EXACT SENSITIVITIES OF TWO-PORT POWER NETWORK ELEMENTS USING THE SHORT-CIRCUIT ADMITTANCE DESCRIPTION

J.W. Bandler, M.A. El-Kady and H.K. Grewal

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Abstract

This paper complements the application of augmented Tellegen theorem developed by Bandler and El-Kady to nonreciprocal two-port elements. The short-circuit admittance description of the two-port elements is utilized in the adjoint network formulation. A consistent modelling of phase-shifting transformer is provided, and sensitivities of a general network function w.r.t. transformer impedance and complex turns ratio are derived in an elegant manner. The transformations necessary for obtaining sensitivities in both rectangular and polar modes are appended. The theoretical results are verified numerically by investigating a simple illustrative example.

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The authors are with the Simulation Optimization Systems Research Laboratory and the Department of Electrical and Computer Engineering, McMaster University, Hamilton, Canada L8S 4L7.

M.A. El-Kady is also with Ontario Hydro, Toronto, Canada.

I. INTRODUCTION

The augmented Tellegen theorem [1]-[4] has facilitated exact a.c. load flow modelling and sensitivity evaluation in power networks. The generalized version of the theorem has been successfully applied to power networks comprising reciprocal transmission elements. Nevertheless, the approach is quite promising and is extended to nonreciprocal two-port elements in this paper.

The two-port elements are characterized by two independent relationships involving four variables, viz. port voltages and port currents [5]. We investigate the short-circuit admittance description of these elements and express the port currents in terms of the independent port voltages. A general network function is considered and an adjoint two-port network is formulated. The sensitivity matrix df/dy is obtained using matrix manipulations [6]. The control variables of phase-shifting transformers are considered in complex mode. The sensitivity matrix is exploited with the aid of conjugate notation, and sensitivities w.r.t. transformer impedance and turns ratio are derived explicitly. The transformations necessary for sensitivities in real modes are also provided. A two-bus sample power system shown in Fig. 1, is exploited to verify the theoretical results listed in Table I. We have considered the load bus voltage magnitude as the function of interest. The adjoint transmission network model of the system is displayed in Fig. 2.

II. APPLICATION OF AUGMENTED TELLEGEN THEOREM TO GENERAL TWO-PORT ELEMENTS

We consider a general two-port network inserted between nodes p and q, and describe it by its short-circuit admittance parameters. The current-voltage relationships for this network are expressed, in matrix form, as

$$\begin{bmatrix} I_p \\ I_q \end{bmatrix} = \begin{bmatrix} y_{pp} & y_{pq} \\ y_{qp} & y_{qq} \end{bmatrix} \begin{bmatrix} V_p \\ V_q \end{bmatrix} , \tag{1}$$

or in more compact notation, we write

$$\mathbf{I} = \mathbf{y}\,\mathbf{V}\,. \tag{2}$$

The vectors **I** and **V** are current and voltage two-component vectors, respectively, and **y** is the short-circuit admittance matrix associated with the two-port network considered. The perturbed form of (2) is expressed as

$$\delta\mathbf{I} = \mathbf{y}\,\delta\mathbf{V} + \delta\mathbf{y}\,\mathbf{V}, \tag{3}$$

which relates the first-order variation of current vector **I** to the first-order changes of voltage vector **V** and the short-circuit admittance matrix **y**.

The augmented Tellegen sum developed by Bandler and El-Kady [1]-[4] is considered next. The terms associated with branches p and q are

$$\hat{I}_p\,\delta V_p + \hat{I}_p^*\,\delta V_p^* - \hat{V}_p\,\delta I_p - \hat{V}_p^*\,\delta I_p^*$$

$$+\ \hat{I}_q\,\delta V_q + \hat{I}_q^*\,\delta V_q^* - \hat{V}_q\,\delta I_q - \hat{V}_q^*\,\delta I_q^*\,. \tag{4}$$

This portion of the Tellegen sum can be expressed in matrix form as

$$\hat{\mathbf{I}}^T\,\delta\mathbf{V} + \hat{\mathbf{I}}^{*T}\,\delta\mathbf{V}^* - \hat{\mathbf{V}}^T\,\delta\mathbf{I} - \hat{\mathbf{V}}^{*T}\,\delta\mathbf{I}^*\,, \tag{5}$$

where $\hat{\mathbf{I}}$ and $\hat{\mathbf{V}}$ are the adjoint current and voltage vectors, respectively. We use (3) and its conjugate in (5). The expression is written as

$$(\hat{\mathbf{I}}^T - \hat{\mathbf{V}}^T\,\mathbf{y})\delta\mathbf{V} - \hat{\mathbf{V}}^T\,\delta\mathbf{y}\,\mathbf{V}$$

$$+\ (\hat{\mathbf{I}}^{*T} - \hat{\mathbf{V}}^{*T}\,\mathbf{y}^*)\,\delta\mathbf{V}^* - \hat{\mathbf{V}}^{*T}\,\delta\mathbf{y}^*\,\mathbf{V}^*. \tag{6}$$

The first-order change of a general real function f is given by

$$\delta f = \sum_b \left(\frac{\partial f}{\partial I_b}\,\delta I_b + \frac{\partial f}{\partial I_b^*}\,\delta I_b^* + \frac{\partial f}{\partial V_b}\,\delta V_b + \frac{\partial f}{\partial V_b^*}\,\delta V_b^*\right). \tag{7}$$

The terms associated with branches p and q in (7) are written in compact form, as

$$\left(\frac{\partial f}{\partial \mathbf{I}}\right)^T \delta\mathbf{I} + \left(\frac{\partial f}{\partial \mathbf{I}}\right)^{*T} \delta\mathbf{I}^* + \left(\frac{\partial f}{\partial \mathbf{V}}\right)^T \delta\mathbf{V} + \left(\frac{\partial f}{\partial \mathbf{V}}\right)^{*T} \delta\mathbf{V}^*\,. \tag{8}$$

Using (3) and its conjugate, we rewrite (8) as

$$\left[\left(\frac{\partial f}{\partial \mathbf{I}}\right)^T \mathbf{y} + \left(\frac{\partial f}{\partial \mathbf{V}}\right)^T\right] \delta\mathbf{V} + \left[\left(\frac{\partial f}{\partial \mathbf{I}}\right)^{*T} \mathbf{y}^* + \left(\frac{\partial f}{\partial \mathbf{V}}\right)^{*T}\right] \delta\mathbf{V}^*$$

$$+\left(\frac{\partial f}{\partial \mathbf{I}}\right)^{T} \delta \mathbf{y}\, \mathbf{V} + \left(\frac{\partial f}{\partial \mathbf{I}}\right)^{*T} \delta \mathbf{y}^{*}\, \mathbf{V}^{*}. \tag{9}$$

We define an adjoint system associated with the branches p and q so that

$$\hat{\mathbf{I}} = \mathbf{y}^{T}\left(\hat{\mathbf{V}} + \frac{\partial f}{\partial \mathbf{I}}\right) + \frac{\partial f}{\partial \mathbf{V}} \quad . \tag{10}$$

Now the first-order change of f reduces to

$$\delta f = \left(\hat{\mathbf{V}} + \frac{\partial f}{\partial \mathbf{I}}\right)^{T} \delta \mathbf{y}\, \mathbf{V} + \left(\hat{\mathbf{V}}^{*} + \frac{\partial f}{\partial \mathbf{I}^{*}}\right)^{T} \delta \mathbf{y}^{*}\, \mathbf{V}^{*} + \text{ other terms}, \tag{11}$$

or from matrix theory [6], we write

$$\frac{df}{d\mathbf{y}} = \left(\hat{\mathbf{V}} + \frac{\partial f}{\partial \mathbf{I}}\right) \mathbf{V}^{T} \ , \tag{12}$$

and

$$\frac{df}{d\mathbf{y}^{*}} = \left(\hat{\mathbf{V}}^{*} + \frac{\partial f}{\partial \mathbf{I}^{*}}\right) \mathbf{V}^{*T} \ . \tag{13}$$

Note that (12) and (13) represent sensitivities of function f w.r.t. short-circuit admittance matrix **y** and its conjugate, respectively, and are basically 2x2 matrices.

III. SENSITIVITY EVALUATION OF NONRECIPROCAL POWER NETWORK ELEMENTS

A two-port element is said to be nonreciprocal when $y_{pq} \neq y_{qp}$. In power networks, the phase shifting transformers with complex turns ratio [7] are categorized as nonreciprocal elements. The y-matrix pertaining to these elements is expressed as

$$\mathbf{y} = \begin{bmatrix} \frac{1}{Z_t a a^{*}} & -\frac{1}{Z_t a^{*}} \\ -\frac{1}{Z_t a} & \frac{1}{Z_t} \end{bmatrix} . \tag{14}$$

The first-order variation of **y** in terms of first-order changes in Z_t, Z^*_t, a and a*, is expressed as

$$\delta \mathbf{y} = \frac{1}{Z_t^2 a a^{*}} \begin{bmatrix} -1 & a \\ a^{*} & -aa^{*} \end{bmatrix} \delta Z_t + \mathbf{O}\, \delta Z_t^{*}$$

$$+ \frac{1}{Z_t a^2 a^{*}} \begin{bmatrix} -1 & 0 \\ a^{*} & 0 \end{bmatrix} \delta a + \frac{1}{Z_t a a^{*2}} \begin{bmatrix} -1 & a \\ 0 & 0 \end{bmatrix} \delta a^{*} . \tag{15}$$

The complex conjugate $\delta\mathbf{y}^*$ from (15) is then written as

$$\delta\mathbf{y}^* = \mathbf{O}\,\delta Z_t + \frac{1}{Z_t^{*2} a a^*}\begin{bmatrix} -1 & a^* \\ a & -aa^* \end{bmatrix}\delta Z_t^*$$

$$+\frac{1}{Z_t^* a^2 a^*}\begin{bmatrix} -1 & a^* \\ 0 & 0 \end{bmatrix}\delta a + \frac{1}{Z_t^* a a^{*2}}\begin{bmatrix} -1 & 0 \\ a & 0 \end{bmatrix}\delta a^* . \tag{16}$$

We substitute (15) and (16) in (11), and rewrite it as

$$\delta f = \left(\hat{\mathbf{V}} + \frac{\partial f}{\partial \mathbf{I}}\right)^T \left(\frac{1}{Z_t^2 a a^*}\begin{bmatrix} -1 & a \\ a^* & -aa^* \end{bmatrix}\delta Z_t + \frac{1}{Z_t a^2 a^*}\begin{bmatrix} -1 & 0 \\ a^* & 0 \end{bmatrix}\delta a\right.$$

$$\left. + \frac{1}{Z_t a a^{*2}}\begin{bmatrix} -1 & a \\ 0 & 0 \end{bmatrix}\delta a^*\right)\mathbf{V}$$

$$+\left(\hat{\mathbf{V}}^* + \frac{\partial f}{\partial \mathbf{I}^*}\right)^T \left(\frac{1}{Z_t^{*2} a a^*}\begin{bmatrix} -1 & a^* \\ a & -aa^* \end{bmatrix}\delta Z_t^* + \frac{1}{Z_t^* a^2 a^*}\begin{bmatrix} -1 & a^* \\ 0 & 0 \end{bmatrix}\delta a\right.$$

$$\left. + \frac{1}{Z_t^* a a^{*2}}\begin{bmatrix} -1 & 0 \\ a & 0 \end{bmatrix}\delta a^*\right)\mathbf{V}^* + \text{ other terms.} \tag{17}$$

Hence, the derivatives of a real function f w.r.t. Z_t and a are

$$\frac{df}{dZ_t} = \frac{1}{Z_t^2 a a^*}\left(\hat{\mathbf{V}} + \frac{\partial f}{\partial \mathbf{I}}\right)^T \begin{bmatrix} -1 & a \\ a^* & -aa^* \end{bmatrix}\mathbf{V}, \tag{18}$$

and

$$\frac{df}{da} = \frac{1}{Z_t a^2 a^*}\left(\hat{\mathbf{V}} + \frac{\partial f}{\partial \mathbf{I}}\right)^T \begin{bmatrix} -1 & 0 \\ a^* & 0 \end{bmatrix}\mathbf{V}$$

$$+\frac{1}{Z_t^* a^2 a^*}\left(\hat{\mathbf{V}}^* + \frac{\partial f}{\partial \mathbf{I}^*}\right)^T \begin{bmatrix} -1 & a^* \\ 0 & 0 \end{bmatrix}\mathbf{V}^* . \tag{19}$$

The expressions in (18) and (19) provide sensitivities in complex mode, however, sensitivities w.r.t. R_t, X_t, a_1 and a_2 are obtained in the following manner

$$\frac{df}{dR_t} = 2\,\mathrm{Re}\left\{\frac{df}{dZ_t}\right\}, \tag{20}$$

$$\frac{df}{dX_t} = -2\,\mathrm{Im}\left\{\frac{df}{dZ_t}\right\}, \tag{21}$$

$$\frac{df}{da_1} = 2\,\mathrm{Re}\left\{\frac{df}{da}\right\} \tag{22}$$

and

$$\frac{df}{da_2} = -2\,\mathrm{Im}\left\{\frac{df}{da}\right\}. \tag{23}$$

The sensitivities w.r.t. $|a|$ and ϕ, the magnitude and phase angle of complex turns ratio of phase-shifting transformer, are of practical interest [7]. These sensitivities are

$$\begin{bmatrix} \dfrac{df}{d|a|} \\ \dfrac{df}{d\phi} \end{bmatrix} = \begin{bmatrix} \dfrac{a_1}{|a|} & \dfrac{a_2}{|a|} \\ -a_2 & a_1 \end{bmatrix} \begin{bmatrix} \dfrac{df}{da_1} \\ \dfrac{df}{da_2} \end{bmatrix}. \tag{24}$$

In order to deduce the sensitivity relations for tap-changing-under-load transformers, we observe the turns ratio as a real quantity. We write (19) as

$$\frac{df}{da} = \frac{1}{Z_t a^3}\left(\hat{\mathbf{V}} + \frac{\partial f}{\partial \mathbf{I}}\right)^T \begin{bmatrix} -2 & a \\ a & 0 \end{bmatrix} \mathbf{V}$$

$$+ \frac{1}{Z_t^* a^3}\left(\hat{\mathbf{V}}^* + \frac{\partial f}{\partial \mathbf{I}^*}\right)^T \begin{bmatrix} -2 & a \\ a & 0 \end{bmatrix} \mathbf{V}^*. \tag{25}$$

Note that these transformers have only one practical control variable associated with their turns ratio. These elements are classified as reciprocal two-port elements because their short-circuit admittance matrix is symmetrical.

The transmission lines are symmetrical two-port elements with unity turns ratio. Their contribution to power network sensitivities is through line and/or shunt conductance and susceptance. The sensitivity relations for this type of element is readily deduced from (18) by substituting $a = a^* = 1.0$, that is

$$\frac{df}{dZ_t} = \frac{1}{Z_t^2}\left(\hat{\mathbf{V}} + \frac{\partial f}{\partial \mathbf{I}}\right)^T \begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix} \mathbf{V} \tag{26}$$

or

$$\frac{df}{dY_t} = \left(\hat{\mathbf{V}} + \frac{\partial f}{\partial \mathbf{I}}\right)^T \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \mathbf{V} . \tag{27}$$

We express (27) in a convenient form [1], [3]

$$\frac{df}{dY_t} = \hat{v}_t \; v_t , \tag{28}$$

where

$$\hat{v}_t = \left(\hat{V}_p + \frac{\partial f}{\partial I_p}\right) - \left(\hat{V}_q + \frac{\partial f}{\partial I_q}\right) \tag{29}$$

and

$$v_t = V_p - V_q . \tag{30}$$

The sensitivity formulas derived in this paper are listed in Table I, where we have used the following notation

$$\hat{\hat{V}}_p = \hat{V}_p + \frac{\partial f}{\partial I_p} \tag{31}$$

and

$$\hat{\hat{V}}_q = \hat{V}_q + \frac{\partial f}{\partial I_q} . \tag{32}$$

We present a small size system to illustrate the theoretical results developed in this paper.

IV. AN ILLUSTRATIVE EXAMPLE

A 2-bus sample power system is shown in Fig. 1. The phase-shifting transformer turns ratio is taken as 0.8 + j 0.6. The nodal admittance matrix of the system is

$$\mathbf{Y} = \begin{bmatrix} 6 - j18 & 7.2 + j19.6 \\ -16.8 + j12.4 & 6 - j17 \end{bmatrix} ,$$

and the load-flow solution for specified S_1 and V_2 is

$$V_1 = 0.46573 - j\,0.60442$$

and

$$S_2 = 5.67052 + j\,1.07059 .$$

We consider the load bus voltage magnitude as a function of interest. The adjoint transmission branches are shown in Fig. 2. The adjoint system of equations [1], [3] is simply

$$\begin{bmatrix} G_{11}+\psi_{11}^{1} & -B_{11}+\psi_{12}^{1} \\ B_{11}+\psi_{21}^{1} & G_{11}+\psi_{22}^{1} \end{bmatrix} \begin{bmatrix} \hat{V}_{11} \\ \hat{V}_{12} \end{bmatrix} = - \begin{bmatrix} \mathrm{Re}\left\{\frac{\partial f}{\partial V_1}\right\} \\ \mathrm{Im}\left\{\frac{\partial f}{\partial V_1}\right\} \end{bmatrix},$$

or

$$\begin{bmatrix} 13.17154 & 11.00949 \\ -24.99051 & -1.17154 \end{bmatrix} \begin{bmatrix} \hat{V}_{11} \\ \hat{V}_{12} \end{bmatrix} = - \begin{bmatrix} .30518 \\ .39606 \end{bmatrix}.$$

The solution of this system is

$$\hat{V}_1 = 0.01817 - j\,0.04945 .$$

We use Table I and observe $\hat{V}_2 = 0$, which reduces the sensitivity expression in (19) to

$$\frac{df}{da} = \frac{1}{a^2 Z_t}\,\hat{V}_1 ,$$

or numerically, we get

$$\frac{df}{da} = -\,0.88025 + j0.66023 .$$

The sensitivity result corresponding to the example considered is $-0.8815 + j\,0.6607$ by small perturbations. Another independent check is made on the sensitivities in polar mode. We use (22)-(23) to obtain sensitivities in rectangular mode and substitute these results in (24) to get the sensitivities of $|V_1|$ w.r.t $|a|$ and ϕ. These sensitivities are -2.20067 and 0.00007, indicating that the function under consideration is independent of ϕ.

V. CONCLUSIONS

The augmented Tellegen theorem has been applied to general two-port elements frequently encountered in power transmission networks. The short-circuit admittance description has been utilized to derive sensitivities of a general network function with respect to practical control variables of transmission elements. These sensitivity formulas have been verified numerically, and a simple illustrative example has been included.

VI. REFERENCES

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TABLE I

SENSITIVITIES OF A REAL FUNCTION

Control Variable	Description	Derivative
Z_t	transformer impedance	† $-I_t \hat{I}_t$
R_t	transformer resistance	$2\ \mathrm{Re}\left\{\frac{df}{dZ_t}\right\}$
X_t	transformer reactance	$-2\ \mathrm{Im}\left\{\frac{df}{dZ_t}\right\}$
a	transformer complex turns ratio	†† $-V_{sp}\hat{I}_p - \hat{V}^*_{sp} I^*_p$
a_1	real component of complex turns ratio	$2\ \mathrm{Re}\left\{\frac{df}{da}\right\}$
a_2	imaginary component of complex turn ratio	$-2\ \mathrm{Im}\left\{\frac{df}{da}\right\}$
$\|a\|$	magnitude of complex turns ratio	$\frac{1}{\|a\|}\left(a_1 \frac{df}{da_1} + a_2 \frac{df}{da_2}\right)$
ϕ	phase angle of complex turns ratio	$-a_2 \frac{df}{da_1} + a_1 \frac{df}{da_2}$

† $I_t = \left(\frac{V_p}{a} - V_q\right)/Z_t, \quad \hat{I}_t = \left(\frac{\hat{V}_p}{a^*} - \hat{V}_q\right)/Z_t$

†† $I_p = \frac{I_t}{a^*}, \quad \hat{I}_p = \frac{\hat{I}_t}{a}, \quad V_{sp} = \frac{V_p}{a}, \quad \hat{V}_{sp} = \frac{\hat{V}_p}{a^*}$

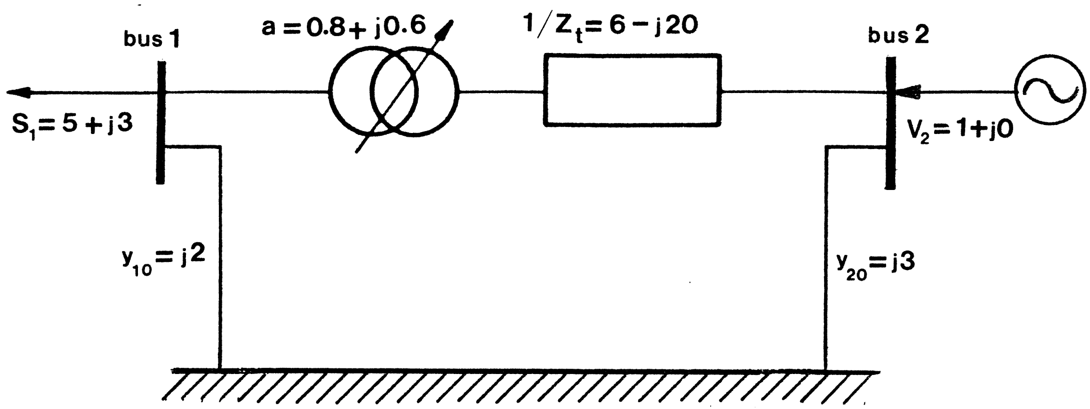

Fig. 1. A 2-bus sample power system.

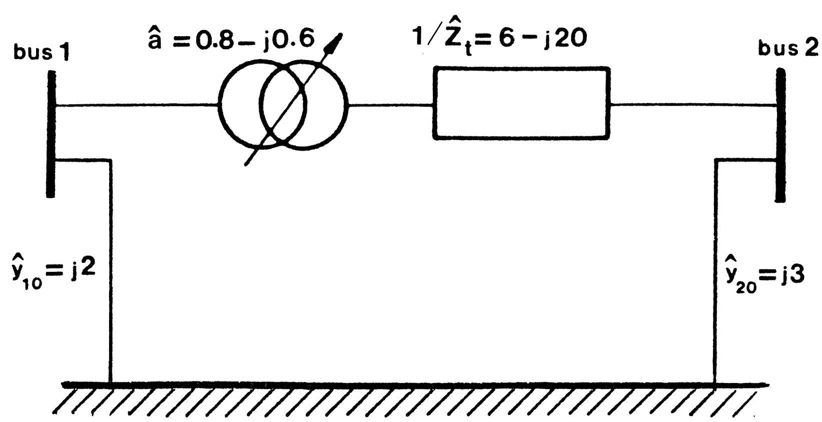

Fig.2 Representation of adjoint transmission branches associated with $|V_1|$.