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A NOVEL APPROACH TO MULTI-COUPLED CAVITY FILTER SENSITIVITY AND GROUP DELAY COMPUTATION

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Abstract

Novel formulas for simulation and sensitivity analysis of multi-coupled cavity filters are presented. Only one real LU factorization and simple utilization of the corresponding original network analysis permit exact first-order sensitivity computations, group delay calculation and loss prediction.

Introduction The frequent use of narrow-bandpass multi-coupled cavity filters in practical satellite communication systems[1] makes them the prime candidates for the application of new theoretical advances in sensitivity analysis. An important result in sensitivity analysis of lossless two-ports stated by Orchard, Temes and Cataltepe[2] has inspired the following presentation for these microwave filters using a general, novel approach based on matrix notation. Exact group delay computation and response prediction of lossy filters based on first-order sensitivity concepts can be easily performed using only the results of the original analysis. A significant advance which we claim is the exploitation of the lossless property not only in obviating any adjoint analysis, but also in utilizing only one real LU factorization.

Solution of the Filter Using the lossless and narrow-band model given by Atia and Williams[1], a multi-cavity microwave filter terminated by a normalized source and a resistive load, as shown in Fig. 1, can be solved by

$$(\mathbf{R} + j\mathbf{Z})\mathbf{I} = \mathbf{e}_1 , \tag{1}$$

where $\mathbf{R} \triangleq \mathrm{diag}\{R_1, 0, \ldots, 0, R_n\}$, $\mathbf{Z} \triangleq s\,\mathbf{1} + \mathbf{M}$, $\mathbf{1}$ denotes an $n \times n$ identity matrix, $s \triangleq (\omega_0/\Delta\omega)(\omega/\omega_0 - \omega_0/\omega)$ the normalized frequency variable, $\mathbf{M}$ the real symmetrical coupling matrix, and $\mathbf{e}_1 \triangleq [1\ 0\ \ldots\ 0]^T$ represents the (voltage) excitation.

Sensitivity Expressions Differentiating eqn. (1) w.r.t. a variable ϕ contained in $\mathbf{Z}$, we have

$$(\mathbf{R} + j\mathbf{Z}) \frac{\partial \mathbf{I}}{\partial \phi} = -j \frac{\partial \mathbf{Z}}{\partial \phi} \mathbf{I}\,. \tag{2}$$

Premultiplying eqn (2) by $\mathbf{I}^T$, knowing that $\mathbf{R}$ and $\mathbf{Z}$ are symmetrical and using eqn. (1), we obtain

$$\mathbf{I}^T(\mathbf{R} + j\mathbf{Z}) \frac{\partial \mathbf{I}}{\partial \phi} = [(\mathbf{R} + j\mathbf{Z})\mathbf{I}]^T \frac{\partial \mathbf{I}}{\partial \phi} = \mathbf{e}_1^T \frac{\partial \mathbf{I}}{\partial \phi}$$

$$= \frac{\partial I_1}{\partial \phi} = -j\mathbf{I}^T \frac{\partial \mathbf{Z}}{\partial \phi} \mathbf{I}\,. \tag{3}$$

Premultiplying eqn. (2) by $(\mathbf{I}^*)^T$, where * stands for the complex conjugate, we obtain

$$(\mathbf{I}^*)^T(\mathbf{R} + j\mathbf{Z}) \frac{\partial \mathbf{I}}{\partial \phi} = [\mathbf{R}\,\mathbf{I}^* + j\mathbf{Z}\,\mathbf{I}^*]^T \frac{\partial \mathbf{I}}{\partial \phi} = -j(\mathbf{I}^*)^T \frac{\partial \mathbf{Z}}{\partial \phi} \mathbf{I}\,. \tag{4}$$

Taking the complex conjugate of eqn. (1), $j\,\mathbf{Z}\mathbf{I}^*$ can be solved to be

$$j\,\mathbf{Z}\mathbf{I}^* = \mathbf{R}\mathbf{I}^* - \mathbf{e}_1\,. \tag{5}$$

Substituting for eqn. (5), eqn. (4) can be rewritten as

$$[2\mathbf{R}\,\mathbf{I}^* - \mathbf{e}_1]^T \frac{\partial \mathbf{I}}{\partial \phi} = (2R_1 I_1^* - 1) \frac{\partial I_1}{\partial \phi} + 2R_n I_n^* \frac{\partial I_n}{\partial \phi}$$

$$= -\rho_1^* \frac{\partial I_1}{\partial \phi} + 2R_n I_n^* \frac{\partial I_n}{\partial \phi} = -j(\mathbf{I}^*)^T \frac{\partial \mathbf{Z}}{\partial \phi} \mathbf{I}\,, \tag{6}$$

where ρ_1 is the input reflection coefficient shown in Fig. 1. Combining eqns. (3) and (6), we get

$$\frac{\partial I_n}{\partial \phi} = \frac{-j}{2R_n I_n^*} (\mathbf{I}^* + \rho_1^* \mathbf{I})^T \frac{\partial \mathbf{Z}}{\partial \phi} \mathbf{I}\,. \tag{7}$$

The sensitivity expressions of various filter responses of interest can be readily derived from the basic formula (7). The sensitivity of the transducer coefficient θ as defined in

Reference 3 is given by

$$\frac{\partial\theta}{\partial\phi} = -\frac{1}{V_n}\frac{\partial V_n}{\partial\phi} = -\frac{1}{I_n}\frac{\partial I_n}{\partial\phi} = \frac{j}{2P_n}(\mathbf{I}^* + \rho_1^*\mathbf{I})^T\frac{\partial\mathbf{Z}}{\partial\phi}\mathbf{I}\,, \tag{8}$$

where P_n is the power in the load, given by $P_n = R_n I_n I^*_n$. Let $\phi = M_{\ell k}$, namely the coupling between the ℓth and kth cavities. By definition of **Z** and taking symmetry of **M** into account, it can be shown, after some simple manipulations, that

$$\frac{\partial\theta}{\partial M_{\ell k}} = \frac{cj}{P_n}[\rho_1^* I_\ell I_k + \mathrm{Re}(I^*_\ell I_k)]\,, \quad c \triangleq \begin{cases} 1/2\,, & \text{if } \ell = k, \\ 1\,, & \text{if } \ell \neq k. \end{cases} \tag{9}$$

By definition, we know that $\partial\mathbf{Z}/\partial\omega = (\partial s/\partial\omega)\mathbf{1}$, where $\partial s/\partial\omega = (1 + \omega^2_0/\omega^2)/\Delta\omega$. It follows that

$$\frac{\partial\theta}{\partial\omega} = \frac{j}{2P_n}\frac{\partial s}{\partial\omega}(\mathbf{I}^* + \rho_1^*\ \mathbf{I})^T\mathbf{I} = \frac{j}{2P_n}\frac{\partial s}{\partial\omega}[\,\|\mathbf{I}\|^2 + \rho_1^*\mathbf{I}^T\mathbf{I}]\,. \tag{10}$$

The imaginary part of (10) gives the group delay formula as

$$T_G = \mathrm{Im}\left[\frac{\partial\theta}{\partial\omega}\right] = \frac{1}{2P_n}\frac{\partial s}{\partial\omega}[\,\|\mathbf{I}\|^2 + \mathrm{Re}(\rho_1^*\mathbf{I}^T\mathbf{I})]\,. \tag{11}$$

Assume the cavities have uniform dissipation, represented by the diagonal elements of **Z**, as $j\mathbf{Z} = r\mathbf{1} + j(s\mathbf{1} + \mathbf{M})$, where $r = 0$ for the nominal, lossless design. Hence,

$$\frac{\partial\theta}{\partial r} = \frac{1}{2P_n}(\mathbf{I}^* + \rho_1^*\ \mathbf{I})^T\mathbf{I} = \frac{1}{2P_n}[\,\|\mathbf{I}\|^2 + \rho_1^*\mathbf{I}^T\mathbf{I}]\,. \tag{12}$$

The effect of dissipation on the transducer loss, as conventionally defined in dB, can be predicted by a first-order change, using the real part of (12) with appropriate coefficients, as

$$\frac{\partial\Delta}{\partial r} = \frac{20}{\ln 10}\mathrm{Re}\left[\frac{\partial\theta}{\partial r}\right] = \frac{10}{P_n \ln 10}[\,\|\mathbf{I}\|^2 + \mathrm{Re}(\rho_1^*\mathbf{I}^T\mathbf{I})]\,. \tag{13}$$

<u>Computational Considerations</u> Consdier the singly terminated system

$$\mathbf{Z}^0\,\mathbf{I}^0 = \mathbf{e}_1\,, \tag{14}$$

where $\mathbf{Z}^0 \triangleq \mathrm{diag}\{0, 0, ..., R_n\} + j\mathbf{Z}$ and assume that the real matrix **Z** has been factorized as $\mathbf{Z} = \mathbf{LU}$, where **L** and **U** are real lower and upper triangular matrices, respectively. It is easily shown that

$$\mathbf{Z}^0 = j\,\mathbf{LU}^0\,, \tag{15}$$

where $\mathbf{U}^0$ is the augmented upper triangular matrix

$$\mathbf{U}^0 = \mathbf{U} + \mathrm{diag}\{0, 0, \ldots, -j\, R_n/L_{nn}\}, \tag{16}$$

L_{nn} being the (n,n) element of **L**. Then, we solve the systems

$$\mathbf{L}\,\mathbf{x} = \mathbf{e}_1\,,\ \mathbf{U}^0\,\mathbf{I}^0 = -j\,\mathbf{x} \tag{17}$$

by forward and backward substitutions, respectively, involving mostly real calculations. Finally, the doubly-terminated complex solution is obtained from the complex $\mathbf{I}^0$ as

$$\mathbf{I} = \mathbf{I}^0/(1 + R_1 I_1^0)\,. \tag{18}$$

Example Consider a 6-pole optimized elliptic function filter. The non-zero elements of the coupling matrix **M** are $M_{12} = M_{56} = 0.81777$, $M_{23} = M_{45} = 0.51110$, $M_{34} = 0.82430$, $M_{16} = 0.09330$, $M_{25} = -0.35710$. The filter has 1% bandwidth and is centred at 4 GHz. A complete program implementing our formulas has produced the responses of Figs. 2 and 3. Equation (13) was used to predict the effect of losses shown in Fig. 4.

Conclusions Modern computer-aided design and computer-assisted tuning and alignment of high order multi-coupled cavity microwave filters require exact sensitivity analysis for application of state-of-the-art optimization procedures. Computational efficiency in the calculation of sensitivities is of great significance. Evaluation of non-ideal effects (dispersion, dissipation, parasitics) and the integration of such filters into optimal multiplexing networks makes our numerical approach extremely attractive.

Acknowledgement

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References

1. Atia, A.E. and Williams, A.E: "New types of waveguide bandpass filters for satellite transponders", COMSAT Technical Review, 1971, 1, pp. 21-43.

2. Orchard, H.J., Temes, G.C. and Cataltepe, T.: "General sensitivity formulas for lossless two-ports", Electron. Lett., 1983, 19, pp. 576-578.

3. Temes, G.C. and Mitra, S.K.: "Modern filter theory and design" (Wiley-Interscience, 1973), Chap. 1.

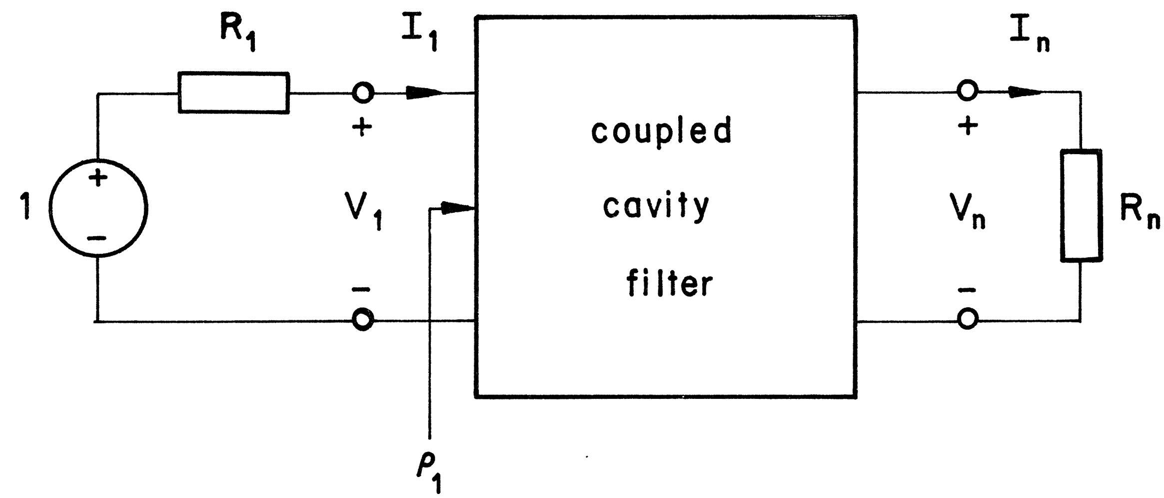

Fig. 1 Doubly terminated multi-coupled cavity microwave filter.

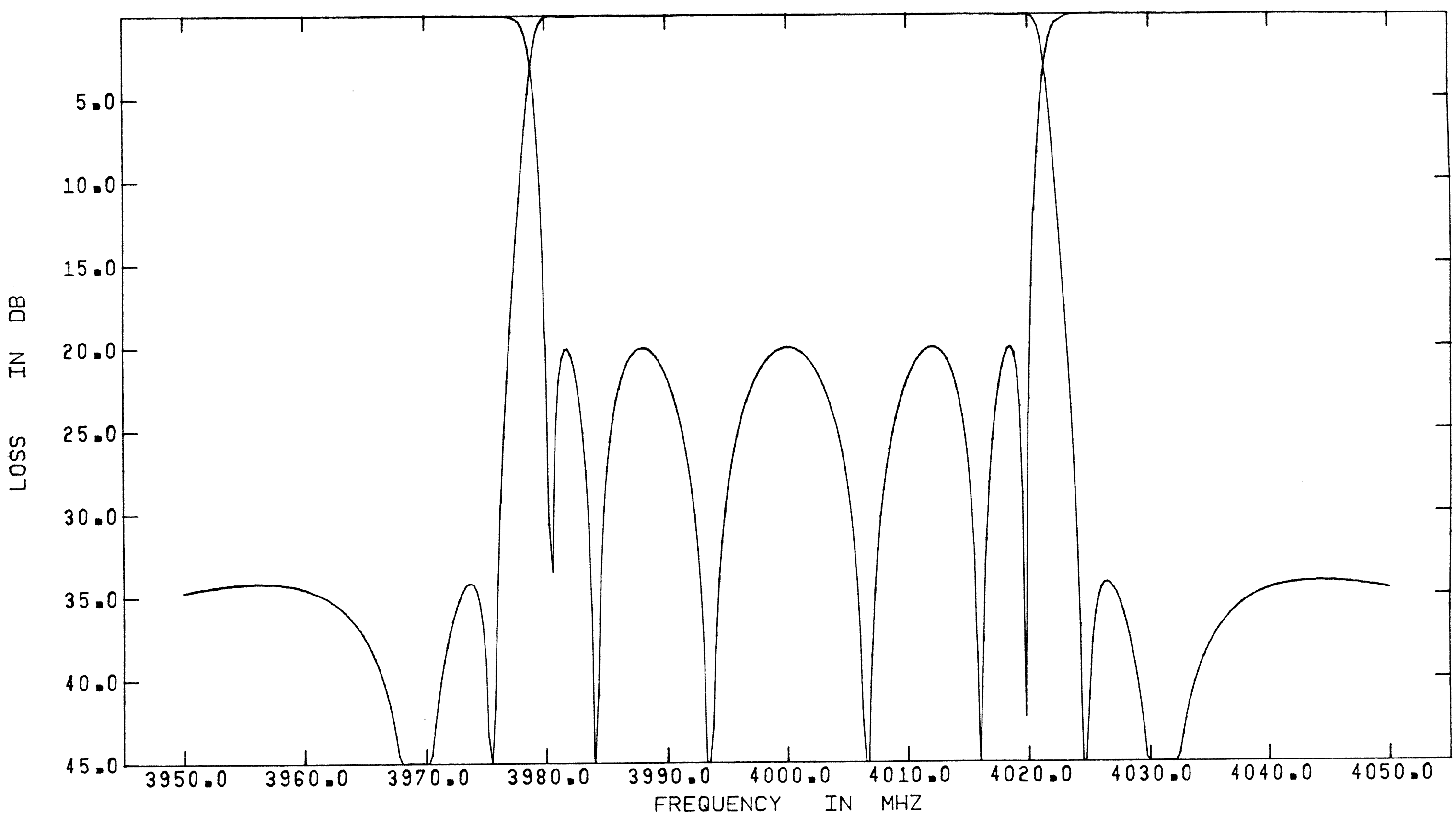

Fig. 2 Return loss and insertion loss for the filter used in the example. Our real analysis approach has been utilized in obtaining the responses.

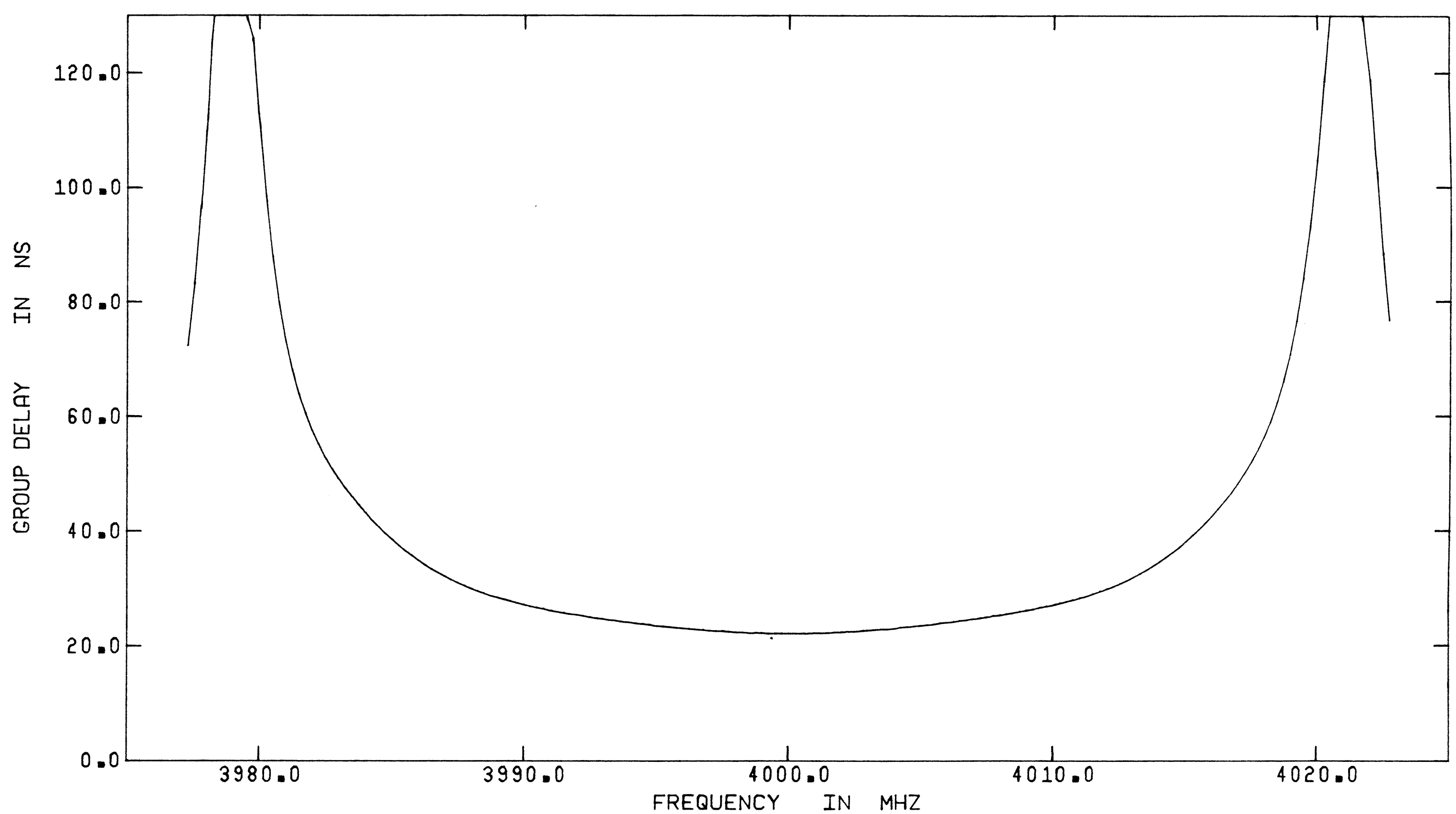

Fig. 3 Exact group delay in the passband of the filter used in the example. Equation (11) has been utilized to obtain the response.

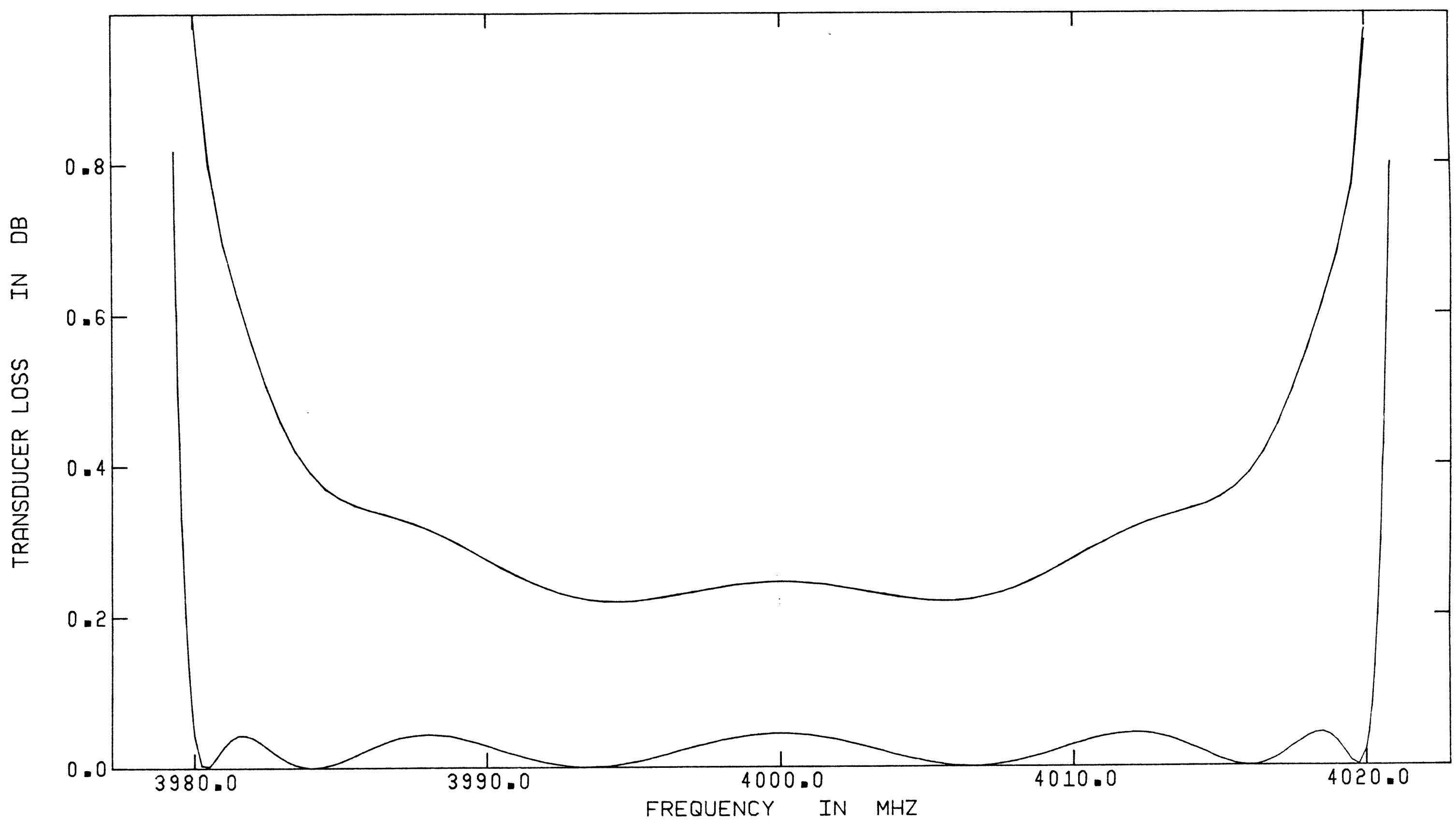

Fig. 4 Predicted passband response of the lossy filter using exact sensitivities based on eqn. (13). Unloaded $Q = 12000$. The result is indistinguishable from the exact simulation of the lossy filter.