SENSITIVITY EVALUATION AND OPTIMIZATION OF ELECTRICAL POWER SYSTEMS WITH EMPHASIS ON NONRECIPROCAL ELEMENTS

H.K. Grewal

SOS-83-17-T

October 1983

SENSITIVITY EVALUATION AND OPTIMIZATION OF

ELECTRICAL POWER SYSTEMS

SENSITIVITY EVALUATION AND OPTIMIZATION
OF ELECTRICAL POWER SYSTEMS
WITH EMPHASIS ON NONRECIPROCAL ELEMENTS

By

HARKIRAT K. GREWAL

A Thesis

Submitted to the School of Graduate Studies

in Partial Fulfilment of the Requirements

for the Degree

Master of Engineering

McMaster University

October 1983

MASTER OF ENGINEERING (1983)
(Electrical and Computer Engineering)

McMASTER UNIVERSITY
Hamilton, Ontario

TITLE: SENSITIVITY EVALUATION AND OPTIMIZATION OF ELECTRICAL POWER SYSTEMS WITH EMPHASIS ON NONRECIPROCAL ELEMENTS

AUTHOR: Harkirat Kaur Grewal
B.Sc.Eng. (Elect. Eng.) (Panjab University)
M.Eng. (Electronics and Comm. Eng.) (University of Roorkee)

SUPERVISOR: J.W. Bandler, Professor,
Department of Electrical and Computer Engineering
B.Sc.(Eng.), Ph.D., D.Sc.(Eng.) (University of London)
D.I.C. (Imperial College)
P.Eng. (Province of Ontario)
C.Eng., F.I.E.E. (United Kingdom)
Fellow, I.E.E.E.

CO-SUPERVISOR: M.A. El-Kady, Assistant Professor (part-time),
Department of Electrical and Computer Engineering
B.Sc. (E.E.), M.Sc. (Cairo University)
Ph.D. (McMaster University)
P. Eng. (Province of Ontario)
Senior Member, I.E.E.E.

NUMBER OF PAGES: viii, 69

ABSTRACT

This thesis complements the recently developed adjoint network approaches to sensitivity evaluation and optimization of electrical power systems. The nonreciprocal power network elements comprising phase-shifting transformers are handled by two different approaches, namely, the Tellegen theorem and the Lagrange multiplier approaches. The method based on the Tellegen theorem incorporates the exact a.c. load flow model and provides compact sensitivity expressions for network controls frequently encountered in relevant power system studies. The complex Lagrangian method accommodates general complex variables, including turns ratio and the internal impedance of phase-shifting transformers. The theoretical results are both exact and computationally practical, and have been verified numerically by investigating different power systems. A minimum-loss problem for the IEEE 118-bus system has been formulated and solved using the computer package MINOS/AUGMENTED.

ACKNOWLEDGEMENTS

The author wishes to gratefully thank Dr. J.W. Bandler and Dr. M.A. El- Kady for their constant encouragement, understanding, expert guidance and supervision during the course of this work.

The financial assistance provided by the Natural Sciences and Engineering Research Council of Canada through grants A7239, A1708 and G0647, and the Department of Electrical and Computer Engineering through a Teaching Assistantship, is gratefully acknowledged.

The author thanks the Faculty of Engineering Word Processing Centre for its diligent efforts in typing this thesis.

TABLE OF CONTENTS

TABLE OF CONTENTS (continued)

LIST OF FIGURES

LIST OF TABLES

CHAPTER 1

INTRODUCTION

Sensitivity calculations play an important role in steady-state computer-aided power system analysis and planning. These sensitivities are very valuable in estimating the effects of transmission system contingencies, generator outages, and other defects expected in power system operations. In the optimal power flow problems, the sensitivities of the cost function with respect to adjustable control parameters directly contribute in gradient computations required by most optimization techniques.

The basic requirements for an efficient technique for sensitivity calculations are simplicity of derivation and formulation, flexibility in modelling different components of the power system and efficiency in computations. The methods based on Tellegen's theorem and the Lagrangian approach are commonly applied in power network sensitivity analysis, however, the choice of a suitable method depends on various factors such as the kind of application considered, the types of elements defined in the power system and the available storage and facilities for computations.

In power system operations and planning, a wide variety of problems are encountered which are frequently formulated as constrained optimization problems. One class of such problems is called the optimal power flow problem, in which a feasible power flow solution is obtained to minimize some cost criteria by adjusting voltage levels, power output of generators, transformer tap settings, phase-shifting transformer angle positions and switchable shunt control elements. The minimum-loss problem falls into this class and is characterized by a cost function which reflects the total active power transmission losses in a given system.

This thesis incorporates a simple notation to describe and develop gradients of real functions subject to equality constraints representing the power flow equations, which are retained in their complex mode. The term sensitivity calculations is used to represent both first-order changes and gradient evaluation. The load flow problem is formulated in Chapter 2 and the methods for sensitivity evaluation are outlined, using a recently developed complex notation. The phase-shifting transformers are described as nonreciprocal two-port networks and their π-equivalent circuit is presented using a voltage-current relationship via short-circuit admittance parameters.

In Chapter 3, the Tellegen theorem is explained and its augmented form is exploited by two different approaches based on the basic and element variable description, and the short-circuit admittance matrix description. A simple derivation of sensitivity expressions is presented which can accommodate general complex branch models of a given system. The sensitivities with respect to adjustable parameters of phase-shifting transformers are obtained in a straightforward manner and presented in a compact tabular form.

The material described in Chapter 4 elaborates on the recently developed complex Lagrangian approach to power systems. The power network is assumed to contain nonreciprocal transmission elements and, therefore, the bus admittance matrix is considered unsymmetrical throughout the analysis. The generalized sensitivity expressions are derived exploiting the complex conjugate notation and these results are extended to establish useful information regarding phase-shifting transformers. Some numerical examples are included to display sensitivities with respect to several network control variables, which have been verified by small perturbations about the nominal point. A minimum-loss problem has been formulated and solved for the IEEE 118-bus system using the optimization package MINOS/AUGMENTED.

CHAPTER 2

COMPLEX POWER FLOW AND SENSITIVITY EVALUATION

2.1 INTRODUCTION

The mathematical formulation of the load flow problem results in a system of nonlinear algebraic equations, called the power flow equations (Van Ness and Griffin 1961). These equations can be established in a bus or loop frame of reference. The bus frame of reference employs the nodal admittance matrix (Gross 1979, Guile 1977, Stagg and El-Abiad 1968) and requires minimal computer storage. This approach also motivates the exploitation of network sparsity by ordered elimination and skillful programming (Duff 1977, Tinney and Walker 1967).

The power flow equations are basically expressed in the complex form and the variables of these equations are generally functions of the network states and control parameters of the system under consideration. These equations are usually separated into real and imaginary parts (Stott 1974), however, the compact complex notation developed by Bandler and El-Kady (1979a) facilitates the formulation in complex mode. Another notation, called complex conjugate notation (Bandler and El-Kady 1982), is also very useful and allows direct handling of the complex functions and constraints. These notations are described in the following sections.

2.2 NOTATION

The power flow equations involve various variables and it is important to adopt and describe a suitable notation at the beginning of the analysis. This facilitates the derivation of theoretical expressions and contributes towards a better understanding of the relevant material.

In general, an n-node power system is considered. The principal notation used in this thesis is described in Table 2.1, however, other notations may be used wherever it is felt necessary.

2.3 COMPLEX SOLUTION OF POWER FLOW EQUATIONS

The power network performance equations are expressed, using the bus frame of reference in the admittance form

$$\mathbf{Y}_T \mathbf{V}_M = \mathbf{I}_M , \tag{2.1}$$

where $\mathbf{Y}_T$ is the complex bus admittance matrix of the network, $\mathbf{V}_M$ is a column vector of the complex bus voltages and $\mathbf{I}_M$ is the corresponding column vector of the complex injected bus currents.

The bus loading equations are expressed in the matrix form

$$\mathbf{E}_M^* \mathbf{I}_M = \mathbf{S}_M^* , \tag{2.2}$$

where

$$\mathbf{E}_M \triangleq \text{diag}\, \mathbf{V}_M , \tag{2.3}$$

$$\mathbf{S}_M \triangleq \mathbf{P}_M + j\mathbf{Q}_M \tag{2.4}$$

and * denotes the complex conjugate. Substituting (2.1) in (2.2), the system of complex nonlinear equations

$$\mathbf{E}_M^* \mathbf{Y}_T \mathbf{V}_M = \mathbf{S}_M^* \tag{2.5}$$

is obtained, which represents the typical load flow problem.

The load flow equations (2.5) are perturbed using the conjugate notation and the resulting linearized equations are solved by Newton-Raphson method in complex mode (El-Kady 1980, Bandler and El-Kady 1982, Bandler, El-Kady, Grewal and Gupta 1982). In

TABLE 2.1

NOTATION FOR THE VARIOUS QUANTITIES

Notation	Description
$a = a_1 + j\,a_2 = \|a\| \angle\phi$	Complex turns ratio of the phase-shifting transformer
$\mathbf{d}$	Right-hand-side vector of the perturbed load flow equations
f	A general scalar function
g	The index of a generator bus
$\mathbf{I}_M$	The bus current vector
$\mathbf{K}, \overline{\mathbf{K}}, \mathbf{K}^S, \overline{\mathbf{K}}^S$	The matrices of cofficients used in defining the perturbed form of the load flow equations
$k_g, \overline{k}_g$	Vectors corresponding to the gth generator bus
ℓ	The index of a load bus
$\mathbf{M}_{11}{}^b, \mathbf{M}_{12}{}^b, \mathbf{M}_{21}{}^b, \mathbf{M}_{22}{}^b,$	The matrices of coefficients used in the Tellegen's theorem approach
n	Total number of buses, also the index of slack bus
n_L	Total number of load buses
n_G	Total number of generator buses
P_i	Real power at the ith node
Q_i	Reactive power at the ith node
S_i	Complex power at the ith node
$\mathbf{S}_M$	Vector of complex powers
$\tilde{S}_g = P_g + j\,\|V_g\|$	Special complex notation for the gth generator bus
$\mathbf{T}^{plr}$	Transformation matrix for the polar formulation

TABLE 2.1 (continued)

NOTATION FOR THE VARIOUS QUANTITIES

Notation	Description
$\mathbf{u}$	Vector of control variables
V_i	Complex voltage at the ith node
$\mathbf{V}_M$	Vector of complex voltages
$\mathbf{x}$	Vector of network states
y_{i0}	Shunt admittance at the ith node
y_{ij}	Line admittance between the nodes i and j
$\mathcal{y}_{ii}$	Short-circuit driving-point admittance at the ith node of a two-port network
$\mathcal{y}_{ij}$	Short-circuit transfer admittance for a two-port network between the nodes i and j
$\mathbf{y}$	Short-circuit admittance matrix of a two-port network
$\mathbf{Y}_T$	The nodal admittance matrix of the power system
Y_{ii}	The ith diagonal element of nodal admittance matrix
Y_{ij}	The off-diagonal element of $\mathbf{Y}_T$ in the ith row and jth column
Z_t	The internal impedance of a phase-shifting transformer
$\boldsymbol{\sigma}_b$, $\boldsymbol{\tau}_b$, $\boldsymbol{\kappa}_b$	The coefficient matrices used in general complex branch modelling
$\boldsymbol{\mu}_i$	Column unit vector with ith unity element and zero other elements
δ	First-order change
*	Distinguishes the complex conjugate
^	Distinguishes the adjoint network variables

order to obtain network sensitivities, we describe two adjoint network approaches, namely, the Tellegen theorem-based method and the method of Lagrange multipliers.

2.4 APPLICATION OF ADJOINT NETWORK APPROACH TO POWER NETWORK SENSITIVITY ANALYSIS

The adjoint network approach, based on Tellegen's theorem (Penfield, Spence and Duinker 1970) is a promising technique for calculating the network sensitivities. It was initially applied to electronic circuits (Director and Rohrer 1969, Bandler and Seviora 1970, Calahan 1968) and has been successfully introduced to power system analysis and design problems. Recently, a generalized version, utilizing a suitable augmented form of Tellegen's theorem has been developed (Bandler and El-Kady 1979b) which allows the sensitivity evaluation on the basis of an exact a.c. load flow model. This has facilitated generalized complex branch modelling and has led to a simple derivation and elegant formulation of exact sensitivity formulas in real and/or complex modes.

In general, the augmented form of Tellegen's theorem is expressed as

$$\sum_{b} (\hat{I}_b \delta V_b + \hat{I}_b^* \delta V_b^* - \hat{V}_b \delta I_b - \hat{V}_b^* \delta I_b^*) = 0 \ , \tag{2.6}$$

where I_b and V_b are the current and voltage associated with branch b of a network, respectively, and the ^ distinguishes the variables associated with the topologically similar adjoint network. The sum expressed in (2.6) is in terms of variations in V_b, $V_b{}^*$, I_b and $I_b{}^*$, which are designated as the basic variables of branch b. Another set of variables, called the element variables are defined, depending upon the type of branch model. These sets of variables are denoted by

$$\mathbf{w}_b = \begin{bmatrix} \mathbf{w}_{bv} \\ \mathbf{w}_{bi} \end{bmatrix} \triangleq \begin{bmatrix} V_b \\ V_b^* \\ I_b \\ I_b^* \end{bmatrix} \tag{2.7}$$

and

$$\mathbf{z}_b = \begin{bmatrix} \mathbf{x}_b \\ \mathbf{u}_b \end{bmatrix} , \tag{2.8}$$

where $\mathbf{x}_b$ and $\mathbf{u}_b$ are 2-component real and/or complex vectors. The variations of the element variables $\mathbf{z}_b$ and the basic variables $\mathbf{w}_b$ are related and are expressed as

$$\delta \mathbf{z}_b = \mathbf{J}_b \, \delta \mathbf{w}_b \ , \tag{2.9}$$

where $\mathbf{J}_b = (\partial \mathbf{z}_b^T / \partial \mathbf{w}_b)^T$ is a transformation matrix containing the conventional and/or formal derivatives of $\mathbf{z}_b$ w.r.t. $\mathbf{w}_b$. The inverse transpose of $\mathbf{J}_b$ is of major interest in the following derivations and in partitioned form, it is denoted by

$$(\mathbf{J}_b^{-1})^T = \begin{bmatrix} \mathbf{M}_{11}^b & \mathbf{M}_{12}^b \\ \mathbf{M}_{21}^b & \mathbf{M}_{22}^b \end{bmatrix} , \tag{2.10}$$

where the submatrices $\mathbf{M}_{11}{}^b$, $\mathbf{M}_{12}{}^b$, $\mathbf{M}_{21}{}^b$, and $\mathbf{M}_{22}{}^b$ are 2x2 Jacobian matrices.

Using (2.7), the augmented Tellegen sum (2.6) is given by

$$\sum_b \hat{\mathbf{f}}_b^T \delta \mathbf{w}_b = 0 \ , \tag{2.11}$$

where $\hat{\mathbf{f}}_b$ is a complex vector the elements of which are, in general, linear functions of the adjoint current and voltage variables and their complex conjugate, and is defined as

$$\hat{\mathbf{f}}_b = \begin{bmatrix} \hat{\mathbf{f}}_{bi} \\ \hat{\mathbf{f}}_{bv} \end{bmatrix} . \tag{2.12}$$

We define a set of transformed adjoint variables $\hat{\boldsymbol{\eta}}_{bx}$ and $\hat{\boldsymbol{\eta}}_{bu}$, given by

$$\hat{\boldsymbol{\eta}}_{bx} = \mathbf{M}_{11}^b \hat{\mathbf{f}}_{bi} + \mathbf{M}_{12}^b \hat{\mathbf{f}}_{bv} \tag{2.13}$$

and

$$\hat{\boldsymbol{\eta}}_{bu} = \mathbf{M}_{21}^b \hat{\mathbf{f}}_{bi} + \mathbf{M}_{22}^b \hat{\mathbf{f}}_{bv} \ . \tag{2.14}$$

The elements of $\hat{\boldsymbol{\eta}}_{bx}$ and $\hat{\boldsymbol{\eta}}_{bu}$ are also linear functions of the adjoint current and voltage variables and their complex conjugate. Using (2.7 - 2.14), the augmented Tellegen sum (2.6) is expressed as

$$\sum_{b} (\hat{\boldsymbol{\eta}}_{bx}^{T} \, \delta \mathbf{x}_{b} + \hat{\boldsymbol{\eta}}_{bu}^{T} \, \delta \mathbf{u}_{b}) = 0 \; . \tag{2.15}$$

The first-order change of a general function f of all the state vectors $\mathbf{x}_b$ and the control vectors $\mathbf{u}_b$ is given by

$$\delta f = \sum_{b} \left[\left(\frac{\partial f}{\partial \mathbf{x}_{b}} \right)^{T} \delta \mathbf{x}_{b} + \left(\frac{\partial f}{\partial \mathbf{u}_{b}} \right)^{T} \delta \mathbf{u}_{b} \right] . \tag{2.16}$$

Assuming a possible consistent modelling of the adjoint system, we define

$$\hat{\boldsymbol{\eta}}_{bx} = \frac{\partial f}{\partial \mathbf{x}_{b}} \; , \tag{2.17}$$

which reduces (2.16), using (2.15), to

$$\delta f = \sum_{b} \left[\left(\frac{\partial f}{\partial \mathbf{u}_{b}} \right)^{T} - \hat{\boldsymbol{\eta}}_{bu}^{T} \right] \delta \mathbf{u}_{b} \; . \tag{2.18}$$

Hence, the reduced gradients of f are given by

$$\frac{df}{d\mathbf{u}_{b}} = \frac{\partial f}{\partial \mathbf{u}_{b}} - \hat{\boldsymbol{\eta}}_{bu} \; . \tag{2.19}$$

It is interesting to observe that (2.19) provides an exact sensitivity expression and accommodates real and/or complex control variables. The general concepts in network modelling are discussed in more detail in the next chapter.

Another powerful method for efficient sensitivity evaluation is the complex Lagrangian approach, which utilizes a compact complex notation and exploits the Jacobian available at the load flow solution (Peschon, Piercy, Tinney and Tveit 1968).

We write the perturbed form of (2.5) in the form

$$\mathbf{K}^{s} \, \delta \mathbf{V}_{M} + \overline{\mathbf{K}}^{s} \, \delta \mathbf{V}_{M}^{*} = \mathbf{d}^{s} \; , \tag{2.20}$$

where

$$\mathbf{d}^{s} = \delta \mathbf{S}_{M}^{*} - \mathbf{E}_{M}^{*} \, \delta \mathbf{Y}_{T} \, \mathbf{V}_{M} \, . \tag{2.21}$$

In order to incorporate equations corresponding to slack and generator buses in (2.21), we write for slack bus

$$\mathbf{k}_n^T \delta \mathbf{V}_M + \bar{\mathbf{k}}_n^T \delta \mathbf{V}_M^* = \delta V_n^* , \tag{2.22}$$

where $\mathbf{k}_n$ is a null vector and $\bar{\mathbf{k}}_n$ is given by

$$\bar{\mathbf{k}}_n = \begin{bmatrix} 0 \\ 0 \\ . \\ . \\ . \\ 1 \end{bmatrix} . \tag{2.23}$$

For generator buses, a complex quantity is defined

$$\tilde{S}_g \triangleq P_g + j|V_g| , \tag{2.24}$$

and its first-order change is expressed as

$$\delta \tilde{S}_g = \delta P_g + j\delta |V_g| . \tag{2.25}$$

Since

$$2P_g = V_g I_g^* + V_g^* I_g , \tag{2.26}$$

the perturbed expression associated with (2.26) is given by

$$2\delta P_g = V_g \delta I_g^* + I_g^* \delta V_g + V_g^* \delta I_g + I_g \delta V_g^* . \tag{2.27}$$

Now the current I_g injected at the gth node is given, using $\mathbf{I}_M = \mathbf{Y}_T \mathbf{V}_M$, by

$$I_g = \mathbf{y}_g^T \mathbf{V}_M . \tag{2.28}$$

where $\mathbf{y}_g{}^T$ represents the corresponding row of the bus admittance matrix $\mathbf{Y}_T$. In perturbed form, (2.28) is given by

$$\delta I_g = \mathbf{y}_g^T \delta \mathbf{V}_M + \mathbf{V}_M^T \delta \mathbf{y}_g . \tag{2.29}$$

The imaginary part in (2.24) is given by

$$|V_g| = (V_g V_g^*)^{1/2} , \tag{2.30}$$

and in perturbed form, we write (2.30) in the form

$$\delta |V_g| = (V_g \delta V_g^* + V_g^* \delta V_g)/(2|V_g|) . \tag{2.31}$$

Using (2.27)-(2.31) in (2.25), the conjugate equation is given by

$$\delta \tilde{S}_g^* = \mathbf{k}_g^T \delta \mathbf{V}_M + \bar{\mathbf{k}}_g^T \delta \mathbf{V}_M^* + V_g^* \mathbf{V}_M^T \delta \mathbf{y}_g/2 + V_g \mathbf{V}_M^{*T} \delta \mathbf{y}_g^*/2 , \tag{2.32}$$

where

$$\mathbf{k}_g \triangleq (V_g^*/2)\,\mathbf{y}_g + [\,\mathbf{y}_g^{*T}\mathbf{V}_M^*/2 - j\,V_g^*/(2|V_g|)]\,\boldsymbol{\mu}_g, \tag{2.33}$$

and

$$\bar{\mathbf{k}}_g \triangleq (V_g/2)\,\mathbf{y}_g^* + [\,\mathbf{y}_g^{T}\mathbf{V}_M/2 - j\,V_g/(2|V_g|)]\,\boldsymbol{\mu}_g. \tag{2.34}$$

Defining

$$d_g \triangleq \delta\tilde{S}_g^* - V_g^*\mathbf{V}_M^T\,\delta\mathbf{y}_g/2 - V_g\mathbf{V}_M^{*T}\,\delta\mathbf{y}_g^*/2, \tag{2.35}$$

the equation (2.32) reduces to

$$\mathbf{k}_g^T\,\delta\mathbf{V}_M + \bar{\mathbf{k}}_g^T\delta\mathbf{V}_M^* = d_g. \tag{2.36}$$

We write (2.20), including (2.22) and (2.36), in the form

$$\mathbf{K}\,\delta\mathbf{V}_M + \bar{\mathbf{K}}\delta\mathbf{V}_M^* = \mathbf{d}. \tag{2.37}$$

The complex conjugate of (2.37) is given by

$$\bar{\mathbf{K}}^*\,\delta\mathbf{V}_M + \mathbf{K}^*\,\delta\mathbf{V}_M^* = \mathbf{d}^*, \tag{2.38}$$

and in compact form (2.37) and (2.38) are written as

$$\begin{bmatrix} \mathbf{K} & \bar{\mathbf{K}} \\ \bar{\mathbf{K}}^* & \mathbf{K}^* \end{bmatrix} \begin{bmatrix} \delta\mathbf{V}_M \\ \delta\mathbf{V}_M^* \end{bmatrix} = \begin{bmatrix} \mathbf{d} \\ \mathbf{d}^* \end{bmatrix}. \tag{2.39}$$

For a general function f, the first-order change is expressed as

$$\delta f = \left(\frac{\partial f}{\partial \mathbf{V}_M}\right)^T \delta\mathbf{V}_M + \left(\frac{\partial f}{\partial \mathbf{V}_M^*}\right)^T \delta\mathbf{V}_M^* + \delta f_\rho, \tag{2.40}$$

where δf_ρ denotes the first-order change in f due to changes in other variables in terms of which f may be explicitly expressed. We let $\hat{\boldsymbol{\mu}}$ be given by

$$\hat{\boldsymbol{\mu}} = \frac{\partial f}{\partial \mathbf{V}_M} \tag{2.41}$$

and, in general, for a real function, use (El-Kady 1980) to write

$$\frac{\partial f}{\partial \mathbf{V}_M^*} = \left(\frac{\partial f}{\partial \mathbf{V}_M}\right)^* = \hat{\boldsymbol{\mu}}^*. \tag{2.42}$$

Hence, substituting (2.41) and (2.42) in (2.40), we write

$$\delta f = [\,\hat{\boldsymbol{\mu}}^T\,\hat{\boldsymbol{\mu}}^{*T}] \begin{bmatrix} \delta\mathbf{V}_M \\ \delta\mathbf{V}_M^* \end{bmatrix} + \delta f_\rho, \tag{2.43}$$

and from (2.39), (2.43) is expressed in the form

$$\delta f = [\, \hat{\boldsymbol{\mu}}^T \hat{\boldsymbol{\mu}}^{*T}] \begin{bmatrix} \mathbf{K} & \bar{\mathbf{K}} \\ \bar{\mathbf{K}}^* & \mathbf{K}^* \end{bmatrix}^{-1} \begin{bmatrix} \mathbf{d} \\ \mathbf{d}^* \end{bmatrix} + \delta f_\rho \tag{2.44}$$

or

$$\delta f = [\, \hat{\mathbf{V}}^T \hat{\mathbf{V}}^{*T}] \begin{bmatrix} \mathbf{d} \\ \mathbf{d}^* \end{bmatrix} + \delta f_\rho , \tag{2.45}$$

where

$$\begin{bmatrix} \mathbf{K}^T & \bar{\mathbf{K}}^{*T} \\ \bar{\mathbf{K}}^T & \mathbf{K}^{*T} \end{bmatrix} \begin{bmatrix} \hat{\mathbf{V}} \\ \hat{\mathbf{V}}^* \end{bmatrix} = \begin{bmatrix} \hat{\boldsymbol{\mu}} \\ \hat{\boldsymbol{\mu}}^* \end{bmatrix} , \tag{2.46}$$

or, simply,

$$[\, \mathbf{K}^T \;\; \bar{\mathbf{K}}^{*T}] \begin{bmatrix} \hat{\mathbf{V}} \\ \hat{\mathbf{V}}^* \end{bmatrix} = \hat{\boldsymbol{\mu}} \, . \tag{2.47}$$

The first-order change of a real function f and its corresponding gradients can be readily evaluated by solving the adjoint system defined in (2.46). We disucss this approach in more detail in Chapter 4, where some compact sensitivities expressions have been developed in a straightforward manner. Various control variables have been described, including the adjustable parameters of phase-shifting transformers.

2.5 NONRECIPROCAL POWER NETWORK ELEMENTS

Phase-shifting transformers are categorized as nonreciprocal power transmission elements owing to their complex turns ratio (Gross 1979, Stagg and El-Abiad 1968). These transformers are capable of changing the complex voltage and current at a particular node to a prescribed value; and therefore, help in delaying the need for future transmission reinforcement (Lyman 1930, Lyman and North 1938, Han 1982).

The short-circuit admittance matrix of a phase-shifting transformer installed between nodes p and q as shown in Fig. 2.1, is given by

$$\mathbf{y} = \begin{bmatrix} \dfrac{1}{Z_t aa^*} & -\dfrac{1}{Z_t a^*} \\ -\dfrac{1}{Z_t a} & \dfrac{1}{Z_t} \end{bmatrix}, \tag{2.48}$$

where a and Z_t are the complex turns ratio and internal impedance of the transformer, respectively. It can be observed that the off-diagonal elements in (2.48) are unequal when a is complex. One possible way of representing this element is shown in Fig. 2.2, where a voltage-controlled current source is indicated at node q.

The phase-shifting transformers are installed in transmission lines as shown in Fig. 2.3, and the voltage at the output terminals of the transformers can be controlled by two independent adjustments. These adjustments are usually carried out in steps and their practical values are

$$0.90 \leq |a| \leq 1.10 \text{ with } \Delta|a| = 0.025 \text{ (p.u.)}$$

$$-10° \leq \phi \leq +10° \text{ with } \Delta\phi = 2.5°.$$

The control actions may be manual or automatic (Han 1982), complete with output sensors and feedback methods, and the device may be used to control real and reactive power flow in a power transmission network.

The sensitivities with respect to turns ratio and transformer impedance are of prime importance and are derived in the following chapters. The knowledge of these sensitivities finds practical utility in power system planning and operations, and helps in providing possible relief in overloaded transmission facilities.

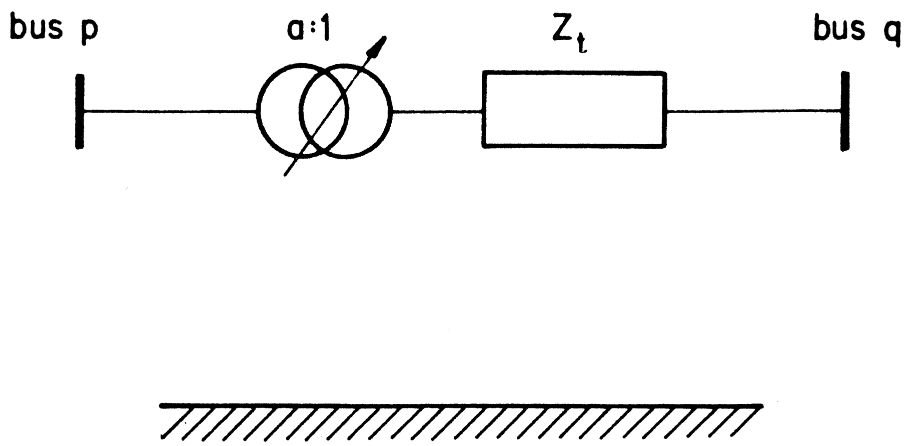


Fig. 2.1 Representation of a phase-shifting transformer

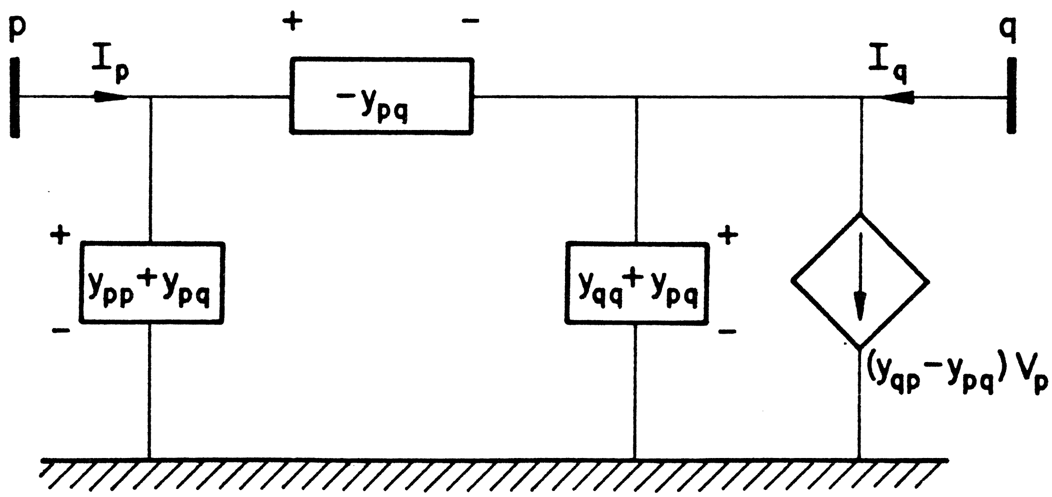


Fig. 2.2 A п-equivalent of phase-shifting transformer

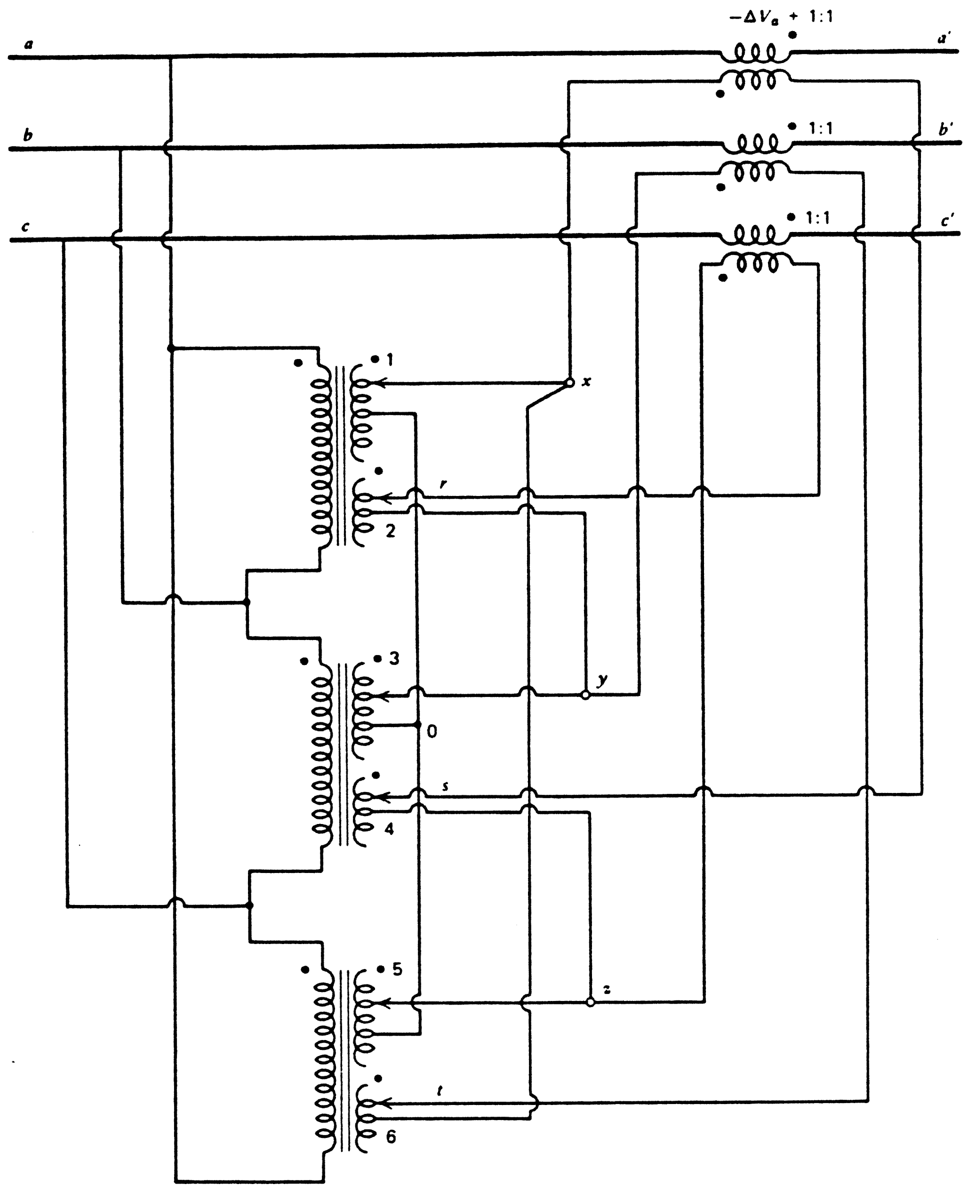


Fig. 2.3 Three-phase phase-shifting transformer (Gross 1979)

CHAPTER 3

TELLEGEN THEOREM APPROACH TO NONRECIPROCAL POWER NETWORKS

3.1 INTRODUCTION

The Tellegen's theorem has been exploited by several algorithms as a powerful tool for calculating the required gradient, using one additional linear network analysis (Bandler and El-Kady 1979, Director 1975, Penfield, Spence and Duinker 1970). The generalized version of the theorem utilizes a suitable augmentation and facilitates sensitivity evaluation on the basis of an exact a.c. load flow model.

The augmented form of the Tellegen's theorem has been successfully applied to the generalized complex branch models of power networks (El-Kady 1980) and has led to a simple derivation and elegant formulation of exact sensitivity formulas in real and/or complex modes. The concept of generalized perturbed complex branch modelling is extended to general two-port networks by implementing a pertinent adjoint technique. The two-port networks may comprise transmission lines, tap-changing-under-load transformers and phase-shifting transformers. These elements can be distinguished by variable a, which signifies the complex turns ratio. Its value is unity, a real quantity and a complex quantity, for the respective elements.

It is interesting to observe that (2.19) provides an exact sensitivity expression and accommodates real and/or complex control variables. The general concepts in network modelling and analysis applicable to systems of general complex branch models are discussed further in the following sections.

3.2 GRADIENT CALCULATIONS CONSIDERING BASIC AND ELEMENT VARIABLES

Consider a general form of perturbed equation relating the basic variables $\mathbf{w}_b$ and element variables $\mathbf{z}_b$ of bth branch, given by

$$\sigma_b \delta I_b + \bar{\sigma}_b \delta I_b^* = \tau_b \delta V_b + \bar{\tau}_b \delta V_b^* + \kappa_b \delta u_b + \bar{\kappa}_b \delta u_b^* . \tag{3.1}$$

Using the conjugate of (3.1) and expressing the general equation in matrix form, we write

$$\begin{bmatrix} \sigma_b & \bar{\sigma}_b \\ \bar{\sigma}_b^* & \sigma_b^* \end{bmatrix} \begin{bmatrix} \delta I_b \\ \delta I_b^* \end{bmatrix} = \begin{bmatrix} \tau_b & \bar{\tau}_b \\ \bar{\tau}_b^* & \tau_b^* \end{bmatrix} \begin{bmatrix} \delta V_b \\ \delta V_b^* \end{bmatrix} + \begin{bmatrix} \kappa_b & \bar{\kappa}_b \\ \bar{\kappa}_b^* & \kappa_b^* \end{bmatrix} \begin{bmatrix} \delta u_b \\ \delta u_b^* \end{bmatrix} . \tag{3.2}$$

In order to handle (3.2) in a convenient way, the matrices of coefficients are denoted by

$$\boldsymbol{\sigma}_b = \begin{bmatrix} \sigma_b & \bar{\sigma}_b \\ \bar{\sigma}_b^* & \sigma_b^* \end{bmatrix} , \tag{3.3}$$

$$\boldsymbol{\tau}_b = \begin{bmatrix} \tau_b & \bar{\tau}_b \\ \bar{\tau}_b^* & \tau_b^* \end{bmatrix} , \tag{3.4}$$

and

$$\boldsymbol{\kappa}_b = \begin{bmatrix} \kappa_b & \bar{\kappa}_b \\ \bar{\kappa}_b^* & \kappa_b^* \end{bmatrix} , \tag{3.5}$$

and we use (2.7) and (2.8) to express (3.2) in the form

$$\boldsymbol{\sigma}_b \delta \mathbf{w}_{bi} = \boldsymbol{\tau}_b \delta \mathbf{w}_{bv} + \boldsymbol{\kappa}_b \delta \mathbf{u}_b . \tag{3.6}$$

For a general branch, we define element variables as

$$\mathbf{z}_b = \begin{bmatrix} I_b \\ I_b^* \\ u_b \\ u_b^* \end{bmatrix} , \tag{3.7}$$

and this leads to a convenient form of the corresponding Jacobian matrix. It is straightforward to express $\mathbf{J}_b$ of (2.9) in a partitioned form, given by

$$\mathbf{J}_b = \begin{bmatrix} \mathbf{0} & \mathbf{1} \\ -\boldsymbol{\kappa}_b^{-1} \boldsymbol{\tau}_b & \boldsymbol{\kappa}_b^{-1} \boldsymbol{\sigma}_b \end{bmatrix} , \tag{3.8}$$

and the inverse of this matrix can be written by using matrix manipulations, in the form

$$\mathbf{J}_b^{-1} = \begin{bmatrix} \boldsymbol{\tau}_b^{-1}\boldsymbol{\sigma}_b & -\boldsymbol{\tau}_b^{-1}\boldsymbol{\kappa}_b \\ \mathbf{1} & \mathbf{0} \end{bmatrix} . \tag{3.9}$$

The transpose of the matrix given in (3.9) is

$$\left(\mathbf{J}_b^{-1}\right)^T = \begin{bmatrix} \boldsymbol{\sigma}_b^T(\boldsymbol{\tau}_b^{-1})^T & \mathbf{1} \\ -\boldsymbol{\kappa}_b^T(\boldsymbol{\tau}_b^{-1})^T & \mathbf{0} \end{bmatrix} , \tag{3.10}$$

and, using (2.10), we get the submatrices

$$\mathbf{M}_{11}^b = \boldsymbol{\sigma}_b^T(\boldsymbol{\tau}_b^{-1})^T , \tag{3.11}$$

$$\mathbf{M}_{12}^b = \mathbf{1} , \tag{3.12}$$

$$\mathbf{M}_{21}^b = -\boldsymbol{\kappa}_b^T(\boldsymbol{\tau}_b^{-1})^T \tag{3.13}$$

and

$$\mathbf{M}_{22}^b = \mathbf{0} . \tag{3.14}$$

The transformed adjoint variables defined in (2.13) and (2.14) are given by

$$\hat{\boldsymbol{\eta}}_{bx} = \mathbf{M}_{11}^b \hat{\mathbf{w}}_{bi} - \hat{\mathbf{w}}_{bv} \tag{3.15}$$

and

$$\hat{\boldsymbol{\eta}}_{bu} = \mathbf{M}_{21}^b \hat{\mathbf{w}}_{bi} . \tag{3.16}$$

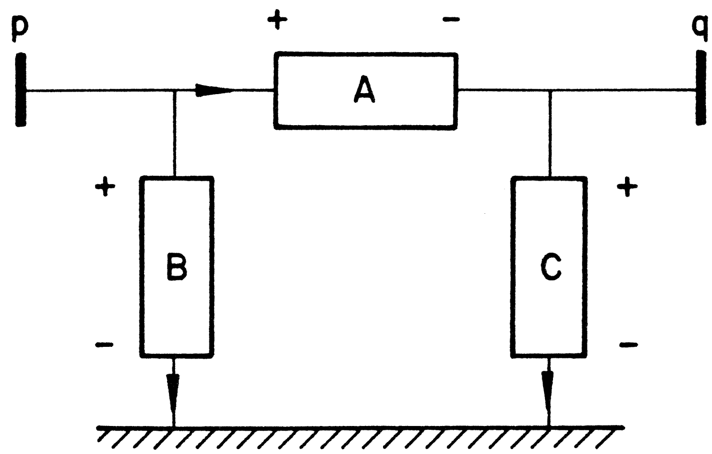
Note that $\hat{\mathbf{w}}_{bi}$ and $\hat{\mathbf{w}}_{bv}$ are basic variables of an adjoint system, and from (2.6), (2.11) and (2.12), it is obvious that

$$\hat{\mathbf{w}}_{bi} = \hat{\mathbf{f}}_{bi} = \begin{bmatrix} \hat{I}_b \\ \hat{I}_b^* \end{bmatrix} \tag{3.17}$$

and

$$\hat{\mathbf{w}}_{bv} = -\hat{\mathbf{f}}_{bv} = \begin{bmatrix} \hat{V}_b \\ \hat{V}_b^* \end{bmatrix} . \tag{3.18}$$

The first-order change of a general function stated in (2.16) can be used and an adjoint system can be defined as in (2.17) to yield the required gradients expressed in (2.19) by using



$$I_A - a^* I_A^* = \frac{1}{Z_t a^*} V_A - \frac{a^*}{Z_t^* a} V_A^*$$

$$I_B - a^* I_B^* = \frac{1}{Z_t a^*} \left(\frac{1}{a} - 1\right) V_B - \frac{a^*}{Z_t^* a} \left(\frac{1}{a^*} - 1\right) V_B^*$$

$$I_C + a I_C^* = \frac{-1}{Z_t} \left(\frac{1}{a} - 1\right) V_C - \frac{a}{Z_t^*} \left(\frac{1}{a^*} - 1\right) V_C^*$$

Fig. 3.1 Modelling of phase-shifting transformers.

(3.15) and (3.16). Table 3.1 displays the coefficient matrices associated with various power system elements.

3.2.1 Gradient Calculations of Phase-Shifting Transformers

The phase-shifting transformers having complex turns ratio can be modelled by using general branch models (El-Kady 1980) and one possible construction is represented in Fig. 3.1. The various coefficient matrices are summarized in Table 3.2a-f, considering two sets of control variables, namely, associated with internal admittance Z_t and complex turns ratio a.

TABLE 3.1

COEFFICIENT MATRICES USING GENERALIZED BASIC AND ELEMENT VARIABLES

Coefficient matrix	Load branch	Generator branch	Slack generator	Transmission line
$\mathbf{u}_b$	$[S_\ell \quad S_\ell^*]^T$	$[S_g \quad S_g^*]^T$	$[V_n \quad V_n^*]^T$	$[Y_t \quad Y_t^*]^T$
$\boldsymbol{\sigma}_b$	$\begin{bmatrix} 0 & V_\ell \\ V_\ell^* & 0 \end{bmatrix}$	$\begin{bmatrix} V_g^* & V_g \\ V_g^* & V_g \end{bmatrix}$	$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
$\boldsymbol{\tau}_b$	$\begin{bmatrix} -I_\ell^* & 0 \\ 0 & -I_\ell \end{bmatrix}$	$\begin{bmatrix} -(I_g^*+jV_g^*) & -(I_g+jV_g) \\ -(I_g^*-jV_g^*) & -(I_g-jV_g) \end{bmatrix}$	$\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$	$\begin{bmatrix} Y_t & 0 \\ 0 & Y_t^* \end{bmatrix}$
$\boldsymbol{\kappa}_b$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} V_t & 0 \\ 0 & V_t^* \end{bmatrix}$
$\mathbf{M}_{11}^b$	$\begin{bmatrix} 0 & -\dfrac{V_\ell^*}{I_\ell} \\ -\dfrac{V_\ell}{I_\ell^*} & 0 \end{bmatrix}$	† $\dfrac{-1}{2\Delta} \begin{bmatrix} jV_g V_g^* & jV_g^{*2} \\ -jV_g^2 & -jV_g V_g^* \end{bmatrix}$	$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$	$\begin{bmatrix} \dfrac{1}{Y_t} & 0 \\ 0 & \dfrac{1}{Y_t^*} \end{bmatrix}$
$\mathbf{M}_{21}^b$	$\begin{bmatrix} \dfrac{1}{I_\ell^*} & 0 \\ 0 & \dfrac{1}{I_\ell} \end{bmatrix}$	† $\dfrac{-1}{4\Delta} \begin{bmatrix} -(I_g - j\,V_g) & (I_g^* - j\,V_g^*) \\ I_g + j\,V_g & -(I_g^* + j\,V_g^*) \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} \dfrac{-V_t}{Y_t} & 0 \\ 0 & \dfrac{-V_t^*}{Y_t^*} \end{bmatrix}$

† $\Delta = I_{g1} V_{g2} - I_{g2} V_{g1}$

TABLE 3.2a

MATRICES $\mathbf{u}_b$, $\boldsymbol{\sigma}_b$, $\boldsymbol{\tau}_b$, $\boldsymbol{\kappa}_b$ FOR PHASE-SHIFTING TRANSFORMERS

Matrix	Branch A
$\mathbf{u}_b^1$	$[Z_t \quad Z_t^*]^T$
$\mathbf{u}_b^2$	$[a \quad a^*]^T$
$\boldsymbol{\sigma}_b$	$\begin{bmatrix} 1 & -a^* \\ -a & 1 \end{bmatrix}$
$\boldsymbol{\tau}_b$	$\begin{bmatrix} \frac{1}{Z_t a^*} & \frac{-a^*}{Z_t^* a} \\ \frac{-a}{Z_t a^*} & \frac{1}{Z_t^* a} \end{bmatrix}$
$\boldsymbol{\kappa}_b^1$	$\begin{bmatrix} \frac{-V_A}{Z_t^2 a^*} & \frac{a^* V_A^*}{Z_t^{*2} a} \\ \frac{a V_A}{Z_t^2 a^*} & \frac{-V_A^*}{Z_t^{*2} a} \end{bmatrix}$
$\boldsymbol{\kappa}_b^2$	$\begin{bmatrix} \frac{a^* V_A^*}{Z_t^* a^2} & I_A^* - \frac{V_A}{Z_t a^{*2}} - \frac{V_A^*}{Z_t^* a} \\ I_A - \frac{V_A}{Z_t a} - \frac{V_A^*}{Z_t^* a^2} & \frac{a V_A}{Z_t a^{*2}} \end{bmatrix}$

TABLE 3.2b

MATRICES $\mathbf{u}_b$, σ_b, τ_b, κ_b FOR PHASE-SHIFTING TRANSFORMERS

Matrix	Branch B
$\mathbf{u}_b^1$	$[Z_t \quad Z_t^*]^T$
$\mathbf{u}_b^2$	$[a \quad a^*]^T$
σ_b	$\begin{bmatrix} 1 & -a^* \\ -a & 1 \end{bmatrix}$
τ_b	$\begin{bmatrix} \dfrac{1-a}{Z_t a a^*} & -\dfrac{1-a^*}{Z_t^* a} \\ -\dfrac{1-a}{Z_t a^*} & \dfrac{1-a^*}{Z_t^* a a^*} \end{bmatrix}$
κ_b^1	$\begin{bmatrix} \dfrac{-V_B(1-a)}{Z_t^2 a a^*} & \dfrac{V_B^*(1-a^*)}{Z_t^{*2} a} \\ \dfrac{V_B(1-a)}{Z_t^2 a^*} & \dfrac{-V_B^*(1-a^*)}{Z_t^{*2} a a^*} \end{bmatrix}$
κ_b^2	$\begin{bmatrix} \dfrac{-V_B}{Z_t a^2 a^*} + \dfrac{(1-a^*)V_B^*}{Z_t^* a^2} & I_B^* - \dfrac{(1-a)V_B}{Z_t a a^{*2}} + \dfrac{V_B^*}{Z_t^* a} \\ I_B - \dfrac{(1-a^*)V_B^*}{Z_t^* a^2 a^*} + \dfrac{V_B}{Z_t a^*} & \dfrac{(1-a)V_B}{Z_t a^{*2}} - \dfrac{V_B^*}{Z_t^* a a^{*2}} \end{bmatrix}$

TABLE 3.2c

MATRICES $\mathbf{u}_b$, σ_b, τ_b, κ_b FOR PHASE-SHIFTING TRANSFORMERS

Matrix	Branch C
$\mathbf{u}_b^1$	$[Z_t \quad Z_t^*]^T$
$\mathbf{u}_b^2$	$[a \quad a^*]^T$
σ_b	$\begin{bmatrix} 1 & a \\ a^* & 1 \end{bmatrix}$
τ_b	$\begin{bmatrix} \frac{a-1}{Z_t a} & \frac{a(a^*-1)}{Z_t^* a^*} \\ \frac{a^*(a-1)}{Z_t a} & \frac{a^*-1}{Z_t^* a^*} \end{bmatrix}$
κ_b^1	$\begin{bmatrix} \frac{1-a}{Z_t^2 a} V_C & \frac{a(1-a^*)}{Z_t^{*2} a^*} V_C^* \\ \frac{a^*(1-a)}{Z_t^2 a} V_C & \frac{1-a^*}{Z_t^{*2} a^*} V_C^* \end{bmatrix}$
κ_b^2	$\begin{bmatrix} \frac{V_C}{a^2 Z_t} - I_C^* - \frac{(1-a^*)}{a^* Z_t^*} V_C^* & \frac{a V_C^*}{Z_t^* a^{*2}} \\ \frac{a^* V_C}{Z_t a^2} & \frac{V_C^*}{a^{*2} Z_t^*} - I_C - \frac{(1-a) V_C}{a Z_t} \end{bmatrix}$

3.3 GRADIENT CALCULATIONS CONSIDERING SHORT-CIRCUIT ADMITTANCE MATRIX

Consider a general two-port netowrk inserted between nodes p and q and let the short-circuit admittance parameters be represented by y_{pp}, y_{pq}, y_{qp} and y_{qq}, respectively. Without loss of generality, we assume that $y_{pq} \neq y_{qp}$ and the current-voltage relationships for the network are given, in matrix form (Desoer and Kuh 1969), by

$$\begin{bmatrix} I_p \\ I_q \end{bmatrix} = \begin{bmatrix} y_{pp} & y_{pq} \\ y_{qp} & y_{qq} \end{bmatrix} \begin{bmatrix} V_p \\ V_q \end{bmatrix} . \tag{3.22}$$

In compact notation, (3.22) can be written as

$$\mathbf{I} = \mathbf{y}\,\mathbf{V}, \tag{3.23}$$

where **I** and **V** are current and voltage two-component vectors, respectively, associated with a two-port network.

The form expressed in (3.23) can be perturbed and is given by

$$\delta\mathbf{I} = \mathbf{y}\,\delta\mathbf{V} + \delta\mathbf{y}\,\mathbf{V}, \tag{3.24}$$

which relates the first-order variation of current vector **I** to first-order changes of voltage vector **V** and short-circuit admittance matrix **y** .

We write the augmented terms in (2.6) associated with branches p and q, given by

$$\hat{I}_p \delta V_p + \hat{I}_p^* \delta V_p^* - \hat{V}_p \delta I_p - \hat{V}_p^* \delta I_p^*$$

$$+ \hat{I}_q \delta V_q + \hat{I}_q^* \delta V_q^* - \hat{V}_q \delta I_q - \hat{V}_q^* \delta I_q^* . \tag{3.25}$$

This portion of the Tellegen sum can be expressed in matrix form, given by

$$\hat{\mathbf{I}}^T \delta\mathbf{V} + \hat{\mathbf{I}}^{*T} \delta\mathbf{V}^* - \hat{\mathbf{V}}^T \delta\mathbf{I} - \hat{\mathbf{V}}^{*T} \delta\mathbf{I}^* , \tag{3.26}$$

where the adjoint current and voltage vectors denote

$$\hat{\mathbf{I}} = \begin{bmatrix} \hat{I}_p \\ \hat{I}_q \end{bmatrix} , \tag{3.27}$$

$$\hat{\mathbf{V}} = \begin{bmatrix} \hat{V}_p \\ \hat{V}_q \end{bmatrix} . \tag{3.28}$$

From (3.24) and its conjugate for $\delta \mathbf{I}^*$, we write (3.26) as

$$(\hat{\mathbf{I}}^T - \hat{\mathbf{V}}^T \mathbf{y})\delta\mathbf{V} - \hat{\mathbf{V}}^T \delta\mathbf{y}\,\mathbf{V}$$
$$+ (\hat{\mathbf{I}}^{*T} - \hat{\mathbf{V}}^{*T} \mathbf{y}^*)\,\delta\mathbf{V}^* - \hat{\mathbf{V}}^{*T}\delta\mathbf{y}^*\,\mathbf{V}^* . \tag{3.29}$$

The first-order change of a general real function f is given by

$$\delta f = \sum_b \left(\frac{\partial f}{\partial I_b}\delta I_b + \frac{\partial f}{\partial I_b^*}\delta I_b^* + \frac{\partial f}{\partial V_b}\delta V_b + \frac{\partial f}{\partial V_b^*}\delta V_b^*\right) , \tag{3.30}$$

and the corresponding terms associated with branches p and q are obtained by putting b = p and q, which can be written in compact form, given by

$$\left(\frac{\partial f}{\partial \mathbf{I}}\right)^T \delta\mathbf{I} + \left(\frac{\partial f}{\partial \mathbf{I}}\right)^{*T} \delta\mathbf{I}^* + \left(\frac{\partial f}{\partial \mathbf{V}}\right)^T \delta\mathbf{V} + \left(\frac{\partial f}{\partial \mathbf{V}}\right)^{*T} \delta\mathbf{V}^* . \tag{3.31}$$

Again, using (3.24) and its conjugate in (3.31), we write it as

$$\left[\left(\frac{\partial f}{\partial \mathbf{I}}\right)^T \mathbf{y} + \left(\frac{\partial f}{\partial \mathbf{V}}\right)^T\right]\delta\mathbf{V} + \left[\left(\frac{\partial f}{\partial \mathbf{I}}\right)^{*T} \mathbf{y}^* + \left(\frac{\partial f}{\partial \mathbf{V}}\right)^{*T}\right]\delta\mathbf{V}^*$$

$$+ \left(\frac{\partial f}{\partial \mathbf{I}}\right)^T \delta\mathbf{y}\;\mathbf{V} + \left(\frac{\partial f}{\partial \mathbf{I}}\right)^{*T} \delta\mathbf{y}^*\,\mathbf{V}^* . \tag{3.32}$$

We define an adjoint system associated with branches p and q so that

$$\hat{\mathbf{I}} = \mathbf{y}^T\left(\hat{\mathbf{V}} + \frac{\partial f}{\partial \mathbf{I}}\right) + \frac{\partial f}{\partial \mathbf{V}} , \tag{3.33}$$

and using (3.29) and (3.34) in (3.30), it is straightforward to express

$$\delta f = \left(\hat{\mathbf{V}} + \frac{\partial f}{\partial \mathbf{I}}\right)^T \delta\mathbf{y}\;\mathbf{V} + \left(\hat{\mathbf{V}}^* + \frac{\partial f}{\partial \mathbf{I}^*}\right)^T \delta\mathbf{y}^*\;\mathbf{V}^* + \text{other terms.} \tag{3.34}$$

From (3.34), we write

$$\frac{df}{d\mathbf{y}} = \left(\hat{\mathbf{V}} + \frac{\partial f}{\partial \mathbf{I}}\right)\mathbf{V}^T , \tag{3.35}$$

and

$$\frac{df}{dy^*} = \left(\hat{\mathbf{V}}^* + \frac{\partial f}{\partial \mathbf{I}^*}\right) \mathbf{V}^{*T} . \tag{3.36}$$

Note that (3.35) and (3.36) represent sensitivities of function f w.r.t. short-circuit admittance matrix **y** and its conjugate, respectively, and basically are 2x2 matrices.

3.3.1 Gradient Calculations of Phase-Shifting Transformers

The short-circuit admittance matrix of a two-port network comprising phase-shifting transformer is given by (2.48) and the first-order variation of **y** can be expressed in terms of first-order changes in Z_t, Z_t^*, a and a^*. We write

$$\delta \mathbf{y} = \frac{1}{Z_t^2 a a^*} \begin{bmatrix} -1 & a \\ a^* & -aa^* \end{bmatrix} \delta Z_t + \mathbf{O}\,\delta Z_t^*$$

$$+ \frac{1}{Z_t a^2 a^*} \begin{bmatrix} -1 & 0 \\ a^* & 0 \end{bmatrix} \delta a + \frac{1}{Z_t a a^{*2}} \begin{bmatrix} -1 & a \\ 0 & 0 \end{bmatrix} \delta a^* , \tag{3.37}$$

and using complex conjugate notation, we express $\delta \mathbf{y}^*$ from (3.37), given by

$$\delta \mathbf{y}^* = \mathbf{O}\,\delta Z_t + \frac{1}{Z_t^{*2} a a^*} \begin{bmatrix} -1 & a^* \\ a & -aa^* \end{bmatrix} \delta Z_t^*$$

$$+ \frac{1}{Z_t^* a^2 a^*} \begin{bmatrix} -1 & a^* \\ 0 & 0 \end{bmatrix} \delta a + \frac{1}{Z_t^* a a^{*2}} \begin{bmatrix} -1 & 0 \\ a & 0 \end{bmatrix} \delta a^* . \tag{3.38}$$

Substituting (3.37) and (3.38) into (3.34), we obtain

$$\delta f = \left(\hat{\mathbf{V}} + \frac{\partial f}{\partial \mathbf{I}}\right)^T \left(\frac{1}{Z_t^2 a a^*} \begin{bmatrix} -1 & a \\ a^* & -aa^* \end{bmatrix} \delta Z_t + \frac{1}{Z_t a^2 a^*} \begin{bmatrix} -1 & 0 \\ a^* & 0 \end{bmatrix} \delta a \right.$$

$$\left. + \frac{1}{Z_t a a^{*2}} \begin{bmatrix} -1 & a \\ 0 & 0 \end{bmatrix} \delta a^* \right) \mathbf{V}$$

$$+\left(\hat{\mathbf{V}}^* + \frac{\partial f}{\partial \mathbf{I}^*}\right)^T \left(\frac{1}{Z_t^{*2} a a^*} \begin{bmatrix} -1 & a^* \\ a & -aa^* \end{bmatrix} \delta Z_t^* + \frac{1}{Z_t^* a^2 a^*} \begin{bmatrix} -1 & a^* \\ 0 & 0 \end{bmatrix} \delta a \right.$$

$$\left. + \frac{1}{Z_t^* a a^{*2}} \begin{bmatrix} -1 & 0 \\ a & 0 \end{bmatrix} \delta a^* \right) \mathbf{V}^* + \text{other terms}. \qquad (3.39)$$

Hence, the derivatives of f w.r.t. Z_t, Z_t^*, a and a^* are given, using (3.39), by

$$\frac{df}{dZ_t} = \frac{1}{Z_t^2 a a^*} \left(\hat{\mathbf{V}} + \frac{\partial f}{\partial \mathbf{I}}\right)^T \begin{bmatrix} -1 & a \\ a^* & -aa^* \end{bmatrix} \mathbf{V}, \qquad (3.40)$$

$$\frac{df}{dZ_t^*} = \frac{1}{Z_t^{*2} a a^*} \left(\hat{\mathbf{V}}^* + \frac{\partial f}{\partial \mathbf{I}^*}\right)^T \begin{bmatrix} -1 & a^* \\ a & -aa^* \end{bmatrix} \mathbf{V}^*, \qquad (3.41)$$

$$\frac{df}{da} = \frac{1}{Z_t a^2 a^*} \left(\hat{\mathbf{V}} + \frac{\partial f}{\partial \mathbf{I}}\right)^T \begin{bmatrix} -1 & 0 \\ a^* & 0 \end{bmatrix} \mathbf{V}$$

$$+ \frac{1}{Z_t^* a^2 a^*} \left(\hat{\mathbf{V}}^* + \frac{\partial f}{\partial \mathbf{I}^*}\right)^T \begin{bmatrix} -1 & a^* \\ 0 & 0 \end{bmatrix} \mathbf{V}^* \qquad (3.42)$$

and

$$\frac{df}{da^*} = \frac{1}{Z_t a a^{*2}} \left(\hat{\mathbf{V}} + \frac{\partial f}{\partial \mathbf{I}}\right)^T \begin{bmatrix} -1 & a \\ 0 & 0 \end{bmatrix} \mathbf{V}$$

$$+ \frac{1}{Z_t^* a a^{*2}} \left(\hat{\mathbf{V}}^* + \frac{\partial f}{\partial \mathbf{I}^*}\right)^T \begin{bmatrix} -1 & 0 \\ a & 0 \end{bmatrix} \mathbf{V}^*. \qquad (3.43)$$

The expressions (3.40)-(3.43) can be used to obtain sensitivities w.r.t. practical control variables of phase-shifting transformer, namely, R_t, X_t, a_1 and a_2 in a straightforward manner. We have used the following notations to display simplified expressions in Table 3.3

$$\hat{\hat{V}}_p = \hat{V}_p + \frac{\partial f}{\partial I_p} \qquad (3.44)$$

and

$$\hat{\hat{V}}_q = \hat{V}_q + \frac{\partial f}{\partial I_q}. \qquad (3.45)$$

TABLE 3.3

SENSITIVITIES OF A REAL FUNCTION

Control Variable	Description	Derivative
Z_t	transformer impedance	$\dfrac{\left(\hat{\bar{V}}_q - \dfrac{\hat{\bar{V}}_p}{a^*}\right)\left(\dfrac{V_p}{a} - V_q\right)}{Z_t^2}$
R_t	transformer resistance	$2 \text{ Re } \left\{\dfrac{df}{dZ_t}\right\}$
X_t	transformer reactance	$-2 \text{ Im } \left\{\dfrac{df}{dZ_t}\right\}$
a	transformer complex turns ratio	$\dfrac{\dfrac{V_p}{Z_t}\left(\hat{\bar{V}}_q - \dfrac{\hat{\bar{V}}_p}{a^*}\right) + \dfrac{\hat{\bar{V}}_p^*}{Z_t^*}\left(V_q^* - \dfrac{V_p^*}{a^*}\right)}{a^2}$
a_1	real component of complex turns ratio	$2 \text{ Re } \left\{\dfrac{df}{da}\right\}$
a_2	imaginary component of complex turn ratio	$-2 \text{ Im } \left\{\dfrac{df}{da}\right\}$
$\|a\|$	magnitude of complex turns ratio	$\dfrac{1}{\|a\|}\left(a_1 \dfrac{df}{da_1} + a_2 \dfrac{df}{da_2}\right)$
ϕ	phase angle of complex turns ratio	$-a_2 \dfrac{df}{da_1} + a_1 \dfrac{df}{da_2}$

3.4 CONCLUSIONS

The Tellegen's theorem has been applied to a.c. power flow models in general, and exact network sensitivity formulas have been derived for various network branches. A special treatment of phase-shifting transformers has been empasized to provide sensitivities with respect to their adjustable parameters, namely, internal resistance and reactance, and turns ratio in rectangular and polar modes. We have considered basic/element variable approach and short-circuit admittance description of general two-port network and applied to nonreciprocal power networks.

CHAPTER 4

COMPLEX LAGRANGIAN APPROACH TO NONRECIPROCAL POWER NETWORKS

4.1 INTRODUCTION

The recently developed complex Lagrangian approach (Bandler and El-Kady 1982) has been successfully applied to power system analysis and design. The complex Lagrange multipliers are obtained by solving a set of adjoint equations in complex mode, exploiting the Jacobian matrix of the load flow problem. A compact complex notation (Bandler and El-Kady 1980) is employed to handle complex functions and constraints directly. The approach is extended to power networks comprising phase-shifting and tap-changing-under-load transformers. The sensitivity expressions are derived and presented in a compact complex mode, and in order to obtain sensitivites in rectangular or polar modes, suitable transformations have been derived.

4.2 GRADIENT EVALUATION IN COMPLEX MODE

The first-order change δf can be calculated from (2.45) in complex mode and throughout the analysis, the nodal admittance matrix $\mathbf{Y}_T$ shall be assumed unsymmetrical. This assumption facilitates direct handling of phase-shifting transformers and the sensitivites with respect to complex turns ratio and transformer impedance can be obtained in a straightforward manner.

4.2.1 General Derivation

We consider an n-node power system comprising n_L loads, n_G generators and one slack generator. The buses are ordered such that subscripts $\ell = 1, 2, ..., n_L$ identify load buses, $g = n_L + 1, ..., n_L + n_G$ identify generator buses and n identifies the slack bus.

The vector $\mathbf{d}$ of (2.45) can be partitioned into subvectors associated with the sets of load, generator and slack buses of dimensions n_L, n_G and 1, respectively, in the form

$$\mathbf{d} = \begin{bmatrix} \mathbf{d}_L \\ \mathbf{d}_G \\ d_n \end{bmatrix}, \tag{4.1}$$

where $\mathbf{d}_L$ constitutes elements d_ℓ given, from (2.21), by

$$d_\ell = \delta S_\ell^* - V_\ell^* \mathbf{V}_M^T \delta \mathbf{y}_\ell , \tag{4.2}$$

$\mathbf{d}_G$ has elements d_g given by (2.35) and d_n is simply $\delta V_n{}^*$ by (2.22). The adjoint vector $\hat{\mathbf{V}}$ of (2.45) can also be partitioned in a similar way and is expressed as

$$\hat{\mathbf{V}} = \begin{bmatrix} \hat{\mathbf{V}}_L \\ \hat{\mathbf{V}}_G \\ \hat{V}_n \end{bmatrix}. \tag{4.3}$$

The first-order change of a real function f, expressed in (2.45) can be written in the form

$$\delta f = \hat{\mathbf{V}}_L^T \mathbf{d}_L + \hat{\mathbf{V}}_G^T \mathbf{d}_G + \hat{V}_n d_n + \left(\frac{\partial f}{\partial \boldsymbol{\rho}} \right)^T \delta \boldsymbol{\rho}$$

$$+ \hat{\mathbf{V}}_L^{*T} \mathbf{d}_L^* + \hat{\mathbf{V}}_G^{*T} \mathbf{d}_G^* + \hat{V}_n^* d_n^* + \left(\frac{\partial f}{\partial \boldsymbol{\rho}} \right)^{*T} \delta \boldsymbol{\rho}^* . \tag{4.4}$$

The first term of (4.4) is written in expanded form, using (4.2), as

$$\hat{\mathbf{V}}_L^T \mathbf{d}_L = \sum_{\ell=1}^{n_L} \hat{V}_\ell d_\ell$$

$$= \sum_{\ell=1}^{n_L} (\hat{V}_\ell \delta S_\ell^*) - \sum_{\ell=1}^{n_L} \sum_{m=1}^{n} (\hat{V}_\ell V_\ell^* V_m \delta Y_{\ell m}), \tag{4.5}$$

where $Y_{\ell m}$ is an element of $\mathbf{Y}_T$ located in the ℓth row and mth column, and in general is not equal to $Y_{m\ell}$.

The second term of (4.4) is corresponding to generator buses and using (2.35) can be expressed in the form

$$\hat{\mathbf{V}}_G^T \mathbf{d}_G = \sum_{g=n_L+1}^{n-1} \hat{V}_g d_g$$

$$= \sum_{g=n_L+1}^{n-1} (\hat{V}_g \delta \tilde{S}_g^*)$$

$$- \sum_{g=n_L+1}^{n-1} \sum_{m=1}^{n} [\hat{V}_g (V_g^* V_m \delta Y_{gm} + V_g V_m^* \delta Y_{gm}^*)/2] \tag{4.6}$$

and Y_{gm} is an element of $\mathbf{Y}_T$ associated with the gth row and mth column. For an unsymmetrical $\mathbf{Y}_T$, we have $Y_{gm} \neq Y_{mg}$. The third term of (4.4) is corresponding to slack bus and can be written, using (2.22), as

$$\hat{V}_n d_n = \hat{V}_n \delta V_n^* . \tag{4.7}$$

The fourth term of (4.4) is simply the first-order change of f due to variations in other variables $\boldsymbol{\rho}$ in terms of which the function f may be explicitly expressed.

The equations derived above provide useful information for gradient evaluation since they are directly related to changes in the control variables, e.g., bus quantities and transmission network quantities. The derivatives of the function f with respect to both types of control variables are derived in the following sections, and temporarily the function has been assumed to be independent of $\boldsymbol{\rho}$.

4.2.2 Derivatives of a Real Function w.r.t. Bus Type Variables

The derivatives of function f w.r.t. the demand S_ℓ and its conjugate S_ℓ^*, associated with the ℓth load bus are given, using (4.5), by

$$\frac{df}{dS_\ell} = \hat{V}_\ell^* \tag{4.8}$$

and the complex conjugate of (4.8) is expressed as

$$\frac{df}{dS_\ell^*} = \hat{V}_\ell . \tag{4.9}$$

For a generator bus, we use (4.6) and its conjugate equation to provide the derivatives, given by

$$\frac{df}{d\tilde{S}_g} = \hat{V}_g^*, \tag{4.10}$$

and

$$\frac{df}{d\tilde{S}_g^*} = \hat{V}_g, \tag{4.11}$$

where $\tilde{S}_g$ is a special notation (El-Kady 1980) used to incorporate the generator buses in the generalized formulation of load flow problem and sensitivity evaluation.

The derivatives of f w.r.t. slack bus voltage V_n and its conjugate V_n^*, are obtained from (4.7), given by

$$\frac{df}{dV_n} = \hat{V}_n^*, \tag{4.12}$$

and

$$\frac{df}{dV_n^*} = \hat{V}_n . \tag{4.13}$$

However, in practice the phase angle of the slack bus voltage is referred to be zero, which leads to

$$V_n = V_n^*, \tag{4.14}$$

and $\hat{V}_n$ of the adjoint system turns out to be real. This indicates that there is only one practical control variable associated with the slack bus.

4.2.3 Derivatives of a Real Function w.r.t. Short-Circuit Admittance Parameters

The transmission network of a power system comprises lines, power transformers, tap-changing-under-load transformers and phase-shifting transformers. For a general analysis, we assume these elements to be nonreciprocal two-port networks, however, by appropriate selection of turns ratio, the formulas derived can be applied to any particular element.

Consider a transmission element connected between nodes m (= ℓ, g or n) and m′ (= ℓ, g or n), where m ≠ m′. The diagonal and off-diagonal elements of the nodal admittance matrix are given, in terms of short-circuit admittance parameters, by

$$Y_{mm} = \sum_{m'=1}^{n} y_{mm}^{m'}, \tag{4.15}$$

where $y_{mm}{}^{m'}$ represents the short-circuit driving-point admittance at node m, of a two-port network inserted between nodes m and m′, and

$$Y_{mm'} = y_{mm'}, \tag{4.16}$$

respectively, where $y_{mm'}$ represents the short-circuit transfer admittance. Note that for an unsymmetrical $\mathbf{Y}_T$, we consider $Y_{mm'} \neq Y_{m'm}$.

The derivatives of a real function f for an element connected between a load bus and any other bus are expressed in a compact form, using (4.4), (4.5), (4.15) and (4.16), given by

$$\frac{df}{dy_{\ell m}} = -\hat{V}_\ell V_\ell^* V_m, \tag{4.17}$$

and

$$\frac{df}{dy_{\ell m}^*} = -\hat{V}_\ell^* V_\ell V_m^*, \tag{4.18}$$

where m stands for any node of the power network under consideration, i.e., m = 1, ..., n.

The derivatives associated with a generator bus can be obtained by considering (4.6) and its conjugate equation. It is interesting to note that δY_{gm} appears in $\hat{\mathbf{V}}_G^T \mathbf{d}_G$ as well as in $\hat{\mathbf{V}}_G^{*T} \mathbf{d}_G^*$ and it is convenient to write the expressions in a combined manner, given by

$$-\sum_{g=n_L+1}^{n-1} \sum_{m=1}^{n} (\hat{V}_g + \hat{V}_g^*)(V_g^* V_m \delta Y_{gm} + V_g V_m^* \delta Y_{gm}^*)/2, \tag{4.19}$$

or

$$-\sum_{g=n_L+1}^{n-1} \sum_{m=1}^{n} \hat{V}_{g1}(V_g^* V_m \delta Y_{gm} + V_g V_m^* \delta Y_{gm}^*), \tag{4.20}$$

where $\hat{V}_{g1}$ represents the real part of $\hat{V}_g$.

Using (4.15), (4.16) and (4.20), the derivatives of f for an element connected between a generator bus and any other bus are given by

$$\frac{df}{dy_{gm}} = -\hat{V}_{g1} V_g^* V_m, \tag{4.21}$$

and

$$\frac{df}{dy_{gm}^*} = -\hat{V}_{g1} V_g V_m^*. \tag{4.22}$$

For a special case when m = g, (4.21) and (4.22) become identical suggesting that a function is independent of the shunt susceptance at a generator node. Therefore, generator buses have only one practical real shunt control variable.

4.2.4 Derivatives of a Real Function w.r.t. Line Variables

In general, we have four variables y_{ij}, y_{ji}, y_{i0} and y_{j0} associated with an element inserted between nodes i and j. These variables are called line control variables and for an unsymmetrical $\mathbf{Y}_T$, we consider $Y_{ij} \neq Y_{ji}$ in our analysis.

The term associated with $i = \ell$ is expressed in (4.5), which can be expressed as

$$\hat{\mathbf{V}}_L^T \mathbf{d}_L = \sum_{\ell=1}^{n_L} (\hat{V}_\ell \delta S_\ell^*) + \sum_{\ell=1}^{n_L} \sum_{\substack{m=1 \\ m \neq \ell}}^{n} \hat{V}_\ell V_\ell^* (V_m - V_\ell) \delta y_{\ell m}$$

$$- \sum_{\ell=1}^{n_L} (\hat{V}_\ell V_\ell^* V_\ell \ \delta y_{\ell 0}). \tag{4.23}$$

Using (4.4), (4.5) and (4.23), we write

$$\frac{df}{dy_{\ell m}} = - \hat{V}_\ell V_\ell^* (V_\ell - V_m) \tag{4.24}$$

and

$$\frac{df}{dy_{\ell m}^*} = - \hat{V}_\ell^* V_\ell (V_\ell^* - V_m^*) , \tag{4.25}$$

which also incorporate $y_{\ell 0}$ and $y_{\ell 0}^*$, respectively, and for these variables V_m stands for ground voltage, and is equal to zero.

The derivatives associated with i=g can be obtained in a similar manner by using (4.6). We express $\hat{\mathbf{V}}_G{}^T \mathbf{d}_G$ and $\hat{\mathbf{V}}_G{}^{*T} \mathbf{d}_G{}^*$ together, appearing in (4.4), in the form

$$\hat{\mathbf{V}}_G^T \mathbf{d}_G + \hat{\mathbf{V}}_G^{*T} \mathbf{d}_G^* = \sum_{g=n_L+1}^{n-1} \sum_{\substack{m=1 \\ m \neq g}}^{n} \hat{V}_{g1} [V_g^* (V_m - V_g) \delta y_{gm} + V_g (V_m^* - V_g^*) \delta y_{gm}^*]$$

$$- \sum_{g=n_L+1}^{n-1} \hat{V}_{g1} V_g^* V_g (\delta y_{g0} + \delta y_{g0}^*) + \sum_{g=n_L+1}^{n-1} (\hat{V}_g^* \delta \tilde{S}_g + \hat{V}_g \delta \tilde{S}_g^*) \tag{4.26}$$

and from (4.26), we write

$$\frac{df}{dy_{gm}} = - \hat{V}_{g1} V_g^* (V_g - V_m) \tag{4.27}$$

and

$$\frac{df}{dy_{gm}^*} = - \hat{V}_{g1} V_g (V_g^* - V_m^*). \tag{4.28}$$

It is interesting to note that (4.27) and (4.28) are valid for y_{g0} and $y_{g0}{}^*$ where we have to set $V_m = 0$ and the expressions turn out to be identical, which indicates that function f is independent of shunt susceptance at a generator node.

The sensitivity expressions derived for a real function f are summarized in Table 4.1 and it can be observed that the results for $\mathcal{y}$ and y variables are related via a simple transformation (Appendix). In order to deduce these expressions for reciprocal networks, the algebraic sum of the derivatives associated with $\mathcal{y}_{ij}$ and $\mathcal{y}_{ji}$, and y_{ij} and y_{ji}, respectively, leads to the results derived in the related previous work (El-Kady 1980).

Another simplification can be achieved by defning a vector $\hat{\mathbf{V}}^R$ given by

$$\hat{\mathbf{V}}^R = \begin{bmatrix} \hat{V}_L \\ \hat{V}_{G1} \end{bmatrix}, \tag{4.29}$$

where $\hat{\mathbf{V}}_L$ is a vector of length n_L associated with load buses (4.3), and $\hat{\mathbf{V}}_{G1}$ is a vector of real quantities given by

$$\hat{\mathbf{V}}_{G1} = \mathrm{Re}\,\{\hat{\mathbf{V}}_G\}\,. \tag{4.30}$$

The sensitivity expressions (4.17), (4.18), (4.21) and (4.22) are given by

$$\frac{df}{d\mathcal{y}_{mm'}} = -\hat{V}^R_m V^*_m V_{m'} \tag{4.31}$$

and

$$\frac{df}{d\mathcal{y}^*_{mm'}} = -\hat{V}^{R^*}_m V_m V^*_{m'}\,, \tag{4.32}$$

where $m = \ell, g$ and $m' = \ell, g$ or n.

The expressions derived for line variables (4.24), (4.25), (4.27) and (4.28), can also be expressed, using (4.29) in the form

$$\frac{df}{dy_{mm'}} = -\hat{V}^R_m V^*_m (V_m - V_{m'}) \tag{4.33}$$

TABLE 4.1

SENSITIVITIES OF A REAL FUNCTION IN COMPLEX MODE

Variable	Description	Sensitivity formula
S_ℓ	demand power	$\hat{V}_\ell^*$
$\tilde{S}_g$	generator complex quantity ($Re\{\tilde{S}_g\} = P_g$ and $Im\{\tilde{S}_g\} = \lvert V_g \rvert$)	$\hat{V}_g^*$
V_n	slack bus voltage	$\hat{V}_n^*$
$y_{\ell m}$	short-circuit admittance parameter associated with a load and any other bus	$-\hat{V}_\ell V_\ell^* V_m$
y_{gm}	short-circuit admittance parameter associated with a generator and any other bus	$-\hat{V}_{g1} V_g^* V_m$
† $y_{\ell m}$	line variable associated with a load and any other bus	$-\hat{V}_\ell V_\ell^* (V_\ell - V_m)$
† y_{gm}	line variable associated with a generator and any other bus	$-\hat{V}_{g1} V_g^* (V_g - V_m)$

† $m = 0$ for shunt variables and $V_m = 0$, the ground potential.

and

$$\frac{df}{dy^*_{mm'}} = -\hat{V}^{R^*}_m V_m (V^*_m - V^*_{m'}), \tag{4.34}$$

where m = ℓ, g and m′ = ℓ, g, n or 0.

4.2.5 Derivatives of a Real Function w.r.t Phase-shifting Transformer Control Variables

The controllable parameters of a phase-shifting transformer in complex mode are its complex turns ratio and impedance. Using the conjugate notation, we express the control vector **u** for this element in the form

$$\mathbf{u} = \begin{bmatrix} \mathbf{u}^1 \\ \mathbf{u}^2 \end{bmatrix} = \begin{bmatrix} a \\ a^* \\ Z_t \\ Z^*_t \end{bmatrix}, \tag{4.35}$$

where $\mathbf{u}^1$ and $\mathbf{u}^2$ represent the first and second sets of variables in complex mode. The derivatives of short-circuit admittance parameters and line variables w.r.t. control variables specified in (4.35) are summarized in Tables 4.2 and 4.3, respectively.

The first-order change of function f can be expressed, using conjugate notation, in the form

$$\delta f = \frac{\partial f}{\partial y_{pp}} \delta y_{pp} + \frac{\partial f}{\partial y_{pq}} \delta y_{pq} + \frac{\partial f}{\partial y_{qp}} \delta y_{qp} + \frac{\partial f}{\partial y_{qq}} \delta y_{qq}$$

$$+ \frac{\partial f}{\partial y^*_{pp}} \delta y^*_{pp} + \frac{\partial f}{\partial y^*_{pq}} \delta y^*_{pq} + \frac{\partial f}{\partial y^*_{qp}} \delta y^*_{qp} + \frac{\partial f}{\partial y^*_{qq}} \delta y^*_{qq}\,. \tag{4.36}$$

In general, the first-order variation of a short-circuit parameter y_{ij}, where i,j = p,q, is given by

$$\delta y_{ij} = \frac{\partial y_{ij}}{\partial a} \delta a + \frac{\partial y_{ij}}{\partial a^*} \delta a^* + \frac{\partial y_{ij}}{\partial Z_t} \delta Z_t + \frac{\partial y_{ij}}{\partial Z^*_t} \delta Z^*_t\,. \tag{4.37}$$

TABLE 4.2

SHORT-CIRCUIT ADMITTANCE PARAMETRS AND CONTROL

VARIABLES OF A PHASE-SHIFTING TRANSFORMER

Parameter	Expression	Derivative w.r.t. a	Derivative w.r.t. a^*	Derivative w.r.t. Z_t	Derivative w.r.t. Z_t^*
y_{pp}	$\frac{1}{Z_t a a^*}$	$\frac{-1}{Z_t a^2 a^*}$	$\frac{-1}{Z_t a a^{*2}}$	$\frac{-1}{Z_t^2 a a^*}$	0
y_{pq}	$\frac{-1}{Z_t a^*}$	0	$\frac{1}{Z_t a^{*2}}$	$\frac{1}{Z_t^2 a^*}$	0
y_{qp}	$\frac{-1}{Z_t a}$	$\frac{1}{Z_t a^2}$	0	$\frac{1}{Z_t^2 a}$	0
y_{qq}	$\frac{1}{Z_t}$	0	0	$\frac{-1}{Z_t^2}$	0

TABLE 4.3

LINE VARIABLES AND CONTROL VARIABLES OF

A PHASE-SHIFTING TRANSFORMER

Line Variable	Expression	Derivative w.r.t. a	Derivative w.r.t. a^*	Derivative w.r.t. Z_t	Derivative w.r.t. Z_t^*
y_{p0}	$\frac{1}{Z_t a^*}\left(\frac{1}{a}-1\right)$	$\frac{-1}{Z_t a^2 a^*}$	$\frac{-1}{Z_t a^{*2}}\left(\frac{1}{a}-1\right)$	$\frac{-1}{Z_t^2 a^*}\left(\frac{1}{a}-1\right)$	0
y_{pq}	$\frac{1}{Z_t a^*}$	0	$\frac{-1}{Z_t a^{*2}}$	$\frac{-1}{Z_t^2 a^*}$	0
y_{qp}	$\frac{1}{Z_t a}$	$\frac{-1}{Z_t a^2}$	0	$\frac{-1}{Z_t^2 a}$	0
y_{q0}	$\frac{1}{Z_t}\left(1-\frac{1}{a}\right)$	$\frac{1}{Z_t a^2}$	0	$\frac{-1}{Z_t^2}\left(1-\frac{1}{a}\right)$	0

Using (4.37) and its conjugate, and Table 4.2, we write (4.36) in the form

$$\delta f = \frac{-1}{a^2}\left[\frac{1}{a^*}\left(\frac{1}{Z_t}\frac{\partial f}{\partial y_{pp}} + \frac{1}{Z_t^*}\frac{\partial f}{\partial y_{pp}^*}\right) - \left(\frac{1}{Z_t}\frac{\partial f}{\partial y_{qp}} + \frac{1}{Z_t^*}\frac{\partial f}{\partial y_{pq}^*}\right)\right]\delta a$$

$$- \frac{1}{a^{*2}}\left[\frac{1}{a}\left(\frac{1}{Z_t}\frac{\partial f}{\partial y_{pp}} + \frac{1}{Z_t^*}\frac{\partial f}{\partial y_{pp}^*}\right) - \left(\frac{1}{Z_t}\frac{\partial f}{\partial y_{pq}} + \frac{1}{Z_t^*}\frac{\partial f}{\partial y_{qp}^*}\right)\right]\delta a^*$$

$$- \frac{1}{Z_t^2}\left(\frac{1}{aa^*}\frac{\partial f}{\partial y_{pp}} - \frac{1}{a^*}\frac{\partial f}{\partial y_{pq}} - \frac{1}{a}\frac{\partial f}{\partial y_{qp}} + \frac{\partial f}{\partial y_{qq}}\right)\delta Z_t$$

$$- \frac{1}{Z_t^{*2}}\left(\frac{1}{aa^*}\frac{\partial f}{\partial y_{pp}^*} - \frac{1}{a}\frac{\partial f}{\partial y_{pq}^*} - \frac{1}{a^*}\frac{\partial f}{\partial y_{qp}^*} + \frac{\partial f}{\partial y_{qq}^*}\right)\delta Z_t^* . \quad (4.38)$$

The derivatives of f w.r.t. control variables of phase-shifting transformers can be written, directly from (4.38). Using Table 4.2, we summarize the sensitivity formulas in complex mode, in a compact, practical form as presented in Table 4.4a. Note that we have defined a vector $\hat{\mathbf{V}}^R$ (4.29) in order to display a more general formula which takes care of the adjoint voltages associated with a generator node.

A similar treatment can be carried on with sensitivity expressions derived for line variables and the results can be summarized as shown in Table 4.4b.

4.3 GRADIENT EVALUATION IN RECTANGULAR AND POLAR MODES

For a general real function f, which may be function of complex vector ζ and its conjugate ζ^*, the sensitivities in complex mode are given by $df/d\zeta$ and $df/d\zeta^*$. However, the practical variables are given in Table 4.5 and we derive a transformation which can be used to give sensitivities in the rectangular mode.

TABLE 4.4a

SENSITIVITES OF A REAL FUNCTION W.R.T. PHASE-SHIFTING

TRANSFORMER CONTROL VARIABLES

Control Variable	Sensitivity Expression
a	$\frac{1}{a^2}\left[-\frac{1}{a^*}\left(\frac{1}{Z_t}\frac{\partial f}{\partial y_{pp}}+\frac{1}{Z_t^*}\frac{\partial f}{\partial y_{pp}^*}\right)+\left(\frac{1}{Z_t}\frac{\partial f}{\partial y_{qp}}+\frac{1}{Z_t^*}\frac{\partial f}{\partial y_{pq}^*}\right)\right]$
a^*	$\frac{1}{a^{*2}}\left[-\frac{1}{a}\left(\frac{1}{Z_t}\frac{\partial f}{\partial y_{pp}}+\frac{1}{Z_t^*}\frac{\partial f}{\partial y_{pp}^*}\right)+\left(\frac{1}{Z_t}\frac{\partial f}{\partial y_{pq}}+\frac{1}{Z_t^*}\frac{\partial f}{\partial y_{qp}^*}\right)\right]$
Z_t	$\frac{1}{Z_t^2}\left[-\left(\frac{1}{aa^*}\frac{\partial f}{\partial y_{pp}}+\frac{\partial f}{\partial y_{qq}}\right)+\left(\frac{1}{a^*}\frac{\partial f}{\partial y_{pq}}+\frac{1}{a}\frac{\partial f}{\partial y_{qp}}\right)\right]$
Z_t^*	$\frac{1}{Z_t^{*2}}\left[-\left(\frac{1}{aa^*}\frac{\partial f}{\partial y_{pp}^*}+\frac{\partial f}{\partial y_{qq}^*}\right)+\left(\frac{1}{a}\frac{\partial f}{\partial y_{pq}^*}+\frac{1}{a^*}\frac{\partial f}{\partial y_{qp}^*}\right)\right]$

TABLE 4.4b

SENSITIVITES OF A REAL FUNCTION W.R.T. PHASE-SHIFTING

TRANSFORMER CONTROL VARIABLES

Control Variable	Sensitivity Expression
a	$\frac{1}{a^2}\left[-\frac{1}{Z_t}\left(\frac{1}{a^*}\frac{\partial f}{\partial y_{p0}}+\frac{1}{\partial y_{qp}}-\frac{\partial f}{\partial y_{q0}}\right)-\frac{1}{Z_t^*}\left\{\left(\frac{1}{a^*}-1\right)\frac{\partial f}{\partial y^*_{p0}}+\frac{\partial f}{\partial y^*_{pq}}\right\}\right]$
a^*	$\frac{1}{a^{*2}}\left[-\frac{1}{Z_t}\left\{\left(\frac{1}{a}-1\right)\frac{\partial f}{\partial y_{p0}}+\frac{\partial f}{\partial y_{pq}}\right\}-\frac{1}{Z_t^*}\left(\frac{1}{a}\frac{\partial f}{\partial y^*_{p0}}+\frac{\partial f}{\partial y^*_{qp}}-\frac{\partial f}{\partial y^*_{q0}}\right)\right]$
Z_t	$\frac{1}{Z_t^2}\left[-\frac{1}{a^*}\left(\frac{1}{a}-1\right)\frac{\partial f}{\partial y_{p0}}-\frac{1}{a^*}\frac{\partial f}{\partial y_{pq}}-\frac{1}{a}\frac{\partial f}{\partial y_{qp}}-\left(1-\frac{1}{a}\right)\frac{\partial f}{\partial y_{q0}}\right]$
Z_t^*	$\frac{1}{Z_t^{*2}}\left[-\frac{1}{a}\left(\frac{1}{a^*}-1\right)\frac{\partial f}{\partial y^*_{p0}}-\frac{1}{a}\frac{\partial f}{\partial y^*_{pq}}-\frac{1}{a^*}\frac{\partial f}{\partial y^*_{qp}}-\left(1-\frac{1}{a^*}\right)\frac{\partial f}{\partial y^*_{q0}}\right]$

TABLE 4.5

PRACTICAL CONTROL VARIABLES OF A POWER NETWORK

Variable	Description
P_ℓ	demand real power
Q_ℓ	demand reactive power
P_g	generator real power
$\lvert V_g \rvert$	generator bus voltage magnitude
V_{n1}	real component of the slack bus voltage
G_t	line conductance of a transmission line
B_t	line susceptance of a transmission line
G_{sh}	shunt conductance of a transmission line
B_{sh}	shunt susceptance of a transmission line
a_1	real component of the complex turns ratio
a_2	imaginary component of the complex turns ratio
$\lvert a \rvert$	magnitude of the complex turns ratio
ϕ	phase angle of the complex turns ratio
R_t	resistance of a phase-shifting transformer
X_t	reactance of a phase-shifting transformer

We have

$$\boldsymbol{\zeta} = \boldsymbol{\zeta}_1 + j\boldsymbol{\zeta}_2 , \tag{4.39}$$

where $\boldsymbol{\zeta}_1$ and $\boldsymbol{\zeta}_2$ are vectors of real quantities, and a complex quantity ζ_1 and its conjugate, are given by

$$\zeta_1 = \zeta_{11} + j\zeta_{12} \tag{4.40}$$

and

$$\zeta_1^* = \zeta_{11} - j\zeta_{12} , \tag{4.41}$$

respectively. Then the first-order changes of ζ_1 and ζ_1^* can be compactly expressed in the form, using (4.40) and (4.41)

$$\begin{bmatrix} \delta\zeta_1 \\ \delta\zeta_1^* \end{bmatrix} = \begin{bmatrix} 1 & j \\ 1 & -j \end{bmatrix} \begin{bmatrix} \delta\zeta_{11} \\ \delta\zeta_{12} \end{bmatrix} , \tag{4.42}$$

or

$$\begin{bmatrix} \delta\zeta_{11} \\ \delta\zeta_{12} \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 1 & 1 \\ -j & j \end{bmatrix} \begin{bmatrix} \delta\zeta_1 \\ \delta\zeta_1^* \end{bmatrix} . \tag{4.43}$$

Therefore, the transformation for sensitivity in rectangular mode is given by

$$\begin{bmatrix} \dfrac{df}{d\zeta_{11}} \\ \dfrac{df}{d\zeta_{12}} \end{bmatrix} = \left(\frac{1}{2} \begin{bmatrix} 1 & 1 \\ -j & j \end{bmatrix} \right)^{-T} \begin{bmatrix} \dfrac{df}{d\zeta_1} \\ \dfrac{df}{d\zeta_1^*} \end{bmatrix} \tag{4.44}$$

or

$$\begin{bmatrix} \dfrac{df}{d\zeta_{11}} \\ \dfrac{df}{d\zeta_{12}} \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ j & -j \end{bmatrix} \begin{bmatrix} \dfrac{df}{d\zeta_1} \\ \dfrac{df}{d\zeta_1^*} \end{bmatrix} . \tag{4.45}$$

A direct way of obtaining (4.45) from (4.42) is to simply transpose the matrix of coefficient in (4.42) as the perturbed equations are linear. As we have considered only real functions in this thesis, we use $df/d\zeta_1^* = (df/d\zeta_1)^*$, and (4.45) yields a simple relation

$$\frac{df}{d\zeta_{11}} = 2 \,\mathrm{Re} \left\{ \frac{df}{d\zeta_1} \right\} \tag{4.46}$$

and

$$\frac{df}{d\zeta_{12}} = -2\,\mathrm{Im}\left\{ \frac{df}{d\zeta_1} \right\} \quad . \tag{4.47}$$

These relations can be used to obtain sensitivities w.r.t. real control variables listed in Table 4.5.

The practical control variables associated with the complex turns ratio of a phase-shifting transformer are $|a|$ and ϕ, described in section 2.5 and we derive another transformation of practical interest. The rectangular mode of complex turns ratio can be expressed in the form

$$a_1 = |a| \cos \phi \tag{4.48}$$

and

$$a_2 = |a| \sin \phi \, , \tag{4.49}$$

and their first-order variations are given, using (4.48) and (4.49), by

$$\delta a_1 = \cos \phi \, \delta|a| - |a| \sin \phi \; \delta\phi \tag{4.50}$$

and

$$\delta a_2 = \sin \phi \, \delta|a| + |a| \cos \phi \; \delta\phi \, . \tag{4.51}$$

In matrix form, we write (4.50) and (4.51) as

$$\begin{bmatrix} \delta a_1 \\ \delta a_2 \end{bmatrix} = \begin{bmatrix} \cos \phi & -|a| \sin \phi \\ \sin \phi & |a| \cos \phi \end{bmatrix} \begin{bmatrix} \delta|a| \\ \delta\phi \end{bmatrix} , \tag{4.52}$$

representing a linear relation between δa_1, δa_2 and $\delta|a|$, $\delta\phi$, respectively. Taking the transpose of inverse of the coefficient matrix in (4.52), we obtain a transformation matrix $\mathbf{T}^{plr}$, which relates the sensitivities of a given real function in two different real modes, namely, polar mode and rectangular mode. The required transformation for polar mode is given by

$$\begin{bmatrix} \dfrac{df}{d|a|} \\ \dfrac{df}{d\phi} \end{bmatrix} = \begin{bmatrix} \cos\phi & \sin\phi \\ -|a|\sin\phi & |a|\cos\phi \end{bmatrix} \begin{bmatrix} \dfrac{df}{da_1} \\ \dfrac{df}{da_2} \end{bmatrix}, \tag{4.53}$$

or

$$\begin{bmatrix} \dfrac{df}{d|a|} \\ \dfrac{df}{d\phi} \end{bmatrix} = \begin{bmatrix} \dfrac{a_1}{|a|} & \dfrac{a_2}{|a|} \\ -a_2 & a_1 \end{bmatrix} \begin{bmatrix} \dfrac{df}{da_1} \\ \dfrac{df}{da_2} \end{bmatrix}. \tag{4.54}$$

The sensitivity expressions derived in this chapter are verified numerically by considering a 6-bus sample power system, a 26-bus system, and the IEEE 118-bus system (Bandler, El-Kady, Grewal and Wojciechowski 1983).

4.4 NUMERICAL EXAMPLES

4.4.1 6-Bus Sample System

The 6-bus power system (El-Kady 1980) has been augmented by inserting a phase-shifting transformer with a large phase angle ($\phi = 36.8°$) between nodes 1 and 4, as indicated in the single-line diagram shown in Fig. 4.1. The nodal admittance matrix of the system and the a.c. load flow solution in rectangular and polar modes are displayed in Tables 4.6a and b. The Jacobian available at the solution is utilized to determine sensitivities of the voltage magnitude at load bus ($\ell = 1$). The right-hand-side vector of the adjoint system constitutes the partial derivatives of $f = |V_1|$ w.r.t. complex bus voltages and their complex conjugates. The nonzero elements of this vector are

$$\frac{df}{dV_1} = \frac{0.5\, V_1^*}{|V_1|} \tag{4.55}$$

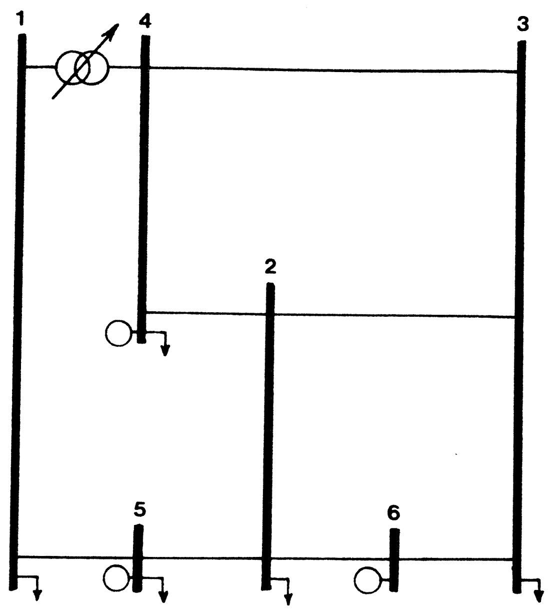


Fig. 4.1 6-bus sample system

and

$$\frac{df}{dV_1^*} = \frac{0.5\, V_1}{|V_1|} \ . \tag{4.56}$$

The adjoint solution vector is used in Tables 4.1-4.5 and the sensitivities are tabulated as shown in Table 4.7.

4.4.2 26-Bus Power System

We consider a network function of the system shown in Fig. 4.2, given by

$$f = \delta_{20} = \tan^{-1}\left[\frac{j(V_{20}^* - V_{20})}{V_{20} + V_{20}^*}\right] \tag{4.57}$$

TABLE 4.6a

NODAL ADMITTANCE MATRIX YT

The sequence in each row is: Column Index, Real(YT), Imag(YT)

Bus No. 1

1:	3.529412	−14.117647	4:	−3.764706	3.058824
5:	−2.352941	9.411765			

Bus No. 2

2:	5.490196	−21.960784	3:	−.588235	2.352941
4:	−.588235	2.352941	5:	−1.176471	4.705882
6:	−3.137255	12.549020			

Bus No. 3

3:	2.549020	−10.196078	2:	−.588235	2.352941
4:	−.392157	1.568627	6:	−1.568627	6.274510

Bus No. 4

4:	2.156863	−8.627451	1:	1.882353	4.470588
2:	−.588235	2.352941	3:	−.392157	1.568627

Bus No. 5

5:	3.529412	−14.117647	1:	−2.352941	9.411765
2:	−1.176471	4.705882			

Bus No. 6

6:	4.705882	−18.823529	2:	−3.137255	12.549020
3:	−1.568627	6.274510			

TABLE 4.6b

LOAD FLOW SOLUTION OF THE 6-BUS SAMPLE SYSTEM

Bus Index	Rectangular Coordinates		Polar Coordinates	
1	.88812	−j .40719	.97702	−.42989
2	.91190	−j .27668	.95295	−.29459
3	.82475	−j .29829	.87703	−.34704
4	.69671	−j .74498	1.02000	−.81887
5	.98821	−j .32411	1.04000	−.31693
6	1.04000	−j 0	1.04000	0

TABLE 4.7

SENSITIVITIES OF $|V_1|$ OF THE 6-BUS SYSTEM

Load Bus Quantities -- Total Derivatives

Bus	Real Power	Reactive Power	Shunt Conductance	Shunt Susceptance
1	.030383	.070579	−.029003	.067373
2	.000012	−.000317	−.000011	−.000288
3	−.000889	−.000647	.000684	−.000497

Generator Bus Quantities -- Total Derivatives

Bus	Real Power	Voltage Magnitude	Shunt Conductance	Shunt Susceptance
4	−.004743	.370314	.004935	0.000000
5	.002579	.713495	−.002790	0.000000

Line Quantities -- Total Derivatives

Line Index	Element	Line Conductance	Line Susceptance
2	1,5	−.006597	−.007071
3	2,3	−.000045	.000059
4	2,4	.000789	.002293
5	2,5	−.000240	−.000031
6	2,6	.000092	.000009

Phase Shifter Quantities -- Total Derivatives

Element	Turns Ratio Magnitude	Turns Ratio Phase Angle	Internal Resistance	Internal Reactance
1,4	.332271	−.011565	−.459281	−.008697

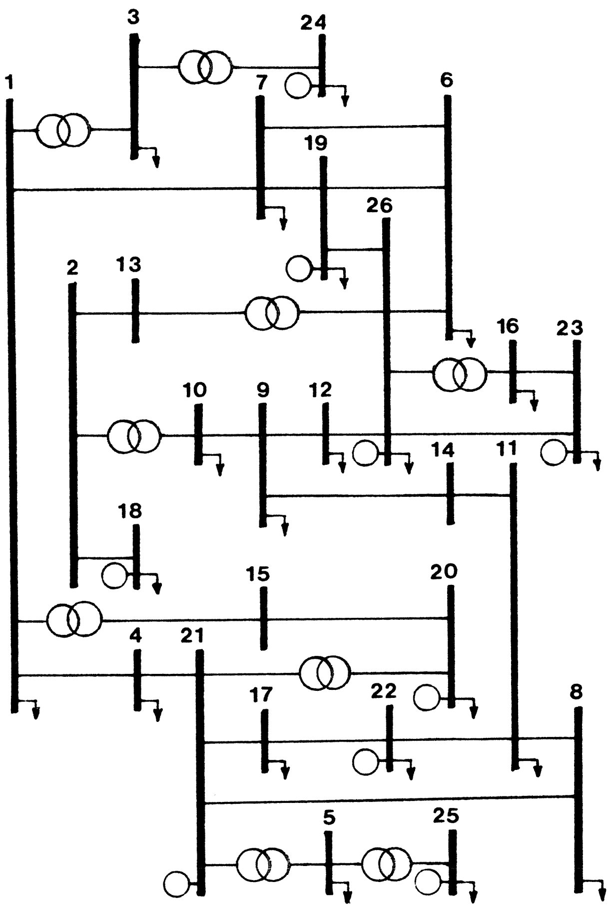


Fig. 4.2 26-bus power system

and the partial derivatives of this function with respect to complex voltages have a nonzero element, expressed as

$$\frac{df}{dV_{20}} = \frac{-j\,0.5}{V_{20}} \; . \tag{4.58}$$

The sensitivities of the function defined in (4.57) are tabulated in Table 4.8.

TABLE 4.8

SENSITIVITIES OF δ_{20} OF THE 26-BUS SYSTEM

Load Bus Quantities -- Total Derivatives

Bus	Real Power	Reactive Power	Shunt Conductance	Shunt Susceptance
1	.267581	−.007722	−.286998	−.008283
2	.008245	.001363	−.009414	.001557
3	.267275	−.006188	−.291205	−.006742
4	.290383	−.002045	−.285035	−.002007
5	.296479	.000000	−.301324	.000000
6	.080168	−.000773	−.085701	−.000826
7	.216789	−.004509	−.222613	−.004630
8	.269938	−.009039	−.241050	−.008071
9	.056949	−.002912	−.053309	−.002726
10	.009824	.002211	−.010611	.002388
11	.222100	−.005921	−.181376	−.004835
12	.025652	.000475	−.024130	.000447
13	.001374	.000267	−.001504	.000292
14	.133023	−.004979	−.118771	−.004446
15	.276482	−.005600	−.243064	−.004868
16	0.000000	0.000000	0.000000	0.000000
17	.272539	−.003772	−.236823	−.003278

Generator Bus Quantities -- Total Derivatives

Bus	Real Power	Voltage Magnitude	Shunt Conductance	Shunt Susceptance
18	.007581	.025614	−.008680	0.000000
19	.080415	−.039806	−.088657	0.000000
20	.307280	−.354038	−.307280	0.000000
21	.296479	−.219182	−.308457	0.000000
22	.255422	−.053318	−.202320	0.000000
23	0.000000	0.000000	0.000000	0.000000
24	.267232	−.045456	−.267232	0.000000
25	.296479	.000000	−.296479	0.000000

TABLE 4.8 (continued)

SENSITIVITIES OF δ_{20} OF THE 26-BUS SYSTEM

Line Quantities -- Total Derivatives

Line Index	Element	Line Conductance	Line Susceptance
3	16,23	0.000000	0.000000
4	23,26	0.000000	0.000000
6	9,10	.003064	−.008097
7	9,12	.000136	−.001057
8	12,26	.000889	−.001936
9	9,14	.001597	−.000024
10	11,14	.003088	.000301
11	19,26	−.003739	.007844
12	6,26	−.002147	.004467
13	6,19	−.000122	.000023
14	7,19	.004441	−.010615
15	6,7	.002757	−.005050
16	11,22	.000393	.000206
17	8,11	−.007535	.005931
18	17,22	−.005194	.001658
19	8,21	−.011203	.005182
20	17,21	−.014022	.004697
21	1,4	.000448	.000296
22	4,21	−.005019	.000829
25	2,13	−.000102	.000594
26	1,7	−.002295	.002933
27	15,20	.007868	.004280
28	2,18	−.000436	−.000098

TCUL Transformer Quantities -- Total Derivatives

Element	Turns Ratio	Internal Resistance	Internal Reactance
13,26	.022095	−.025511	.133014
26,16	0.000000	0.000000	0.000000
2,10	−.061836	−.122407	−.153065
15,1	.186552	−1.571927	1.408706
1,3	−.049465	−.149478	−.011940
24,3	.046384	−.009770	−.007182
5,21	.000000	−.011448	.000000
5,25	.000000	−.126153	−.000000

4.4.3 The IEEE 118-Bus System

This system is a standard average size test network shown in Fig. 4.3 and has been extensively used in a variety of steady-state analysis and planning studies (Burchett and Happ 1983, Burchett, Happ and Wirgau 1982). The load flow solution of this system has been investigated by implementing different algorithms (Bandler, El-Kady, Grewal and Gupta 1982, Bandler, El-Kady and Wojciechowski 1982, Bandler, El-Kady, Kellermann and Zuberek 1983, Bandler, El-Kady and Grewal 1983).

We consider a network state associated with the slack bus, namely, the real power and express it in the form

$$f = P_n = \mathrm{Re}\left\{ V_n \sum_{j=1}^{n} Y_{nj}^* V_j^* \right\}. \tag{4.59}$$

The nonzero elements of df/d**V** are given by

$$\frac{df}{dV_k} = 0.5\left(Y_{nk} V_n^* + \delta_{kn} \sum_{j=1}^{n} Y_{nj}^* V_j^* \right)$$

where

$$\delta_{kn} = \begin{cases} 0 & \text{for} \quad k \neq n \\ 1 & \text{for} \quad k = n \end{cases} . \tag{4.60}$$

The sensitivities of the function considered are shown in Table 4.9 and have been verified by small perturbation about the nominal point.

We formulate a minimum-loss problem (Dommel and Tinney 1968, Sasson 1969) for the IEEE 118-bus system and express the cost function in the form,

$$f = \sum_{i=1}^{n} P_i \tag{4.61}$$

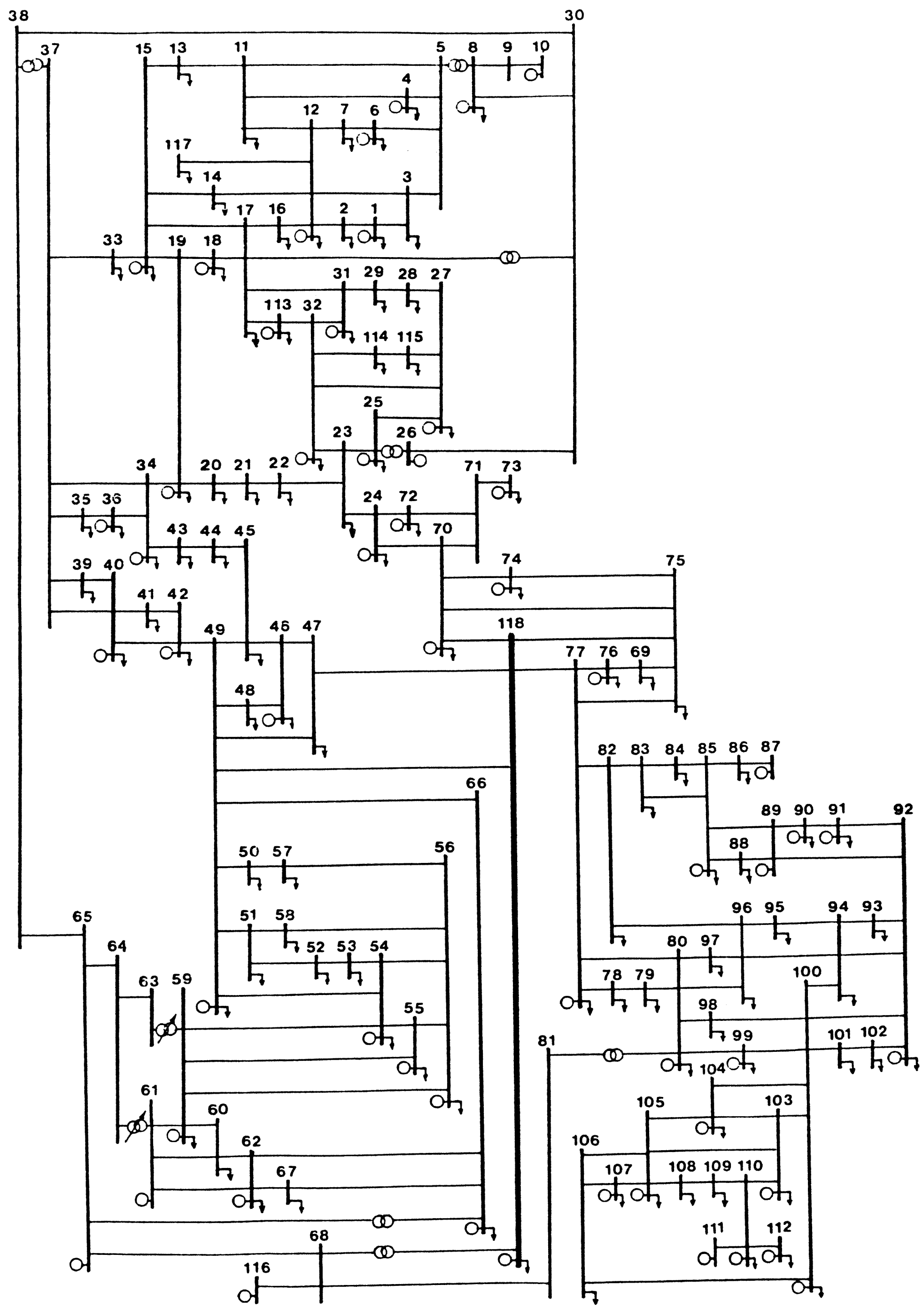


Fig. 4.3 The IEEE 118-bus system

TABLE 4.9

SENSITIVITIES OF P_n OF THE 118-BUS SYSTEM

Load Bus Quantities -- Total Derivatives

Bus	Real Power	Reactive Power	Shunt Conductance	Shunt Susceptance
2	−1.039480	−.001776	1.222015	−.002087
5	−.995079	−.007493	1.152184	−.008676
9	−.977067	.000778	1.172358	.000933
17	−1.005144	.002108	.968617	.002032
20	−1.033789	−.004819	.903283	−.004211
21	−1.028398	−.008006	.872155	−.006790
22	−1.014626	−.007771	.850805	−.006516
23	−.983963	−.001731	.832173	−.001464
38	−1.004240	−.002092	1.088326	−.002267
39	−1.061038	−.005893	.981905	−.005454
41	−1.090438	−.001984	1.084327	−.001973
52	−1.031752	−.008662	.939432	−.007887
53	−1.025027	−.007288	.909682	−.006468
57	−1.020321	−.000690	.954047	−.000645
78	−.982543	−.007871	.982164	−.007868
79	−.975549	−.007183	.987800	−.007273
81	−.952472	.001083	.960059	.001093
82	−.947823	−.005311	.928035	−.005200
83	−.927885	−.004839	.900134	−.004694
84	−.885244	−.005147	.854794	−.004970
86	−.863121	−.001071	.844713	−.001048
106	−.977820	−.003407	.904207	−.003150
108	−.982431	−.000472	.917336	−.000441
117	−1.037589	−.003583	1.116755	−.003856

Phase Shifter Quantities -- Total Derivatives

Element	Turns Ratio Magnitude	Turns Ratio Phase Angle	Internal Resistance	Internal Reactance
63,59	.039796	.239011	1.860750	.312149
64,61	.029692	−.142203	.000229	−.009722

TABLE 4.9 (continued)

SENSITIVITIES OF P_n OF THE 118-BUS SYSTEM

Generator Bus Quantities -- Total Derivatives

Bus	Real Power	Voltage Magnitude	Shunt Conductance	Shunt Susceptance
1	−1.050857	.882783	1.389758	0.000000
4	−1.032740	.726316	1.249615	0.000000
19	−1.030086	−.162855	.949328	0.000000
24	−.995664	−.377848	.824509	0.000000
25	−.940147	.297842	.884584	0.000000
34	−1.033093	−.198182	.950116	0.000000
36	−1.031219	−.001704	.956321	0.000000
40	−1.077351	−.769418	1.013680	0.000000
42	−1.092801	1.205544	1.322289	0.000000
59	−.987729	.229232	.968073	0.000000
61	−.965655	−.071162	.965655	0.000000
62	−.976510	−.071795	.976510	0.000000
65	−.959474	−.587420	.959474	0.000000
76	−1.018810	−.456361	.909822	0.000000
77	−.975753	−.110255	.995366	0.000000
80	−.945021	.463286	1.022135	0.000000
85	−.858860	−.313478	.841768	0.000000
87	−.856185	.013011	.873394	0.000000
99	−.935353	−.084126	.954153	0.000000
100	−.919584	.230646	.956735	0.000000
103	−.945492	.251899	.964496	0.000000
113	−1.012186	.595070	.992044	0.000000
116	−.989912	−.101326	.989912	0.000000

TCUL Transformer Quantities -- Total Derivatives

Element	Turns Ratio	Internal Resistance	Internal Reactance
8,5	.316354	11.930163	−.084332
26,25	−.004820	.529850	−.131223
30,17	−.058476	5.266261	.337658
38,37	.094365	5.157105	.443463
66,65	−.000316	9.261379	−.008021
81,80	.037468	.371101	.078608

TABLE 4.10

RESULTS OF THE MINIMUM-LOSS PROBLEM

Iteration	Objective Function	Iteration	Objective Function
1	1.874029	31	1.113729
2	1.791540	32	1.106405
3	1.788261	33	1.099119
4	1.742582	34	1.098632
5	1.728534	35	1.097584
6	1.675286	36	1.077108
7	1.650821	37	1.069434
8	1.646664	38	1.058328
9	1.602012	39	1.057547
10	1.532712	40	1.039549
11	1.474493	41	1.026739
12	1.438996	42	1.012004
13	1.430586	43	1.001770
14	1.426693	44	0.998249
15	1.417182	45	0.993113
16	1.392310	46	0.991415
17	1.354504	47	0.990480
18	1.339082	48	0.984823
19	1.321483	49	0.971396
20	1.305068	50	0.969115
21	1.265463	51	0.962826
22	1.265322	52	0.941750
23	1.257992	53	0.935488
24	1.253278	54	0.931943
25	1.252076	55	0.927768
26	1.249091	56	0.922292
27	1.199212	57	0.914366
28	1.163750	58	0.910232
29	1.156131	59	0.902772
30	1.134962	60	0.898881

where P_i is the net real power injected at the ith node (El-Hawary and Christensen 1979) and n is the total number of buses in the system. We consider the generator bus-type control variables and minimize the function given by (4.61), subject to load flow equations and allowable range of the control variables given by

$$0 \leq P_g \leq P_{max}$$

$$0.95 \leq |V_g| \leq 1.05$$

where $P_{max} = 1.30\ P_g$ (nominal). An optimization package, called MINOS Version 4 or MINOS/AUGMENTED (Murtagh and Saunders 1980) is used to solve the problem and to verify the gradients of the objective function. A summary of the results from the package is displayed in Table 4.10.

4.5 CONCLUSIONS

A unified study for the complex Lagrangian method as applied to power system sensitivity analysis has been presented. Generalized sensitivity expressions have been derived in a simple, straightforward manner, and tabulated in compact form, using the recently developed special complex notation. The transformations required for sensitivities in rectangular and polar modes have been obtained. The adjustable control parameters of phase-shifting transformers have been described and investigated with the help of line control variables and short-circuit admittance description. The theoretical results have been verified numerically for some practical power systems.

CHAPTER 5

CONCLUSIONS

The material presented in this thesis is based on the recently developed methodology for adjoint network approaches to sensitivity evaluation and optimization of electrical power systems. The methodology exploits the complex conjugate notation and facilitates the exact power system steady-state component modelling. The notation allows compact computation and easier handling of complex variables involved in power flow problems.

The load flow equations of power system comprising nonreciprocal two-port elements are established in the standard compact complex form using a bus frame of reference. The nodal admittance matrix has been assumed to have unequal off-diagonal elements associated with nodes where phase-shifting transformers are installed. The complex turns ratio and impedance of these transformers have been considered as control variables and some useful sensitivity relations have been derived and described in a simple, elegant manner.

The phase-shifting transformers have been described with the help of their short-circuit admittance matrix description in Chapter 2. The practical control variables of these elements are the magnitude and phase angle of the complex turns ratio, however, the resistance and reactance may also contribute in controlling the network functions. Basically, these transformers are employed to control power flow in transmission network and can be used to delay the future expansion plans.

We have considered an augmented form of Tellegen's theorem in Chapter 3 and have presented generalized basic and element variables for various types of network branches. The concept has been extended to nonreciprocal two-port elements and the short-circuit admittance matrix description has been used to develop exact and compact sensitivity expressions for a general network function.

The complex Lagrangian method has been employed to handle complex dependent variables defined in a particular problem in Chapter 4. The unsymmetrical nodal admittance matrix has been exploited and general sensitivity expressions common to all relevant power system studies have been derived and tabulated. Some numerical examples have been provided to illustrate and verify the sensitivity relations. An optimization problem for the IEEE 118-bus system has been formulated and solved by MINOS/AUGMENTED.

The work presented in this thesis provides promising research directions in the area of modelling and analysis of nonreciprocal power network elements. The difficulties previously encountered in handling phase-shifting transformers have been overcome and exact sensitivities can be utilized effectively in transmission planning and power system operation. The methodology affords a very convenient way of treating exact element models in power networks and can be appied to large scale power systems.

APPENDIX

DERIVATION OF TRANSFORMATION MATRIX RELATING SHORT-CIRCUIT ADMITTANCE PARAMETER AND LINE VARIABLE SENSITIVITIES

Consider a general two-port network connected between nodes p and q. Its short-circuit admittance parameters are y_{pp}, y_{pq}, y_{qp} and y_{qq}. The line variables for the same netwrok can be represented by y_{p0}, y_{pq}, y_{qp} and y_{q0}, and we assume, in general, the network to be nonreciprocal i.e., $y_{pq} \neq y_{qp}$ and also $\mathrm{y}_{pq} \neq \mathrm{y}_{qp}$. We can express short-circuit admittance parameters in terms of line variables, in matrix form, given by

$$\begin{bmatrix} y_{pp} \\ y_{pq} \\ y_{qp} \\ y_{qq} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} \mathrm{y}_{p0} \\ \mathrm{y}_{pq} \\ \mathrm{y}_{qp} \\ \mathrm{y}_{q0} \end{bmatrix} , \qquad \text{(A.1)}$$

and from elementary matrix manipulations, we write

$$\begin{bmatrix} \dfrac{df}{dy_{pp}} \\ \dfrac{df}{dy_{pq}} \\ \dfrac{df}{dy_{qp}} \\ \dfrac{df}{dy_{qq}} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \dfrac{df}{d\mathrm{y}_{p0}} \\ \dfrac{df}{d\mathrm{y}_{pq}} \\ \dfrac{df}{d\mathrm{y}_{qp}} \\ \dfrac{df}{d\mathrm{y}_{q0}} \end{bmatrix} . \qquad \text{(A.2)}$$

REFERENCES

J.W. Bandler and M.A. El-Kady (1979a), "A new, a.c. approach to power system sensitivity analysis and planning", Department of Electrical and Computer Engineering, McMaster University, Hamilton, Canada, Report SOS-79-5-R.

J.W. Bandler and M.A. El-Kady (1979b), "A unified approach to power system sensitivity analysis and planning, part I: family of adjoint systems", Department of Electrical and Computer Engineering, McMaster University, Hamilton, Canada, Report SOS-79-8-C.

J.W. Bandler and M.A. El-Kady (1979c), "A unified approach to power system sensitivity analysis and planning, part II: special class of adjoint systems", Department of Electrical and Computer Engineering, McMaster University, Hamilton, Canada, Report SOS-79-9-C.

J.W. Bandler and M.A. El-Kady (1980a), "A unified approach to power system sensitivity analysis and planning, part III: consistent selection of adjoining coefficients", Department of Electrical and Computer Engineering, McMaster University, Hamilton, Canada, Report SOS-80-1-R.

J.W. Bandler and M.A. El-Kady (1982), "A complex Lagrangian approach with applications to power network sensitivity analysis", IEEE Trans. Circuit and Systems, vol. 29, pp. 1-6.

J.W. Bandler, M.A. El-Kady and H.K. Grewal (1983), "Sensitivity evaluation and optimization of the IEEE 118-bus power system", Department of Electrical and Computer Engineering, McMaster University, Hamilton, Canada, Report SOS-83-19-R.

J.W. Bandler, M.A. El-Kady, H.K. Grewal and H. Gupta (1982), "XLF2 - A program for analysis and sensitivity evaluation of complex load flows by the complex Lagrangian method", Department of Electrical and Computer Engineering, McMaster University, Hamilton, Canada, Report SOS-82-7-U.

J.W. Bandler, M.A. El-Kady, H.K. Grewal and J. Wojciechowski (1983), "XLF3 - A Fortran implementation of the complex Lagrangian method to power system analysis and design", Department of Electrical and Computer Engineering, McMaster University, Hamilton, Canada, Report SOS-83-18-U.

J.W. Bandler, M.A. El-Kady, W. Kellermann and W.M. Zuberek (1983), "LFLNR – A Fortran implementation of the Newton-Raphson load flow technique", Department of Electrical and Computer Engineering, McMaster University, Hamilton, Canada, Report SOS-83-7-U.

J.W. Bandler, M.A. El-Kady and J. Wojciechowski (1983), "TTM1 - A Fortran implementation of the Tellegen theorem method to power system simulation and design", Department of Electrical and Computer Engineering, McMaster University, Hamilton, Canada, Report SOS-82-12-U.

J.W. Bandler and R.E. Seviora (1970), "Current trends in network optimization", IEEE Trans. Microwave Theory Tech., vol. MTT-18, pp. 1159-1170.

R.C. Burchett and H.H. Happ (1983), "Large scale security dispatching: an exact model", IEEE/PES Winter Meeting, New York, paper no. 83 WM 078-3.

R.C. Burchett, H.H. Happ and K.A. Wirgau (1982), "Large scale optimal power flow", IEEE Trans. Power Apparatus and Systems, vol. 101, pp. 3722-3732.

D.A. Calahan (1968), Computer-aided Network Design. New York: McGraw-Hill.

C.A. Desoer and E.S. Kuh (1969), Basic Circuit Theory. New York: McGraw-Hill.

S.W. Director (1975), Circuit Theory - A Computational Approach. New York: Wiley.

S.W. Director and R.A. Rohrer (1969), "Generalized adjoint network and network sensitivities", IEEE Trans. Circuit Theory, vol. 16, pp. 318-323.

H.W. Dommel and W.F. Tinney (1968), "Optimal power flow solutions", IEEE Trans. Power Apparatus and Systems, vol. 87, pp. 1866-1876.

I.S. Duff (1977), "A survey of sparse matrix research", Proc. IEEE, vol. 65, pp. 500-535.

M.A. El-Kady (1980), "A unified approach to generalized network sensitivities with applications to power system analysis and planning", Ph.D. Thesis, Department of Electrical and Computer Engineering, McMaster University, Hamilton, Canada.

M.E. El-Hawary and G.S. Christenen (1979), Optimal Economic Operation of Electric Power Systems. New York: Academic Press.

C.A. Gross (1979), Power System Analysis. New York: Wiley.

A.E. Guile (1977), Electrical Power Systems. New York: Pergamon Press.

Z.X. Han (1982), "Phase shifter and power flow control", IEEE Trans. Power Apparatus and Systems, vol. 101, pp. 3790-3795.

W.J. Lyman (1930), "Controlling power flow with phase shifting equipment", AIEE Trans., vol. 49, pp. 825-831.

W.J. Lyman and J.R. North (1938), "Application of large phase-shifting transformer on an interconnected system loop", AIEE Trans., vol. 57, pp. 579-587.

B.A. Murtagh and M.A. Saunders (1980), "MINOS/AUGMENTED user's manual", Department of Operations Research, Stanford University, Stanford, California, Technical Report SOL 80-14.

P. Penfield, Jr., R. Spence and S. Duinker (1970), "A generalized form of Tellegen's theorem", IEEE Trans. Circuit Theory, vol. 17, pp. 302-305.

J. Peschon, D.S. Piercy, W.F. Tinney and O.J. Tveit (1968), "Sensitivity in power systems", IEEE Trans. Power Apparatus and Systems, vol. 87, pp. 1687-1696.

A.M. Sasson (1969), "Nonlinear programming solutions for load-flow, minimum-loss, and economic dispatching problems", IEEE Power Apparatus and Systems, vol. 88, pp. 399-408.

G.W. Stagg and A.H. El-Abiad (1968), Computer Methods in Power System Analysis. New York: McGraw-Hill.

B. Stott (1974), "Review of load-flow calculation methods", Proc. IEEE, vol. 62, pp. 916-926.

W.F. Tinney and J.W. Walker (1967), "Direct solutions of sparse network equations by optimally ordered triangular factorization", Proc. IEEE, vol. 55, pp. 1801- 1809.

J.E. Van Ness and J.H. Griffin (1961), "Elimination methods for load flow studies", AIEE Trans. Power Apparatus and Systems, vol. 80, pp. 299-302.

AUTHOR INDEX

– AUTHOR INDEX (continued)