EXACT SENSITIVITIES OF GENERAL POWER NETWORK COMPONENTS VIA COMPLEX BRANCH MODELLING

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Abstract

This paper complements the application of augmented Tellegen theorem approach to sensitivity evaluation in power networks. We have considered the y-matrix description of nonreciprocal two-port elements. The generalized, steady-state complex branch modelling technique accomplished by Bandler and El-Kady is applied to phase-shifting transformers. The adjoint model of these transformers is envisaged in an elegant and straightforward manner. The derivation of reduced gradients pertaining to transformer impedance and complex turns ratio is fully delineated. The theoretical results are verified numerically on a 6-bus sample example by using the existing software systems.

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I. INTRODUCTION

The power network sensitivities play an important role in numerous steady-state power system analysis and design problems [1], [2]. Large power systems inherently require load exchanges at various disparate locations to be regulated accurately, and phase-shifting transformers are installed to facilitate the required adjustments in real and/or reactive power flows [3].

We present the two-port network theory to investigate the phase-shifting transformers which are categorized as nonreciprocal power network elements. We utilize the generalized complex branch modelling technique developed by Bandler and El-Kady [4] to derive sensitivity expressions pertaining to these transformers. The generalized technique apparently manifests the line responses while preserving the advantages of compactness, sparsity and simplicity of the adjoint system. Moreover, there are no restrictive assumptions on the branch models. The sensitivities of a general network function w.r.t. practical control variables of the transformers are derived in an elegant manner. Both the series impedance and complex turns ratio are adequately included in the adjoint formulation. An illustrative example on a 6-bus sample system elucidates the theoretical results developed in this paper. We have considered a load bus voltage magnitude as the function of interest and checked the results by small perturbations at the nominal point.

II. APPLICATION OF COMPLEX BRANCH MODELLING TO TWO-PORT NETWORKS

We consider a two-port element inserted between nodes p and q as shown in Fig. 1. In general, we assume the turns ratio a_t to be complex. The port currents are expressed in terms of port voltages as

$$\mathbf{I} = \mathbf{y}\,\mathbf{V}, \tag{1}$$

where $\mathbf{I} \triangleq [I_p \; I_q]^T$, $\mathbf{V} \triangleq [V_p \; V_q]^T$, and $\mathbf{y}$ is the short-circuit admittance matrix of the element considered.

The basic variable vector $\mathbf{w}_b$ associated with the two-port branch is an 8-element vector defined as

$$\mathbf{w}_b = \begin{bmatrix} \mathbf{w}_{bV} \\ \mathbf{w}_{bI} \end{bmatrix} \triangleq \begin{bmatrix} \mathbf{V}^c \\ \mathbf{I}^c \end{bmatrix} , \tag{2}$$

where $\mathbf{V}^c \triangleq [V_p\ V_q\ V_p^*\ V_q^*]^T$ and $\mathbf{I}^c \triangleq [I_p\ I_q\ I_p^*\ I_q^*]^T$. The element variables are judiciously selected, that is

$$\mathbf{z}_b = \begin{bmatrix} \mathbf{x}_b \\ \mathbf{u}_b \end{bmatrix} \triangleq \begin{bmatrix} \mathbf{w}_{bV} \\ \mathbf{u}_b \end{bmatrix} , \tag{3}$$

where $\mathbf{u}_b \triangleq [Z_t\ Z_t^*\ a_t\ a_t^*]^T$. Using (1)- (3), we write [6]

$$\delta\mathbf{w}_{bI} = \boldsymbol{\tau}\,\delta\mathbf{x}_b + \boldsymbol{\kappa}\,\delta\mathbf{u}_b \tag{4}$$

and

$$\delta\mathbf{w}_{bV} = \mathbf{1}\,\delta\mathbf{x}_b + \mathbf{0}\,\delta\mathbf{u}_b. \tag{5}$$

We combine (4) and (5), and write in more compact form

$$\delta\mathbf{w}_b = \begin{bmatrix} \mathbf{1} & \mathbf{0} \\ \boldsymbol{\tau} & \boldsymbol{\kappa} \end{bmatrix} \delta\mathbf{z}_b . \tag{6}$$

The augmented Tellegen sum [5] is expressed as

$$\sum_b \hat{\mathbf{w}}_b^T \begin{bmatrix} \mathbf{0} & -\mathbf{1} \\ \mathbf{1} & \mathbf{0} \end{bmatrix} \delta\mathbf{w}_b = 0 , \tag{7}$$

where b stands for branches ranging from a load bus to a nonreciprocal element. Note that $\hat{\mathbf{w}}_b$ comprises adjoint port variables similar to those of $\mathbf{w}_b$. We rewrite (7) as

$$\sum_b \hat{\mathbf{w}}_b^T \bar{\mathbf{1}}\ \delta\mathbf{w}_b = 0 , \tag{8}$$

where

$$\bar{\mathbf{1}} \triangleq \begin{bmatrix} \mathbf{0} & -\mathbf{1} \\ \mathbf{1} & \mathbf{0} \end{bmatrix} . \tag{9}$$

The matrix $\bar{\mathbf{1}}$ is 4 x 4 for two-terminal branches and 8 x 8 for two-port branches. Substituting (6) in (8), we get

$$\sum_b \hat{\mathbf{w}}_b^T \begin{bmatrix} -\boldsymbol{\tau} & -\boldsymbol{\kappa} \\ \mathbf{1} & \mathbf{0} \end{bmatrix} \delta\mathbf{w}_b = 0 . \tag{10}$$

We use the transformed adjoint two-port variables [4], and express (10) as

$$\sum_{b} \left(\hat{\eta}_{bx}^{T}\, \delta \mathbf{x}_b + \hat{\eta}_{bu}^{T}\, \delta \mathbf{u}_b \right) = 0 \ , \tag{11}$$

where

$$\hat{\eta}_{bx} = \hat{\mathbf{w}}_{bI} - \boldsymbol{\tau}^{T} \hat{\mathbf{w}}_{bV} \tag{12}$$

and

$$\hat{\eta}_{bu} = -\boldsymbol{\kappa}^{T} \hat{\mathbf{w}}_{bV} \, . \tag{13}$$

The transformed adjoint variable vector $\hat{\eta}_{bx}$ is set equal to $\partial f/\partial \mathbf{x}_b$, where f is the network function whose reduced gradients [5], [6] are required. These gradients are simply

$$\frac{df}{d\mathbf{u}_b} = \frac{\partial f}{\partial \mathbf{u}_b} + \boldsymbol{\kappa}^{T} \hat{\mathbf{w}}_{bV} \, . \tag{14}$$

We use equations (12) and (14) in the next section to derive some useful sensitivity relations.

III. SENSITIVITY EVALUATION OF NONRECIPROCAL POWER NETWORKS

A power network comprising phase shifting transformers is said to be a nonreciprocal power network. These transformers [6] are characterized by their unsymmetrical y-matrix. For a general transmission element, we consider

$$\mathbf{y} = \begin{bmatrix} \dfrac{1}{Z_t a_t a_t^{*}} & -\dfrac{1}{Z_t a_t^{*}} \\ -\dfrac{1}{Z_t a_t} & \dfrac{1}{Z_t} \end{bmatrix} . \tag{15}$$

The port currents in (1) are written using (15), in an alternative convenient form

$$\mathbf{I} = \frac{1}{Z_t}\left(\frac{V_p}{a_t} - V_q \right) \begin{bmatrix} \dfrac{1}{a_t^{*}} \\ -1 \end{bmatrix} . \tag{16}$$

The coefficient matrices $\boldsymbol{\tau}$ and $\boldsymbol{\kappa}$ involved in (4) are obtained from (1) and (16) as

$$\boldsymbol{\tau} = \begin{bmatrix} \mathbf{y} & 0 \\ 0 & \mathbf{y}^{*} \end{bmatrix} \tag{17}$$

and

$$\boldsymbol{\kappa} = \begin{bmatrix} \boldsymbol{\kappa}_1 & 0 & \boldsymbol{\kappa}_2 & \bar{\boldsymbol{\kappa}}_2 \\ 0 & \boldsymbol{\kappa}_1^{*} & \bar{\boldsymbol{\kappa}}_2^{*} & \boldsymbol{\kappa}_2^{*} \end{bmatrix} , \tag{18}$$

where κ_1, κ_2 and $\overline{\kappa}_2$ are formal partial derivatives of **I** w.r.t. Z_t, a_t and a_t^*, respectively.

The adjoint transformer model has the same original series impedance, however, its complex turns ratio is simply

$$\hat{a}_t = a_t^* . \tag{19}$$

The excitations at nodes p and q in the adjoint model for a real function f are $\partial f/\partial I_p$ and $\partial f/\partial I_q$, respectively.

We use (14), (16) and (18) to obtain df/dZ_t and df/da_t. These expressions are

$$\frac{df}{dZ_t} = \frac{\partial f}{\partial Z_t} - \frac{1}{Z_t^2}\left(\frac{V_p}{a_t} - V_q\right) \cdot \left(\frac{\hat{V}_p}{a_t^*} - \hat{V}_q\right) \tag{20}$$

and

$$\frac{df}{da_t} = \frac{\partial f}{\partial a_t} - \frac{1}{a_t^2}\left\{ \frac{V_p}{Z_t}\left(\frac{\hat{V}_p}{a_t^*} - \hat{V}_q\right) + \frac{\hat{V}_p^*}{Z_t^*}\left(\frac{V_p^*}{a_t^*} - V_q^*\right)\right\} , \tag{21}$$

respectively. We use $V_s = V_p/a_t$ as the secondary voltage, and $V_t = V_s - V_q$ as the voltage across impedance Z_t in the expressions (20) and (21). These equations are rewritten as

$$\frac{df}{dZ_t} = \frac{\partial f}{\partial Z_t} - I_t \hat{I}_t \tag{22}$$

and

$$\frac{df}{da_t} = \frac{\partial f}{\partial a_t} - \mathbf{V}_s^T \mathbf{1}^R \mathbf{I}_p^* , \tag{23}$$

where $I_t \triangleq V_t/Z_t$, $\mathbf{V}_s \triangleq [V_s \ \hat{V}_s^*]^T$, $\mathbf{I}_p \triangleq [I_p \ \hat{I}_p^*]^T$ and $\mathbf{1}^R$ is the rotation matrix. These theoretical results are investigated in the following section by considering a sample example [6].

IV. NUMERICAL EXAMPLE

We consider a 6-bus sample power system (Fig. 2) and utilize the existing software systems [7]-[9] in a manner shown in Figs. 3 and 4. The data relevant to the power system is provided in the Appendix. The nodal admittance matrix is obtained using the XLF3 package [7]. The load flow solution is determined by using the fast decoupled method [8]. We formulate an adjoint system of equations [8] for $f = |V_1|$. The coefficient matrix, RHS vector and solution vector of the adjoint system are displayed in Table I. We use the sensitivity relations (22), (23) to obtain df/dZ_t and df/da_t. Suitable transformations [6] are utilized to determine sensitivities in the real mode. These results are summarised as

$$\frac{df}{d\Phi_t} = \begin{bmatrix} -.459281 \\ -.008697 \\ .332271 \\ -.011565 \end{bmatrix},$$

where $\Phi_t \triangleq [R_t \; X_t \; |a_t| \; \Phi_t]^T$; and have been checked by small perturbations at the nominal point.

V. CONCLUSIONS

We have utilized the generalized complex branch modelling technique based on the exact a.c. load flow model and extended its domain to cover nonreciprocal power network elements. Our approach preserves all the esoteric features of the previous related work. It also facilitates much simpler and smaller manipulations to interpret the adjoint element models. The exact sensitivities of network functions of interest can be investigated for a larger range of power system control capabilities. The power system problems, namely, overloading of cables, upsetting contractual exchange values and future transmission reinforcements, can be handled efficiently by exploiting the phase-shifting transformers in a coordinated manner.

VI. REFERENCES

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APPENDIX

DATA PERTAINING TO THE 6-BUS SAMPLE SYSTEM

BUS DATA

BUS INDEX	! BUS TYPE	!! U
1	ℓ	−2.4 +j0.
2	ℓ	−2.4 +j0.
3	ℓ	−1.6 −j0.4
4	g	−0.3 +j1.02
5	g	1.25+j1.04
6	n	1.04+j0.

TRANSMISSION NETWORK DATA

ELEMENT	R_t	X_t	a_t
1,4	.0500	.200	.8+j.6
1,5	.0250	.100	1.0
2,3	.1000	.400	1.0
2,4	.1000	.400	1.0
2,5	.0500	.200	1.0
2,6	.01875	.075	1.0
3,4	.1500	.600	1.0
3,6	.0375	.150	1.0

! ℓ LOAD BUS
g GENERATOR BUS
n SLACK BUS

!! $P_\ell + jQ_\ell$
$P_g + j|V_g|$
$V_{n1} + jV_{n2}$

STRUCTURAL DETAILS OF MATRICES AND VECTORS INVOLVED IN SENSITIVITY EVALUATION

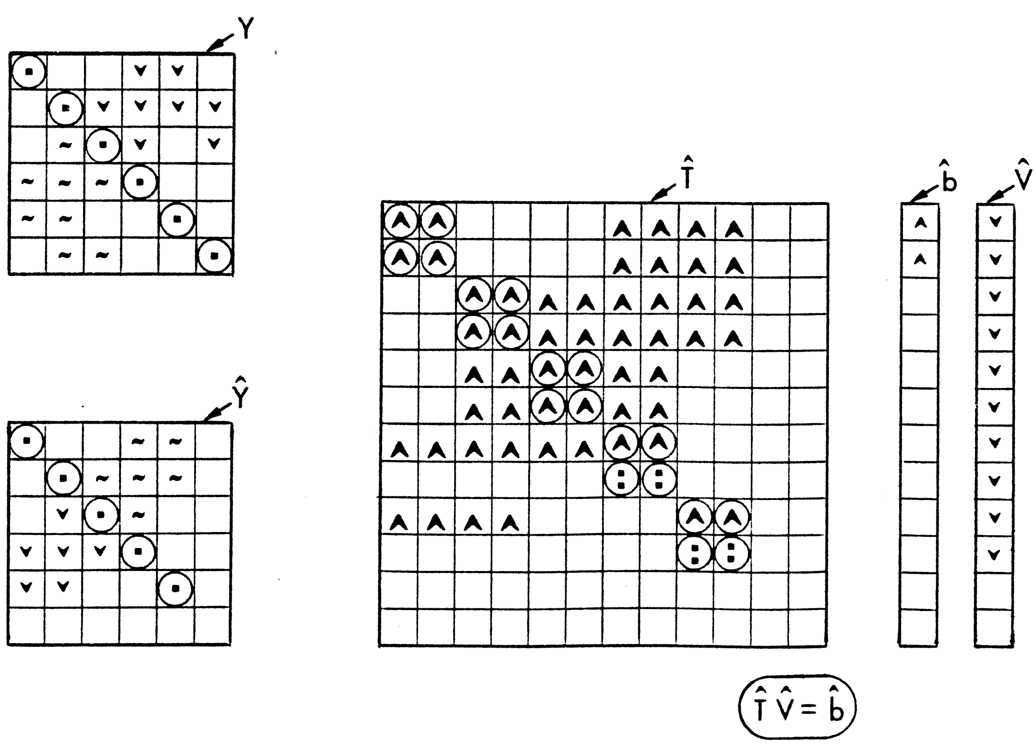


TABLE I

SUMMARY OF THE ADJOINT SYSTEM

```
ELEMENTS OF THE RHS VECTOR
------------------------------

   1: -.4545053E+00     2: -.2083864E+00     3: 0.               4: 0.
   5: 0.                6: 0.                7: 0.               8: 0.
   9: 0.               10: 0.               11: 0.              12: 0.


NONZERO ELEMENTS OF THE COEFFICIENT MATRIX
------------------------------------------------
(VALUE OF AN ELEMENT IS PRECEDED BY THE ROW AND COLUMN INDICES)

BUS NO.1
--------
   1, 1: .1889E+01     2, 1:-.1602E+02     1, 2: .1221E+02     2, 2: .5170E+01
   1, 7: .1882E+01     2, 7: .4471E+01     1, 8:-.4471E+01     2, 8: .1882E+01
   1, 9:-.2353E+01     2, 9: .9412E+01     1,10:-.9412E+01     2,10:-.2353E+01

BUS NO.2
--------
   3, 3: .3293E+01     4, 3:-.2343E+02     3, 4: .2049E+02     4, 4: .7687E+01
   3, 5:-.5882E+00     4, 5: .2353E+01     3, 6:-.2353E+01     4, 6:-.5882E+00
   3, 7:-.5882E+00     4, 7: .2353E+01     3, 8:-.2353E+01     4, 8:-.5882E+00
   3, 9:-.1176E+01     4, 9: .4706E+01     3,10:-.4706E+01     4,10:-.1176E+01

BUS NO.3
--------
   5, 5: .1283E+01     6, 5:-.1193E+02     5, 6: .8466E+01     6, 6: .3815E+01
   5, 3:-.5882E+00     6, 3: .2353E+01     5, 4:-.2353E+01     6, 4:-.5882E+00
   5, 7:-.3922E+00     6, 7: .1569E+01     5, 8:-.1569E+01     6, 8:-.3922E+00

BUS NO.4
--------
   7, 7:-.6923E+01     7, 8:-.3751E+01     7, 1: .4936E+01     7, 2:-.3441E+00
   7, 3: .2078E+01     7, 4: .1343E+01     7, 5: .1385E+01     7, 6: .8954E+00
   8, 7:-.7450E+00     8, 8: .6967E+00

BUS NO.5
--------
   9, 9:-.1393E+02     9,10:-.1972E+01     9, 1: .1006E+02     9, 2: .7253E+00
   9, 3: .5032E+01     9, 4: .3626E+00    10, 9:-.3241E+00    10,10: .9882E+00


SOLUTION VECTOR IN COMPLEX MODE
--------------------------------

BUS NO.1:      .877710E-03 ,  -.375274E-01
BUS NO.2:     -.494961E-04 ,   .142684E-03
BUS NO.3:      .270136E-03 ,   .399208E-03
BUS NO.4:      .165236E-02 ,   .176685E-02
BUS NO.5:     -.127451E-02 ,  -.418017E-03
BUS NO.6:     0.           ,  0.

SENSITIVITIES OF VM1:
---------------------
     1:-.4592806E+00
     2:-.8697112E-02
     3: .3322709E+00
     4:-.1156535E-01
```

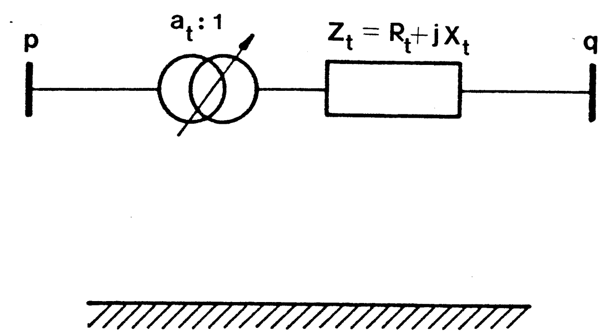


Fig. 1 Representation of a nonreciprocal two-port element.

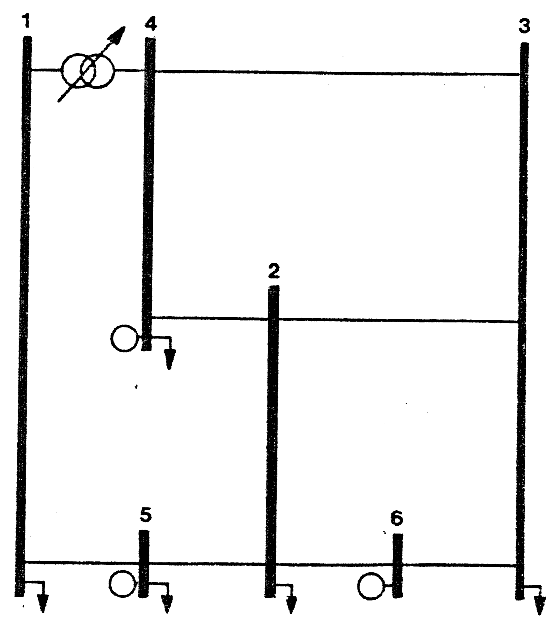
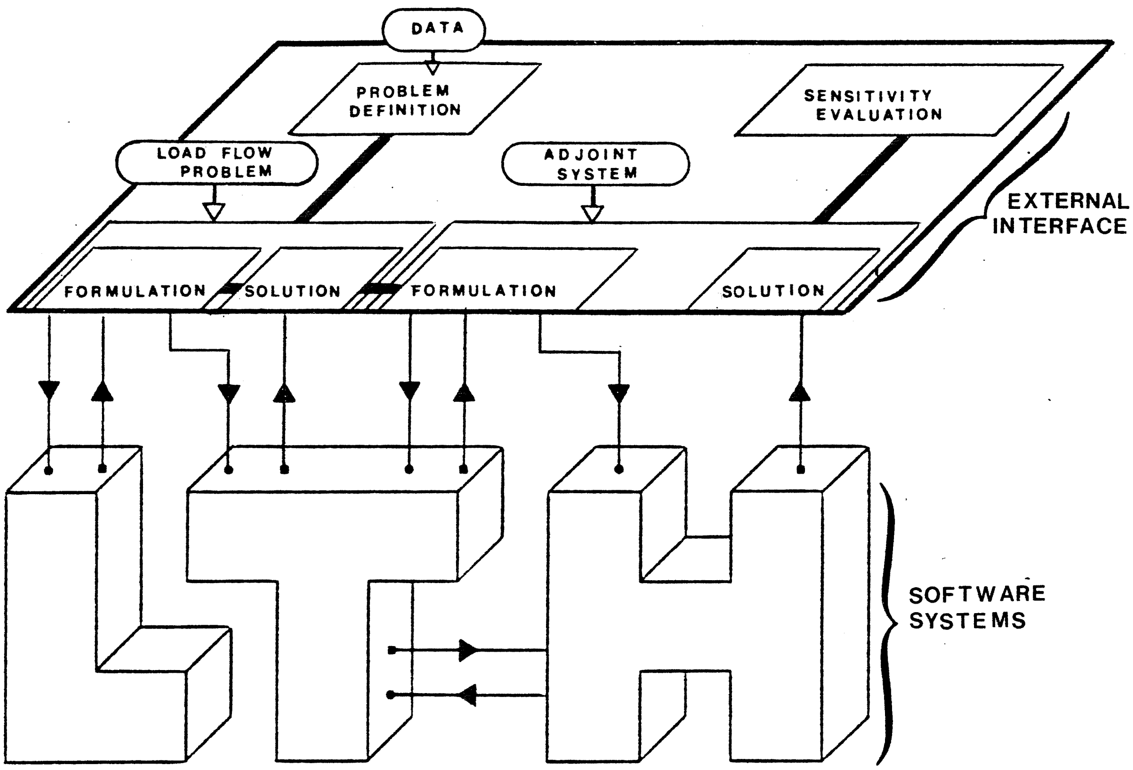


Fig. 2 6-bus sample power system.

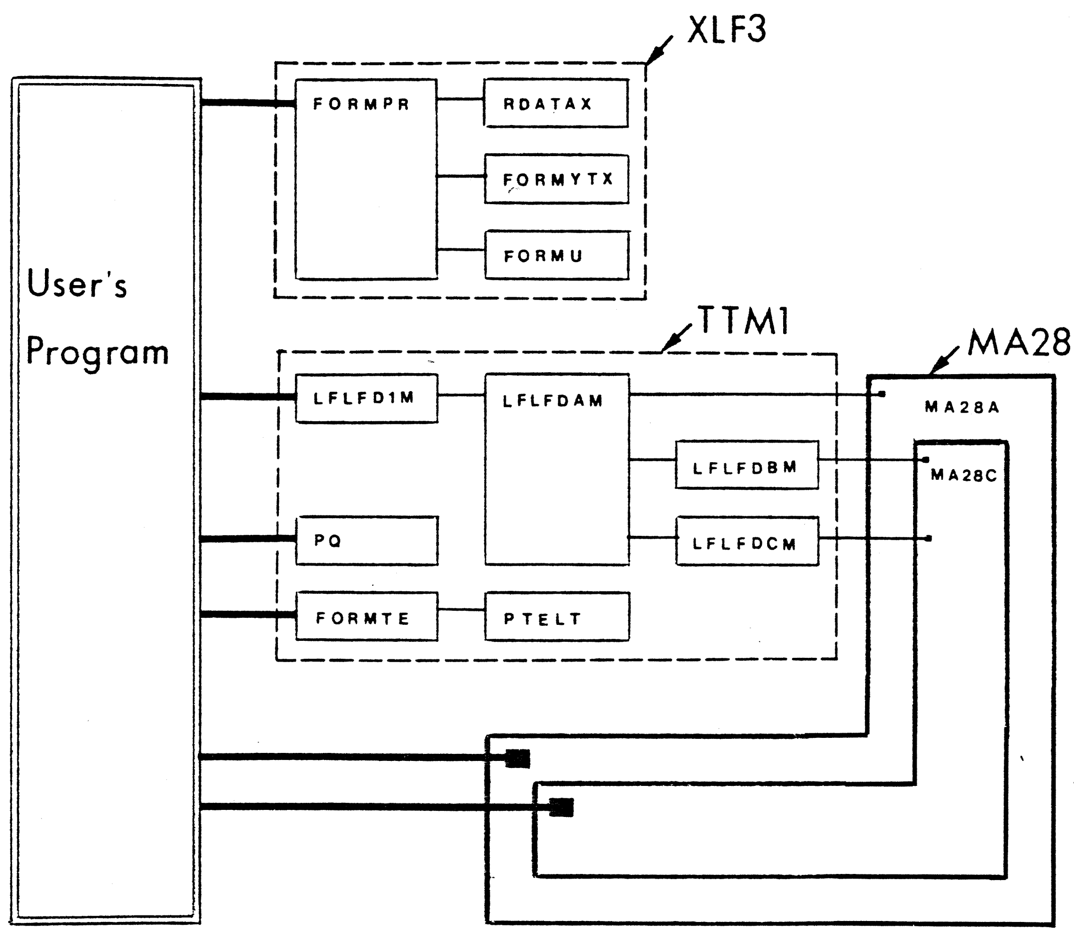


L – XLF3 package [7]

T – TTM1 package [8]

H – MA28 Harwell package [9]

Fig. 3 Conceptual structure for sensitivity evaluation of nonreciprocal power networks.



XLF3 package [7]

TTM1 package [8]

MA28 Harwell package [9]

Fig. 4 Schematic details for general sensitivity evaluation.