APPROXIMATE SIMULATION AND SENSITIVITY EVALUATION OF MULTIPLEXER CHANNEL FILTERS

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Abstract

An approximate method, based on the Neumann series expansion, of calculating the input impedance and its sensitivities of multi-coupled cavity channel filters is described. Its application to the simulation and sensitivity evaluation of microwave multiplexers leads to significant computational savings.

Introduction Microwave multiplexers consisting of multi-cavity channel filters have become an important subject of satellite communication practice[1,2]. Figure 1 shows a block representation of an ℓ-channel multiplexer, where the kth channel filter, characterized by an impedance matrix $\mathbf{Z}_k$, is terminated at the output end by a load R_k and, at the other end, merges into the waveguide manifold. Each channel output is active over only a portion of the operating frequency band of the multiplexer. Suppose that the kth channel output is of interest at a particular frequency. We can represent the other channels by their input impedances as seen from the waveguide manifold, as illustrated in Fig. 2. This being done, the network is basically of a cascaded structure to which the theory of cascaded analysis, as developed by Bandler et al.[3], can be applied directly. To do this, however, the input impedance of each channel and its sensitivities with respect to design variables associated with that channel have to be made available. In this paper, we first describe an exact method to show that the channel input impedance and its sensitivities can be evaluated utilizing one solution of the filter. However, carrying such an exact analysis through the whole frequency band of a multiplexer becomes expensive and unnecessary. To improve computational efficiency, a novel approximate solution is presented.

<u>An Exact Method</u> Consider a lumped model of an nth order singly terminated multi-coupled cavity filter[2] as

$$\mathbf{Z}\,\mathbf{I} = \mathbf{V}\,, \tag{1}$$

where

$$\mathbf{Z} \triangleq j\,(s\,\mathbf{1} + \mathbf{M}) + \mathrm{diag}\,\{0 \ldots 0\; R\}\,. \tag{2}$$

In eqn. (2), $\mathbf{1}$ denotes an nxn identity matrix, $\mathbf{M}$ is the real symmetrical coupling matrix. R is the load and s is the normalized frequency variable defined by

$$s \triangleq \frac{\omega_0}{\Delta\omega}\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right), \tag{3}$$

where ω_0 and $\Delta\omega$ are the center frequency and the bandwidth, respectively.

Letting $\mathbf{e}_1 \triangleq [1\;\;0\;\;\ldots\;\;0]^T$ represent a unit voltage excitation, we can solve eqn. (1) for the currents as

$$\mathbf{I} = \mathbf{Z}^{-1}\,\mathbf{e}_1\,. \tag{4}$$

The input impedance Z_{in} is obtained by

$$Z_{in} = \frac{1}{I_1} = \frac{1}{\mathbf{e}_1^T \mathbf{Z}^{-1}\,\mathbf{e}_1} \tag{5}$$

and its sensitivity with respect to a variable ϕ contained in $\mathbf{Z}$ is given by

$$\frac{\partial Z_{in}}{\partial\phi} = -\frac{1}{I_1^2}\frac{\partial I_1}{\partial\phi} = -\frac{1}{I_1^2}\frac{\partial}{\partial\phi}\,(\mathbf{e}_1^T \mathbf{Z}^{-1}\,\mathbf{e}_1)$$

$$= \frac{1}{I_1^2}\,\mathbf{e}_1^T \mathbf{Z}^{-1}\,\frac{\partial \mathbf{Z}}{\partial\phi}\,\mathbf{Z}^{-1}\,\mathbf{e}_1 = \frac{1}{I_1^2}\,\mathbf{I}^T\,\frac{\partial \mathbf{Z}}{\partial\phi}\,\mathbf{I}\,. \tag{6}$$

Formulas (5) and (6) show that the evaluation of Z_{in} and its sensitivities at one frequency requires one solution of (4). When a large number of frequency points are considered, the matrix analyses such as LU factorization involved in obtaining exact solutions can be very time-consuming. In order to improve computational efficiency, we present an approximate solution of eqn. (4).

An Approximate Method We assume that the frequency ω under consideration is sufficiently far away from the center frequency ω_0. This will be described more precisely as we proceed. Another assumption is that at such frequencies the input impedance of the filter is virtually purely reactive, i.e., $\mathrm{Re}\,(Z_{in}) \approx 0$. The second assumption leads to an approximation of (4) as

$$\mathbf{I} \approx -j\,(s\,\mathbf{1} + \mathbf{M})^{-1}\mathbf{e}_1 = -j\,\frac{1}{s}\,(\mathbf{1} + \frac{1}{s}\,\mathbf{M})^{-1}\mathbf{e}_1 . \tag{7}$$

From matrix theory we have the Neumann series[4] as

$$(\mathbf{1} + \mathbf{A})^{-1} = \mathbf{1} - \mathbf{A} + \mathbf{A}^2 - \mathbf{A}^3 + \ldots \tag{8}$$

which is valid for

$$1 > \max_{1 \le i \le n} |\lambda_{ai}| , \tag{9}$$

where λ_{ai} is an eigenvalue of the nxn matrix $\mathbf{A}$.

Taking a truncated Neumann expansion of $(\mathbf{1} + (1/s)\,\mathbf{M})^{-1}$, we obtain an approximate solution of (7) as

$$\mathbf{I} \approx -j\,\frac{1}{s}\,[\mathbf{1} - \frac{1}{s}\,\mathbf{M} + \frac{1}{s^2}\,\mathbf{M}^2 - \ldots + (-\frac{1}{s})^m\,\mathbf{M}^m]\,\mathbf{e}_1$$

$$= -j\,\frac{1}{s}\,[\mathbf{e}_1 + \sum_{i=1}^{m} (-\frac{1}{s})^i\,\mathbf{b}_i] , \tag{10}$$

where the coefficients $\mathbf{b}_i$ are calculated recursively by

$$\mathbf{b}_i = \mathbf{M}\,\mathbf{b}_{i-1}, \quad i = 2, \ldots, m \tag{11}$$

and $\mathbf{b}_1$ is the first column vector of $\mathbf{M}$. Such an approximation is valid if, following (9),

$$1 > \max_{1 \le i \le n} \left|\frac{\lambda_i}{s}\right| , \quad \text{i.e.,} \quad |s| > \max_{1 \le i \le n} |\lambda_i| , \tag{12}$$

where λ_i is an eigenvalue of $\mathbf{M}$. Condition (12) describes more precisely our assumptions.

Approximate Sensitivity Analysis The accuracy of the approximate Z_{in} and its sensitivities depends on the factor m in formula (10). Here, some interesting results are obtained by taking $m = 1$, which leads to a rough approximation as

$$I_1 \approx -j\,\frac{1}{s}\,(1 - \frac{1}{s}\,M_{11}) = -j\,\frac{1}{s}\,, \tag{13}$$

where $M_{11} = 0$ for a synchronously tuned filter. Also, we have

$$I_k \approx j\,\frac{1}{s^2}\,M_{k1} \text{ for } k \neq 1\,. \tag{14}$$

From equation (6) we have, using the above results,

$$\frac{\partial Z_{in}}{\partial M_{k1}} = \frac{1}{I_1^2}\,\mathbf{I}^T\,\frac{\partial \mathbf{Z}}{\partial M_{k1}}\,\mathbf{I} = j\,\frac{2}{I_1^2}\,(I_1 I_k) \approx -j\,\frac{2}{s}\,M_{k1} \tag{15}$$

and

$$\frac{\partial Z_{in}}{\partial M_{k\ell}} = j\,\frac{2}{I_1^2}\,(I_\ell I_k) \approx j\,\frac{2}{s^2}\,M_{k1}\,M_{\ell 1} = -\,\frac{M_{\ell 1}}{s}\,\frac{\partial Z_{in}}{\partial M_{k1}}\,, \quad \ell, k \neq 1\,. \tag{16}$$

Formulas (15) and (16) show that the couplings related to the first cavity (input cavity), namely M_{k1} and $M_{\ell 1}$, play an important role in sensitivity analysis, particularly when ω is far away from ω_0. Also, for a large $|s|$, we have

$$\left|\frac{\partial Z_{in}}{\partial M_{k1}}\right| >> \left|\frac{\partial Z_{in}}{\partial M_{k\ell}}\right|\,, \quad \ell \neq 1\,. \tag{17}$$

The implication is well-known, namely that the couplings closest to the input dominate the out-of-band performance of the filter. This is also verified numerically in Table I.

Examples and Computational Advantages A 6th-order filter is chosen to illustrate our method. The filter is centered at 4 GHz and has a 40 MHz bandwidth. Its input reactance (i.e., $\mathrm{Im}(Z_{in})$) is shown in Fig. 3. Some typical sensitivity results are given in Table I.

The coefficients $\mathbf{b}_i$, as given by (11), are frequency independent and need be computed only once, which requires $(m-1)\,n^2$ multiplications. Then at each frequency $m \times n$ multiplications are needed to obtain the approximate solution by (10), whereas the exact solution requires $n^3/3 + n^2 - n/3$ multiplications. The saving in computational effort is best illustrated by considering a multiplexer example.

Assume that a contiguous-band multiplexer consists of 5 channels and each channel contains a 6th-order multi-cavity filter of 40 MHz bandwidth. The exact simulation for each channel at one frequency requires 106 multiplications, whereas an approximate solution with $m=4$ involves only 24 multiplications. Suppose we want to simulate such a multiplexer over a 240 MHz band. Taking a 1 MHz interval, each channel filter has to be solved for at 240 frequency points. The approximations can be applied to 180 out-of-band points per channel. Therefore, using the approximate method leads to a saving of 5x (180x(106–24)–108) = 73260 multiplications, which is equivalent to more than 57% of the total multiplications. Notice that 108 multiplications are needed for the recursive calculation of the coefficients $\mathbf{b}_i$. For multiplexers consisting of more channels for which each channel is active over a smaller portion of the total frequency band, the computational saving becomes even more significant.

An important and realistic case, namely, the solution of a lossy filter, is readily accommodated by replacing s in (10) by s–jr (i.e., replacing js by js + r), where r represents the uniform cavity dissipation. Also, it should be noted that the approximate method can be generalized to the case of dispersive filters, for which the coupling matrix and consequently the coefficients $\mathbf{b}_i$ are frequency dependent.

TABLE I SOME TYPICAL RESULTS OF SENSITIVITIES

Frequency (MHz)	$\partial[\mathrm{Im}(Z_{in})]/\partial M_{12}$		$\partial[\mathrm{Im}(Z_{in})]/\partial M_{34}$	
	Exact	Approximate*	Exact	Approximate*
3930	0.477	0.474	0.0004	0.0004
3940	0.564	0.556	0.0009	0.0008
3950	0.692	0.673	0.0026	0.0019
3960	0.903	0.845	0.0096	0.0056

*m = 4

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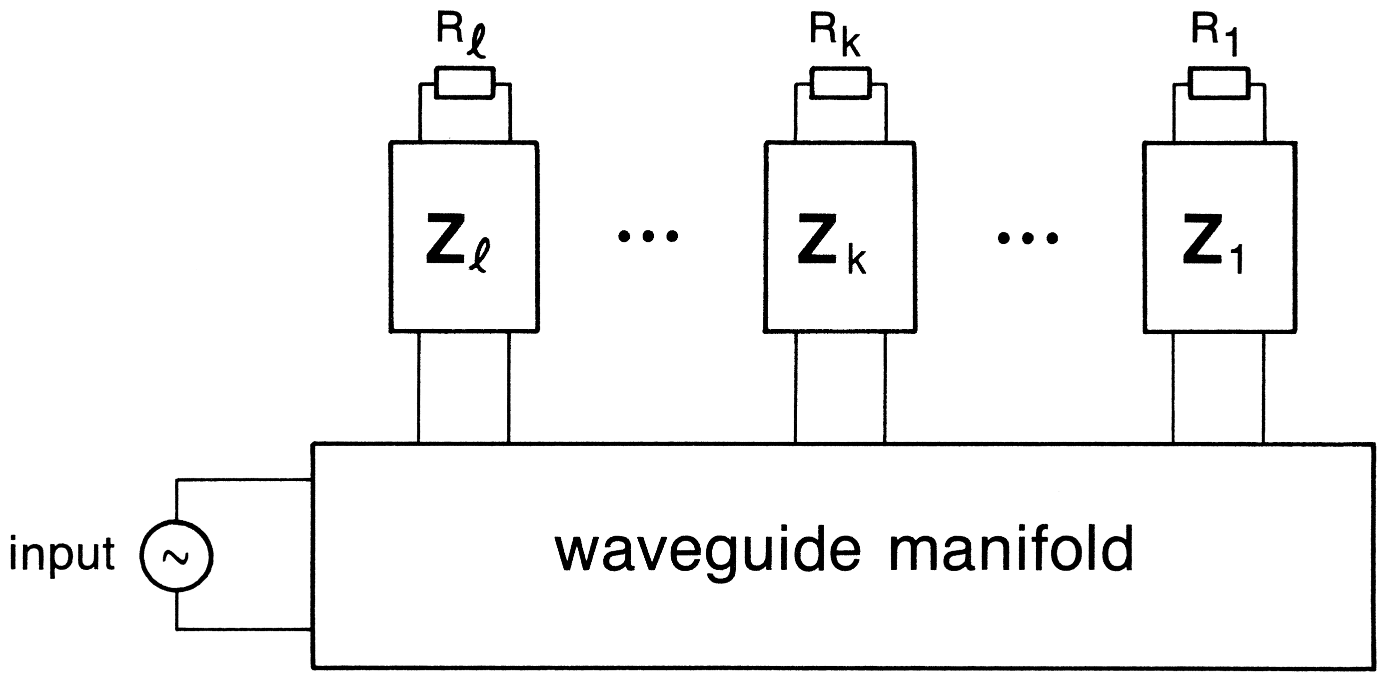

Fig. 1 Block representation of a multiplexer consisting of ℓ channels.

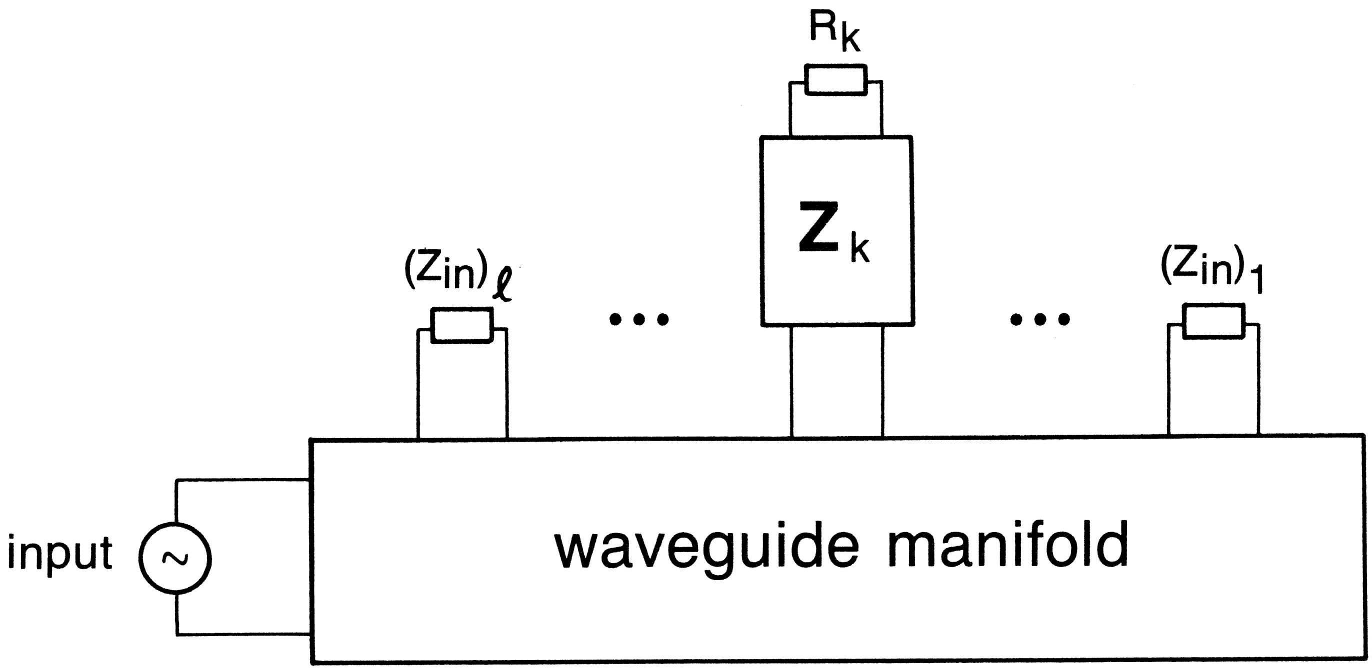

Fig. 2 The reduced network. A channel filter is represented by its input impedance Z_{in}.

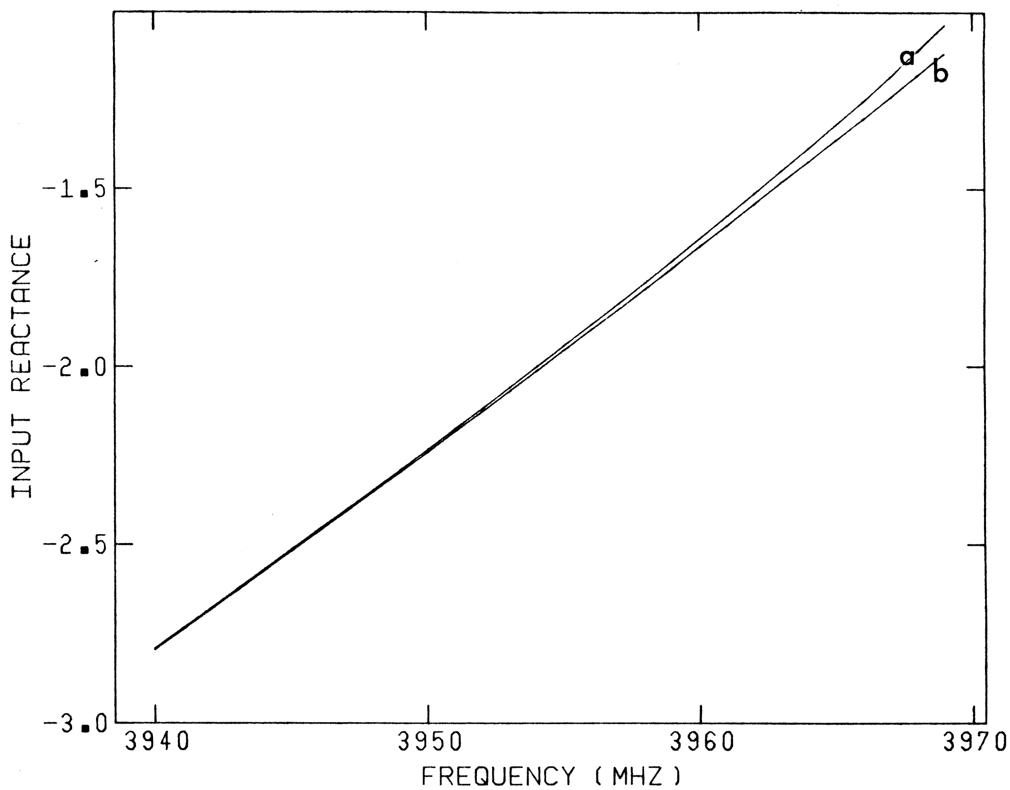

Fig. 3 The input reactance ($Im(Z_{in})$) of a 6th order filter.

a) By exact method. $Re(Z_{in}) \leq 0.0005$ (not shown).

b) By approximate method with $m=4$.