REVIEW OF THE WARD CLASS OF EXTERNAL EQUIVALENTS FOR POWER SYSTEMS

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SOS-83-23-R

October 1983

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REVIEW OF THE WARD CLASS OF EXTERNAL EQUIVALENTS

FOR POWER SYSTEMS

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Abstract

Steady state equivalents are of great importance for the study of the static characteristics of a large power system. There exist three general classes of external equivalents - Ward, REI and Linearization. This paper reviews the basic WARD-type equivalencing methods. We present the four basic Ward methods, namely, the standard Ward method, the Ward method with buffer zone, the extended Ward method, and the simplified Ward method. All these methods are well-conditioned, have standard load model, and are often used in load flow studies of large power systems. Several numerical examples are also included in this paper to illustrate Ward methods.

This work was supported by the Natural Sciences and Engineering Research Council of Canada under Grants A7239,A1708 and G0647.

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I. INTRODUCTION

Recently, a great deal of attention has been devoted to problems of external equivalents. Equivalencing methods for power systems were originated by Ward [1]. In his work, the external power system is substituted by a fixed equivalent admittance network obtained after Gaussian elimination of the external buses of the system admittance matrix. The external equivalents have been used for many years in off-line load flow studies to reduce the problem of the size and hence computer time and storage. Recently, attention has been given to the problems of using external equivalents in on-line static security analysis. Many different approaches for steady state and dynamic power network equivalents have been proposed. Three approaches, namely the Ward approach [1-6], the REI approach [2,6-9] and Linearization [2,3] are extensively used for determination of external equivalents. In this paper, we consider the Ward class approach. In Section II, we present the concept of the external equivalent and the basic type of the Ward equivalents. In Section III, Ward-type equivalent methods are summarized by flow-diagrams. In Section IV, several numerical examples using the existing test power systems [10-14], are presented to show the application of the methods.

II. THE WARD TYPE OF EXTERNAL EQUIVALENTS

Generally, the problem of steady state equivalents can be stated as follows. Given a solved load flow model of an interconnected power system as in Fig. 1, find a new equivalent load flow reduced model that has a smaller number of buses and branches than the interconnected power system such that for changes of the operating conditions in the internal system, the responses of the reduced system are close to those of the interconnected system.

Standard Ward Equivalent

According to Ward methods [1-6], the external system can be represented by a fixed admittance equivalent network connected to the internal system at the boundary buses and by equivalent injections at those buses. The external equivalent is obtained by eliminating the external voltages $\mathbf{V}_E$ from the nodal equations describing the interconnected power system that can be written in the form:

$$\begin{bmatrix} \mathbf{I}_I \\ \mathbf{I}_B \\ \mathbf{I}_E \end{bmatrix} = \begin{bmatrix} \mathbf{Y}_{II} & \mathbf{Y}_{IB} & \mathbf{0} \\ \mathbf{Y}_{BI} & \mathbf{Y}^I_{BB} + \mathbf{Y}^E_{BB} & \mathbf{Y}_{BE} \\ \mathbf{0} & \mathbf{Y}_{EB} & \mathbf{Y}_{EE} \end{bmatrix} \begin{bmatrix} \mathbf{V}_I \\ \mathbf{V}_B \\ \mathbf{V}_E \end{bmatrix}, \tag{1}$$

where $\mathbf{V}_I$, $\mathbf{V}_B$, $\mathbf{V}_E$ are nodal complex voltages at a particular operating point and the indices I, B, E refer to internal, boundary and external buses, respectively. From (1), we obtain

$$\mathbf{V}_E = \mathbf{Y}_{EE}^{-1}(\mathbf{I}_E - \mathbf{Y}_{EB}\mathbf{V}_B). \tag{2}$$

Eliminating the external buses with voltages $\mathbf{V}_E$ from (1), we obtain the Ward standard model given by

$$\begin{bmatrix} \mathbf{I}_I \\ \mathbf{I}_B - \mathbf{Y}_{BE}\mathbf{Y}_{EE}^{-1}\mathbf{I}_E \end{bmatrix} = \begin{bmatrix} \mathbf{Y}_{II} & \mathbf{Y}_{IB} \\ \mathbf{Y}_{BI} & \mathbf{Y}^I_{BB} + \mathbf{Y}^E_{BB} - \mathbf{Y}_{BE}\mathbf{Y}_{EE}^{-1}\mathbf{Y}_{EB} \end{bmatrix} \begin{bmatrix} \mathbf{V}_I \\ \mathbf{V}_B \end{bmatrix}, \tag{3}$$

with power injections at the boundary buses

$$\mathbf{S}_B^{eq} = \mathbf{E}_B \cdot \mathbf{I}_{eq}^*, \tag{4}$$

where $\mathbf{E}_B$ is a diagonal matrix of components of $\mathbf{V}_B$ in corresponding order, and

$$\mathbf{I}_{eq} = \mathbf{I}_B - \mathbf{Y}_{BE}\mathbf{Y}_{EE}^{-1}\mathbf{I}_E. \tag{5}$$

The external network in this model is represented by the equivalent nodal admittance matrix $\mathbf{Y}_{eq}$, given by

$$\mathbf{Y}_{eq} = \mathbf{Y}^E_{BB} - \mathbf{Y}_{BE}\mathbf{Y}_{EE}^{-1}\mathbf{Y}_{EB}. \tag{6}$$

Fig. 2 illustrates the internal system with the external equivalent to be attached at boundary buses. In order to obtain the model of the reduced system described by (3), it is quite unnecessary to know the external bus currents, when the boundary bus voltages are given. In fact, knowing the base case study-system bus voltage magnitudes $|V^0|$ and angles δ^0, it is easy to compute the equivalent injection. For each ith boundary bus, we have

$$P_i^{eq} = |V_i^0| \sum_{1 \le k \le n} |V_k^0| \left(G_{ik}\cos(\delta_i^0 - \delta_k^0) + B_{ik}\sin(\delta_i^0 - \delta_k^0)\right), \tag{7a}$$

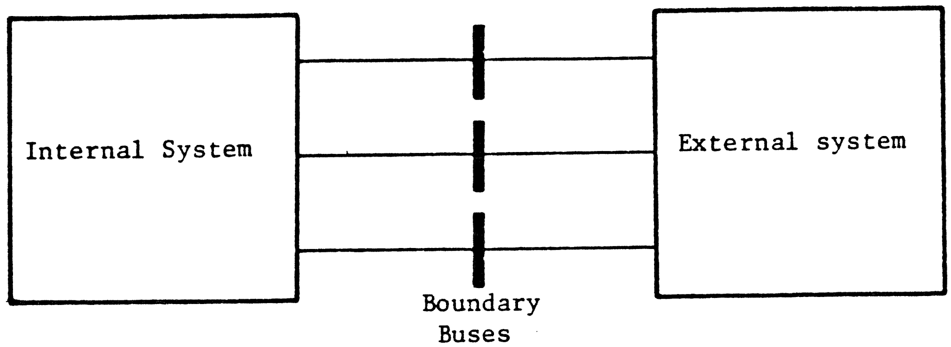

Fig.1 Interconnected power system.

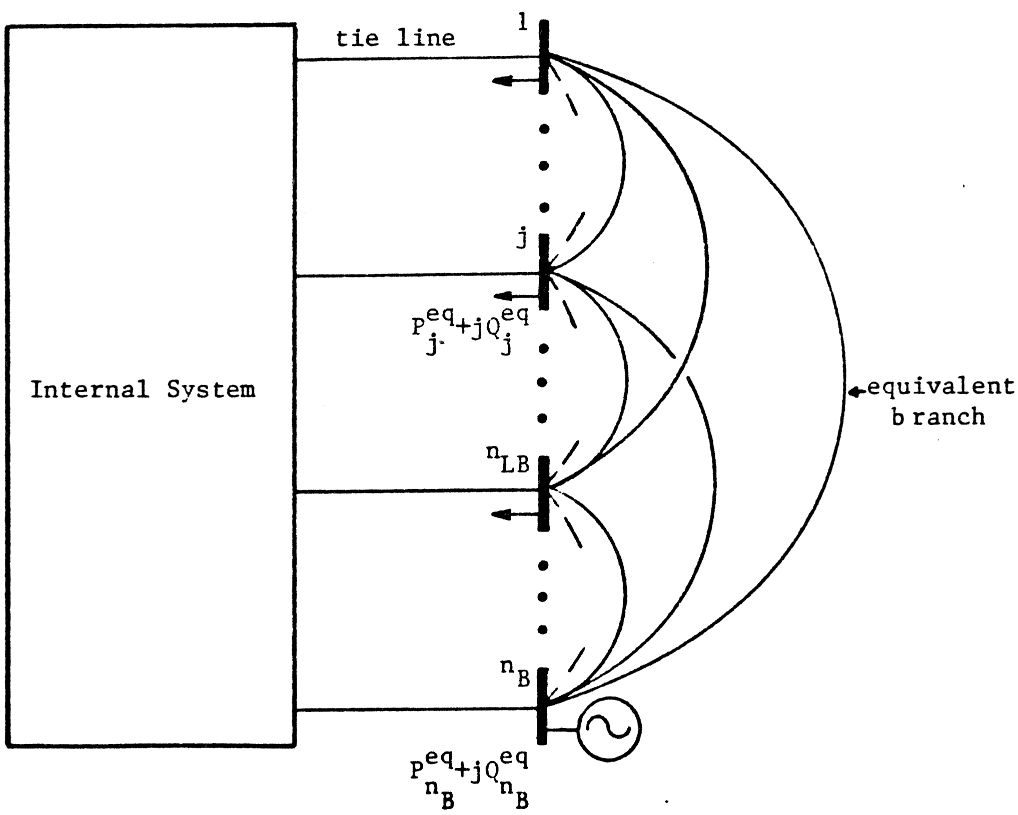

Fig.2 Internal system with the external equivalent.

$$Q_j^{eq} = |V_j^0| \sum_{1 \le k \le n} |V_k^0| (G_{jk} \sin(\delta_j^0 - \delta_k^0) - B_{jk} \cos(\delta_j^0 - \delta_k^0)), \tag{7b}$$

where P_i^{eq} and Q_j^{eq} are equivalent active and reactive power injections for the boundary bus i, $i = 1, ..., n_B$ and bus j, $j = 1, ..., n_{LB}$, respectively, n_B is the number of boundary buses, n_{LB} is the number of the load boundary buses, n is the number of buses of the reduced system and $G_{ik} + jB_{ik}$ is the (i,k)th element of the complex bus admittance matrix of the reduced power system.

An alternative form of this equivalent that is often used is the equivalent without shunts in the external system [2,5]. Prior to reduction, the shunts are converted to the extra bus injections, so that only the series external network is reduced.

Ward derived his equivalent under the assumption that the base-case equivalent injections, expressed in active and reactive powers, will be constant during changes in the internal system. As far as the active power flows are concerned, this way of equivalencing gives good results. The main disadvantage of this equivalent is the fact that there is no reactive power support from the external system. The real and reactive powers, as is known, are fixed for load buses, but the reactive power injection of a generator bus is allowed to vary in order to keep the voltage at that bus constant, so the assumption that the reactive power generation of a generator bus remains constant is not always valid.

Ward Equivalent with Buffer Zone [2,4]

As mentioned above, the main problem with the standard Ward equivalent is that no reactive power can be imported from an external system if the outaged case so requires. If instead of eliminating all buses from the external system, a few generator buses, which are able to support large amounts of reactive powers will be kept, the quality of the equivalent will be better. The problem is to find a sufficient number of generator buses to be retained in order to provide sufficient reactive power. From the experiments performed [4], it seems that:

i) the generator buses with the maximum reactive power capability should be kept. That is, keep the generator buses for which the difference between the base case reactive power production and reactive limits is the largest [4].

ii) all the generator buses which do not produce enough real or reactive power to cover their own local requirments should be eliminated.

Extended Ward Equivalent [2,5]

A simple extension to the standard Ward equivalent is the extended Ward equivalent as presented [2,5]. The nodal equations of the reduced power system can be written for the extended Ward method in the form:

$$\begin{bmatrix} \mathbf{I}_I \\ \mathbf{I}_{eq} - \hat{\mathbf{Y}}\Delta|\mathbf{V}_B| \end{bmatrix} = \begin{bmatrix} \mathbf{Y}_{II} & \mathbf{Y}_{IB} \\ \mathbf{Y}_{BI} & \mathbf{Y}^I_{BB} + \mathbf{Y}_{eq} \end{bmatrix} \begin{bmatrix} \mathbf{V}_I \\ \mathbf{V}_B \end{bmatrix}, \tag{8}$$

where

$$\hat{\mathbf{Y}} = \begin{bmatrix} \hat{y}_1 & & & & 0 \\ & \ddots & & & \\ & & \hat{y}_i & & \\ & & & \ddots & \\ 0 & & & & \hat{y}_{n_B} \end{bmatrix}, \tag{9}$$

and

$$\hat{y}_i \begin{cases} = 0 \text{ if i corresponds to PV bus,} \\ \neq 0 \text{ if i corresponds to PQ bus,} \end{cases}$$

$i = 1, \ldots, n_B$, where n_B is the number of the boundary buses,

$\Delta|V_B|$ is the change of the magnitude of the voltages on the boundary buses corresponding to a change in operating conditions in the internal system,

$\mathbf{Y}_{eq}$ is the equivalent nodal admittance matrix of the external system without shunts.

From (3) and (8), it can be seen that the difference between the standard and extended Ward equivalent is that in the extended Ward method we have an additional power injection at each PQ boundary bus. This injection is a function of the corresponding boundary voltage only. The extended Ward equivalent is illustrated in Fig. 3. It has been obtained from the standard Ward equivalent by attaching new k fictitious PV buses, $k = n_B+1, \dots, n_B+j, \dots, n_B+n_{LB}$, with the active power $P_k = 0$ and $|V_k| = |V^0_{k-nB}|$ to each boundary PQ bus j, $j = 1, \dots, n_{LB}$, through a branch of the admittance $\hat{y}_j$. The new fictitious PV buses contribute no active and reactive power in the base case. For a change of the operating conditions in the internal system, the additional injection is

$$\Delta P_j + j\Delta Q_j = V_j(V^*_{n_B+j} - V^*_j)\hat{y}^*_j . \tag{10}$$

Assuming that

$$\hat{y}_j = j\hat{b}_j \tag{11}$$

and

$$\delta_j \approx \delta_{n_B+j}, \tag{12}$$

the additional reactive injection ΔQ_j in the jth boundary PQ bus can be expressed as

$$\Delta Q_j = |V_j|(|V_j| - |V_{n_B+j}|)\hat{b}_j = |V_j|\Delta|V_j|\hat{b}_j . \tag{13}$$

The algorithm for determining the branch admittances $\hat{y}_j$ has been presented in [5]. The fictitious branch admittances $\hat{y}_j$ are obtained in the following manner:

i) extract the external system with its shunts from the interconnected system,

ii) ground all PV buses in the external system and determine the admittance matrix **Y** of this network,

iii) perform elimination of the external buses from **Y**; as a result, we obtain the nodal admittance matrix $\mathbf{Y}_{eq}$ of the network containing only the boundary PQ buses,

iv) for $j = 1, \dots, n_{LB}$ determine the admittance $\hat{y}_j$ as follows

$$\hat{y}_j = \sum_{1 \le k \le n_{LB}} y^{eq}_{jk} , \tag{14}$$

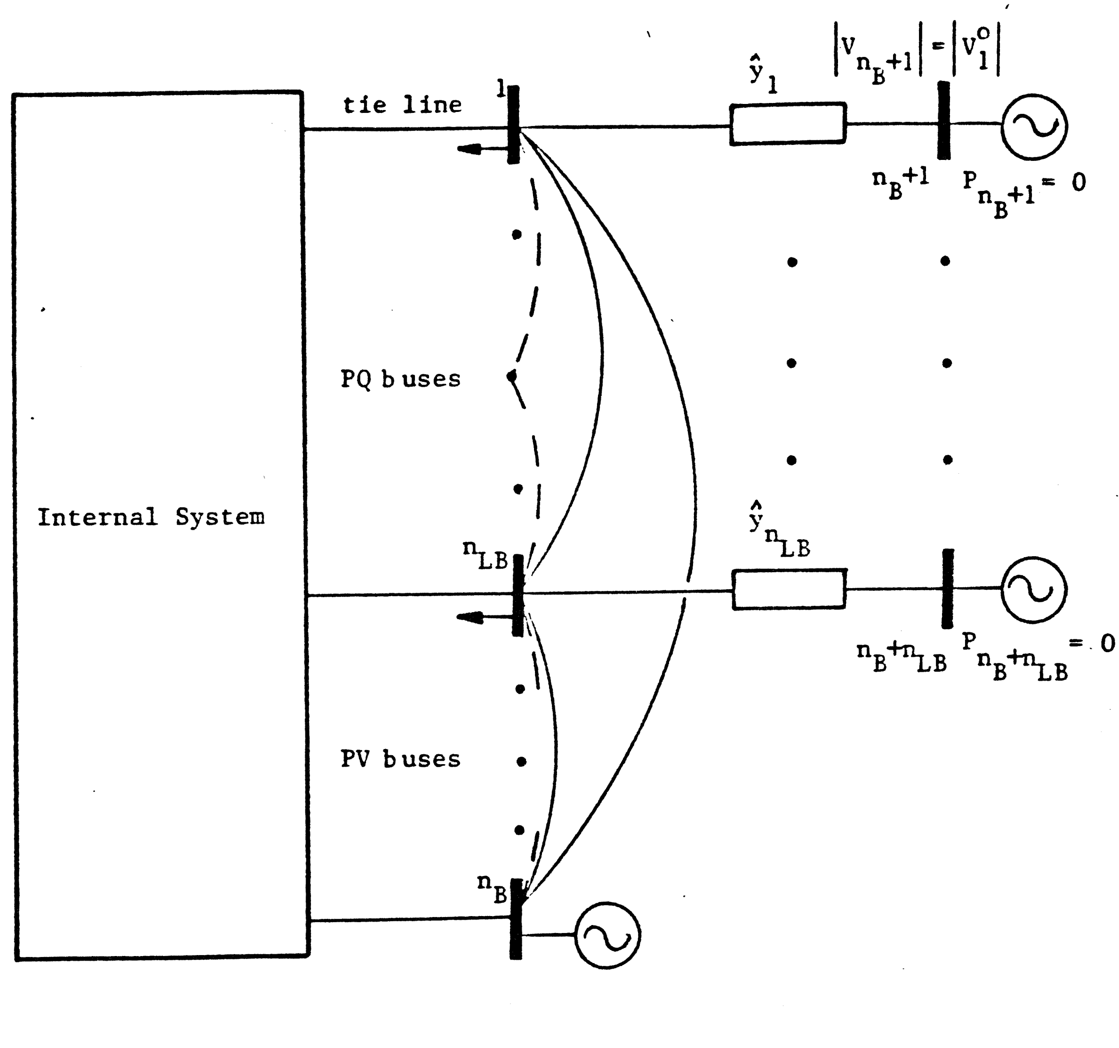

Fig. 3 Internal system with the external Ward equivalent.

where y_{jk}^{eq} is the (j,k)th element of the matrix $\mathbf{Y}_{eq}$.

Simplified Extended Ward Equivalent [5]

In this version of the Ward equivalent, the external system is represented soley by matrix $\mathbf{0} + j\mathbf{B}_{eq}$ instead of the matrix $\mathbf{Y}_{eq}$. If the extended version is used, the additional fictitious branches are obtained from the load flow matrix $\mathbf{B}''_{eq}$, where

$$\mathbf{B}''_{eq} = \mathbf{B}''_{BB} - \mathbf{B}''_{BE}\,\mathbf{B}''^{-1}_{EE}\,\mathbf{B}''_{EB} \tag{15}$$

and the matrices $\mathbf{B}''_{BB}$, $\mathbf{B}''_{BE}$, $\mathbf{B}''_{EB}$ and $\mathbf{B}''_{EE}$ are the elements of the load-flow matrix $\mathbf{B}''$ [15,16] of the external system, i.e.,

$$\mathbf{B}'' = \begin{bmatrix} \mathbf{B}''_{BB} & \mathbf{B}''_{BE} \\ \mathbf{B}''_{EB} & \mathbf{B}''_{EE} \end{bmatrix} . \tag{16}$$

The admittances $\hat{y}_j$, $j = 1, \ldots, n_{LB}$ of the fictitious branches are computed as follows

$$\hat{y}_j = -j \sum_{1 \le k \le n_{LB}} b_{jk}^{eq} , \tag{17}$$

where b_{jk}^{eq} is the (j,k)th element of the matrix $\mathbf{B}''_{eq}$.

III. FLOW DIAGRAMS OF THE WARD TYPE EQUIVALENT METHODS

Ward type equivalent methods that have been presented in the Section II can be summarized by the flow-diagrams shown in Figs. 4, 5, 6, 7 and 8, where the major steps of each method are outlined.

IV. EXAMPLES OF THE EXTERNAL EQUIVALENTS

In this section, we present seven examples of the Ward type external equivalents. The numerical examples of this paper are based on the 26-bus [10-12] and the IEEE 118-bus [13] power systems. The detailed data of these systems are presented in [10,11]. All the presented examples have been computed using WARDEQ package [14].

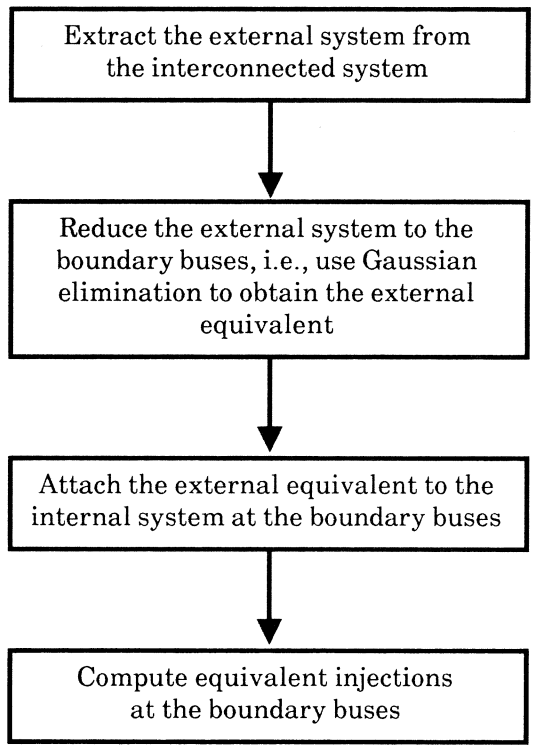

Fig. 4 Flow diagram of the standard Ward method - the model with shunts in the external system.

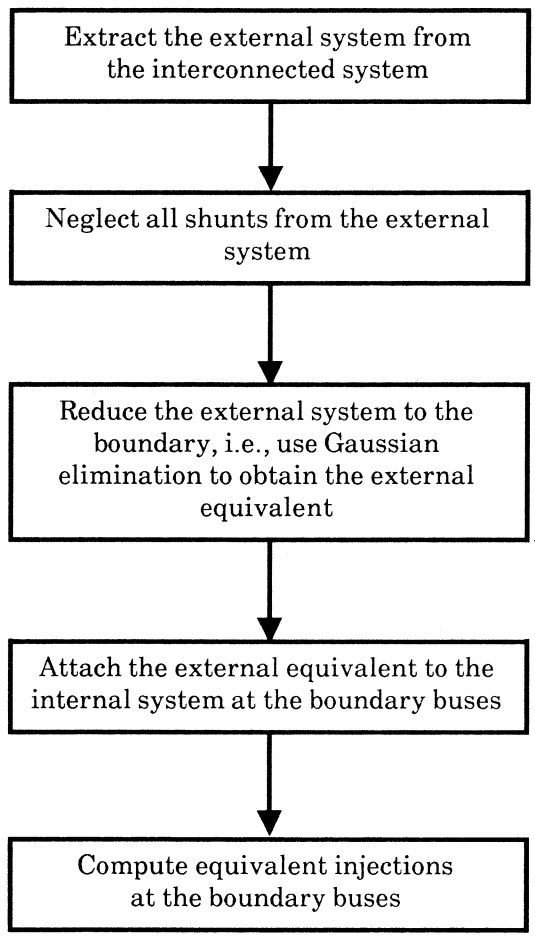

Fig. 5 Flow diagram of the standard Ward method - the model without shunts in the external system.

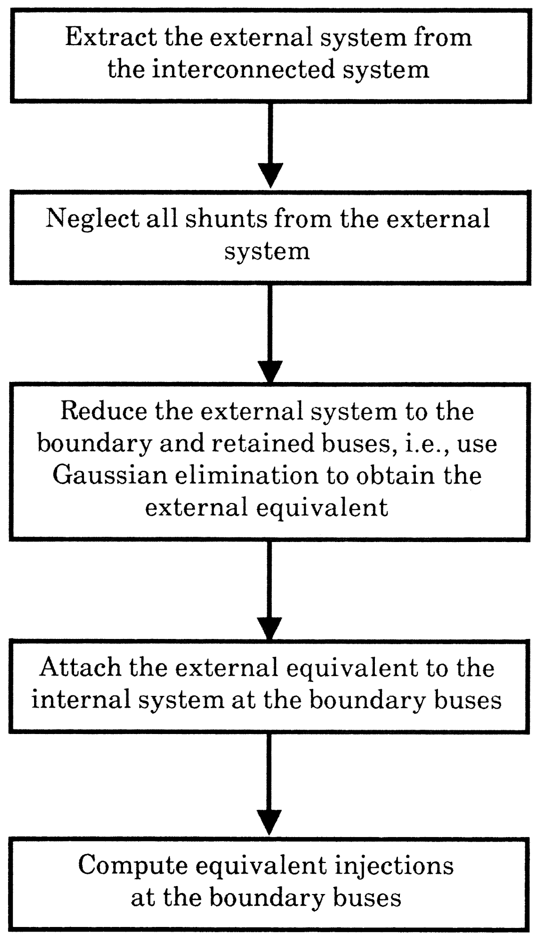

Fig. 6 Flow diagram of the Ward method with buffer zone.

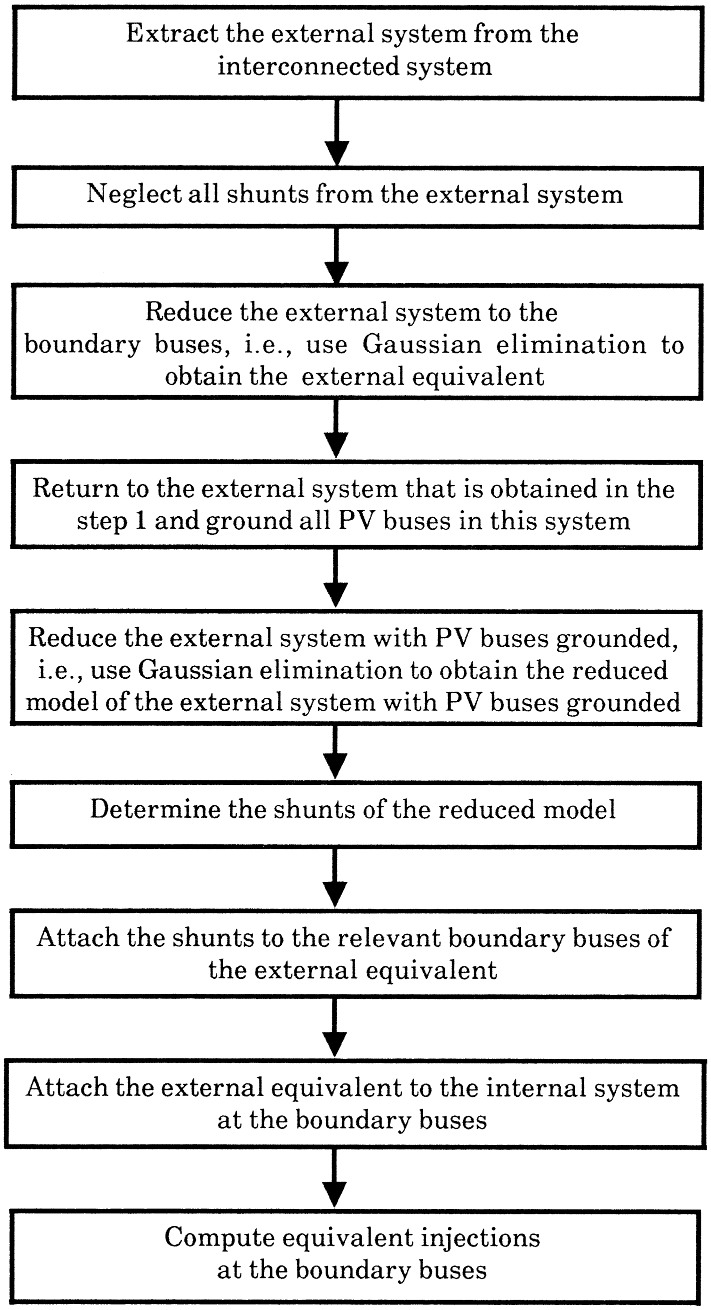

Fig. 7 Flow diagram of the extended Ward method.

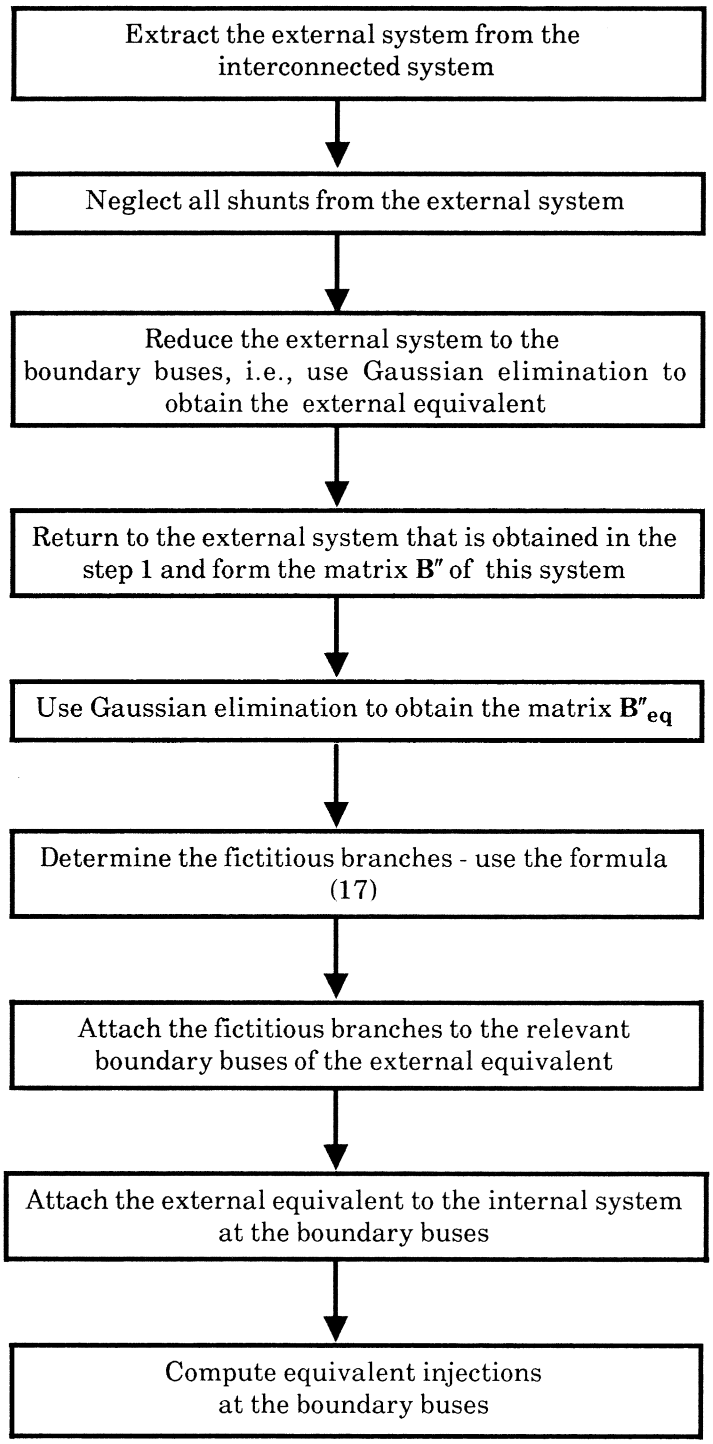

Fig. 8 Flow diagram of the simplified Ward method.

The 26-bus system

This power system (Saskatchewan Power Corporation System) has been considered in some studies on steady-state power system analysis. The single line diagram of this system is shown in Fig. 9. The load flow solution and injected powers are tabulated in Table I.

The 26-bus system has been divided into two subsystems as follows:

- buses in the external system: 2, 5, 8, 10, 11, 14, 17, 18, 22, 25,
- boundary buses: 9, 13, 21,
- buses of the internal system, namely, all the buses not mentioned above.

The single line diagram of the external system is shown in Fig. 10. From this system, five different types of the external equivalents have been formed. Tables II, III, IV, V and VI show the parameters of these external equivalents, i.e., the equivalent injections, admittances of the branches of the external equivalent, shunt admittances at the boundary buses and the fictitious admittances for the extended and simplified external equivalents. The structures of the standard and extended equivalents are shown in Figs. 11 and 12, respectively. All the external equivalents have been formed at the operating point shown in Table I.

The IEEE 118-bus system

This system has been considered in [3,5,9]. The detailed data and load flow solution are presented in [11,13].

The 118-bus system has been divided into two subsystems as follows:

- buses in the external system: 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 38, 113, 114, 115, 117,
- boundary buses: 24, 37, 43, 65,
- buses of the internal system, namely, all the buses not mentioned above.

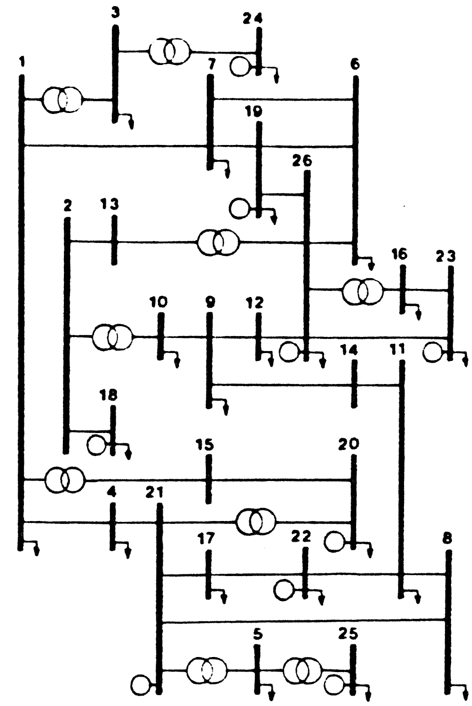

Fig. 9 The 26-bus power system.

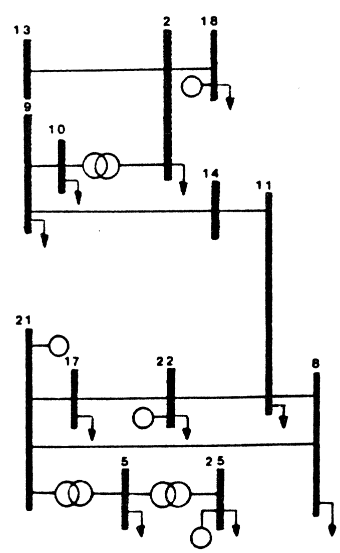

Fig. 10 External system.

TABLE I

LOAD FLOW SOLUTION AND INJECTED POWER
FOR THE 26-BUS POWER SYSTEM

	Injected Power		Bus Voltage	
Bus No.	P_m	Q_m	ReV_m	ImV_m
1	−0.82	−0.21	1.0328	0.0773
2	0.0	0.0	1.0644	0.0943
3	−0.57	−0.17	1.0424	0.0549
4	−0.48	−0.21	0.9859	0.0979
5	−0.43	−0.11	0.9741	0.2598
6	−0.40	−0.10	1.0324	0.0544
7	−1.11	−0.27	1.0132	0.0181
8	−0.23	−0.06	0.9441	0.0403
9	−0.67	−0.21	0.9614	−0.1088
10	−1.02	−0.27	1.0370	0.0692
11	−0.43	−0.14	0.8982	−0.0992
12	−0.43	−0.12	0.9670	−0.0741
13	0.0	0.0	1.0463	0.0157
14	0.0	0.0	0.9388	−0.1071
15	0.0	0.0	0.9273	0.0970
16	−1.31	−0.30	1.0353	−0.0471
17	−0.03	−0.01	0.9313	0.0278
18	2.80	−0.4004	1.0397	0.2528
19	1.45	0.1872	1.0455	0.0966
20	2.80	0.7795	0.9706	0.2408
21	1.10	−0.294	0.9938	0.2295
22	−0.56	−0.1775	0.8856	−0.0885
23	−0.04	−0.1144	0.9996	−0.0265
24	−0.05	−0.1645	0.9989	0.0458
25	0.63	−0.1691	0.9359	0.3522
26	0.1344	−0.0513	1.01	0.0

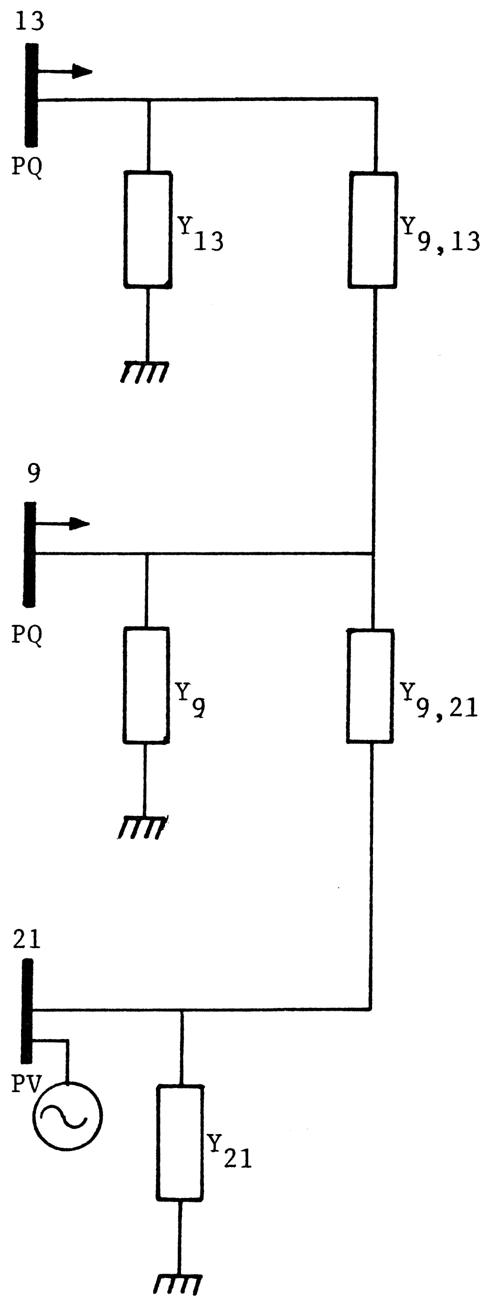

Fig. 11 The structure of the standard Ward equivalent associated with the 26-bus test power system.

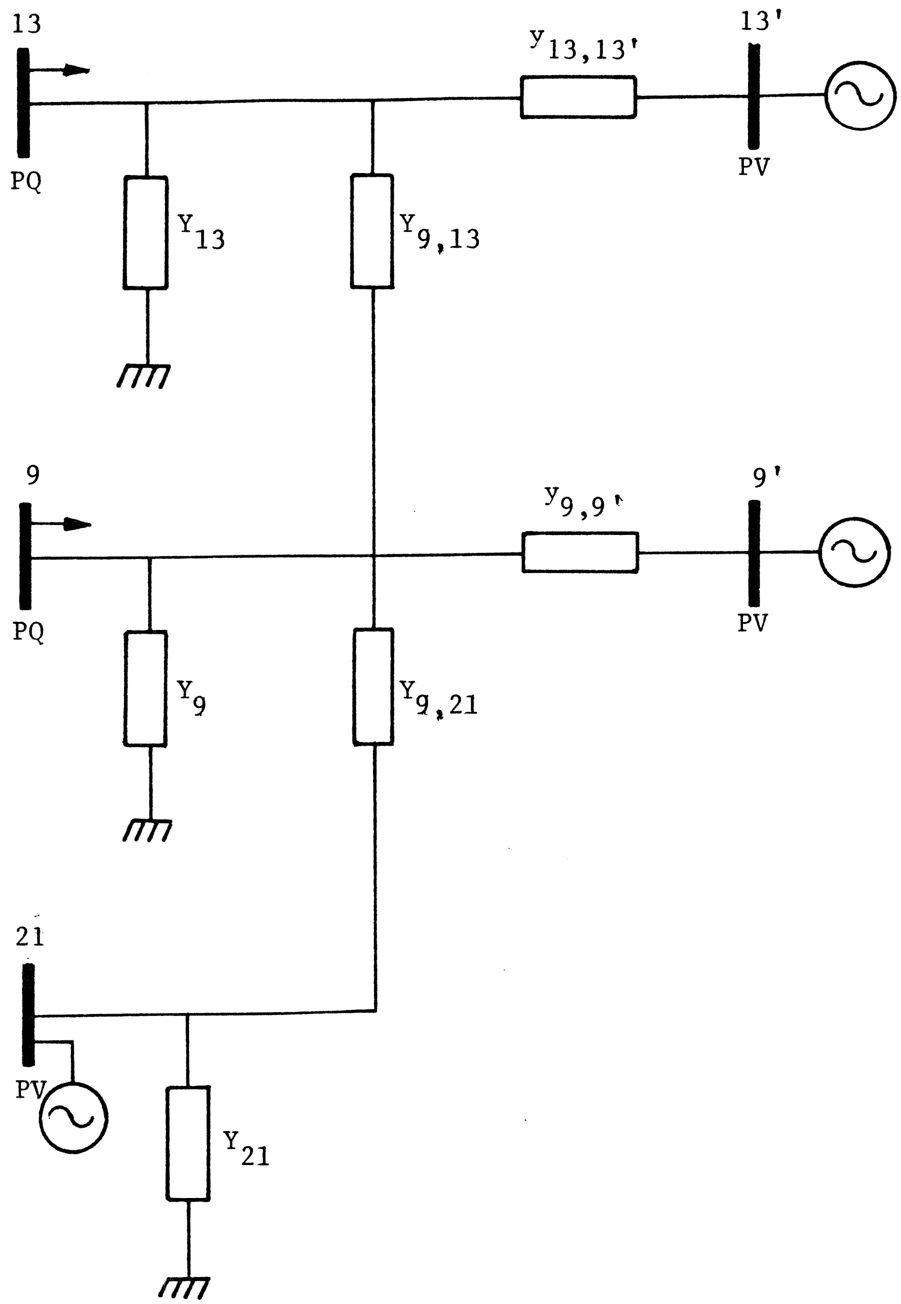

Fig. 12 The structure of the extended Ward equivalent associated with the 26-bus test power system.

TABLE II

STANDARD WARD EXTERNAL EQUIVALENT

WITH SHUNTS IN THE EXTERNAL SYSTEM

Boundary Bus No.	Equivalent Injected Power		Bus Voltage		Shunt Admittance	
	P_m	Q_m	ReV_m	ImV_m	Conductance	Susceptance
9	−0.7603	−0.6310	0.9614	−0.1088	−0.0287	0.7298
13	0.4029	−1.5118	1.0463	0.0157	0.0430	1.4702
21	0.4325	--	0.9938	0.2295	0.0050	0.3233

Admittance $Y_{mm'}$ between buses m and m′

Buses		Admittance	
m	m′	Conductance	Susceptance
9	13	0.79075	−2.20255
9	21	0.27580	−1.20770

TABLE III

STANDARD WARD EXTERNAL EQUIVALENT WITHOUT SHUNTS IN THE EXTERNAL SYSTEM

Boundary Bus No.	Equivalent Injected Power		Bus Voltage		Shunt Admittance	
	P_m	Q_m	ReV_m	ImV_m	Conductance	Susceptance
9	−0.6902	−0.3604	0.9614	−0.1088	0.0	0.4439
13	0.3361	−0.2416	1.0463	0.0157	0.0	0.3017
21	0.4034	--	0.9938	0.2295	0.0	0.1343

Admittance $Y_{mm'}$ between buses m and m′

Buses		Admittance	
m	m′	Conductance	Susceptance
9	13	0.76741	−2.06594
9	21	0.27560	−1.13505

TABLE IV

WARD EQUIVALENT WITH THE BUFFER ZONE

Boundary & Buffer Buses No.	Equivalent Injected Power		Bus Voltage		Shunt Admittance	
	P_m	Q_m	ReV_m	ImV_m	Conductance	Susceptance
9	–0.4285	–0.3409	0.9614	–0.1088	0.0	0.0485
13	0.3360	–0.2416	1.0463	0.0157	0.0	0.4439
21	1.0506	--	0.9938	0.2295	0.0	0.3017
22	–0.8700	--	0.8855	–0.0885	0.0	0.1343

Admittance $Y_{mm'}$ between buses m and m′

Buses		Admittance	
m	m′	Conductance	Susceptance
9	13	0.76741	–2.06594
9	21	0.09188	–0.36033
9	22	0.26696	–1.06312
21	22	0.51446	–2.84600

TABLE V

EXTENDED WARD EXTERNAL EQUIVALENT

Boundary Bus No.	Equivalent Injected Power		Bus Voltage		Shunt Admittance	
	P_m	Q_m	ReV_m	ImV_m	Conductance	Susceptance
9	−0.6902	−0.3604	0.9614	−0.1088	0.0	0.4439
13	0.3361	−0.2416	1.0463	0.0157	0.0	0.3017
21	0.4034	--	0.9938	0.2295	0.0	0.1343

Admittance $Y_{mm'}$ between buses m and m′

Buses		Admittance	
m	m′	Conductance	Susceptance
9	13	0.76741	−2.06594
9	21	0.27560	−1.13505

Fictitious admittance $Y_{mm'}$ between buses m and m′

Buses		Admittance	
m	m′	Conductance	Susceptance
9	9′	0.84092	−2.5804
13	13′	0.69040	−6.7273

Fictitious buses

Bus No.	Bus Voltage $\|V_m\|$	Injected Power P_m
9′	0.96751	0.0
13′	1.04645	0.0

TABLE VI

SIMPLIFIED WARD EXTENDED EQUIVALENT

Boundary Bus No.	Equivalent Injected Power		Bus Voltage		Shunt Admittance	
	P_m	Q_m	ReV_m	ImV_m	Conductance	Susceptance
9	−0.6348	−0.5476	0.9614	−0.1088	0.0	0.4439
13	0.2619	−0.1458	1.0463	0.0157	0.0	0.3017
21	0.3729	--	0.9938	0.2295	0.0	0.1343

Admittance $Y_{mm'}$ between buses m and m′

Buses		Admittance	
m	m′	Conductance	Susceptance
9	13	0.0	−2.06594
9	21	0.0	−1.13505

Fictitious admittance $Y_{mm'}$ between buses m and m′

Buses		Admittance	
m	m′	Conductance	Susceptance
9	9′	0.0	−2.5804
13	13′	0.0	−6.7273

Fictitious buses

Bus No.	Bus Voltage $\|V_m\|$	Injected Power P_m
9′	0.96751	0.0
13′	1.04645	0.0

From the external system, two different types of external equivalents have been formed. Tables VII and VIII show parameters of the standard and extended Ward equivalents, i.e., the equivalent injections, the admittances of the branches of the external equivalent, shunt admittances at the boundary buses and the fictitious admittances for the extended version. The structures of these external equivalents are shown in Figs. 13 and 14, respectively.

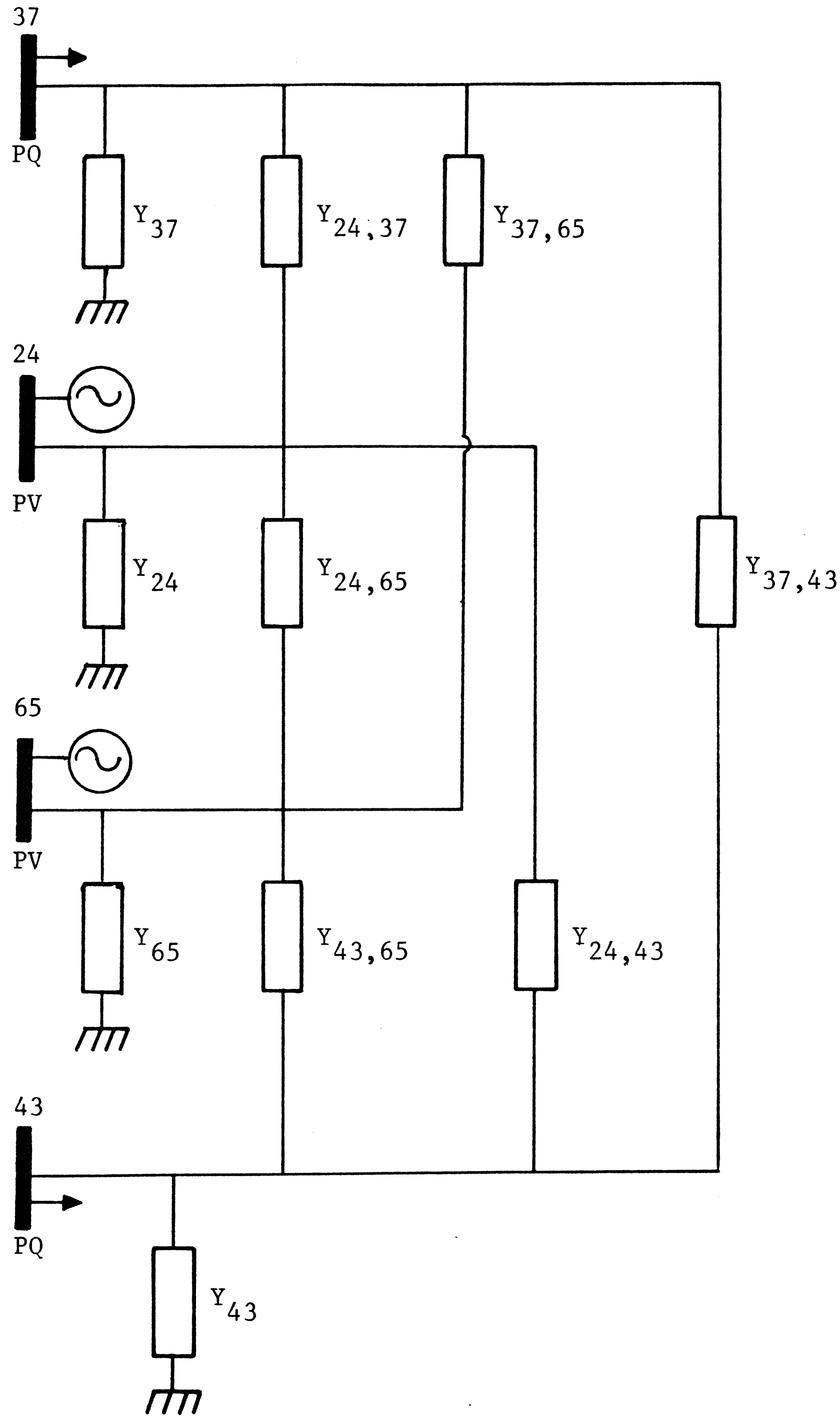

Fig. 13 The structure of the standard Ward equivalent associated with the IEEE 118-bus system.

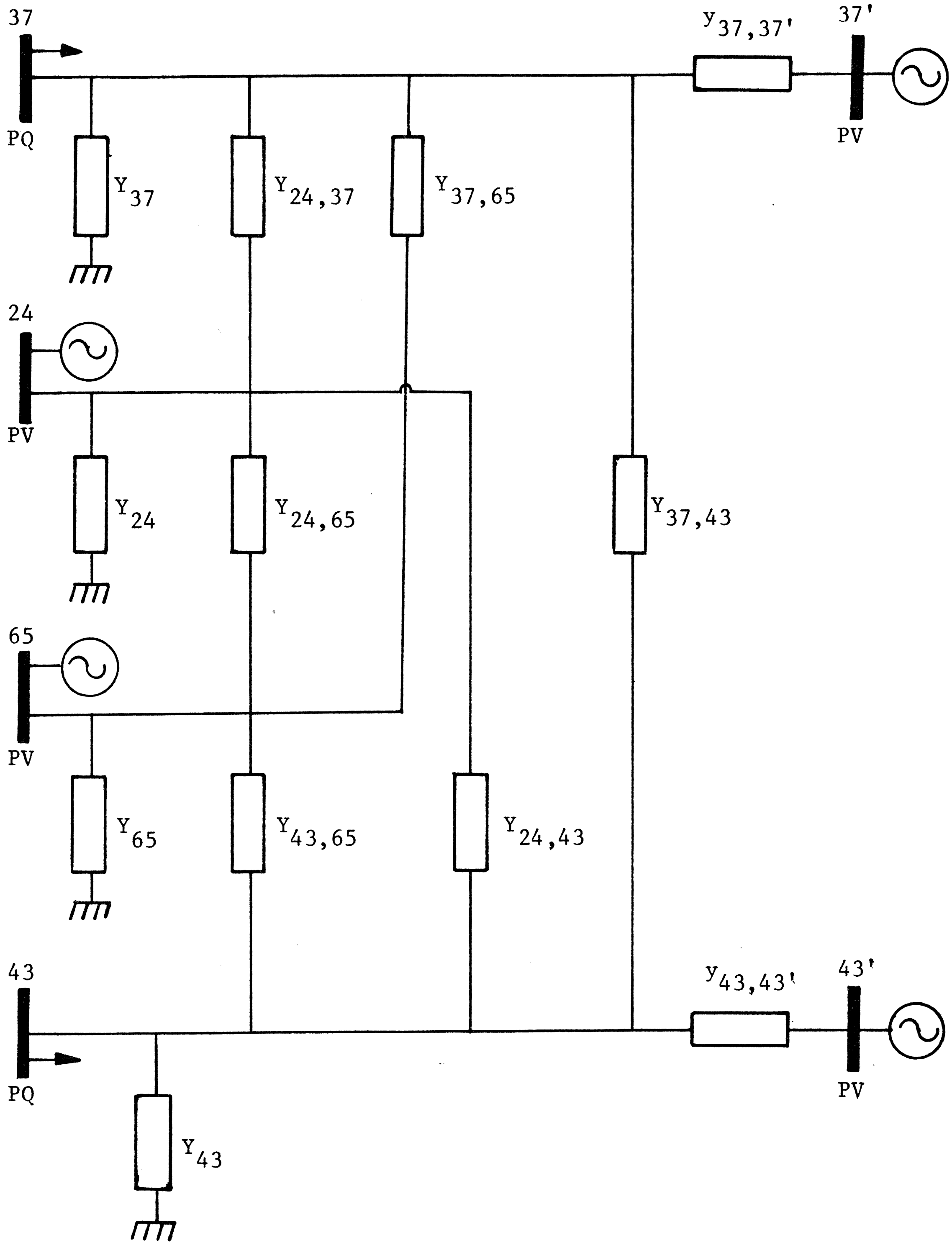

Fig. 14 The structure of the extended Ward equivalent associated with the IEEE 118-bus system.

TABLE VII

STANDARD WARD EXTERNAL EQUIVALENT

WITH SHUNTS IN THE EXTERNAL SYSTEM

Boundary Bus No.	Equivalent Injected Power		Bus Voltage		Shunt Admittance	
	P_m	Q_m	ReV_m	ImV_m	Conductance	Susceptance
24	0.5708	--	0.8943	−0.1682	0.0070	2.5512
37	−0.2569	−0.3222	0.9061	−0.3313	0.4137	4.3208
43	−0.1288	−0.2637	0.8804	−0.3324	0.0026	0.2017
65	2.6503	--	0.9995	−0.0299	0.0512	2.2525

Admittance $Y_{mm'}$ between buses m and m′

Buses m	Buses m′	Conductance	Susceptance	Buses m	Buses m′	Conductance	Susceptance
24	37	0.76992	−1.53017	37	43	1.14815	−4.57428
24	43	0.04096	−0.20505	37	65	0.35996	−7.68098
24	65	0.16914	−1.28726	43	65	0.01018	−0.08053

TABLE VIII

EXTENDED WARD EQUIVALENT

Boundary Bus No.	Equivalent Injected Power		Bus Voltage		Shunt Admittance	
	P_m	Q_m	ReV	ImV	Conductance	Susceptance
24	0.4142	--	0.8943	−0.1682	0.0	0.0249
37	−2.4496	2.3409	0.9061	−0.3313	0.0	−1.7147
43	−0.1171	−0.1031	0.8803	−0.3324	0.0	0.0211
65	2.2234	--	0.9995	−0.0299	0.0	0.5230

Admittance $Y_{mm'}$ between buses m and m′

Buses		Admittance		Buses		Admittance	
m	m′	Conductance	Susceptance	m	m′	Conductance	Susceptance
24	37	0.80516	−3.93513	37	43	1.15178	−4.46530
24	43	0.04658	−0.16447	37	65	0.41279	−6.66626
24	65	0.16773	−0.91068	43	65	0.01016	−0.05755

Fictitious admittance $Y_{mm'}$ between buses m and m′

Buses		Admittance	
m	m′	Conductance	Susceptance
24	24′	12.110	51.110
65	65′	27.567	11.220

Fictitious buses

Bus No.	Bus Voltage $\lvert V_m \rvert$	Injected Power P_m
37′	0.96808	0.0
43′	0.94101	0.0

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