1.0000000E+00  -1.0000000E+00  -1.0000000E+00
  9.4795061E-01   1.0000000E+00   1.0000000E+00  -9.9030126E-01   6.0495400E-01
  1.0000000E+00

ROW ERROR AFTER RELINEARIZATION =    5.7981E-05
RELATIVE CHANGE IN MULTIPLIERS  =    5.0609E-02

FACTORIZE    5    DEMAND    0    ITERATION    83    INFEAS    0    OBJECTV   2.639288426E+01

ITN   83 -- FEASIBLE SOLUTION.  OBJECTIVE =   2.639283965E+01

 ITN PH PP NOPT  DJ/RG  +SBS -SBS  -BS   STEP     PIVOT    NSPK    L    U NINF  SINF/OBJECTIVE   NFG  NSB  R1M H-COND
  84  4  1    0  3.7E-03    0    0    0  1.0E+00  0.          4   28    5    0  2.63928367E+01   184    8  4 4 8.7E+04
  85  4  1    0  7.1E-06    0    0    0  9.9E-01  0.          4   28    5    0  2.63928367E+01   186    8  4 4 8.5E+04

RG TOLS REDUCED.   TOLRG =     2.399E-06

  86  4  1    0  1.7E-06    0    0    0  8.3E-01  0.          4   28    5    0  2.63928367E+01   188    8  4 4 8.6E+04

BIGGEST DJ =  0.            NORM RG =   1.694E-06    NORM PI =   2.399E+00

END OF MAJOR ITN   5  -  OPTIMAL SOLN AT MINOR ITN   3  -  TOTAL ITNS =    86
```

```
START OF MAJOR ITN    6  -  PENALTY PARAMETER =    1.00E+01

NONLINEAR VARIABLES
-------------------
  2.0669578E+00   9.0758145E-01   2.0669579E+00   3.7221596E-01   1.2835139E-01
  3.7221598E-01   0.              1.4912539E-01   0.              1.0000000E+00
  1.0000000E+00  -1.8571922E-01  -1.0000000E+00  -1.0000000E+00  -1.0000000E+00
  9.4821844E-01   1.0000000E+00   1.0000000E+00  -9.9061381E-01   6.0476382E-01
  1.0000000E+00

ROW ERROR AFTER RELINEARIZATION =    6.8367E-08
RELATIVE CHANGE IN MULTIPLIERS  =    3.0149E-04

PENALTY PARAMETER DECREASED TO    0.

FACTORIZE    6    DEMAND    0    ITERATION   86    INFEAS    0    OBJECTV   2.639283670E+01

ITN   86 -- FEASIBLE SOLUTION.  OBJECTIVE =   2.639283670E+01

 ITN PH PP NOPT  DJ/RG  +SBS -SBS  -BS    STEP    PIVOT   NSPK    L    U NINF  SINF/OBJECTIVE   NFG  NSB  R1M H-COND
  87  4  1    0  6.6E-07   0    0    0  1.2E+00  0.         4    28    5    0  2.63928367E+01   191    8  4 4 8.6E+04

BIGGEST DJ =  0.           NORM RG =  6.586E-07   NORM PI =  2.399E+00

END OF MAJOR ITN    6  -  OPTIMAL SOLN AT MINOR ITN   1  -  TOTAL ITNS =    87
```

```
EXIT -- OPTIMAL SOLUTION FOUND.
NO. OF ITERATIONS                        87     OBJECTIVE VALUE          2.6392836703310E+01
NO. OF MAJOR ITERATIONS                   6     LINEAR OBJECTIVE         0.
OBJECTIVE  FUNCN AND GRADIENT CALLS     185     NONLINEAR OBJECTIVE      2.6392836703310E+01
CONSTRAINT FUNCN AND GRADIENT CALLS     191     PENALTY PARAMETER                   0.000000
NORM OF X                         2.067E+00     NORM OF PI                         2.399E+00
NO. OF SUPERBASICS                        8     NORM OF REDUCED GRADIENT           6.586E-07
EXECUTION TIME (IN SECONDS)          16.094
```

PROBLEM NAME	LC FILTE	OBJECTIVE VALUE	2.6392836703E+01
STATUS	OPTIMAL SOLN	ITERATION 87	SUPERBASICS 8
OBJECTIVE	OBJECT (MIN)		
RHS	RHS		
RANGES			
BOUNDS			

SECTION 1 - ROWS

	NUMBER	...ROW..	AT	...ACTIVITY...	SLACK ACTIVITY	..LOWER LIMIT.	..UPPER LIMIT.	.DUAL ACTIVITY	..I
	23	F1	SBS	.02050	-.02050	0.00000	NONE	.00000	1
	24	F2	LL	0.00000	0.00000	0.00000	NONE	3.26936	2
	25	F3	BS	.06096	-.06096	0.00000	NONE	0.00000	3
	26	F4	LL	0.00000	0.00000	0.00000	NONE	2.05310	4
	27	F5	LL	0.00000	0.00000	0.00000	NONE	1.63480	5
	28	F6	BS	.96551	-.96551	0.00000	NONE	0.00000	6
	29	F7	BS	.96049	-.96049	0.00000	NONE	0.00000	7
	30	F8	BS	.98899	-.98899	0.00000	NONE	0.00000	8
	31	F9	BS	.58081	-.58081	0.00000	NONE	0.00000	9
	32	F10	SBS	2.02884	-2.02884	0.00000	NONE	-.00000	10
	33	OBJECT	BS	0.00000	0.00000	NONE	NONE	-1.00000	11

SECTION 2 - COLUMNS

	NUMBER	.COLUMN.	AT	...ACTIVITY...	.OBJ GRADIENT.	..LOWER LIMIT.	..UPPER LIMIT.	REDUCED GRADNT	M+J
	1	X1	SBS	2.06696	2.68661	1.00000	100.00000	.00000	12
	2	X2	BS	.90758	-1.26101	.10000	100.00000	-.00000	13
	3	X3	BS	2.06696	2.68661	1.00000	100.00000	-.00000	14
	4	X4	BS	.37222	-14.91907	0.00000	100.00000	0.00000	15
	5	X5	BS	.12835	-55.09148	0.00000	100.00000	-.00000	16
	6	X6	SBS	.37222	-14.91907	0.00000	100.00000	-.00000	17
	7	X7	LL	0.00000	24.19014	0.00000	100.00000	12.52822	18
	8	X8	BS	.14913	55.09147	0.00000	100.00000	0.00000	19
	9	X9	LL	0.00000	24.19014	0.00000	100.00000	10.41161	20
A	10	X10	UL	1.00000	0.00000	-1.00000	1.00000	0.00000	21
	11	X11	UL	1.00000	-.00000	-1.00000	1.00000	-1.19741	22
	12	X12	SBS	-.18572	0.00000	-1.00000	1.00000	0.00000	23
A	13	X13	LL	-1.00000	0.00000	-1.00000	1.00000	0.00000	24
	14	X14	LL	-1.00000	.00000	-1.00000	1.00000	4.20179	25
A	15	X15	LL	-1.00000	0.00000	-1.00000	1.00000	0.00000	26
	16	X16	SBS	.94822	0.00000	-1.00000	1.00000	0.00000	27
	17	X17	UL	1.00000	-.00000	-1.00000	1.00000	-2.81633	28
A	18	X18	UL	1.00000	0.00000	-1.00000	1.00000	0.00000	29
	19	X19	SBS	-.99061	0.00000	-1.00000	1.00000	0.00000	30
	20	X20	SBS	.60476	.00000	-1.00000	1.00000	.00000	31

A	21	X21	UL	1.00000	0.00000	-1.00000	1.00000	0.00000	32
A	22	RHS	EQ	-1.00000	0.00000	-1.00000	-1.00000	0.00000	33

ENDRUN

V. DISCUSSION OF RESULTS

The cost function in (31) contains a constant c. The problem has been solved for three values of c; c = 50, c = 20, and c = 10, using all three packages. Tables IV, V and VI summarize the results.

Comparison of results shows that all three packages have succeeded in solving the problem and have produced identical results. The observed discrepancies in values of slack variables ρ_1 and ρ_3 are insignificant since they correspond to settings of tuning parameters for which tuning is zero.

Tables of results include information about the number of function evaluations and the computation time required to reach the solution. These are the most common criteria used to evaluate the relative effectiveness of programming methods [3].

The advantage gained in the general formulation of the problem, in which tolerancing and tuning parameters are assigned for all three components, is that the optimization automatically chooses the most appropriate component for tuning. This is C in the objective of (31).

The choice of starting point for MMLC package is important. Although the starting point could be reasonably far from the solution, only symmetrical starting points (identical values for L_1 and L_2 parameters) tend to result in symmetrical solutions. Also, for the case c = 10, the solution for MINOS can be obtained only with strict bounds and appropriately chosen starting values for variables in the MPS data. In other cases, there is a reasonable flexibility in selection of starting points.

VI. CONCLUSIONS

A wide range of practical engineering design problems can be solved using the three packages presented. The LC filter problem not only introduces the concepts of design centering, tolerancing and tuning, but also could serve the purpose of familiarization with the three packages. Apart from the basic descriptions of the three packages, which are

TABLE IV

OPTIMAL TUNING DESIGN OF THE LC LOW-PASS FILTER FOR c = 50

Solution	MFNC	MMLC	MINOS
$L_1^0 = L_2^0$	2.06696	2.06696	2.06696
C^0	0.90758	0.90758	0.90758
$100\,\varepsilon_1/L_1^0 = 100\,\varepsilon_3/L_2^0$	18.01%	18.01%	18.01%
$100\,\varepsilon_2/C^0$	14.14%	14.14%	14.14%
$100\,t_1/L_1^0 = 100\,t_3/L_2^0$	0.00%	0.00%	0.00%
$100\,t_2/C^0$	16.43%	16.43%	16.43%
$\rho_1(6)$	0.38823	-0.99986	1.00000
$\rho_2(6)$	1.00000	1.00000	1.00000
$\rho_3(6)$	-1.00000	-0.99986	-0.18572
$\rho_1(8)$	-0.99158	-0.99924	-1.00000
$\rho_2(8)$	-1.00000	-1.00000	-1.00000
$\rho_3(8)$	-1.00000	-0.99924	-1.00000
$\rho_1(1)$	0.98138	0.99842	0.94822
$\rho_2(1)$	1.00000	1.00000	1.00000
$\rho_3(1)$	0.97401	0.99842	1.00000
$\rho_1(3)$	-0.97288	0.99982	-0.99061
$\rho_2(3)$	0.63562	-0.72139	0.60476
$\rho_3(3)$	1.00000	0.99982	1.00000
Cost Function	26.39284	26.39284	26.39284
Number of Function Evaluations	21	42	185 (obj.) 191 (con.)
Time(s)#	29.9	11.3	16.1

Execution time (seconds) on CYBER 170/730.

TABLE V

OPTIMAL TUNING DESIGN OF THE LC LOW-PASS FILTER FOR c = 20

Solution	MFNC	MMLC	MINOS
$L_1^0 = L_2^0$	2.18011	2.18011	2.18011
C^0	0.91970	0.91970	0.91970
$100\,\varepsilon_1/L_1^0 = 100\,\varepsilon_3/L_2^0$	28.03%	28.03%	28.03%
$100\,\varepsilon_2/C^0$	22.36%	22.36%	22.36%
$100\,t_1/L_1^0 = 100\,t_3/L_2^0$	0.00%	0.00%	0.00%
$100\,t_2/C^0$	37.42%	37.42%	37.42%
$\rho_1(6)$	0.72368	-0.99986	1.00000
$\rho_2(6)$	1.00000	1.00000	1.00000
$\rho_3(6)$	-1.00000	-0.99986	-1.00000
$\rho_1(8)$	-0.99522	-0.99974	-1.00000
$\rho_2(8)$	-1.00000	-1.00000	-1.00000
$\rho_3(8)$	-0.99164	-0.99974	-1.00000
$\rho_1(1)$	0.99511	0.99974	1.00000
$\rho_2(1)$	1.00000	1.00000	1.00000
$\rho_3(1)$	0.94228	0.99974	0.41190
$\rho_1(3)$	-0.84897	0.99984	-1.00000
$\rho_2(3)$	0.40254	-0.19504	0.43813
$\rho_3(3)$	0.70258	0.99984	-0.36889
Cost Function	19.09262	19.09262	19.09262
Number of Function Evaluations	22	33	143 (obj.) 149 (con.)
Time(s)#	38.7	11.5	13.7

Execution time (seconds) on CYBER 170/730.

TABLE VI

OPTIMAL TUNING DESIGN OF THE LC LOW-PASS FILTER FOR c = 10

Solution	MFNC	MMLC	MINOS
$L_1^0 = L_2^0$	2.47968	2.47968	2.47969
C^0	0.90831	0.90831	0.90831
$100\,\varepsilon_1/L_1^0 = 100\,\varepsilon_3/L_2^0$	40.98%	40.98%	40.98%
$100\,\varepsilon_2/C^0$	31.62%	31.62%	31.62%
$100\,t_1/L_1^0 = 100\,t_3/L_2^0$	0.00%	0.00%	0.00%
$100\,t_2/C^0$	62.70%	62.70%	62.70%
$\rho_1(6)$	0.56589	-0.99987	1.00000
$\rho_2(6)$	1.00000	1.00000	1.00000
$\rho_3(6)$	-0.94649	-0.99987	-1.00000
$\rho_1(8)$	-0.99994	0.99963	1.00000
$\rho_2(8)$	-1.00000	-1.00000	-1.00000
$\rho_3(8)$	-1.00000	0.99963	1.00000
$\rho_1(1)$	0.99938	0.99936	1.00000
$\rho_2(1)$	1.00000	1.00000	1.00000
$\rho_3(1)$	0.99775	1.00000	1.00000
$\rho_1(3)$	-0.41324	0.99970	-1.00000
$\rho_2(3)$	-0.00869	-0.69162	0.25438
$\rho_3(3)$	0.79659	0.99970	-1.00000
Cost Function	14.31339	14.31339	14.31339
Number of Function Evaluations	25	38	178 (obj.) 186 (con.)
Time(s)#	46.9	12.2	16.5

Execution time (seconds) on CYBER 170/730.

assumed to be available to the reader, this work is self-contained. We feel that this work provides an example of the kind of approach which should be taken in solving difficult circuit design problems using optimization packages.

VII. REFERENCES

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```
      PROGRAM LCMFN(OUTPUT,TAPE6=OUTPUT)
C
C     LC FILTER PROBLEM
C
      DIMENSION X(21), T(2), W(5000)
      EXTERNAL FCD
      DIMENSION A(7,10)
      COMMON /BLK1/ A
      DATA T/10HLC FILTER ,10HPROBLEM   /
      DATA X/2.0,1.0,2.0,3*0.2,3*0.1,2*1.0,4*-1.0,3*1.0,-1.0,2*1.0/
      CALL MMXHDR (2,T)
      N=21
      LEQ=0
      L=43
      MAXF=100
      EPS=1.0E-6
      ICH=6
      IPR=0
      LW=5000
      CALL MFNC1A (FCD,N,L,LEQ,X,EPS,MAXF,W,LW,ICH,IPR,IFLAG)
      STOP
      END
C
C
      SUBROUTINE FCD (N,L,X,F,G,C,D,KK)
      DIMENSION X(N), G(N), C(L), D(KK,L)
      DIMENSION CC(3), GR(21), FI(3), DFI(3,21)
C
C     CC VECTOR REPRESENTS THE CONSTANT COEFFICIENTS
C     IN THE COST FUNCTION
C
      DATA CC/50.0,50.0,50.0/
C
C     LC FILTER PROBLEM
C
      F=0.0
      DO 10 I=1,3
      I3=I+3
      I6=I+6
      F=F+X(I)/X(I3)+CC(I)*X(I6)/X(I)
      G(I)=1./X(I3)-CC(I)*X(I6)*(1./(X(I)*X(I)))
      G(I3)=-X(I)/(X(I3)*X(I3))
      G(I6)=CC(I)/X(I)
   10 CONTINUE
      DO 20 I=10,21
      G(I)=0.0
   20 CONTINUE
C
C     CALCULATION OF NONLINEAR CONSTRAINTS AND THEIR GRADIENTS
C
      NEL=3
      DO 50 I=1,10
      DO 30 LROW=1,NEL
      DO 30 LCOL=1,N
   30 DFI(LROW,LCOL)=0.0
      CALL FIDFI (I,FI,DFI,N,NEL,X)
      CALL SIMGRD (I,FI,DFI,FR,GR,X,N,NEL)
C
      C(I)=FR
      DO 40 J=1,N
      D(J,I)=GR(J)
   40 CONTINUE
   50 CONTINUE
C
C     EVALUATION OF LINEAR CONSTRAINTS AND THEIR GRADIENTS
```

```
C
      DO 60 I=11,43
      DO 60 J=1,21
   60 D(J,I)=0.0
      DO 70 I=1,9
      II=10+I
      C(II)=X(I)
      IF (I.LE.6) C(II)=C(II)-1.E-4
      D(I,II)=1.0
   70 CONTINUE
C
      DO 80 I=10,21
      II=10+I
      C(II)=1.0+X(I)
      D(I,II)=1.0
   80 CONTINUE
C
      DO 90 I=10,21
      JJ=I+22
      C(JJ)=1.0-X(I)
      D(I,JJ)=-1.0
   90 CONTINUE
      RETURN
      END
C
C
      SUBROUTINE FIDFI (I,FI,DFI,N,NEL,X)
C
C     THIS SUBROUTINE CALCULATES VECTOR PHI AND ITS NON-ZERO
C     DERIVATIVES W.R.T. ALL VARIABLES, USING THE CRITICAL VERTICES
C
      DIMENSION FI(NEL), DFI(NEL,N), X(N)
      DIMENSION A(7,10)
      COMMON /BLK1/ A
C
      DO 10 K=1,NEL
      IF (A(1,I).EQ.6.0) J=K+9
      IF (A(1,I).EQ.8.0) J=K+12
      IF (A(1,I).EQ.1.0) J=K+15
      IF (A(1,I).EQ.3.0) J=K+18
      K1=K+1
      K3=K+3
      K6=K+6
      FI(K)=X(K)+X(K3)*A(K1,I)+X(K6)*X(J)
      DFI(K,K)=1.0
      DFI(K,K3)=A(K1,I)
      DFI(K,K6)=X(J)
      DFI(K,J)=X(K6)
   10 CONTINUE
      RETURN
      END
C
C
      SUBROUTINE SIMGRD (I,FI,DFI,FR,GR,X,N,NEL)
C
C     THIS SUBROUTINE PERFORMS A SIMULATION OF ORIGINAL NETWORK
C     AND ITS ADJOINT TO EVALUATE INSERTION LOSS AND SENSITIVITIES
C     AT DIFFERENT FREQUENCIES
C
      DIMENSION FI(NEL), DFI(NEL,N), GR(N), X(N)
      DIMENSION A(7,10), DFR(3)
      COMMON /BLK1/ A
      COMPLEX Y(3,3),V(3),B(3),YY(3,3),VOUT,YADJ(3,3),BADJ(3),AUX
C
      OMEGA=A(5,I)
```

```
      XL1=OMEGA*FI(1)
      XC=OMEGA*FI(2)
      XL2=OMEGA*FI(3)
C
C     FORM ADMITTANCE MATRIX
C
      Y(1,1)=1.0+1.0/CMPLX(0.0,XL1)
      Y(1,2)=-1.0/CMPLX(0.0,XL1)
      Y(1,3)=(0.0,0.0)
      Y(2,1)=Y(1,2)
      Y(2,3)=-1.0/CMPLX(0.0,XL2)
      Y(2,2)=-Y(2,1)-Y(2,3)+CMPLX(0.0,XC)
      Y(3,1)=(0.0,0.0)
      Y(3,2)=Y(2,3)
      Y(3,3)=1.0-Y(2,3)
C
C     FORM EXCITATION VECTOR
C
      DO 10 J=1,3
      B(J)=(0.0,0.0)
   10 CONTINUE
      B(1)=(1.0,0.0)
      DO 20 K=1,3
      DO 20 J=1,3
      YY(K,J)=Y(K,J)
   20 CONTINUE
C
C     SOLVE Y*V=B USING LU FACTORIZATION
C
      CALL SOLLU (3,YY,B)
      VOUT=B(3)
C
C     DEFINE THE ADJOINT EXCITATION
C
      DO 30 K=1,3
      BADJ(K)=(0.0,0.0)
      DO 30 J=1,3
      YADJ(K,J)=Y(J,K)
   30 CONTINUE
      BADJ(3)=(-1.0,0.0)
C
C     SOLVE YADJ*VADJ=BADJ
C
      CALL SOLLU (3,YADJ,BADJ)
C
C     FORM THE I-TH FUNCTION AND ITS DERIVATIVES
C
      SP=A(6,I)
      W=A(7,I)
      H=20.0*ALOG10(0.5)
      HP=EXP(1.0)
      HPT=20.0*ALOG10(HP)
      FR=W*(SP-(H-HPT*ALOG(CABS(VOUT))))
      AUX=(B(1)-B(2))*(BADJ(1)-BADJ(2))/(-XL1*XL1)
      C1VOUT=1.0/CABS(VOUT)
      C2=-W*(-HPT*C1VOUT)
      DFR(1)=C2*C1VOUT*REAL(CONJG(VOUT)*(-CMPLX(0.0,OMEGA)*AUX))
      DFR(2)=C2*C1VOUT*REAL(CONJG(VOUT)*CMPLX(0.0,OMEGA)*B(2)*BADJ(2))
      AUX=(B(2)-B(3))*(BADJ(2)-BADJ(3))/(-XL2*XL2)
      DFR(3)=C2*C1VOUT*REAL(CONJG(VOUT)*(-CMPLX(0.0,OMEGA)*AUX))
      DO 40 J=1,N
      GR(J)=DFR(1)*DFI(1,J)+DFR(2)*DFI(2,J)+DFR(3)*DFI(3,J)
   40 CONTINUE
      RETURN
      END
```

```
C
C
      SUBROUTINE SOLLU (N,A,B)
C
C     CROUT'S ALGORITHM
C
      COMPLEX A(N,N),B(N)
      N2=N-1
      DO 20  I=1,N2
      M=I+1
      DO 20  J=M,N
      A(I,J)=A(I,J)/A(I,I)
      DO 10  K=M,N
      A(K,J)=A(K,J)-A(K,I)*A(I,J)
   10 CONTINUE
   20 CONTINUE
C
C     FORWARD SUBSTITUTION
C
      B(1)=B(1)/A(1,1)
      DO 40  I=2,N
      IM=I-1
      DO 30  K=1,IM
      B(I)=B(I)-A(I,K)*B(K)
   30 CONTINUE
      B(I)=B(I)/A(I,I)
   40 CONTINUE
C
C     BACKWARD SUBSTITUTION
C
      N1=N+1
      DO 60  L=2,N
      I=N1-L
      IP=I+1
      DO 50  K=IP,N
      B(I)=B(I)-A(I,K)*B(K)
   50 CONTINUE
   60 CONTINUE
      RETURN
      END
C
C
      BLOCK DATA
C
C     THIS BLOCK CONTAINS INFORMATION ABOUT CRITICAL VERTICES,
C     SPECIFICATIONS,AND SAMPLE FREQUENCIES
C
      DIMENSION A(7,10)
      COMMON /BLK1/ A
      DATA A/6.0,1.0,-1.0,1.0,0.45,1.5,1.0,
     +        6.0,1.0,-1.0,1.0,0.50,1.5,1.0,
     +        6.0,1.0,-1.0,1.0,0.55,1.5,1.0,
     +        8.0,1.0,1.0,1.0,1.0,1.5,1.0,
     +        1.0,-1.0,-1.0,-1.0,2.5,25.0,-1.0,
     +        3.0,-1.0,1.0,-1.0,0.45,1.5,1.0,
     +        3.0,-1.0,1.0,-1.0,0.50,1.5,1.0,
     +        3.0,-1.0,1.0,-1.0,0.55,1.5,1.0,
     +        3.0,-1.0,1.0,-1.0,1.0,1.5,1.0,
     +        3.0,-1.0,1.0,-1.0,2.5,25.0,-1.0/
      END
```

```
      PROGRAM LCMML(OUTPUT,TAPE6=OUTPUT)
C
C     LC FILTER PROBLEM
C
      DIMENSION X(21), T(2), W(5000), C(33), DC(33,21)
      EXTERNAL FCD
      DIMENSION A(7,10)
      COMMON /BLK1/ A
      DATA T/10HLC FILTER ,10HPROBLEM   /
      DATA X/2.0,1.0,2.0,0.4,0.2,0.4,0.01,0.1,0.01,
     +       -1.0,1.0,4*-1.0,4*1.0,-1.0,1.0/
      CALL MMXHDR (2,T)
      N=21
      M=11
      LEQ=0
      L=33
      IC=33
      DX=0.1
      KEQS=2
      MAXF=100
      EPS=1.0E-6
C
C     FORM THE COEFFICIENT MATRIX FOR CONSTANTS
C
      DO 10 I=1,33
      DO 10 J=1,21
   10 DC(I,J)=0.0
      DO 20 I=1,9
      C(I)=0.0
      DC(I,I)=1.0
   20 CONTINUE
      DO 30 I=10,21
      C(I)=1.0
      DC(I,I)=1.0
   30 CONTINUE
      DO 40 I=10,21
      II=I+12
      C(II)=1.0
      DC(II,I)=-1.0
   40 CONTINUE
      ICH=6
      IPR=0
      LW=5000
      CALL MMLC1A (FCD,N,M,L,LEQ,C,DC,IC,X,DX,EPS,MAXF,KEQS,W,LW,ICH,IPR
     +,IFALL)
      STOP
      END
C
C
      SUBROUTINE FCD (N,M,X,DF,F)
      DIMENSION X(N), F(M), DF(M,N)
      DIMENSION CC(3), GR(21), FI(3), DFI(3,21)
C
C     CC VECTOR REPRESENTS THE CONSTANT COEFFICIENTS
C     IN THE COST FUNCTION
C
      DATA CC/50.0,50.0,50.0/
C
C     THE LAST RESIDUAL FUNCTION REPRESENTS THE COST FUNCTION
C
      FM=0.0
      DO 10 I=1,3
      I3=I+3
      I6=I+6
      FM=FM+X(I)/X(I3)+CC(I)*X(I6)/X(I)
```

```
      F(M)=FM
      DF(M,I)=1./X(I3)-CC(I)*X(I6)*(1./(X(I)*X(I)))
      DF(M,I3)=-X(I)/(X(I3)*X(I3))
      DF(M,I6)=CC(I)/X(I)
   10 CONTINUE
      DO 20 I=10,21
      DF(M,I)=0.0
   20 CONTINUE
C
C     FORM THE REST OF THE RESIDUAL FUNCTIONS AND THEIR GRADIENTS
C
      ALFA=10.0
      NEL=3
      DO 50 I=1,10
      DO 30 LROW=1,NEL
      DO 30 LCOL=1,N
   30 DFI(LROW,LCOL)=0.0
      CALL FIDFI (I,FI,DFI,N,NEL,X)
      CALL SIMGRD (I,FI,DFI,FR,GR,X,N,NEL)
C
      F(I)=FM-ALFA*FR
      DO 40 J=1,N
      DF(I,J)=DF(M,J)-ALFA*GR(J)
   40 CONTINUE
   50 CONTINUE
      RETURN
      END
C
C
      SUBROUTINE FIDFI (I,FI,DFI,N,NEL,X)
C
C     THIS SUBROUTINE CALCULATES VECTOR PHI AND ITS NON-ZERO
C     DERIVATIVES W.R.T. ALL VARIABLES, USING THE CRITICAL VERTICES
C
      DIMENSION FI(NEL), DFI(NEL,N), X(N)
      DIMENSION A(7,10)
      COMMON /BLK1/ A
C
      DO 10 K=1,NEL
      IF (A(1,I).EQ.6.0) J=K+9
      IF (A(1,I).EQ.8.0) J=K+12
      IF (A(1,I).EQ.1.0) J=K+15
      IF (A(1,I).EQ.3.0) J=K+18
      K1=K+1
      K3=K+3
      K6=K+6
      FI(K)=X(K)+X(K3)*A(K1,I)+X(K6)*X(J)
      DFI(K,K)=1.0
      DFI(K,K3)=A(K1,I)
      DFI(K,K6)=X(J)
      DFI(K,J)=X(K6)
   10 CONTINUE
      RETURN
      END
C
C
      SUBROUTINE SIMGRD (I,FI,DFI,FR,GR,X,N,NEL)
C
C     THIS SUBROUTINE PERFORMS A SIMULATION OF ORIGINAL NETWORK
C     AND ITS ADJOINT TO EVALUATE INSERTION LOSS AND SENSITIVITIES
C     AT DIFFERENT FREQUENCIES
C
      DIMENSION FI(NEL), DFI(NEL,N), GR(N), X(N)
      DIMENSION A(7,10), DFR(3)
      COMMON /BLK1/ A
```

```
      COMPLEX Y(3,3),V(3),B(3),YY(3,3),VOUT,YADJ(3,3),BADJ(3),AUX
C
      OMEGA=A(5,I)
      XL1=OMEGA*FI(1)
      XC=OMEGA*FI(2)
      XL2=OMEGA*FI(3)
C
C     FORM ADMITTANCE MATRIX
C
      Y(1,1)=1.0+1.0/CMPLX(0.0,XL1)
      Y(1,2)=-1.0/CMPLX(0.0,XL1)
      Y(1,3)=(0.0,0.0)
      Y(2,1)=Y(1,2)
      Y(2,3)=-1.0/CMPLX(0.0,XL2)
      Y(2,2)=-Y(2,1)-Y(2,3)+CMPLX(0.0,XC)
      Y(3,1)=(0.0,0.0)
      Y(3,2)=Y(2,3)
      Y(3,3)=1.0-Y(2,3)
C
C     FORM EXCITATION VECTOR
C
      DO 10 J=1,3
      B(J)=(0.0,0.0)
   10 CONTINUE
      B(1)=(1.0,0.0)
      DO 20 K=1,3
      DO 20 J=1,3
      YY(K,J)=Y(K,J)
   20 CONTINUE
C
C     SOLVE Y*V=B USING LU FACTORIZATION
C
      CALL SOLLU (3,YY,B)
      VOUT=B(3)
C
C     DEFINE THE ADJOINT EXCITATION
C
      DO 30 K=1,3
      BADJ(K)=(0.0,0.0)
      DO 30 J=1,3
      YADJ(K,J)=Y(J,K)
   30 CONTINUE
      BADJ(3)=(-1.0,0.0)
C
C     SOLVE YADJ*VADJ=BADJ
C
      CALL SOLLU (3,YADJ,BADJ)
C
C     FORM THE I-TH FUNCTION AND ITS DERIVATIVES
C
      SP=A(6,I)
      W=A(7,I)
      H=20.0*ALOG10(0.5)
      HP=EXP(1.0)
      HPT=20.0*ALOG10(HP)
      FR=W*(SP-(H-HPT*ALOG(CABS(VOUT))))
      AUX=(B(1)-B(2))*(BADJ(1)-BADJ(2))/(-XL1*XL1)
      C1VOUT=1.0/CABS(VOUT)
      C2=-W*(-HPT*C1VOUT)
      DFR(1)=C2*C1VOUT*REAL(CONJG(VOUT)*(-CMPLX(0.0,OMEGA)*AUX))
      DFR(2)=C2*C1VOUT*REAL(CONJG(VOUT)*CMPLX(0.0,OMEGA)*B(2)*BADJ(2))
      AUX=(B(2)-B(3))*(BADJ(2)-BADJ(3))/(-XL2*XL2)
      DFR(3)=C2*C1VOUT*REAL(CONJG(VOUT)*(-CMPLX(0.0,OMEGA)*AUX))
      DO 40 J=1,N
      GR(J)=DFR(1)*DFI(1,J)+DFR(2)*DFI(2,J)+DFR(3)*DFI(3,J)
```

```
   40 CONTINUE
      RETURN
      END
C
C
      SUBROUTINE SOLLU (N,A,B)
C
C     CROUT'S ALGORITHM
C
      COMPLEX A(N,N),B(N)
      N2=N-1
      DO 20 I=1,N2
      M=I+1
      DO 20 J=M,N
      A(I,J)=A(I,J)/A(I,I)
      DO 10 K=M,N
      A(K,J)=A(K,J)-A(K,I)*A(I,J)
   10 CONTINUE
   20 CONTINUE
C
C     FORWARD SUBSTITUTION
C
      B(1)=B(1)/A(1,1)
      DO 40 I=2,N
      IM=I-1
      DO 30 K=1,IM
      B(I)=B(I)-A(I,K)*B(K)
   30 CONTINUE
      B(I)=B(I)/A(I,I)
   40 CONTINUE
C
C     BACKWARD SUBSTITUTION
C
      N1=N+1
      DO 60 L=2,N
      I=N1-L
      IP=I+1
      DO 50 K=IP,N
      B(I)=B(I)-A(I,K)*B(K)
   50 CONTINUE
   60 CONTINUE
      RETURN
      END
C
C
      BLOCK DATA
C
C     THIS BLOCK CONTAINS INFORMATION ABOUT CRITICAL VERTICES,
C     SPECIFICATIONS,AND SAMPLE FREQUENCIES
C
      DIMENSION A(7,10)
      COMMON /BLK1/ A
      DATA A/6.0,1.0,-1.0,1.0,0.45,1.5,1.0,
     +       6.0,1.0,-1.0,1.0,0.50,1.5,1.0,
     +       6.0,1.0,-1.0,1.0,0.55,1.5,1.0,
     +       8.0,1.0,1.0,1.0,1.0,1.5,1.0,
     +       1.0,-1.0,-1.0,-1.0,2.5,25.0,-1.0,
     +       3.0,-1.0,1.0,-1.0,0.45,1.5,1.0,
     +       3.0,-1.0,1.0,-1.0,0.50,1.5,1.0,
     +       3.0,-1.0,1.0,-1.0,0.55,1.5,1.0,
     +       3.0,-1.0,1.0,-1.0,1.0,1.5,1.0,
     +       3.0,-1.0,1.0,-1.0,2.5,25.0,-1.0/
      END
```

```
      PROGRAM LCMNS(DATA,OUTPUT,SCRTCH,TAPE1=DATA,TAPE6=OUTPUT,TAPE9=SC
     +RTCH)
C
C     LC FILTER PROBLEM
C
      DIMENSION W(5000), A(7,10)
      COMMON /BLK1/ A
      LW=5000
      CALL MINOS1 (W,LW,1,6,9)
      STOP
      END
C
C
      SUBROUTINE CALCFG (MODE,N,X,F,G,NS,NP)
      DIMENSION X(N), G(N)
      DIMENSION CC(3)
C
C     CC VECTOR REPRESENTS THE CONSTANT COEFFICIENTS
C     IN THE COST FUNCTION
C
      DATA CC/50.0,50.0,50.0/
C
      F=0.0
      DO 10 I=1,3
      I3=I+3
      I6=I+6
      F=F+X(I)/X(I3)+CC(I)*X(I6)/X(I)
      G(I)=1./X(I3)-CC(I)*X(I6)*(1./(X(I)*X(I)))
      G(I3)=-X(I)/(X(I3)*X(I3))
      G(I6)=CC(I)/X(I)
   10 CONTINUE
      DO 20 I=10,21
      G(I)=0.0
   20 CONTINUE
      RETURN
      END
C
C
      SUBROUTINE CALCON (MODE,M,N,NJ,X,F,G,NS,NP)
      DIMENSION X(N), F(M), G(M,N)
      DIMENSION GR(21), FI(3), DFI(3,21)
C
C     FORM NONLINEAR CONSTRAINTS AND THEIR GRADIENTS
C
      NEL=3
      DO 30 I=1,M
      DO 10 LROW=1,NEL
      DO 10 LCOL=1,N
   10 DFI(LROW,LCOL)=0.0
      CALL FIDFI (I,FI,DFI,N,NEL,X)
      CALL SIMGRD (I,FI,DFI,FR,GR,X,N,NEL)
      F(I)=FR
      DO 20 J=1,N
      G(I,J)=GR(J)
   20 CONTINUE
   30 CONTINUE
      RETURN
      END
C
C
      SUBROUTINE FIDFI (I,FI,DFI,N,NEL,X)
C
C     THIS SUBROUTINE CALCULATES VECTOR PHI AND ITS NON-ZERO
C     DERIVATIVES W.R.T. ALL VARIABLES, USING THE CRITICAL VERTICES
C
```

```
      DIMENSION FI(NEL), DFI(NEL,N), X(N)
      DIMENSION A(7,10)
      COMMON /BLK1/ A
C
      DO 10 K=1,NEL
      IF (A(1,I).EQ.6.0) J=K+9
      IF (A(1,I).EQ.8.0) J=K+12
      IF (A(1,I).EQ.1.0) J=K+15
      IF (A(1,I).EQ.3.0) J=K+18
      K1=K+1
      K3=K+3
      K6=K+6
      FI(K)=X(K)+X(K3)*A(K1,I)+X(K6)*X(J)
      DFI(K,K)=1.0
      DFI(K,K3)=A(K1,I)
      DFI(K,K6)=X(J)
      DFI(K,J)=X(K6)
   10 CONTINUE
      RETURN
      END
C
C
      SUBROUTINE SIMGRD (I,FI,DFI,FR,GR,X,N,NEL)
C
C     THIS SUBROUTINE PERFORMS A SIMULATION OF ORIGINAL NETWORK
C     AND ITS ADJOINT TO EVALUATE INSERTION LOSS AND SENSITIVITIES
C     AT DIFFERENT FREQUENCIES
C
      DIMENSION FI(NEL), DFI(NEL,N), GR(N), X(N)
      DIMENSION A(7,10), DFR(3)
      COMMON /BLK1/ A
      COMPLEX Y(3,3),V(3),B(3),YY(3,3),VOUT,YADJ(3,3),BADJ(3),AUX
C
      OMEGA=A(5,I)
      XL1=OMEGA*FI(1)
      XC=OMEGA*FI(2)
      XL2=OMEGA*FI(3)
C
C     FORM ADMITTANCE MATRIX
C
      Y(1,1)=1.0+1.0/CMPLX(0.0,XL1)
      Y(1,2)=-1.0/CMPLX(0.0,XL1)
      Y(1,3)=(0.0,0.0)
      Y(2,1)=Y(1,2)
      Y(2,3)=-1.0/CMPLX(0.0,XL2)
      Y(2,2)=-Y(2,1)-Y(2,3)+CMPLX(0.0,XC)
      Y(3,1)=(0.0,0.0)
      Y(3,2)=Y(2,3)
      Y(3,3)=1.0-Y(2,3)
C
C     FORM EXCITATION VECTOR
C
      DO 10 J=1,3
      B(J)=(0.0,0.0)
   10 CONTINUE
      B(1)=(1.0,0.0)
      DO 20 K=1,3
      DO 20 J=1,3
      YY(K,J)=Y(K,J)
   20 CONTINUE
C
C     SOLVE Y*V=B USING LU FACTORIZATION
C
      CALL SOLLU (3,YY,B)
      VOUT=B(3)
```

```
C
C     DEFINE THE ADJOINT EXCITATION
C
      DO 30 K=1,3
      BADJ(K)=(0.0,0.0)
      DO 30 J=1,3
      YADJ(K,J)=Y(J,K)
   30 CONTINUE
      BADJ(3)=(-1.0,0.0)
C
C     SOLVE YADJ*VADJ=BADJ
C
      CALL SOLLU (3,YADJ,BADJ)
C
C     FORM THE I-TH FUNCTION AND ITS DERIVATIVES
C
      SP=A(6,I)
      W=A(7,I)
      H=20.0*ALOG10(0.5)
      HP=EXP(1.0)
      HPT=20.0*ALOG10(HP)
      FR=W*(SP-(H-HPT*ALOG(CABS(VOUT))))
      AUX=(B(1)-B(2))*(BADJ(1)-BADJ(2))/(-XL1*XL1)
      C1VOUT=1.0/CABS(VOUT)
      C2=-W*(-HPT*C1VOUT)
      DFR(1)=C2*C1VOUT*REAL(CONJG(VOUT)*(-CMPLX(0.0,OMEGA)*AUX))
      DFR(2)=C2*C1VOUT*REAL(CONJG(VOUT)*CMPLX(0.0,OMEGA)*B(2)*BADJ(2))
      AUX=(B(2)-B(3))*(BADJ(2)-BADJ(3))/(-XL2*XL2)
      DFR(3)=C2*C1VOUT*REAL(CONJG(VOUT)*(-CMPLX(0.0,OMEGA)*AUX))
      DO 40 J=1,N
      GR(J)=DFR(1)*DFI(1,J)+DFR(2)*DFI(2,J)+DFR(3)*DFI(3,J)
   40 CONTINUE
      RETURN
      END
C
C
      SUBROUTINE SOLLU (N,A,B)
C
C     CROUT'S ALGORITHM
C
      COMPLEX A(N,N),B(N)
      N2=N-1
      DO 20 I=1,N2
      M=I+1
      DO 20 J=M,N
      A(I,J)=A(I,J)/A(I,I)
      DO 10 K=M,N
      A(K,J)=A(K,J)-A(K,I)*A(I,J)
   10 CONTINUE
   20 CONTINUE
C
C     FORWARD SUBSTITUTION
C
      B(1)=B(1)/A(1,1)
      DO 40 I=2,N
      IM=I-1
      DO 30 K=1,IM
      B(I)=B(I)-A(I,K)*B(K)
   30 CONTINUE
      B(I)=B(I)/A(I,I)
   40 CONTINUE
C
C     BACKWARD SUBSTITUTION
C
      N1=N+1
```

```
      DO 60 L=2,N
      I=N1-L
      IP=I+1
      DO 50 K=IP,N
      B(I)=B(I)-A(I,K)*B(K)
   50 CONTINUE
   60 CONTINUE
      RETURN
      END
C
C
      BLOCK DATA
C
C     THIS BLOCK CONTAINS INFORMATION ABOUT CRITICAL VERTICES,
C     SPECIFICATIONS,AND SAMPLE FREQUENCIES
C
      DIMENSION A(7,10)
      COMMON /BLK1/ A
      DATA A/6.0,1.0,-1.0,1.0,0.45,1.5,1.0,
     +        6.0,1.0,-1.0,1.0,0.50,1.5,1.0,
     +        6.0,1.0,-1.0,1.0,0.55,1.5,1.0,
     +        8.0,1.0,1.0,1.0,1.0,1.5,1.0,
     +        1.0,-1.0,-1.0,-1.0,2.5,25.0,-1.0,
     +        3.0,-1.0,1.0,-1.0,0.45,1.5,1.0,
     +        3.0,-1.0,1.0,-1.0,0.50,1.5,1.0,
     +        3.0,-1.0,1.0,-1.0,0.55,1.5,1.0,
     +        3.0,-1.0,1.0,-1.0,1.0,1.5,1.0,
     +        3.0,-1.0,1.0,-1.0,2.5,25.0,-1.0/
      END
```

```
BEGIN   LC FILTER PROBLEM
*
*     SPECS AND MPS DATA FOR LC FILTER PROBLEM
*
   MINIMIZE
   ROWS                                         100
   COLUMNS                                      100
   NONLINEAR VARIABLES                           21
   NONLINEAR CONSTRAINTS                         10
   JACOBIAN                                   DENSE
   SUPERBASICS LIMIT                             21
   HESSIAN DIMENSION                             21
   MAJOR ITERATIONS                              20
   MINOR ITERATIONS                              25
   ITERATIONS LIMIT                             300
   LOWER BOUNDS                                -1.0
   UPPER BOUNDS                               100.0
   PRINT LEVEL                                   10
END
NAME            LC FILTER PROBLEM
ROWS
 G   F1
 G   F2
 G   F3
 G   F4
 G   F5
 G   F6
 G   F7
 G   F8
 G   F9
 G   F10
 N   OBJECT
COLUMNS
    X1
    X2
    X3
    X4
    X5
    X6
    X7
    X8
    X9
    X10
    X11
    X12
    X13
    X14
    X15
    X16
    X17
```

```
    X18
    X19
    X20
    X21
RHS
BOUNDS
 LO           X1        1.0
 LO           X2        0.1
 LO           X3        1.0
 LO           X4        1.0E-2
 LO           X5        1.0E-2
 LO           X6        1.0E-2
 LO           X7        1.0E-10
 LO           X8        1.0E-10
 LO           X9        1.0E-10
 LO           X10       -1.0
 UP           X10       1.0
 LO           X11       -1.0
 UP           X11       1.0
 LO           X12       -1.0
 UP           X12       1.0
 LO           X13       -1.0
 UP           X13       1.0
 LO           X14       -1.0
 UP           X14       1.0
 LO           X15       -1.0
 UP           X15       1.0
 LO           X16       -1.0
 UP           X16       1.0
 LO           X17       -1.0
 UP           X17       1.0
 LO           X18       -1.0
 UP           X18       1.0
 LO           X19       -1.0
 UP           X19       1.0
 LO           X20       -1.0
 UP           X20       1.0
 LO           X21       -1.0
 UP           X21       1.0
 FX INITIAL   X1        2.0
 FX INITIAL   X2        1.0
 FX INITIAL   X3        2.0
 FX INITIAL   X4        0.2
 FX INITIAL   X5        0.2
 FX INITIAL   X6        0.2
 FX INITIAL   X7        0.1
 FX INITIAL   X8        0.1
 FX INITIAL   X9        0.1
 FX INITIAL   X10       1.0
 FX INITIAL   X11       1.0
```

```
 FX INITIAL    X12        -1.0
 FX INITIAL    X13        -1.0
 FX INITIAL    X14        -1.0
 FX INITIAL    X15        -1.0
 FX INITIAL    X16        1.0
 FX INITIAL    X17        1.0
 FX INITIAL    X18        1.0
 FX INITIAL    X19        -1.0
 FX INITIAL    X20        1.0
 FX INITIAL    X21        1.0
ENDATA
```

```
100 /JOB
110 FILTER,JC2.
120 /USER
130 /CHARGE
140 FETCH(L=LIBRMNS/GR)
150 COPY(L,LIBRMNS)
160 RETURN(L)
170 GET(PROG=LCMNS1,DATA=DLCMNS)
180 MAP(OFF)
190 LIBRARY(LIBRMNS)
200 FTN,I=PROG,L=0.
210 LGO.
220 RENAME,RESMNS=OUTPUT.
230 SAVE,RESMNS.
240 /EOR
```