## AN AUTOMATIC DECOMPOSITION APPROACH TO OPTIMIZATION OF LARGE MICROWAVE SYSTEMS

J.W. Bandler and Q.J. Zhang

SOS-86-11-R

December 1986

© J.W. Bandler and Q.J. Zhang 1986

No part of this document may be copied, translated, transcribed or entered in any form into any machine without written permission. Address enquiries in this regard to Dr. J.W. Bandler. Excerpts may be quoted for scholarly purposes with full acknowledgement of source. This document may not be lent or circulated without this title page and its original cover.

# AN AUTOMATIC DECOMPOSITION APPROACH TO OPTIMIZATION OF LARGE MICROWAVE SYSTEMS

J.W. Bandler, Fellow, IEEE, and Q.J. Zhang, Student Member, IEEE

### **Abstract**

We present a novel and general technique applicable to the optimization of large microwave systems. Using sensitivity information obtained from a suitable Monte-Carlo analysis, we extract possible decomposition properties which could otherwise be deduced only through a detailed physical and topological investigation. The overall problem is automatically separated into a sequence of subproblems, each being characterized by the optimization of a subset of circuit functions w.r.t. variables which are sensitive to the selected responses. The decomposition patterns are dynamically updated until a satisfactory solution is reached. The partitioning approach proposed by Kondoh for FET modelling problems is verified. The technique was successfully tested on large scale optimizations of microwave multiplexers involving 16 channels, 399 nonlinear functions and 240 variables.

This work was supported in part by the Natural Sciences and Engineering Research Council of Canada under Grants A7239 and G1135.

The authors are with the Simulation Optimization Systems Research Laboratory and the Department of Electrical and Computer Engineering, McMaster University, Hamilton, Canada L8S 4L7.

J.W. Bandler is also with Optimization Systems Associates Inc., 163 Watson's Lane, Dundas, Ontario, Canada L9H 6L1.

#### I. INTRODUCTION

A serious challenge to researchers in microwave CAD areas is due to the size of practical microwave systems. Existing CAD techniques, mature enough to handle systems of ordinary size, generally balk at large circuits. The reasons for their failure include prohibitive computer storage and CPU times required. A frequent frustration with large scale optimization is the increased likelihood of stoppping at an undesired local optimum. Other difficulties, especially in prototype and production tuning, are due to human inability to cope with problems involving large numbers of independent variables to be adjusted simultaneously to meet a specified response pattern over a wide frequency range.

Recently, FET modelling[1] and manifold multiplexer design [2] problems were solved using appropriate decomposition schemes. The optimization problems were cleverly treated by systematically or repeatedly selecting and adjusting various small sets of parameters and responses until the system becomes acceptably operational. The success of these efforts motivated us to pursue the generalization and automation of decomposition approaches for microwave optimization problems.

The concept of decomposition has been a traditional mathematically based vehicle for approaching large scale problems. Himmelblau[3] has an excellent collection of surveys from the areas of mathematics, engineering, economics and management sciences. In the present paper, we will be interested in such aspects of decomposition that are beneficial to circuit optimization problems.

Decomposition methods used in mathematical programming theory usually assume certain structures for the objective function and constraints. Theoretical investigations have been performed for linear programming, nonlinear programming and minimax optimizations[3-6].

In circuits and systems, diakoptic analysis, generalized hybrid analysis and network tearing methods have been developed[7-13]. Important to those methods are circuit relations,

especially topological relations. The methods have been used for circuit analysis, design and fault diagnosis[3,7-14].

Decomposition has also been an active subject in electrical power systems since such problems easily result in thousands of variables and equations. Examples can be found in optimal power flow [15,16], state estimation[17] and real and reactive power optimization problems[18]. The decomposition patterns involved are obtained using both physical and analytical investigations of the systems.

Microwave engineers have their own special difficulties. Thorough laboratory experimentation has to be performed before using certain function structures assumed in mathematical programming theory. They do not take advantage of topological analysis often exploited in the areas of circuits and systems since microwave device models are oriented more to physical than topological analysis. Unlike power systems, most microwave responses are much more complicated and highly nonlinear. It is often difficult for microwave engineers to analytically indicate possible decomposition patterns.

The state-of-the-art in large-scale optimization of microwave circuits is still device dependent and based on heuristic judgement[1,2,19]. To our knowledge, there does not exist a general and abstract theory describing a decomposition approach to microwave circuit optimization not requiring particular physical or topological knowledge of the system.

In this paper, we present a novel technique applicable to the optimization of large microwave systems. Using sensitivity information obtained from a suitable Monte-Carlo analysis, we extract possible decomposition properties which could otherwise be deduced only through a physical and topological investigation. The overall problem is automatically separated into a sequence of subproblems, each being characterized by the optimization of a subset of circuit functions w.r.t. variables which are sensitive to the selected responses. Our suggested technique has been successfully tested on microwave multiplexers involving up to 16 channels and 240 variables.

In Section II, we describe the basic concepts of decomposition for circuit optimization problems. Using these concepts, the partitioning approach for FET modelling problems suggested by Kondoh[1] is verified. Section III illustrates the automatic determination of suboptimization problems. An automated decomposition algorithm for large scale microwave optimization is presented in Section IV. In Section V, the method is applied to the optimization of microwave multiplexers. Interesting results demonstrating the whole procedure of automated decomposition for a 5-channel multiplexer are depicted in illustrative graphs. The results of optimizing a 16-channel multiplexer using our approach are provided.

# II. THE DECOMPOSITION APPROACH FOR CIRCUIT OPTIMIZATION PROBLEMS

## Circuit Optimization Problems

Let  $\boldsymbol{\varphi} = [\varphi_1 \ \varphi_2 \ ... \ \varphi_n]^T$  represent the system parameters. The circuit responses, denoted as  $F_k(\boldsymbol{\varphi},\omega)$ ,  $k=1,2,...,n_F$ , are functions of variables  $\boldsymbol{\varphi}$  and frequency  $\omega$ . Fig. 1 gives a graphical explanation of different response functions for a general microwave system. In an optimization problem for circuit design, the objective function usually involves a set of nonlinear error functions  $f_j(\boldsymbol{\varphi})$ , j=1,2,...,m. Typically, the error functions represent the weighted differences between circuit responses and given specifications in the form

$$\begin{split} & w_{Uk}(\omega)(F_k(\pmb{\varphi},\omega) - S_{Uk}(\omega)) \\ & - w_{Lk}(\omega)(F_k(\pmb{\varphi},\omega) - S_{Lk}(\omega)) \\ & \qquad \qquad k \in \{1,2,...,n_F\}, \end{split} \tag{1}$$

where  $S_{Uk}$  and  $S_{Lk}$  are upper and lower specifications, respectively.  $w_{Uk}$  and  $w_{Lk}$  are weighting factors.

Suppose sets I and J are defined as

$$I \stackrel{\Delta}{=} \{1, 2, ..., n\}, \tag{2}$$

$$J \stackrel{\triangle}{=} \{1, 2, ..., m\}. \tag{3}$$

The overall optimization problem, e.g., a minimax optimization, is

minimize 
$$\max_{\Phi_i} f_j(\Phi)$$
. (4)  
 $\Phi_i, i \in I \quad j \in J$ 

## Description of the Decomposition Approach

In a decomposition approach, one attempts to reach the overall solution by solving a sequence of subproblems. A typical subproblem is characterized by

minimize 
$$\max_{\boldsymbol{\varphi}_i,\ i \in I^S} f_j(\boldsymbol{\varphi}),$$
 (5)

where Is and Js are subsets of I and J, respectively.

The basic idea for decomposition is to decouple a variable  $\phi_i$  from a function  $f_j$  if the interaction between them is weak. A subproblem contains only the sensitively related variables and functions. A proper arrangement of the sequence of different subproblems to be solved is often important to ensure convergence and efficiency.

#### Sensitivity Analysis

We perform sensitivity analysis at a set of randomly chosen points  $\Phi^{\ell}$ ,  $\ell=1,2,...$  A measure of the interaction between  $\Phi_i$  and  $f_i$  is defined as

$$S_{ij} \triangleq \sum_{\ell} \left( \frac{\partial f_{j}(\Phi^{\ell})}{\partial \Phi_{i}} \frac{\Phi_{i}^{0}}{f_{i}^{0}} \right)^{2} , \qquad (6)$$

where  $\phi_i^0$  and  $f_j^0$  are used for scaling. All the  $S_{ij}$ , i=1,2,...,n and j=1,2,...,m, constitute a nxm sensitivity matrix S. It is reasonable to conclude that  $\phi_i$  and  $f_j$  can be decoupled if  $S_{ij}$  is very small.

## Grouping of Variables and Functions

The examination of various interaction patterns between  $\phi_i$ ,  $i \in I$ , and  $f_j$ ,  $j \in J$ , results in the breakdown of all variables  $\phi$  into p groups identified by index sets  $I_1$ ,  $I_2$ , ...,  $I_p$ , and all functions f into q groups identified by sets  $J_1, J_2, ..., J_q$ . We have

$$I = I_1 \cup I_2 \cup \dots \cup I_p \tag{7}$$

and

$$J = J_1 \cup J_2 \cup \dots \cup J_q . \tag{8}$$

The partitioning of  $\phi$  or f can be achieved either manually or automatically. The manual procedure corresponds to the manual determination of variable groups and function groups using a priori knowledge. Such knowledge is typically obtained through extensive laboratory experiment and an excellent understanding of the particular device. The automatic procedure corresponds to the computerized partitioning of  $\phi$  or f based upon the sensitivity matrix S. The partitioning of  $\phi$  and f can be performed 1) both manually, 2) manually for  $\phi$  and automatically for f, 3) automatically for  $\phi$  and manually for f, 4) both automatically.

As an example for manual partitioning of f, we consider a N-channel multiplexer. The common port return loss and channel insertion loss responses associated with the same channel can be grouped together since their behavior is similarly affected by variables  $\phi$ . Therefore, we have N groups of functions, i.e., q=N.  $J_{\ell}$  contains indices of error functions related to channel  $\ell$ ,  $\ell=1,2,...,N$ .

## A Procedure for Automatic Partitioning of Variables o

Suppose the function groups have been determined, i.e., J has been decomposed into  $J_{\ell}$ ,  $\ell=1,2,...,q$ . We define a nxq matrix C whose (i,  $\ell$ )th component is

$$C_{i\ell} = \sum_{j \in J_{\ell}} (w_{ij} S_{ij}), \qquad (9)$$

where  $w_{ij}$  is a weighting factor. A very small value of an entry in the C matrix, say,  $C_{i\ell}$ , implies that the ith variable and the  $\ell$ th function group are weakly interconnected.

Let  $C_{ave}$  represent the average value of all components in the C matrix. For a given factor  $\lambda$ ,  $\lambda \geq 0$ , the matrix is made sparse such that  $C_{i\ell}$  is set to zero if it is less than  $\lambda C_{ave}$ . By making C sparse, insensitive variables are eliminated and weak interactions between variables and function groups are decoupled.

Two variables  $\varphi_i$  and  $\varphi_j$  belong to the same group if they interact only with the same groups of functions, i.e., if the ith and the jth rows of C have the same zero/nonzero pattern. A thorough computerized checking of the C matrix results in the automatic determination of index sets  $I_k$ , k=1,2,...,p.

## An Illustrative Example of Matrix C

Consider the fictitious relations between variables and function groups shown in Fig. 2(a). The functions f have been arranged into 5 groups. The C matrix (already made sparse) is

As seen from Fig. 2(a),  $\phi_2$  and  $\phi_3$  both affect only the 2nd function group. In the C matrix, rows 2 and 3 both have only one nonzero located at the 2nd column. Therefore, variables  $\phi_2$  and  $\phi_3$  are grouped together. Similarly, variables  $\phi_4$  and  $\phi_6$  belong to the same group. The resulting index sets for variable groups are  $I_1 = \{9\}$ ,  $I_2 = \{2, 3\}$ ,  $I_3 = \{7\}$ ,  $I_4 = \{5\}$ ,  $I_5 = \{4, 6\}$ ,  $I_6 = \{1\}$  and  $I_7 = \{8\}$ . The index sets have been ordered such that the kth variable group correlates with no more function groups than the (k+1)th variable group does, k=1, 2, ..., 6. Such an arrangement is made to keep subsequent description simple.

#### **Decomposition Dictionary**

To manipulate directly with groups of variables and groups of functions, we construct a  $p \times q$  dictionary decomposition matrix **D**. Define the  $(k, \ell)$ th component of **D** as

$$D_{k\ell} \stackrel{\Delta}{=} \sum_{i \in I_k} \sum_{j \in J_{\ell}} (w_{ij} S_{ij})$$

$$= \sum_{i \in I_k} C_{i\ell}.$$
(11)

If  $D_{k\ell}$  is zero, variables in the kth group are decoupled from functions in the  $\ell$ th group. Otherwise if  $D_{k\ell} \neq 0$ , we say that  $\varphi_i$ ,  $i \in I_k$ , and  $f_j$ ,  $j \in J_\ell$ , are correlated. The decomposition dictionary gives a clear picture of the correlation patterns between groups of variables and functions, facilitating the automatic determination of suboptimization problems. The ideal dictionary is a diagonal matrix where a subproblem simply corresponds to a diagonal element. In this case, only one variable group and one function group is involved in a subproblem. If a diagonal dictionary can be obtained without artificially making C sparse (i.e., using sparse factor  $\lambda = 0$ ), then the system is completely decomposable[20]. For a completely decomposable system, different subproblems can be calculated in parallel.

## An Illustrative Example for Constructing the Decomposition Dictionary

Consider the previous example with the resulting C matrix defined in (10). According to the index sets  $I_k$ , k=1,2,...,7, the decomposition dictionary D can be obtained from C by adding rows 2 and 3, and adding rows 4 and 6, respectively. The relations between groups of variables and functions are shown in Fig. 2(b). The resulting dictionary is

100.	0.	0.	0.	0.	1		
0.	200.	0.	0.	0.			
0.	0.	100.	0.	0.			
0.	0.	0.	0.	100.		,	(12)
0.	0.	180.	180.	0.			
20.	100.	30.	0.	0.	200		
0.	70.	100.	<b>50</b> .	0.			

where each entry has been rounded to multiples of 10.

## Sensitivity Analysis and Decomposition for FET Device Models

Through extensive experiment on practical FET devices, Kondoh[1] summarized 8 suboptimization problems which can be repeatedly solved to yield a FET model with improved accuracy. The equivalent circuit is shown in Fig. 3. The parameter values of a reference point  $\Phi^0$  are listed in Table I. We perform sensitivity analysis at 10 randomly chosen parameter points in the 10% neighborhood of  $\Phi^0$ . The function  $f_j$  used in (6) is defined as the weighted difference between the calculated and the measured values of the modulus or the phase of a particular S parameter. Functions associated with the same S parameter are grouped together. Table II shows the C matrix of (9) before being made sparse, indicating strong as well as weak interconnections between each individual parameter and different groups of functions. Table III provides an example of the decomposition dictionary calculated and normalized from Table II. Table III yields 8 subproblems which agree with and further verify the decomposition scheme proposed in [1]. When the C matrix is made sparse, certain entries, whose values are only slightly less than the dominant ones, are also set to zero. Therefore, as mentioned in [1], repeated cycling and careful ordering of the 8 suboptimizations are necessary. The feasibility of computerized automatic decomposition is demonstrated by this example.

## III. AUTOMATIC DETERMINATION OF SUBOPTIMIZATION PROBLEMS

## The Reference Function Group

Usually, the decomposition dictionary is not diagonal. A suboptimization often involves several function groups and several variable groups. Among the function groups involved, there is a key group which we call the reference group. Such a group typically contains the worst error function. The reference function group is used to initiate a subproblem as described in the subsequent text.

## Candidate Groups of Variables

Suppose the index set  $J_\ell$  indicates the reference function group. The candidate groups of variables to be used for the suboptimization are those which affect  $f_j$ ,  $j \in J_\ell$ .

In the decomposition dictionary, the  $\ell$ th column associates with the reference function group. Rows having a nonzero in the  $\ell$ th column are candidate rows, each corresponding to a candidate variable group. Take Fig. 2(b) as an example. Suppose that the function group associated with index set  $J_2$  is the reference group, i.e.,  $\ell=2$ . The candidate groups of variables are  $I_2$ ,  $I_6$  and  $I_7$  since they correlate with the reference function group. Correspondingly, in the D matrix of (12), rows 2, 6 and 7 are candidate rows since they all have a nonzero in the 2nd column.

## Determination of a Suboptimization Problem

An automatic procedure for the determination of  $I^s$  and  $J^s$  for the suboptimization of (5) has been developed. Suppose  $J_\ell$  indicates the reference function group. For a selected candidate variable group, e.g., the one corresponding to set  $I_k$ , the index set  $J^s$  indicates the union of all function groups which correlate with variable group k.  $I^s$  identifies variables in the kth group, as well as all other variables which correlate with functions only within  $f_j$ ,  $j \in J^s$ . Also,  $I^s$  excludes variables not correlating with any active functions in  $f_j$ ,  $j \in J^s$ . A function  $f_j$  is said to be active if

$$f>0.8M_{\rm f}~$$
 when  $~M_{\rm f}>0$  
$$f>1.25M_{\rm f}~$$
 when  $~M_{\rm f}<0~,$ 

where

$$M_{f} \stackrel{\Delta}{=} \max_{j \in J^{s}} f_{j}. \tag{14}$$

## Priority of Candidate Groups of Variables

It can be seen that a pair of  $(I^s, J^s)$  associate with a pair of  $(I_k, J_\ell)$ . For a selected reference function group, each candidate variable group leads to a subproblem. The sequence of subproblems used to penalize  $f_j$ ,  $j \in J_\ell$ , are determined by the priority of all resulting candidates.

Since each candidate determines the function set  $J^s$  for a suboptimization, the priority of the candidate is based upon the pattern of error functions it will affect, i.e. patterns of  $f_j$ ,  $j \in J^s$ . Firstly, the fewer the number of function groups in  $J^s$ , the higher the priority. Secondly, the worse the overall error functions in  $J^s$ , the higher the priority. The overall error functions in  $J^s$  are ranked by the generalized least pth function (GLP)[21] as

GLP = 
$$\begin{cases} M_{f} \left( \sum_{j \in K} (f_{j}(\Phi)/M_{f})^{q} \right)^{1/q} & \text{if } M_{f} \neq 0 \\ 0 & \text{if } M_{f} = 0 \end{cases},$$
 (15)

where M<sub>f</sub> was defined in (14) and

$$\begin{split} & \text{if}\, M_f > 0 \text{, then}\, K = \{j\, |\, f_j \geq 0 \text{ ,} j \in J^s\} \quad \text{ and } q = p \\ & \text{if}\, M_f < 0 \text{ then}\, K = J^s \qquad \qquad \text{and} \ q = -p \,. \end{split} \label{eq:mass_spectrum}$$

Typically, we choose p = 2.

The priority of candidate variable groups can be similarly determined in the decomposition dictionary. The fewer the number of nonzeros that exist in a candidate row, the higher the priority. For two candidate rows containing an equal number of nonzeros, a

higher priority is given to the candidate having a larger value in its generalized least pth function.

## An Example for Deciding on a Subproblem and Candidate Priority

For the example of Fig. 2, suppose that the maximum error functions within each of the 5 function groups are [3.8 4. 1. -1. 2.]. Suppose that we choose the worst group, i.e., group 2, as the reference function group. According to our previous discussions, the candidate variable groups are  $I_2$ ,  $I_6$  and  $I_7$ .  $I_2$  has the highest priority since it affects fewer (i.e. only one) function groups than  $I_6$  or  $I_7$  does ( $I_6$  and  $I_7$  both affect three function groups). To rank the priority between candidates  $I_6$  and  $I_7$ , we compare the overall error functions they will affect. The functions affected by variables in  $I_6$  (or  $I_7$ ) are  $f_j$ ,  $j \in J^s = J_1 \cup J_2 \cup J_3$  (or  $J^s = J_2 \cup J_3 \cup J_4$ ).  $I_6$  has a higher priority than  $I_7$  since the overall error functions in  $J_1 \cup J_2 \cup J_3$  are worse than that in  $J_2 \cup J_3 \cup J_4$ .

Correspondingly, in the decomposition dictionary of (12), rows 2, 6 and 7 are candidates. Row 2 has the highest priority since it contains fewer nonzeros than others. Row 6 has the second highest priority since its GLP value is obviously larger than the GLP value for row 7.

To formulate a suboptimization problem, i.e., to decide  $I^s$  and  $J^s$ , we choose a pair of  $(I_k, I_\ell)$ , e.g., candidate variable group  $I_6$  and reference function group  $J_2$ . The index set  $J^s = J_1 \cup J_2 \cup J_3$ . The variable index set  $I^s$  includes  $I_6$  (indicating the candidate variable group), as well as  $I_1$ ,  $I_2$  and  $I_3$  (indicating all other variables affecting functions only within  $J^s$ ). Further,  $I_3$  can be excluded from  $I^s$  since variables in  $I_3$  do not affect active functions in  $J^s$ . Therefore, we have  $I^s = I_6 \cup I_1 \cup I_2$ .

### Circuit Responses and Sample Frequencies for a Subproblem

When a subset of error functions  $f_j(\mathbf{\Phi})$ ,  $j \in J^s$ , are included in a subproblem, the necessary circuit response functions  $F_a(\mathbf{\Phi}, \omega_b)$ ,  $a \in \{1, 2, ..., n_F\}$  and frequency points  $\omega_b$ ,

 $b \in \{1, 2, ..., n_{\omega}\}$ , should be selected for circuit simulation programs. This is accomplished using a coding scheme representing the one-to-one correspondence between j and (a, b). We define weighting factor matrices  $\mathbf{W}_U$  (for upper specification) and  $\mathbf{W}_L$  (for lower specification). Both matrices are  $n_F$  by  $n_{\omega}$ . The (a, b)th component of  $\mathbf{W}_U$  and  $\mathbf{W}_L$  are the weighting factors  $\mathbf{w}_{Ua}(\omega_b)$  and  $\mathbf{w}_{La}(\omega_b)$ , respectively, as defined in (1).  $\mathbf{w}_{Ua}(\omega_b)$  or  $\mathbf{w}_{La}(\omega_b)$  is zero if no upper or lower specification is imposed on  $F_a(\Phi, \omega_b)$ . The coding scheme relating the index of  $f_j$  to the indices of nonzeros in  $\mathbf{W}_U$  and  $\mathbf{W}_L$  are constructed by systematically scanning through  $\mathbf{W}_U$  and then  $\mathbf{W}_L$ , respectively.

# IV. AN AUTOMATIC DECOMPOSITION ALGORITHM FOR CIRCUIT OPTIMIZATION

An automatic decomposition algorithm for optimization of microwave systems has been developed and implemented. The algorithm can decide when to update the sensitivity matrix and the decomposition dictionary. The formulation and the sequence of suboptimization problems are dynamically determined. The degree of decomposition is reduced as the system converges to its overall solution. As a special case, if all variables interact with all functions, our approach solves only one subproblem, this being identical to the original overall optimization.

Step 1 Initialize sparse factor λ. Calculate the sensitivity matrix S and the decomposition dictionary D. Calculate f.

<u>Comment</u> The initial sensitivity matrix can be obtained from a suitable Monte-Carlo sensitivity analysis performed off-line. All error functions are calculated in this step.

Step 2 Define  $\ell$  such that

$$\begin{array}{lll} f_{worst} = & max & f_j & = & max & f_j \,. \\ & & j \in J_{\boldsymbol{\ell}} & & j \in J \end{array} \label{eq:fworst}$$

Comment The \$\ell\$th function group contains the worst response. Such a function group will be frequently chosen as the reference group to be penalized.

Step 3 For the given  $\ell$ , determine the sequence of candidate rows in D. Rank the candidates in decreasing priority. Set k=0.

Comment The  $\ell$ th function group is the reference group to be penalized. All variable groups correlating with the  $\ell$ th function group are considered as candidates.

Step 4 If k=0 then set k to the row index of the first candidate, otherwise set k to the row index of the next candidate. If such a candidate does not exist then go to Step 8.

<u>Comment</u> The candidate groups of variables are sequentially selected. Each entry into this step results in a selection of a candidate with a lower priority than the current one.

<u>Step 5</u> Define I<sup>s</sup> and J<sup>s</sup> using the current k,  $\ell$ . If I<sup>s</sup> and J<sup>s</sup> are identical with their previous values then go to Step 4. Solve the suboptimization problem

$$\begin{array}{lll} \text{minimize} & \text{max} & f_j(\pmb{\varphi}) \ . \\ \varphi_i, \ i \! \in \! I^s & j \! \in \! J^s \end{array}$$

Terminate the optimization if

$$\max_{j \notin J^s} \ f_j > \lambda' \ f_{worst} \ .$$

Comment A subproblem is formulated and solved in this step. By checking the functions not covered in the present suboptimization, any significant deterioration in the overall objective function is prevented. The factor  $\lambda'$  can be, e.g., 1.2.

 $\underline{Step \ 6} \qquad \qquad If \ I^s = I \ \ and \ J^s = J \ then \ stop.$ 

<u>Comment</u> The program terminates following the completion of an overall optimization which is considered as the last subproblem.

Step 7 Calculate f. Calculate

$$f_{worst} = \max_{j \in J} f_j .$$

Go to Step 5.

Comment

An overall simulation is performed. By going to Step 5, the current reference function group can be continuously penalized in the next subproblem even if this group does not include the worst error functions.

Step 8

If

$$\begin{array}{lll} \text{max} & f_j & < & \text{max} & f_j \\ j \in J^s & & j \in J \end{array}$$

then go to Step 2. If  $\lambda \approx 0$  then stop otherwise, update S, reduce  $\lambda$ , update dictionary D and go to Step 3.

Comment:

When the selection of a candidate fails, a new sequence of candidates will be defined by going to Steps 2 or 3. By reducing the sparse factor  $\lambda$ , the degree of decomposition is reduced as the overall solution is being approached. The reference function group will be readjusted if the existing one does not contain the maximum error function. For completely decomposable problems, the terminating conditions in Step 6 will not be satisfied and the program will exit from Step 8.

## V. LARGE SCALE OPTIMIZATION OF MULTIPLEXERS

The automatic decomposition technique was tested on the optimization of microwave multiplexers used in satellite communications. Specifications were imposed on the common port return loss and individual channel insertion loss functions. Each suboptimization was solved using a recent minimax algorithm[22]. Until our recent paper on multiplexers[2], the reported design and manufacturing of these devices were limited to 12 channels[23-27].

A contiguous band 5-channel multiplexer was specifically optimized to illustrate the novel process of automatic decomposition, as shown in Fig. 4. Functions associated with the same channel are grouped together. Variables for each channel include 12 coupling parameters, input and output transformer ratios  $(n_1 \text{ and } n_2)$  and the distance measure from

the channel filter to the short circuit main cascade termination. The overall problem involved 75 variables and 124 nonlinear functions. As the parameters approached their solution, weak interactions between variables and functions were also considered. The final subproblem was the overall optimization.

We also tested our approach on a 16-channel multiplexer involving 240 variables and 399 nonlinear functions. The responses at the starting point is shown in Fig. 5. Only 10 suboptimizations were performed before reaching the response of Fig. 6. Then a full optimization is activated resulting in all responses satisfying their specifications as shown in Fig. 7. A comparison between the optimal design with and without decomposition is provided in Table IV. When used to obtain a good starting point for subsequent optimization, the decomposition approach offers considerable reductions in both CPU time and storage. The feasibility of obtaining a near optimum for large problems using computers with memory limitations is observed from the table. However, when close to the desired solution, the sizes of the subproblems may approach those of the overall problem. In this case, the performance of optimization does not differ significantly with or without decomposition, unless the original problem is almost completely decomposable.

## VI. CONCLUSION

We have presented an automated decomposition approach for optimization of large microwave systems. Compared with the existing decomposition methods, the novelty of our approach lies in its generality in terms of device independency and its automation. Advantages of the approach are 1) a very significant saving of CPU time and/or computer storage and 2) efficient decomposition by automation. By partitioning the overall problem into smaller ones, the approach promises to provide a basis for computer-assisted tuning. It contributes positively towards future general computer software for large-scale optimization of microwave systems.

## **ACKNOWLEDGEMENTS**

The authors acknowledge fruitfull interactions with Dr. W. Kellermann, now with Ontario Hydro, Toronto, Canada, and Dr. S. Daijavad, now with the University of California, Berkeley, CA. Mr. S.H. Chen and Ms. M. L. Renault of McMaster University, Hamilton, Canada, provided stimulating suggestions and cooperation throughout this project. Technical discussions with Drs. M.H. Chen of TRW, Redondo Beach, CA, and K. Madsen of the Technical University of Denmark, Lyngby, Denmark, are also appreciated.

#### REFERENCES

- [1] H. Kondoh, "An accurate FET modelling from measured S-parameters", <u>IEEE Int. Microwave Symp. Digest</u> (Baltimore, MD), pp. 377-380, 1986.
- [2] J.W. Bandler, S.H.Chen, S. Daijavad, W. Kellermann, M. Renault and Q.J. Zhang, "Large scale minimax optimization of microwave multiplexers", <u>Proc. European Microwave Conf.</u> (Dublin, Ireland), pp. 435-440, 1986.
- [3] D.M. Himmelblau, Ed., <u>Decomposition of Large-Scale Problems</u>. Amsterdam: North-Holland, 1973.
- [4] A.M. Geoffrion, "Elements of large-scale mathematical programming", <u>Management Science</u>, vol. 16, pp. 652-691, 1970.
- [5] L.S. Lasdon, <u>Optimization Theory for Large Systems</u>. New York, NY: MacMillan, 1970.
- [6] H.P.L. Luna, "A survey on informational decentralization and mathematical programming decomposition", in <u>Mathematical Programming</u>, R.W. Cottle, M.L. Kelmanson and B. Korte Eds. Amsterdam: Elsevier, 1984.
- [7] F. Luccio and M. Sami, "On the decomposition of networks in minimally interconnected subnetworks", <u>IEEE Trans. Circuit Theory</u>, vol. CT-16, pp. 184-188, 1969.
- [8] L.O. Chua and L.K. Chen, "Diakoptic and generalized hybrid analysis", <u>IEEE Trans. Circuits Syst.</u>, vol. CAS-23, pp. 694-705, 1976.
- [9] F.F. Wu, "Solution of large-scale networks by tearing", <u>IEEE Trans. Circuits Syst.</u>, vol. CAS-23, pp. 706-713, 1976.
- [10] M.D. Tong and W.K. Chen, "Hybrid analysis of a large-scale network by node-tearing", Proc. IEEE Int. Symp. Circuits and Systems (San Jose, CA), pp. 178-181, 1986.
- [11] H. Asai, M. Urano and M. Tanaka, "A new simulation technique for a large scale transistor circuit based on relaxation method using network tearing", <u>Proc. IEEE Int. Symp. Circuits and Systems</u> (San Jose, CA), pp. 161-164, 1986.
- [12] J. Pospisil, "Linear network analysis using scattering matrix decomposition", <u>Proc. IEEE Int. Symp. Circuits and Systems</u> (San Jose, CA), pp. 193-196, 1986.
- [13] A. Sangiovanni-Vincentelli, L.K. Chen and L.O. Chua, "An efficient heuristic cluster algorithm for tearing large-scale networks", <u>IEEE Trans. Circuits Syst.</u>, vol. CAS-24, pp. 709-717, 1977.
- [14] A.E. Salama, J.A. Starzyk and J.W. Bandler, "A unified decomposition approach for fault location in large analog circuits", <u>IEEE Trans. Circuits Syst.</u>, vol. CAS-31, pp. 609-622, 1984.
- [15] S.N. Talukdar, T.C. Giras and V.K. Kalyan, "Decompositions for optimal power flows", IEEE Trans. Power Apparatus Syst., vol. PAS-102, pp. 3877-3884, 1983.

- [16] G.C. Contaxis, C. Delkis and G. Korres, "Decoupled optimal load flow using linear or quadratic programming", <u>IEEE Trans. Power Syst.</u>, vol. PWRS-1, pp. 1-7, 1986.
- [17] K.L. Lo and Y.M. Mahmoud, "A decoupled linear programming technique for power system state estimation", <u>IEEE Trans. Power Syst.</u>, vol. PWRS-1, pp. 154-160, 1986.
- [18] R. Billinton and S.S. Sachdeva, "Real and reactive power optimization by suboptimum techniques", IEEE Trans. Power Apparatus Syst., vol. PAS-92, pp. 950-956, 1973.
- [19] C. Tsironis and R. Meierer, "Microwave wide-band model of GaAs dual gate MESFET's", IEEE Trans. Microwave Theory Tech., vol. MTT-30, pp. 243-251, 1982.
- [20] P.J. Courtois, Decomposability. New York, NY: Academic Press, 1977.
- [21] J.W. Bandler and M.R.M. Rizk, "Optimization of electrical circuits", <u>Mathematical</u> Programming Study on Engineering Optimization, vol. 11, pp. 1-64, 1979.
- [22] J.W. Bandler, W. Kellermann and K. Madsen, "A superlinearly convergent minimax algorithm for microwave circuit design", <u>IEEE Trans. Microwave Theory Tech.</u>, vol. MTT-33, pp. 1519-1530, 1985.
- [23] M.H. Chen, "Current state-of-the-art technology on contiguous band multiplexer", Proc. IEEE Int. Symp. Circuits and Systems (Kyoto, Japan), pp. 1583-1586, 1985.
- [24] R.G. Egri, A.E. Williams and A.E. Atia, "A contiguous-band multiplexer design", IEEE Int. Microwave Symp. Digest (Boston, MA), pp.86-88, 1983.
- [25] R. Tong and D. Smith, "A 12-channel contiguous-band multiplexer for satellite application", <u>IEEE Int. Microwave Symp. Digest</u> (San Francisco, CA), pp. 297-298, 1984.
- [26] S.C. Holme, "A 12 GHz 12 channel multiplexer for satellite applications", <u>IEEE Int.</u> Microwave Symp. Digest (San Francisco, CA), pp. 295-296, 1984.
- [27] J.W. Bandler, S. Daijavad and Q.J. Zhang, "Exact simulation and sensitivity analysis of multiplexing networks", <u>IEEE Trans. Microwave Theory Tech.</u>, vol. MTT-34, pp. 93-102, 1986.

TABLE I  $PARAMETER \ VALUES \ FOR \ \Phi^0 \ FOR \ THE \ FET \ CIRCUIT \ MODEL \ (from \ [1])$ 

i	Parameter $\phi_i$	Unit	Value for $\phi_i$
1	gm	mS	50.0
2	τ	ps	3.0
3	$\mathrm{C}_{\mathrm{gs}}$	pF	0.25
4	$\mathrm{C}_{\mathrm{ds}}$	pF	0.08
5	$\mathrm{C_{dg}}$	pF	0.025
6	$\mathrm{R}_{\mathbf{g}}$	Ohm	4.0
7	$ m R_s$	Ohm	4.0
8	$R_d$	Ohm	3.0
9	$ m R_{ds}$	Ohm	250.
10	$ m R_i$	Ohm	0.2
11	$ m L_{g}$	pН	60.0
12	$ m L_d$	pH pH	25.0 15.0
13	$\mathrm{L_{s}}$	pri	15.0

TABLE II

THE C MATRIX FOR THE FET MODEL

(a) FUNCTION GROUPS INVOLVING THE ENTIRE FREQUENCY BAND

(1.5 GHZ TO 26.5 GHZ)

	Function Groups				
Variables	$S_{11}$ Entire Freq. Band	$S_{21}\mathrm{Entire}$ Freq. Band	$S_{12}$ Entire Freq. Band	$S_{22}$ Entire Freq. Band	
$g_{\mathrm{m}}$	18.55	100.00	87.55	68.33	
$\mathrm{C_{gs}}$	100.00	89.74	67.98	62.25	
$\mathrm{C}_{\mathrm{ds}}$	4.88	67.74	45.73	100.00	
$\mathrm{C_{dg}}$	4.24	48.88	100.00	81.27	
$R_s$	35.53	37.14	100.00	5.88	
$R_{ds}$	17.44	97.68	70.51	100.00	

Each row of the table has been scaled.

TABLE II (continued)

THE C MATRIX FOR THE FET MODEL

(b) FUNCTION GROUPS INVOLVING ONLY THE UPPER HALF FREQUENCY BAND

(14.0 GHZ TO 26.5 GHZ)

Function Groups				
$S_{11} { m Upper}$ Freq. Band	$S_{21} { m Upper}$ Freq. Band	$S_{12}$ Upper Freq. Band	$S_{22} { m Upper}$ Freq. Band	
31.91	100.00	36.61	59.31	
100.00	50.67	24.87	29.89	
34.65	74.31	85.85	100.00	
100.00	65.63	88.43	39.53	
100.00	87.85	57.16	37.44	
9.99	97.88	61.78	100.00	
62.94	31.31	100.00	21.99	
	31.91 100.00 34.65 100.00 100.00 9.99	$S_{11}$ Upper Freq. Band $S_{21}$ Upper Freq. Band $S_{22}$ Upper Fr	$S_{11}$ Upper $S_{21}$ Upper Freq. Band $S_{12}$ Upper Freq. Band $S_{10}$ Upper Freq. Band $S$	

Each row of the table has been scaled

.

TABLE III  $NORMALIZED\ DECOMPOSITION\ DICTIONARY\ D$  (a) CORRESPONDING TO THE SENSITIVITY ANALYSIS OF TABLE II(a)

	Function Groups			
Variable Groups	$S_{11} { m Entire}$ Freq. Band	$S_{21}\mathrm{Entire}$ Freq. Band	$S_{12}\mathrm{Entire}$ Freq. Band	$S_{22} { m Entire}$ Freq. Band
$R_{ds}$ , $C_{ds}$	0.00	0.00	0.00	1.00
$\mathrm{C}_{\mathrm{gs}}$	1.00	0.00	0.00	0.00
$C_{dg}$ , $R_s$	0.00	0.00	1.00	0.00
$g_{m}$	0.00	1.00	0.00	0.00

# (b) CORRESPONDING TO THE SENSITIVITY ANALYSIS OF TABLE II(b)

	Function Groups			
Variable Groups	$S_{11} { m Upper}$ Freq. Band	$S_{21}\mathrm{Upper}$ Freq. Band	$S_{12}\mathrm{Upper}$ Freq. Band	$S_{22}\mathrm{Upper}$ Freq. Band
$R_d$ , $L_d$	0.00	0.00	0.00	1.00
$R_g, R_i, L_g$	1.00	0.00	0.00	0.00
$L_s$	0.00	0.00	1.00	0.00
τ	0.00	1.00	0.00	0.00

Purpose of Optimization+	Reduction in Objective Function	Criteria for Comparison	With Decomp.	Without Decomp.
	from	CPU time *	99	250
to provide a good starting point for	13.46	working space needed <sup>†</sup>	2,197	483,036
further opti- mization	to 2.4	number of suboptimizations	10	_
to obtain a	from 13.46	CPU time *	651	553
near optimum solution	13.40	working space needed <sup>†</sup>	73,972	483,036
	to 0.32	number of suboptimizations	51	_
to obtain	from	CPU time *	1045	1289
optimum solution	13.46	working space needed <sup>†</sup>	483,036	483,036
	to -0.09	number of suboptimizations	11	_

<sup>+</sup> different sparse factors  $\lambda$  have been used to control the degree of decomposition for the three different purposes.

<sup>\*</sup> seconds on the FPS-264 mainframe.

of machine memory units (one unit per real number) required by the minimax optimization package[22].

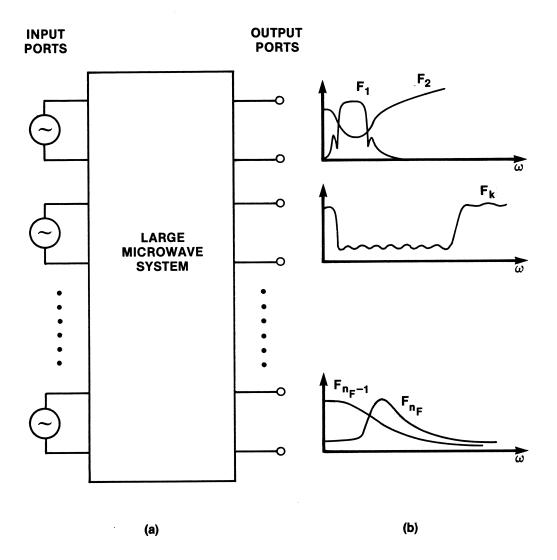


Fig. 1 A general representation of multi-input and multi-output microwave system.  $F_k$ ,  $k=1,2,...,n_F$  are responses being measured, monitored or used as outputs subject to design specifications. Different types of responses (e.g., return loss, insertion loss, S parameters) may exist at the same output port.

(a) System representation.

(b) Responses corresponding to each output port.

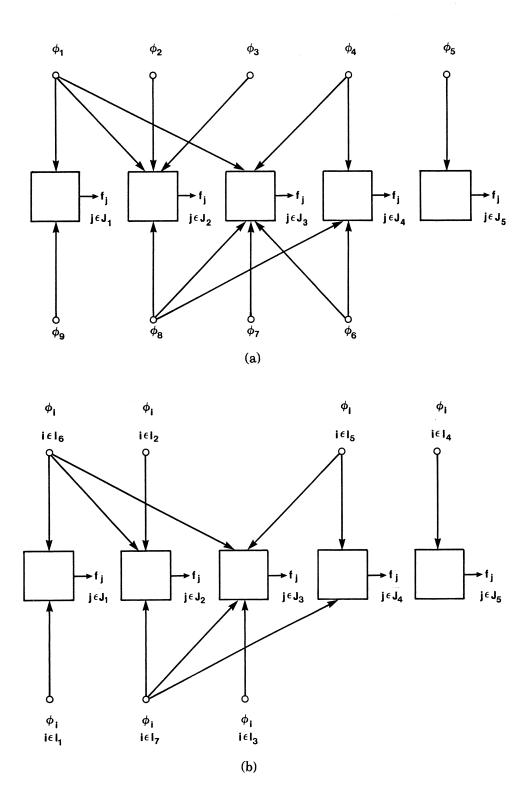
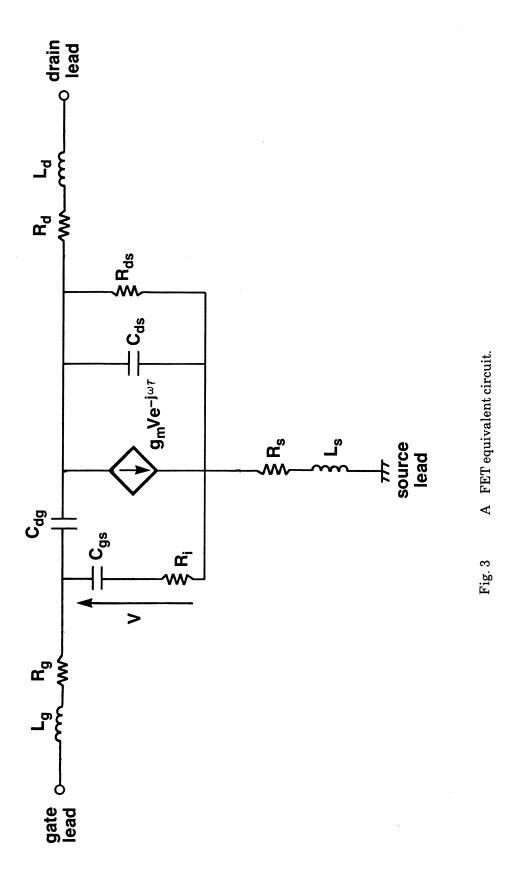


Fig. 2 A fictitious example showing only the strong interconnections between variables and function groups.

- (a) System configuration corresponding to matrix C.
- (b) System configuration corresponding to matrix  ${\bf D}$ .



- Fig. 4 Return and insertion loss responses of the 5-channel multiplexer for each suboptimization. The 20 dB specification line indicates which channel(s) is to be optimized in the next subproblem. The variables to be selected are indicated in the graph, e.g., 35 representing coupling  $M_{35}$ , d representing the distance of the corresponding channel filter from the short circuit main cascade termination. The previously optimized channels are highlighted by thick response curves.
  - (a) Responses at the starting point.
  - (b)-(k) Responses for each suboptimization.
  - $(\ell)$  Responses at the final solution.

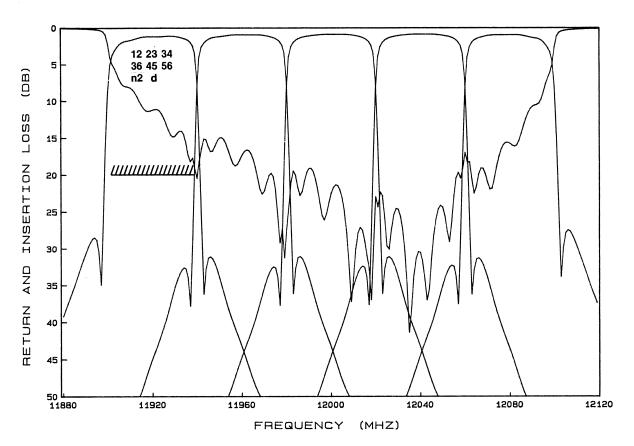


Fig. 4(a)

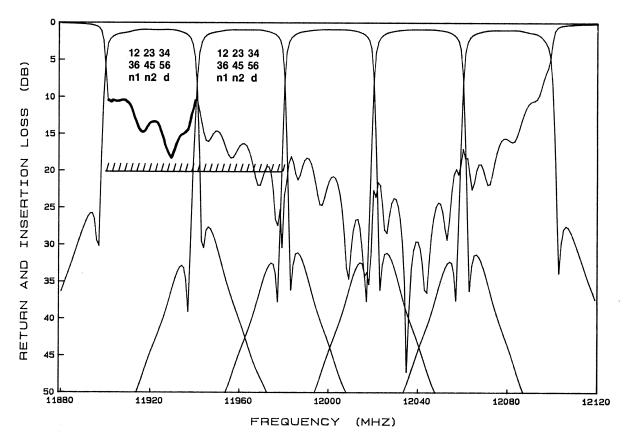


Fig. 4(b)

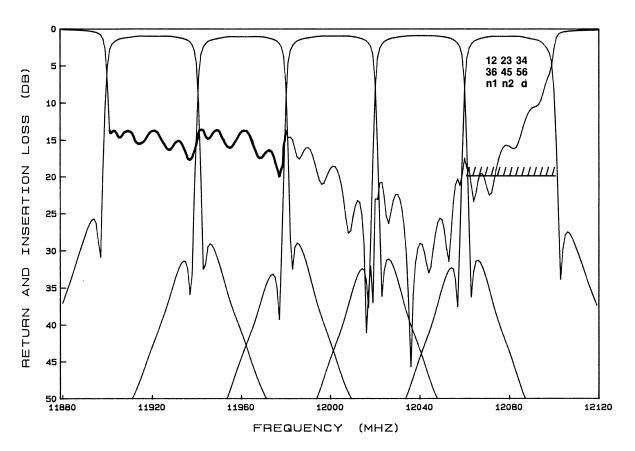


Fig. 4(c)

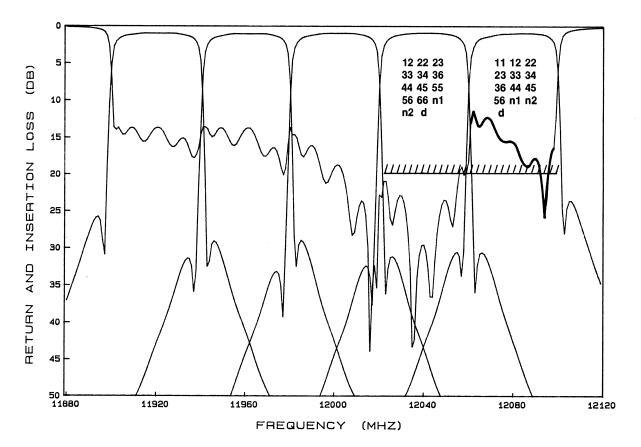


Fig. 4(d)

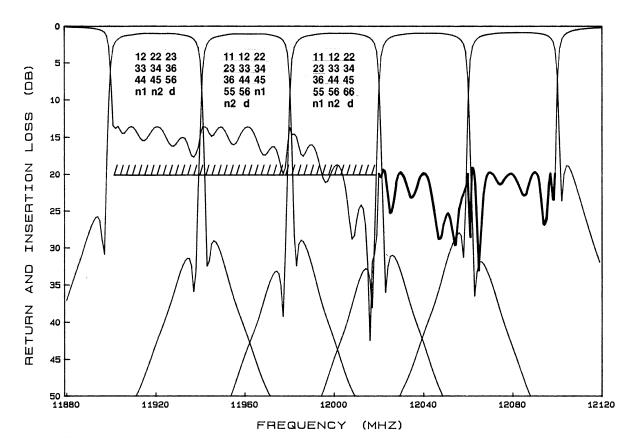


Fig. 4(e)

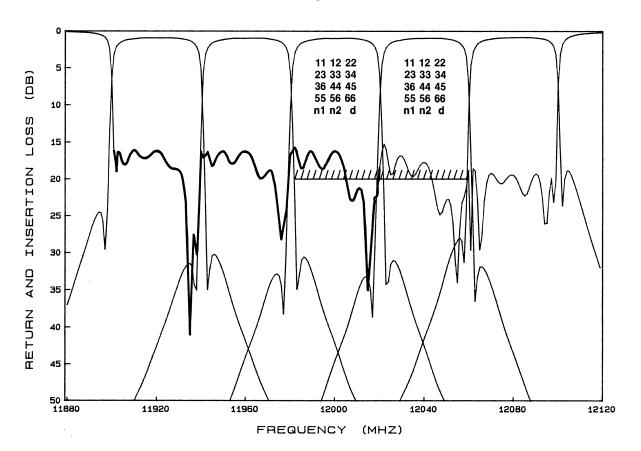


Fig. 4(f)

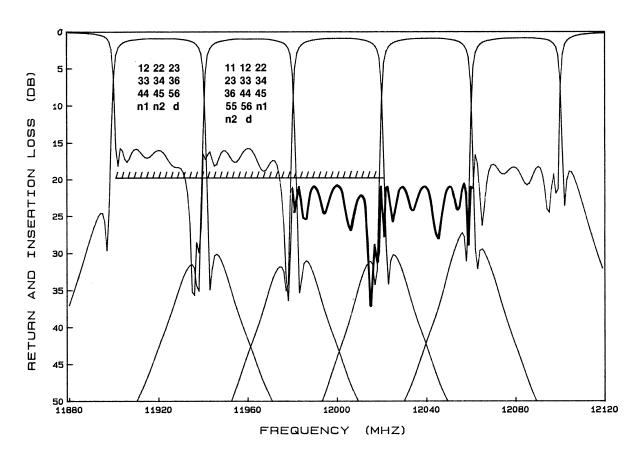


Fig. 4(g)

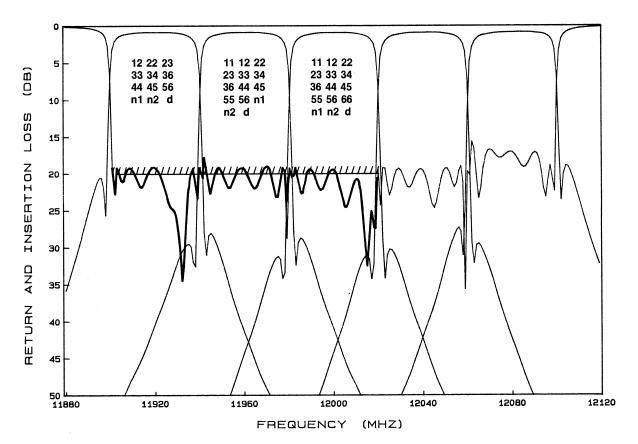
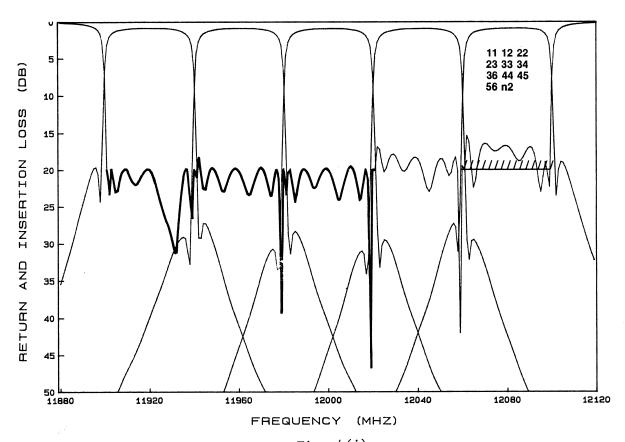


Fig. 4(h)





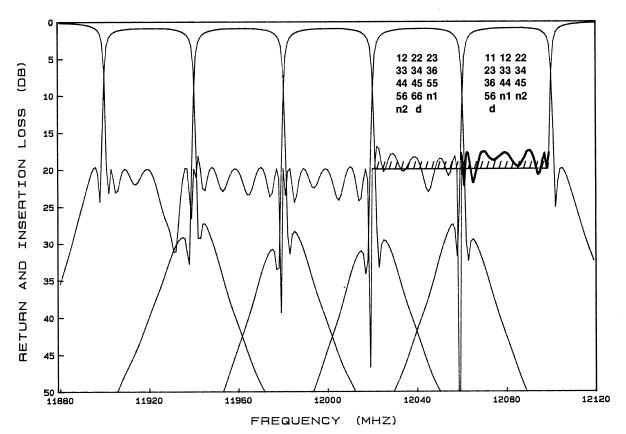


Fig. 4(j)

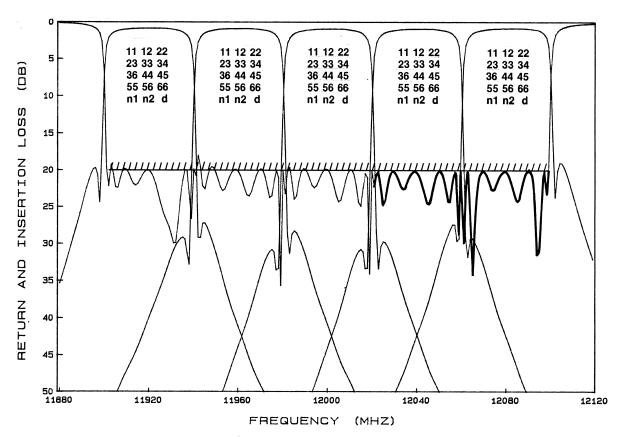


Fig. 4(k)

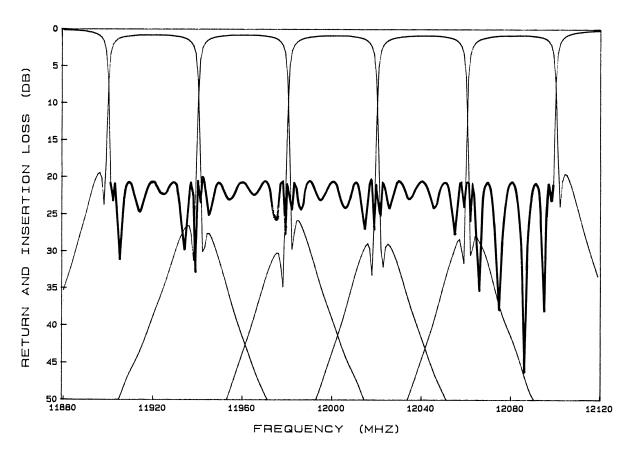
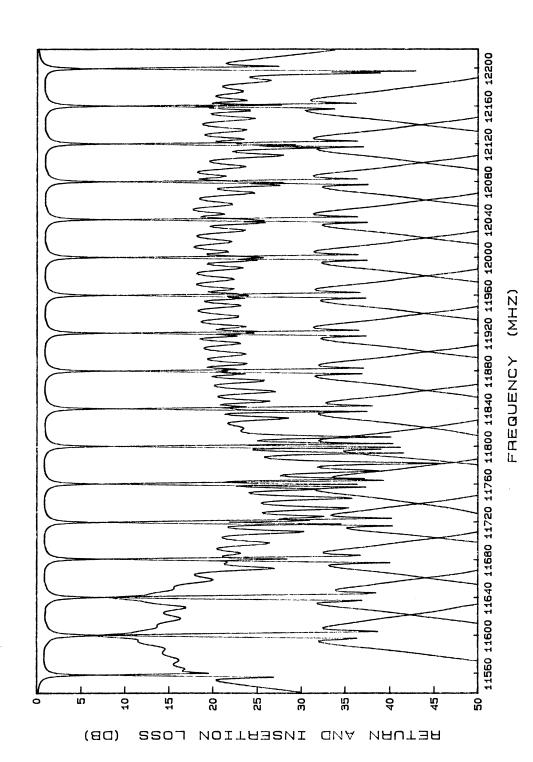
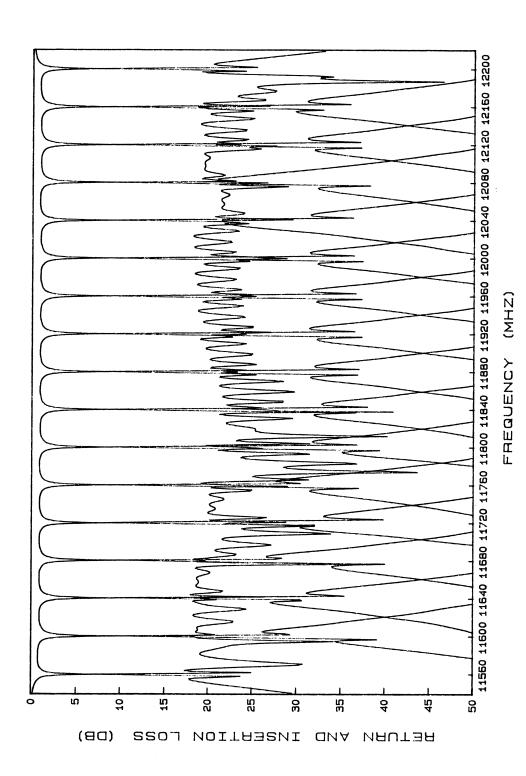


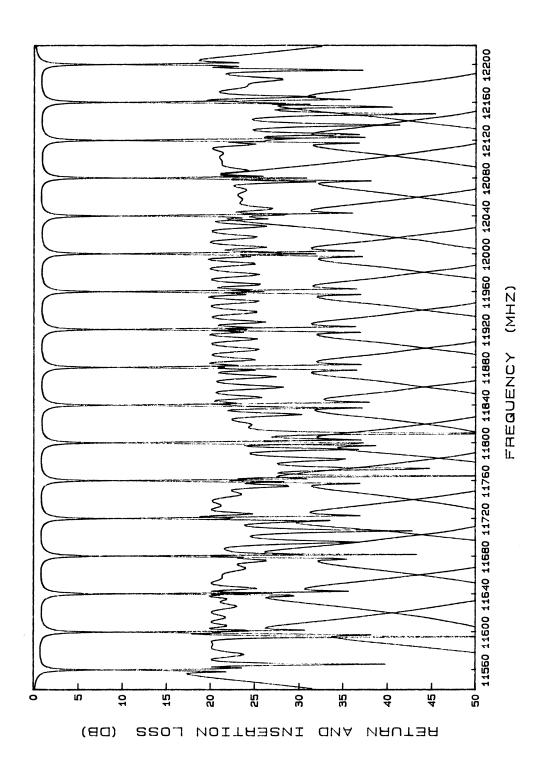
Fig. 4(**l**)



Return and insertion loss responses of the 16-channel multiplexer before optimization. Fig. 5



Return and insertion loss responses of the 16-channel multiplexer after 10 suboptimizations. Each of the 10 suboptimizations involved responses associated with only one channel and no more than 15 variables. Fig. 6



Return and insertion loss responses of the 16-channel multiplexer at the overall solution. All design specifications are satisfied. Fig. 7