APPLICATION OF THE COMPLEX LAGRANGIAN APPROACH TO NONRECIPROCAL POWER NETWORKS

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SOS-83-22-R

October 1983

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Abstract

The nonreciprocal two-port elements frequently encountered in power transmission network are briefly described in this paper. The nodal admittance matrix of the power system is assumed unsymmetrical. The load flow equations are retained in the complex mode and the Jacobian at the load flow solution is utilized to obtain sensitivites of a general network function with respect to several control variables. We have considered both bus-type and transmission element-type control variables. The short-circuit admittance description of the two-port elements is emphasized and the sensitivity relations for phase-shifting transformers are derived in complex, rectangular and polar modes. The theoretical results are numerically verified by investigating a 6-bus sample system, a 26-bus system and the IEEE 118-bus system.

This work was supported by the Natural Sciences and Engineering Research Council of Canada under Grants A7239, A1708 and G0647.

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I. INTRODUCTION

The phase-shifting transformers are frequently installed to control bulk power flow in transmission networks [1]-[3]. These transformers are basically nonreciprocal two-port elements [4], [5] having complex turns ratio [6], [7]. The voltage at the output terminals of these transformers is usually controlled by two independent adjustments, namely, the magnitude $|a|$ and phase-angle ϕ of the complex turns ratio. The control actions may be manual or automatic and require a knowledge of sensitivities w.r.t. the practical control variables.

In this paper, we apply the complex Lagrangian approach accomplished by Bandler and El-Kady [8]-[11] to nonreciprocal power networks. We consider the short-circuit admittance description of general two-port network elements. The sensitivities of a general network function with respect to the internal impedance and complex turns ratio of phase-shifting transformers are derived in a straight-forward manner. However, suitable transformations are presented facilitating the other modes of formulation. We consider a 6-bus sample system, a 26-bus system and the IEEE 118-bus system to verify the sensitivity relations numerically [12], [13].

II. NONRECIPROCAL POWER NETWORK ELEMENTS

Phase-shifting transformers are categorized as nonreciprocal power transmission elements owing to their complex turns ratio [6], [7]. These transformers are capable of changing the complex voltage and current at a particular node to a prescribed value; and, therefore, help in delaying the need for future transmission reinforcement.

The short-circuit admittance matrix of a phase-shifting transformer installed between nodes p and q is expressed as

$$\mathbf{y} = \begin{bmatrix} \dfrac{1}{Z_t a_t a_t^*} & -\dfrac{1}{Z_t a_t^*} \\ -\dfrac{1}{Z_t a_t} & \dfrac{1}{Z_t} \end{bmatrix}, \tag{1}$$

where a_t and Z_t are complex turns ratio and series impedance of the transformer, respectively. The off-diagonal elements in (1) are unequal when a_t is complex, however, the tap-changing-under-load transformers have a real turns ratio and the short-circuit admittance matrix associated with such transmission elements is symmetrical. The voltage at the output terminals of the phase shifting transformers is controlled by two independent adjustments. These adjustments are usually carried out in steps and their practical values are

$$0.90 \leq |a_t| \leq 1.10 \text{ with } \Delta|a_t| = 0.025 \text{ (p.u.)}$$

$$-10° \leq \phi_t \leq +10° \text{ with } \Delta\phi_t = 2.5°.$$

The control actions may be manual or automatic [3], complete with output sensors and feedback methods, and the device may be used to control real and reactive power flow in a power transmission network.

The sensitivities with respect to turns ratio and transformer impedance are of prime importance and are derived in the following sections, by implementing the complex Lagrangian approach [13]. The knowledge of these sensitivities finds practical utility in power system planning and operations, and helps in providing possible relief in overloaded transmission facilities.

III. COMPLEX LAGRANGIAN APPROACH TO POWER NETWORK SENSITIVITIES

Bandler and El-Kady have demonstrated the method of complex Lagrange multipliers [8]-[10] which utilizes a compact complex notation and exploits the Jacobian available at the load flow solution. The transformations relating different ways of formulating power network equations have been described [14],[15]. The basic form of these equations is

$$\mathbf{E}_M^* \mathbf{Y}_T \mathbf{V}_M = \mathbf{S}_M^* , \tag{2}$$

where $\mathbf{Y}_T$ is the complex bus admittance matrix of the network comprising transmission lines and regulating transformers, $\mathbf{V}_M$ is a column vector of the complex bus voltages, $\mathbf{E}_M \triangleq \text{diag}\, \mathbf{V}_M$, $\mathbf{S}_M \triangleq \mathbf{P}_M + j\,\mathbf{Q}_M$ and * denotes the complex conjugate. The equations associated with generator buses have been successfully accommodated in (2) by expressing the known quantities (i.e. P_g, $|V_g|$) on the right-hand side. Hence, we write (2) as

$$\mathbf{S}^* = f(\mathbf{V}_M , \mathbf{V}_M^*) \tag{3}$$

or in perturbed form

$$\mathbf{K}\ \delta\mathbf{V}_M + \overline{\mathbf{K}}\ \delta\mathbf{V}_M^* = \mathbf{d}\ , \tag{4}$$

where $\mathbf{d} \triangleq \delta\mathbf{S}^* - \mathbf{E}_M^*\ \delta\mathbf{Y}_T\ \mathbf{V}_M$, the matrices $\mathbf{K}$ and $\overline{\mathbf{K}}$ constitute the formal partial derivatives of $\mathbf{S}^*$ of (2) with respect to $\mathbf{V}_M$ and $\mathbf{V}_M^*$, respectively. Combining the complex conjugate of (4), we write

$$\begin{bmatrix} \mathbf{K} & \overline{\mathbf{K}} \\ \overline{\mathbf{K}}^* & \mathbf{K}^* \end{bmatrix} \begin{bmatrix} \delta\mathbf{V}_M \\ \delta\mathbf{V}_M^* \end{bmatrix} = \begin{bmatrix} \mathbf{d} \\ \mathbf{d}^* \end{bmatrix} \tag{5}$$

which represents the linearized load flow equations in complex mode . The problem is solved by implementing the complex Newton's method for some test power systems [11,12]. Table I displays a summary of the exact a.c. load flow solutions using the flat voltage profile. The advantage of the exact a.c. load flow solution is that the Jacobian (irrespective of the mode of formulation) can be utilized in solving the adjoint systems.

We express the first-order change of a real function f in the form

$$\delta f = [\hat{\mathbf{V}}^T \quad \hat{\mathbf{V}}^{*T}] \begin{bmatrix} \mathbf{d} \\ \mathbf{d}^* \end{bmatrix}\ , \tag{6}$$

where $\hat{\mathbf{V}}$ is a solution vector of an adjoint system whose coefficient matrix is the transpose of the Jacobian available at the load flow solution and the right-hand side vector contains the formal partial derivatives of function f with respect to the bus voltages. The equation (6), basically represents a linear relationship between the first-order change of f and first-order changes in complex quantities $\mathbf{S}$, $\mathbf{S}^*$, $\mathbf{Y}_T$ and $\mathbf{Y}_T^*$. Hence, we write [13], [16]

$$\frac{df}{d\mathbf{S}} = \hat{\mathbf{V}}^* \tag{7}$$

and

$$\frac{df}{d\mathbf{Y}_T} = -\mathbf{E}_M^*\ \hat{\mathbf{V}}^R\ \mathbf{V}_M^T, \tag{8}$$

where $\hat{V}^R_i \triangleq \hat{V}_{i1} + j\, a_i\, \hat{V}_{i2}$, $a_i = 1$ for load buses and 0 for generator buses. The nxn sensitivity matrix represented in (8) displays the sensitivities of f with respect to the complex bus admittance matrix $\mathbf{Y}_T$ and is unsymmetrical for nonreciprocal power networks. We derive the relations for the two-port transmission elements, namely, lines and regulating transformers, by considering the short-circuit admittance description of these elements. For a general two-port network connected between nodes p and q, we write

$$\frac{df}{d\mathbf{y}} = -\begin{bmatrix} V_p^* & 0 \\ 0 & V_q^* \end{bmatrix} \begin{bmatrix} \hat{V}^R_p \\ \hat{V}^R_q \end{bmatrix} [V_p \;\; V_q],$$

or

$$\frac{df}{dy_{ij}} = -\, V_i^*\, \hat{V}^R_i\, V_j, \tag{9}$$

where y_{ij} represents an element of the short-circuit admittance matrix (e.g. for $i = j = p$, y_{ij} is the driving point admittance at node p of the two-port network considered). We consider the nonreciprocal two-port elements characterized by their complex turns ratio a_t and series impedance Z_t (or $Y_t = 1/Z_t$) in the per unit system. Using the complex conjugate notation, the first-order change of f is expressed as

$$\delta f = \sum_{\substack{i=p,q \\ j=p,q}} \left(\frac{\partial f}{\partial y_{ij}}\, \delta y_{ij} + \frac{\partial f}{\partial y^*_{ij}}\, \delta y^*_{ij} \right). \tag{10}$$

Using Table II, we write the sensitivities of f with respect to a_t and Z_t as

$$\frac{df}{da_t} = \frac{1}{a_t^2}\left[-\frac{1}{a_t^*}\left(\frac{1}{Z_t}\frac{\partial f}{\partial y_{pp}} + \frac{1}{Z_t^*}\frac{\partial f}{\partial y^*_{pp}} \right) + \frac{1}{Z_t}\frac{\partial f}{\partial y_{qp}} + \frac{1}{Z_t^*}\frac{\partial f}{\partial y^*_{pq}} \right] \tag{11}$$

and

$$\frac{df}{dZ_t} = \frac{1}{Z_t^2}\left[-\left(\frac{1}{a_t a_t^*}\frac{\partial f}{\partial y_{pp}} + \frac{\partial f}{\partial y_{qq}} \right) + \frac{1}{a_t}\frac{\partial f}{\partial y_{qp}} + \frac{1}{a_t^*}\frac{\partial f}{\partial y_{pq}} \right]. \tag{12}$$

The expressions in (11) and (12) are simplified by implementing (9) appropriately, that is

$$\frac{df}{da_t} = \frac{1}{a_t^2}\left[\frac{V_p}{Z_t}\left(\frac{V_p^*}{a_t^*}\hat{V}^R_p - V_q^*\hat{V}^R_q \right) + \frac{V_p}{Z_t^*}\hat{V}^{R*}_p\left(\frac{V_p^*}{a_t^*} - V_q^* \right) \right] \tag{13}$$

and

$$\frac{df}{dZ_t} = \frac{1}{Z_t^2}\left(\frac{V_p}{a_t} - V_q\right)\left(\frac{V_p^*}{a_t^*}\,\hat{V}_p^R - V_q^*\,\hat{V}_q^R\right). \tag{14}$$

The expressions derived so far are associated with complex control variables, however, the practical power system design variables are real quantities as shown in Table III. The sensitivities in rectangular mode are obtained using

$$\frac{df}{d\zeta_{i1}} = 2\,\mathrm{Re}\left\{\frac{df}{d\zeta_i}\right\} \tag{15}$$

and

$$\frac{df}{d\zeta_{i2}} = -\,2\,\mathrm{Im}\left\{\frac{df}{d\zeta_i}\right\}, \tag{16}$$

where $\zeta_i \triangleq \zeta_{i1} + j\,\zeta_{i2.}$ The phase-shifting transformers are usually known for their control actions in polar mode,and it is imperative to derive the required transformations. We express a_{t1} and a_{t2} as

$$a_{t1} = |a_t|\cos\phi_t, \tag{17}$$

and

$$a_{t2} = |a_t|\sin\phi_t. \tag{18}$$

The perturbed form of (17) and (18) is written in matrix form as

$$\begin{bmatrix} \delta a_{t1} \\ \delta a_{t2} \end{bmatrix} = \begin{bmatrix} \cos\phi_t & -|a_t|\sin\phi_t \\ \sin\phi_t & |a_t|\cos\phi_t \end{bmatrix} \begin{bmatrix} \delta|a_t| \\ \delta\phi_t \end{bmatrix}. \tag{19}$$

Equation (19) bears a linear relation between the first-order changes of complex turns ratio in rectangular and polar modes, therefore, the sensitivities in polar mode are simply

$$\begin{bmatrix} \dfrac{df}{d|a_t|} \\ \dfrac{df}{d\phi_t} \end{bmatrix} = \begin{bmatrix} \cos\phi_t & \sin\phi_t \\ -|a_t|\sin\phi_t & |a_t|\cos\phi_t \end{bmatrix} \begin{bmatrix} \dfrac{df}{da_{t1}} \\ \dfrac{df}{da_{t2}} \end{bmatrix}, \tag{20}$$

or

$$\begin{bmatrix} \dfrac{df}{d|a_t|} \\ \dfrac{df}{d\phi_t} \end{bmatrix} = \begin{bmatrix} \dfrac{a_{t1}}{|a_t|} & \dfrac{a_{t2}}{|a_t|} \\ -\,a_{t2} & a_{t1} \end{bmatrix} \begin{bmatrix} \dfrac{df}{da_{t1}} \\ \dfrac{df}{da_{t2}} \end{bmatrix}. \tag{21}$$

The tap-changing-under-load transformers have real turns ratio and equation (15) provides the required sensitivity expression. The transmission lines have unity turns ratio and contribute solely through their series and/or shunt admittances. We write the expression in (14) for transmission lines as

$$\frac{df}{dY_t} = -(V_p - V_q)(V_p^* \hat{V}_p^R - V_q^* \hat{V}_q^R). \tag{22}$$

For the shunt elements, we simply observe $V_q = \hat{V}_q^R = 0$, and the expression (22) reduces to

$$\frac{df}{dY_{sh}} = -V_p V_p^* \hat{V}_p^R. \tag{23}$$

The sensitivity of f with respect to generator bus shunt admittance turns out to be a real quantity, which indicates that

$$\frac{df}{dB_{sh}} = 0, \tag{24}$$

or in other words, the generator buses have only one practical shunt control parameter [10].

The sensitivity expressions derived above have been implemented in a computer package, XLF3 [12]; and some test power systems are considered in the following section.

IV. NUMERICAL EXAMPLES

A 6-Bus Sample Power System

The 6-bus power system has been augmented to include a phase-shifting transformer with a large phase angle ϕ_t. This impractical value is chosen for analysis purpose only. A one-line diagram of the system is shown in Fig. 1. The nodal admittance matrix of the power network is shown in Table IVa, and the exact a.c. load flow solution is displayed in Table IVb. The Jacobian available at the solution is utilized for the sensitivity evaluation of a network state associated with a load bus, namely, $|V_1|$. We express the function of interest as

$$f = |V_1| = (V_1 V_1^*)^{1/2}.$$

The non-zero element in df/d**V** is simply

$$\frac{df}{dV_1} = \frac{0.5\, V_1^*}{|V_1|}.$$

The sensitivity results are displayed in Table IVc and have been verified by small perturbations about the nominal point.

A 26-Bus System

This power system (Saskatchewan Power Corporation System) has been considered [10]-[13] in some relevant studies on steady-state power system analysis. The single line diagram of this system is shown in Fig. 2. We consider a generator bus state, namely, the voltage angle at $g = 20$ and express it as

$$f = \delta_{20} = \tan^{-1}\left[\frac{V_{20} - V_{20}^*}{j(V_{20} + V_{20}^*)} \right] .$$

The non-zero element in $df/d\mathbf{V}$ is

$$\frac{df}{dV_{20}} = \frac{-j0.5}{V_{20}} .$$

The sensitivities with respect to various network control variables are shown in Table V.

The IEEE 118-Bus System

This system has been extensively used in various moderate size power flow problems. Fig. 3 shows a one-line diagram of the system comprising load buses ($n_L = 64$), generator buses ($n_G = 53$), a slack bus and transmission elements ($n_T = 179$). There are 2 phase-shifting transformers, 7 tap-changing-under-load transformers and 170 lines. Apart from this, there are 14 switchable shunt elements. We consider the slack bus real power P_n and express it as

$$f = P_n = \frac{S_n + S_n^*}{2} ,$$

where S_n is the slack bus complex power. The non-zero elements in $df/d\mathbf{V}$ are

$$\frac{df}{dV_k} = 0.5\,(Y_{nk} V_n^* + \delta_{kn} \sum_{j=1}^{n} Y_{nj}^* V_j^*) ,$$

where

$$\delta_{kn} = \begin{cases} 0 & \text{for } k \neq n \\ 1 & \text{for } k = n \end{cases} .$$

The sensitivities of this function are displayed in Table VI and have been verified by small perturbations about the nominal point.

V. CONCLUSIONS

We have provided a brief description of the nonreciprocal two-port elements frequently encountered in power transmission networks. The generalized sensitivity expressions are derived and tabulated in an elegant manner by considering the short-circuit admittance description. The sensitivity relations pertaining to complex turns ratio and impedance of phase-shifting transformers are provided in complex and real modes. These results can prove very useful in solving optimal power flow problems with gradient-based optimization routines.

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TABLE I

SUMMARY OF THE EXACT A.C. LOAD-FLOW SOLUTION

Power System	Number of Iterations	Accuracy	CPU in secs.
6-bus	5	.2276 E−06	0.139
23-bus	5	.2714 E−08	0.597
26-bus	5	.2203 E−09	0.551
118-bus	6	.5012 E−09	5.315

TABLE II

SHORT-CIRCUIT ADMITTANCE PARAMETRS AND CONTROL

VARIABLES OF A PHASE-SHIFTING TRANSFORMER

Parameter	Expression	Derivative w.r.t. a_t	Derivative w.r.t. a_t^*	Derivative w.r.t. Z_t	Derivative w.r.t. Z_t^*
y_{pp}	$\frac{1}{Z_t a_t a_t^*}$	$\frac{-1}{Z_t a_t^2 a_t^*}$	$\frac{-1}{Z_t a_t a_t^{*2}}$	$\frac{-1}{Z_t^2 a_t a_t^*}$	0
y_{pq}	$\frac{-1}{Z_t a_t^*}$	0	$\frac{1}{Z_t a_t^{*2}}$	$\frac{1}{Z_t^2 a_t^*}$	0
y_{qp}	$\frac{-1}{Z_t a_t}$	$\frac{1}{Z_t a_t^2}$	0	$\frac{1}{Z_t^2 a_t}$	0
y_{qq}	$\frac{1}{Z_t}$	0	0	$\frac{-1}{Z_t^2}$	0

TABLE III

PRACTICAL CONTROL VARIABLES OF A POWER NETWORK

Variable	Description
P_ℓ	demand real power
Q_ℓ	demand reactive power
P_g	generator real power
$\|V_g\|$	generator bus voltage magnitude
V_{n1}	real component of the slack bus voltage
G_t	line conductance of a transmission line
B_t	line susceptance of a transmission line
G_{sh}	shunt conductance of a transmission line
B_{sh}	shunt susceptance of a transmission line
a_{t1}	real component of the complex turns ratio
a_{t2}	imaginary component of the complex turns ratio
$\|a_t\|$	magnitude of the complex turns ratio
Φ_t	phase angle of the complex turns ratio
R_t	resistance of a phase-shifting transformer
X_t	reactance of a phase-shifting transformer

TABLE IVa

NODAL ADMITTANCE MATRIX YT

The sequence in each row is: Column Index, Real(YT), Imag(YT)

Bus No. 1

1:	3.529412	−14.117647	4:	−3.764706	3.058824
5:	−2.352941	9.411765			

Bus No. 2

2:	5.490196	−21.960784	3:	−.588235	2.352941
4:	−.588235	2.352941	5:	−1.176471	4.705882
6:	−3.137255	12.549020			

Bus No. 3

3:	2.549020	−10.196078	2:	−.588235	2.352941
4:	−.392157	1.568627	6:	−1.568627	6.274510

Bus No. 4

4:	2.156863	−8.627451	1:	1.882353	4.470588
2:	−.588235	2.352941	3:	−.392157	1.568627

Bus No. 5

5:	3.529412	−14.117647	1:	−2.352941	9.411765
2:	−1.176471	4.705882			

Bus No. 6

6:	4.705882	−18.823529	2:	−3.137255	12.549020
3:	−1.568627	6.274510			

TABLE IVb

LOAD FLOW SOLUTION OF THE 6-BUS SAMPLE SYSTEM

Bus Index	Rectangular Coordinates		Polar Coordinates	
1	.88812	−j .40719	.97702	−.42989
2	.91190	−j .27668	.95295	−.29459
3	.82475	−j .29829	.87703	−.34704
4	.69671	−j .74498	1.02000	−.81887
5	.98821	−j .32411	1.04000	−.31693
6	1.04000	−j 0	1.04000	0

TABLE IVc

SENSITIVITIES OF $|V_1|$ OF THE 6-BUS SYSTEM

Load Bus Quantities -- Total Derivatives

Bus	Real Power	Reactive Power	Shunt Conductance	Shunt Susceptance
1	.030383	.070579	−.029003	.067373
2	.000012	−.000317	−.000011	−.000288
3	−.000889	−.000647	.000684	−.000497

Generator Bus Quantities -- Total Derivatives

Bus	Real Power	Voltage Magnitude	Shunt Conductance	Shunt Susceptance
4	−.004743	.370314	.004935	0.000000
5	.002579	.713495	−.002790	0.000000

Line Quantities -- Total Derivatives

Line Index	Element	Line Conductance	Line Susceptance
2	1,5	−.006597	−.007071
3	2,3	−.000045	.000059
4	2,4	.000789	.002293
5	2,5	−.000240	−.000031
6	2,6	.000092	.000009

Phase Shifter Quantities -- Total Derivatives

Element	Turns Ratio Magnitude	Turns Ratio Phase Angle	Internal Resistance	Internal Reactance
1,4	.332271	−.011565	−.459281	−.008697

TABLE V

SENSITIVITIES OF δ_{20} OF THE 26-BUS SYSTEM

Load Bus Quantities -- Total Derivatives

Bus	Real Power	Reactive Power	Shunt Conductance	Shunt Susceptance
1	.267581	−.007722	−.286998	−.008283
2	.008245	.001363	−.009414	.001557
3	.267275	−.006188	−.291205	−.006742
4	.290383	−.002045	−.285035	−.002007
5	.296479	.000000	−.301324	.000000
6	.080168	−.000773	−.085701	−.000826
7	.216789	−.004509	−.222613	−.004630
8	.269938	−.009039	−.241050	−.008071
9	.056949	−.002912	−.053309	−.002726
10	.009824	.002211	−.010611	.002388
11	.222100	−.005921	−.181376	−.004835
12	.025652	.000475	−.024130	.000447
13	.001374	.000267	−.001504	.000292
14	.133023	−.004979	−.118771	−.004446
15	.276482	−.005600	−.243064	−.004868
16	0.000000	0.000000	0.000000	0.000000
17	.272539	−.003772	−.236823	−.003278

Generator Bus Quantities -- Total Derivatives

Bus	Real Power	Voltage Magnitude	Shunt Conductance	Shunt Susceptance
18	.007581	.025614	−.008680	0.000000
19	.080415	−.039806	−.088657	0.000000
20	.307280	−.354038	−.307280	0.000000
21	.296479	−.219182	−.308457	0.000000
22	.255422	−.053318	−.202320	0.000000
23	0.000000	0.000000	0.000000	0.000000
24	.267232	−.045456	−.267232	0.000000
25	.296479	.000000	−.296479	0.000000

TABLE V (continued)

SENSITIVITIES OF δ_{20} OF THE 26-BUS SYSTEM

Line Quantities -- Total Derivatives

Line Index	Element	Line Conductance	Line Susceptance
3	16,23	0.000000	0.000000
4	23,26	0.000000	0.000000
6	9,10	.003064	−.008097
7	9,12	.000136	−.001057
8	12,26	.000889	−.001936
9	9,14	.001597	−.000024
10	11,14	.003088	.000301
11	19,26	−.003739	.007844
12	6,26	−.002147	.004467
13	6,19	−.000122	.000023
14	7,19	.004441	−.010615
15	6,7	.002757	−.005050
16	11,22	.000393	.000206
17	8,11	−.007535	.005931
18	17,22	−.005194	.001658
19	8,21	−.011203	.005182
20	17,21	−.014022	.004697
21	1,4	.000448	.000296
22	4,21	−.005019	.000829
25	2,13	−.000102	.000594
26	1,7	−.002295	.002933
27	15,20	.007868	.004280
28	2,18	−.000436	−.000098

TCUL Transformer Quantities -- Total Derivatives

Element	Turns Ratio	Internal Resistance	Internal Reactance
13,26	.022095	−.025511	.133014
26,16	0.000000	0.000000	0.000000
2,10	−.061836	−.122407	−.153065
15,1	.186552	−1.571927	1.408706
1,3	−.049465	−.149478	−.011940
24,3	.046384	−.009770	−.007182
5,21	.000000	−.011448	.000000
5,25	.000000	−.126153	−.000000

TABLE VI

SENSITIVITIES OF P_n OF THE 118-BUS SYSTEM

Load Bus Quantities -- Total Derivatives

Bus	Real Power	Reactive Power	Shunt Conductance	Shunt Susceptance
2	−1.039480	−.001776	1.222015	−.002087
5	−.995079	−.007493	1.152184	−.008676
9	−.977067	.000778	1.172358	.000933
17	−1.005144	.002108	.968617	.002032
20	−1.033789	−.004819	.903283	−.004211
21	−1.028398	−.008006	.872155	−.006790
22	−1.014626	−.007771	.850805	−.006516
23	−.983963	−.001731	.832173	−.001464
38	−1.004240	−.002092	1.088326	−.002267
39	−1.061038	−.005893	.981905	−.005454
41	−1.090438	−.001984	1.084327	−.001973
52	−1.031752	−.008662	.939432	−.007887
53	−1.025027	−.007288	.909682	−.006468
57	−1.020321	−.000690	.954047	−.000645
78	−.982543	−.007871	.982164	−.007868
79	−.975549	−.007183	.987800	−.007273
81	−.952472	.001083	.960059	.001093
82	−.947823	−.005311	.928035	−.005200
83	−.927885	−.004839	.900134	−.004694
84	−.885244	−.005147	.854794	−.004970
86	−.863121	−.001071	.844713	−.001048
106	−.977820	−.003407	.904207	−.003150
108	−.982431	−.000472	.917336	−.000441
117	−1.037589	−.003583	1.116755	−.003856

Phase Shifter Quantities -- Total Derivatives

Element	Turns Ratio Magnitude	Turns Ratio Phase Angle	Internal Resistance	Internal Reactance
63,59	.039796	.239011	1.860750	.312149
64,61	.029692	−.142203	.000229	−.009722

TABLE VI (continued)

SENSITIVITIES OF P_n OF THE 118-BUS SYSTEM

Generator Bus Quantities -- Total Derivatives

Bus	Real Power	Voltage Magnitude	Shunt Conductance	Shunt Susceptance
1	−1.050857	.882783	1.389758	0.000000
4	−1.032740	.726316	1.249615	0.000000
19	−1.030086	−.162855	.949328	0.000000
24	−.995664	−.377848	.824509	0.000000
25	−.940147	.297842	.884584	0.000000
34	−1.033093	−.198182	.950116	0.000000
36	−1.031219	−.001704	.956321	0.000000
40	−1.077351	−.769418	1.013680	0.000000
42	−1.092801	1.205544	1.322289	0.000000
59	−.987729	.229232	.968073	0.000000
61	−.965655	−.071162	.965655	0.000000
62	−.976510	−.071795	.976510	0.000000
65	−.959474	−.587420	.959474	0.000000
76	−1.018810	−.456361	.909822	0.000000
77	−.975753	−.110255	.995366	0.000000
80	−.945021	.463286	1.022135	0.000000
85	−.858860	−.313478	.841768	0.000000
87	−.856185	.013011	.873394	0.000000
99	−.935353	−.084126	.954153	0.000000
100	−.919584	.230646	.956735	0.000000
103	−.945492	.251899	.964496	0.000000
113	−1.012186	.595070	.992044	0.000000
116	−.989912	−.101326	.989912	0.000000

TCUL Transformer Quantities -- Total Derivatives

Element	Turns Ratio	Internal Resistance	Internal Reactance
8,5	.316354	11.930163	−.084332
26,25	−.004820	.529850	−.131223
30,17	−.058476	5.266261	.337658
38,37	.094365	5.157105	.443463
66,65	−.000316	9.261379	−.008021
81,80	.037468	.371101	.078608

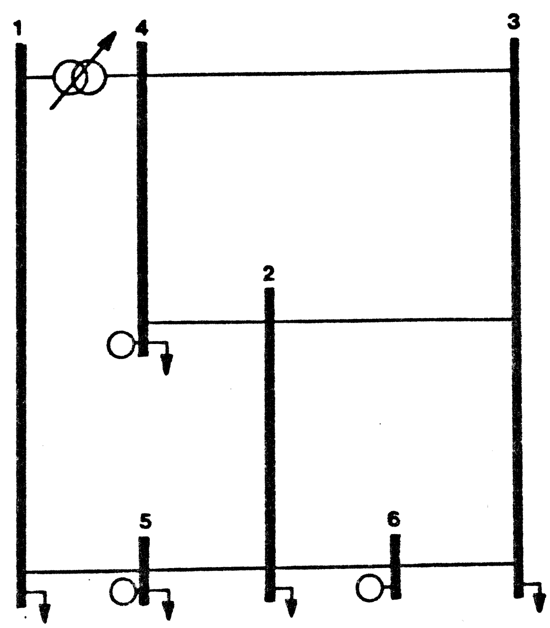

Fig. 1 6-bus sample power system

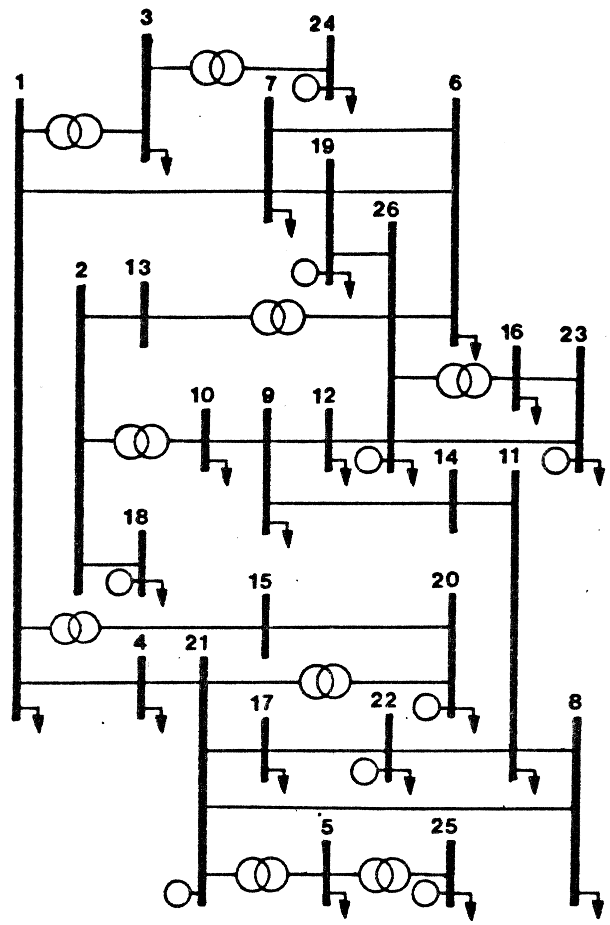

Fig. 2 26-bus power system

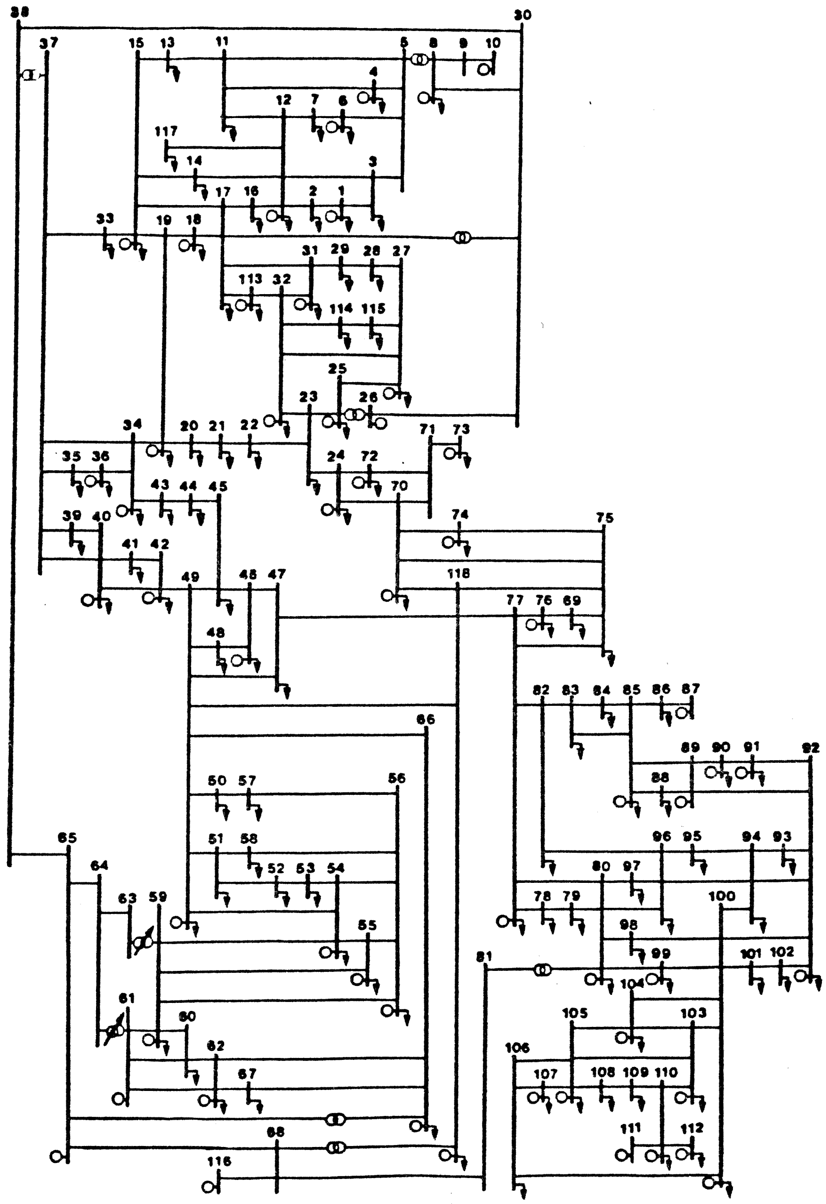

Fig.3 The IEEE 118-bus system