SENSITIVITY EVALUATION OF PHASE-SHIFTING TRANSFORMERS USING THE COMPLEX LAGRANGIAN METHOD

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J.W. BANDLER, M.A. EL-KADY AND H.K. GREWAL

*Simulation Optimization Systems Research Laboratory and the Department of Electrical and Computer Engineering, McMaster University, Hamilton, Canada L8S 4L7**

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* M.A. El-Kady is also with Ontario Hydro, Toronto, Canada.

INTRODUCTION

In References 1 and 2, Bandler and El-Kady have presented the complex Lagrangian method which exploits the Jacobian matrix of the load flow solution in conjunction with a compact conjugate notation. The first-order changes of general network functions have been elaborately investigated to yield generalized sensitivity expressions for practical control parameters pertaining to various two-terminal power system components. The purpose of this letter is to present an extension of the previous work[1,2] to cover the domain of nonreciprocal two-port transmission elements.

We consider a class of phase-shifting transformers which are modelled as an ideal transformer (having complex turns ratio) in series with an equivalent impedance[3,4]. The y-matrix description of this model involves the complex turns ratio and the transformer impedance, and is investigated as a fragment of bus admittance matrix. Exact first-order sensitivity formulae for the transformer control parameters are derived in an elegant manner. Numerical results[4] for the IEEE 118-bus system are also provided.

THE COMPLEX LAGRANGIAN METHOD

Consider an n-node power system containing n_L loads, n_G generators and a slack generator. The buses are ordered in a manner that subscripts $\ell = 1, 2, ..., n_L$ identify load buses, $g = n_L + 1, ..., n_L + n_G$ identify generator buses, and n identifies the slack bus. The power flow equations are expressed compactly as[1,2,4]

$$\mathbf{S}_M^* - \mathbf{E}_M^* \mathbf{Y}_T \mathbf{V}_M = \mathbf{0}\,, \tag{1}$$

where $\mathbf{S}_M$ is a vector of the bus powers, $\mathbf{V}_M$ is a vector of the bus voltages, i.e., $[\mathbf{V}_L^T\ \mathbf{V}_G^T\ V_n]^T$, $\mathbf{Y}_T$ is the nxn bus admittance matrix, and $\mathbf{E}_M$ is a diagonal matrix of components of $\mathbf{V}_M$ in the corresponding order. In order to accomodate the unsymmetric y-matrix of phase-shifting transfomers[3,4], $\mathbf{Y}_T$ is assumed unsymmetrical. Furthermore, by using a quasi-complex power at generator bus g, i.e., $\tilde{S}_g \triangleq P_g + j\,|V_g|$, the standard complex form of the perturbed power flow equations[1,2,4] is written as

$$\begin{bmatrix} \mathbf{K} & \overline{\mathbf{K}} \\ \overline{\mathbf{K}}^* & \mathbf{K}^* \end{bmatrix} \begin{bmatrix} \delta\mathbf{V}_M \\ \delta\mathbf{V}_M^* \end{bmatrix} = \begin{bmatrix} \mathbf{d} \\ \mathbf{d}^* \end{bmatrix}, \tag{2}$$

where the ℓth and gth elements of $\mathbf{d} \triangleq [\mathbf{d}_L^{\ T}\ \mathbf{d}_G^{\ T}\ d_n]^T$ are

$$d_\ell = \delta S_\ell^* - V_\ell^*\ \mathbf{V}_M^T\ \delta\mathbf{y}_\ell, \tag{3}$$

and

$$d_g = \delta\tilde{S}_g^* - (V_g^*\mathbf{V}_M^T\ \delta\mathbf{y}_g + V_g\ \mathbf{V}_M^{*T}\ \delta\mathbf{y}_g^*)/2, \tag{4}$$

respectively, and $\mathbf{y}_i^{\ T}$ is the ith row of $\mathbf{Y}_T$. Observe that d_n is simply $\delta V_n^{\ *}$.

For a general network function f, we write the adjoint equations following the complex Lagrangian method[1,2,4]

$$\begin{bmatrix} \mathbf{K} & \overline{\mathbf{K}} \\ \overline{\mathbf{K}}^* & \mathbf{K}^* \end{bmatrix}^T \begin{bmatrix} \hat{\mathbf{V}} \\ \hat{\mathbf{V}}^* \end{bmatrix} = \begin{bmatrix} \partial f/\partial\mathbf{V}_M \\ \partial f/\partial\mathbf{V}_M^* \end{bmatrix}, \tag{5}$$

where the right-hand side vector contains the formal partial derivatives of f w.r.t. the bus voltages as well as conjugate of the bus voltages. However, for a real f, we observe $\partial f/\partial\mathbf{V}_M = (\partial f/\partial\mathbf{V}_M^{\ *})^*$. The coefficient matrix involved in (5) is the transpose of the Jacobian available at the load flow solution. Partitioning $\hat{\mathbf{V}}$ into subquantities $\hat{\mathbf{V}}_L$, $\hat{\mathbf{V}}_G$ and $\hat{V}_n$, we exploit the first-order change of f expressed in the form[1,2,4]

$$\delta f = \hat{\mathbf{V}}_L^T\mathbf{d}_L + \hat{\mathbf{V}}_G^T\mathbf{d}_G + \hat{V}_n d_n + \hat{\mathbf{V}}_L^{*T}\mathbf{d}_L^* + \hat{\mathbf{V}}_G^{*T}\mathbf{d}_G^* + \hat{V}_n^* d_n^*, \tag{6}$$

where f is assumed explicitly independent of the system control variables. Using (3), (4) and (6), the sensitivity formula for the ith element in the ℓth row vector $\mathbf{y}_\ell^{\ T}$ is obtained as

$$\frac{df}{dy_{\ell i}} = -\hat{V}_\ell V_\ell^* V_i, \tag{7a}$$

and that in the gth row vector $\mathbf{y}_g^{\ T}$ is

$$\frac{df}{dy_{gi}} = -(\hat{V}_g + \hat{V}_g^*)V_g^* V_i/2. \tag{7b}$$

Note that $df/dy_{ni} = 0$ as d_n is independent of the nth row vector of $\mathbf{Y}_T$. Also observe that for the unsymmetric $\mathbf{Y}_T$, we have $y_{\ell i} \neq y_{i\ell}$ and $y_{gi} \neq y_{ig}$. Hence, for the y-parameter y_{pq} of a transmission element connecting buses p and q, we write a general sensitivity expression as

$$\frac{df}{dy_{pq}} = -\hat{V}_p^R\ V_p^*\ V_q, \tag{8}$$

where

$$\hat{V}_p^R \triangleq \begin{cases} \hat{V}_\ell & \text{for } p = \ell \\ \mathrm{Re}\{\hat{V}_g\} & \text{for } p = g \\ 0 & \text{for } p = n \end{cases} \quad . \tag{9}$$

SENSITIVITY EVALUATION OF PHASE-SHIFTING TRANSFORMERS

The y-matrix of the transformer model[3,4] depicted in Fig. 1 is expressed in a matrix form as

$$\mathbf{y} = \frac{1}{Z_t} \begin{bmatrix} \dfrac{1}{a_t^*} \\ -1 \end{bmatrix} \begin{bmatrix} \dfrac{1}{a_t} & -1 \end{bmatrix}, \tag{10}$$

where Z_t and a_t are the transformer impedance and complex turns ratio, respectively. The formal partial derivatives of the y-parameters of (10) with respect to a_t, a_t^*, Z_t and Z_t^* are listed in the Table. Using the chain rule and the derivatives together with (8), we obtain the sensitivity formulae for a_t and Z_t as

$$\frac{df}{da_t} = \frac{1}{a_t^2} \left[\frac{V_p}{Z_t} \left(\frac{V_p^*}{a_t^*} \hat{V}_p^R - V_q^* \hat{V}_q^R \right) + \left\{ \frac{V_p^*}{Z_t} \hat{V}_p^R \left(\frac{V_p}{a_t} - V_q \right) \right\}^* \right] \tag{11}$$

and

$$\frac{df}{dZ_t} = \frac{1}{Z_t^2} \left(\frac{V_p}{a_t} - V_q \right) \left(\frac{V_p^*}{a_t^*} \hat{V}_p^R - V_q^* \hat{V}_q^R \right), \tag{12}$$

respectively. In order to get the sensitivities in polar and Cartesian modes, the transformations are available[1,2,4] and are easy to derive.

A NUMERICAL EXAMPLE[4]

A one-line diagram of the IEEE 118-bus power system is shown in Fig. 2. The system has two phase-shifting transformers, one connecting buses 59 and 63 (t=188) and the other connecting buses 61 and 64 (t=192). We consider the slack bus real power P_n as the function of interest. The kth element of the right-hand vector $\partial f/\partial \mathbf{V}_M$ of (5) pertaining to this function is

$$\frac{\partial f}{\partial V_k} = (Y_{nk} V_n^* + \delta_{kn} \sum_{j=1}^{n} Y_{nj}^* V_j^*)/2 \,,$$

where δ_{kn} stands for the Kronecker delta. The numerical results for $\zeta_t \triangleq [|a_t| \; \phi_t \; R_t \; X_t]^T$ are summarized as

$$\frac{df}{d\zeta_{188}} = [0.039796 \;\; 0.239011 \;\; 1.86075 \;\; 0.312149]^T$$

and

$$\frac{df}{d\zeta_{192}} = [0.029692 \;\; -0.142203 \;\; 0.000229 \;\; -0.009722]^T$$

where $|a_t|$, ϕ_t, R_t and X_t are, respectively, transformer turns ratio magnitude, phase-angle, resistance and reactance. These results have been verified by small perturbations at the nominal point.

CONCLUSIONS

We have presented a useful extension of the complex Lagrangian method to obtain unified sensitivity formulae which are applicable to phase-shifting transformers. The sensitivity formulae have been derived using the y-matrix of a transformer model frequently used in the load flow studies. These formulae have been verified for practical purposes and are capable of providing optimal settings for the real and reactive power flow control in interconnected power systems.

ACKNOWLEDGEMENT

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TABLE

SHORT-CIRCUIT ADMITTANCE PARAMETERS AND THEIR PARTIAL DERIVATIVES

Parameter	Expression	Derivative w.r.t. a_t	Derivative w.r.t. a_t^*	Derivative w.r.t. Z_t	Derivative w.r.t. Z_t^*
y_{pp}	$\frac{1}{Z_t a_t a_t^*}$	$\frac{-1}{Z_t a_t^2 a_t^*}$	$\frac{-1}{Z_t a_t a_t^{*2}}$	$\frac{-1}{Z_t^2 a_t a_t^*}$	0
y_{pq}	$\frac{-1}{Z_t a_t^*}$	0	$\frac{1}{Z_t a_t^{*2}}$	$\frac{1}{Z_t^2 a_t^*}$	0
y_{qp}	$\frac{-1}{Z_t a_t}$	$\frac{1}{Z_t a_t^2}$	0	$\frac{1}{Z_t^2 a_t}$	0
y_{qq}	$\frac{1}{Z_t}$	0	0	$\frac{-1}{Z_t^2}$	0

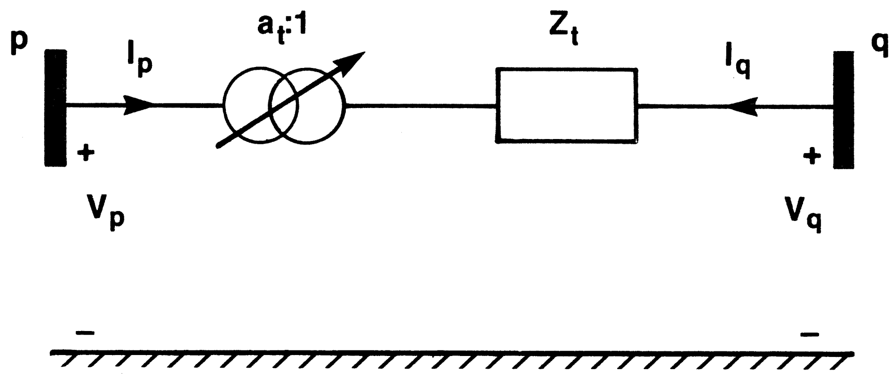

Fig. 1 A phase-shifting transformer model.

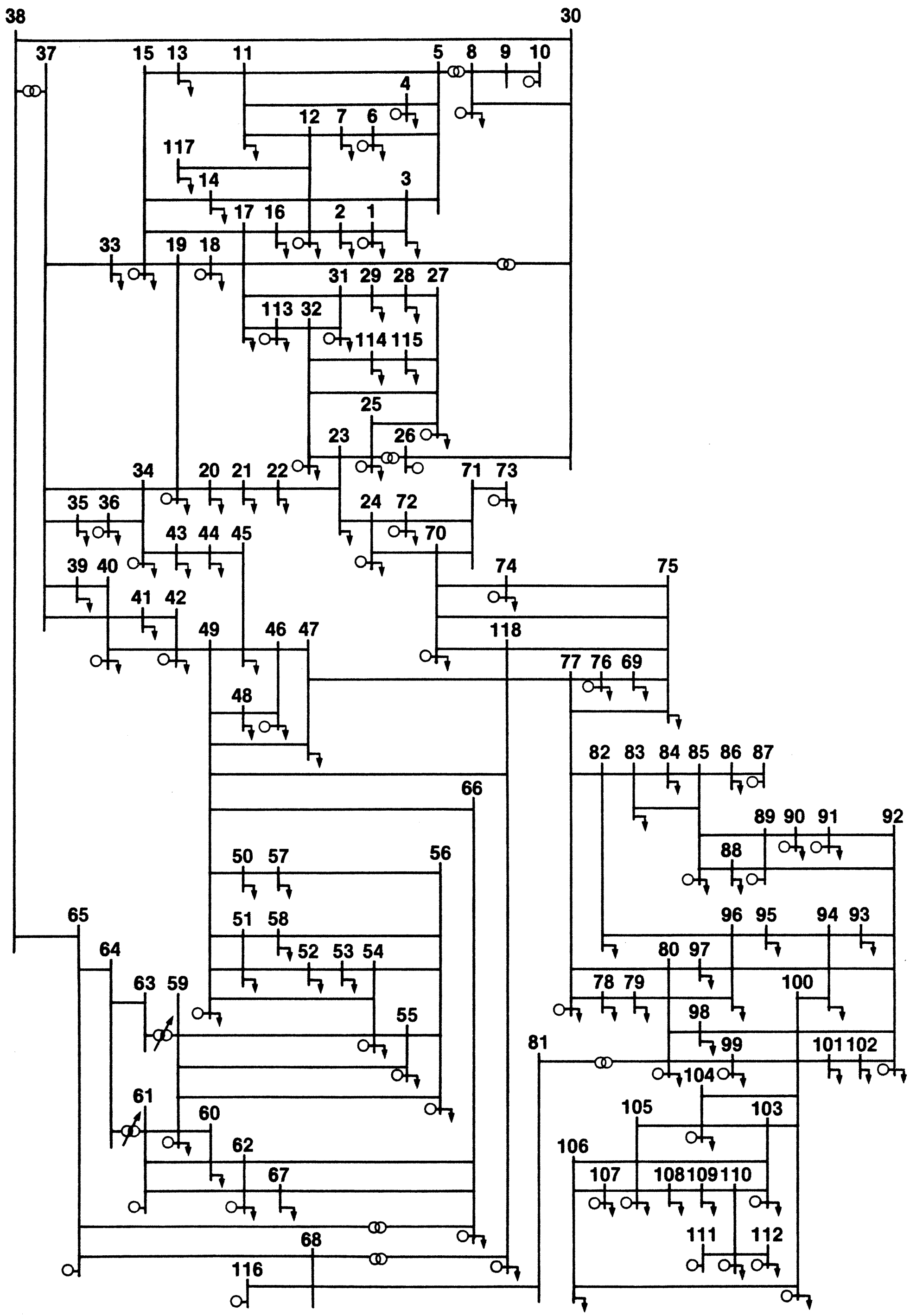

Fig. 2 The IEEE 118-bus system.