AN EFFICIENT APPROACH TO CONSTRUCT A MULTIPORT MATRIX FOR HARMONIC BALANCE ANALYSIS

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Abstract This paper presents an approach to construct a multiport circuit matrix. The approach is simple and therefore very efficient. For a linear circuit with m external ports and b internal nodes, it only needs some matrix row and column additions, and b+m-1 matrix column eliminations, where the order of the matrix is b + (total number of terminals between the linear circuit and m external ports). The approach is suitable for the harmonic balance simulation technique, especially for the optimization of nonlinear circuits using the harmonic balance technique.

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I. INTRODUCTION

The harmonic balance technique is highly efficient for the simulation of nonlinear circuits with periodic steady-state responses [1]. The harmonic balance technique separates a nonlinear circuit into linear and nonlinear parts according to the nature of the circuit elements. The linear part of the circuit may be simplified to a multi-terminal circuit and simulated efficiently in the frequency domain. The nonlinear part of the circuit, on the other hand, is simulated in the time domain and then the time domain response is transformed into the frequency domain using the discrete Fourier transformation. Due to its efficiency in nonlinear circuit simulation, the harmonic balance technique has been successfully used in nonlinear circuit optimization, e.g., [2-3].

In circuit optimization, however, the variable design parameter values change in every iteration, which in turn updates the circuit in every iteration. An efficient way to set up the multi-terminal circuit matrix corresponding to the linear subnetwork is desirable. Different ways of constructing the matrix are available, e.g., the partial elimination of the internal nodes from the general nodal matrix of the circuit, etc. An alternative is to use multiport matrix which takes into account Kirchhoff's current law (KCL) and eliminates one simulation variable for each nonlinear subnetwork or port, therefore speeding up the simulation. In Section II, the approach to set up the matrix under certain conditions is described. This approach is much simpler than the general approach in [4]. An example is demonstrated in Section III.

II. MULTIPORT FORMULATION

Assume that the nonlinear circuit contains just voltage-controlled nonlinear elements. The circuit can be illustrated by Fig. 1, where L represents the linear part of the circuit, N_i, i=1, .., m, the nonlinear part of the circuit. Since the linear subcircuit treats circuit excitations similarly to nonlinear elements, we do not distinguish circuit excitations and nonlinear elements, i.e., the circuit excitations are among the N_i, i=1, .., m. Without loss of generality, we also assume that N_i and N_j, $i \neq j$, do not have any common node. Subscript *ij* is used to indicate the *j*th node connecting N_i to L, and *i0* the relative reference node of N_i. In the following, only one harmonic is considered, since the derivations for other harmonics are similar. Therefore, the harmonic index is omitted to simplify the notation.

Let the internal nodes of L be numbered *rj*, j=1, .., b. The indefinite nodal equation can be written as

$$Y \begin{bmatrix} V_{10} \\ V_{11} \\ : \\ V_{20} \\ V_{21} \\ : \\ V_{m0} \\ V_{m1} \\ : \\ V_{r1} \\ : \\ V_{rb} \end{bmatrix} = \begin{bmatrix} I_{10} \\ I_{11} \\ : \\ I_{20} \\ I_{21} \\ : \\ I_{m0} \\ I_{m1} \\ : \\ 0 \\ : \\ 0 \end{bmatrix}$$

or

$$\mathbf{Y}\ \mathbf{V} = \mathbf{I} \tag{1}$$

where Y is the indefinite nodal matrix.

First, we express Y in its column vector form

$$Y = [Y_{10}\ Y_{11}\ ..\ Y_{20}\ Y_{21}\ ..\ Y_{m0}\ Y_{m1}\ ..\ Y_{r1}\ ..\ Y_{rb}]\ . \tag{2}$$

From (1) we can get

$$Y' \begin{bmatrix} V_{10} \\ V_{11}-V_{10} \\ : \\ V_{20} \\ V_{21}-V_{20} \\ : \\ V_{m0} \\ V_{m1}-V_{m0} \\ : \\ V_{r1} \\ : \\ V_{rb} \end{bmatrix} = \begin{bmatrix} I_{10} \\ I_{11} \\ : \\ I_{20} \\ I_{21} \\ : \\ I_{m0} \\ I_{m1} \\ : \\ 0 \\ : \\ 0 \end{bmatrix}$$

or

$$\mathbf{Y'}\ \mathbf{V'} = \mathbf{I'} \tag{3}$$

where $Y' = [\Sigma Y_{1j}\ Y_{11}\ ..\ \Sigma Y_{2j}\ Y_{21}\ ..\ \Sigma Y_{mj}\ Y_{m1}\ ..\ Y_{r1}\ ..\ Y_{rb}]$, ΣY_{ij} is the summation over all the columns corresponding to N_i, and $\mathbf{I}' = \mathbf{I}$.

Second, we rewrite (3) with Y′ in its row vector form

$$\begin{bmatrix} Y'_{10} \\ Y'_{11} \\ : \\ Y'_{20} \\ Y'_{21} \\ : \\ Y'_{m0} \\ Y'_{m1} \\ : \\ Y'_{r1} \\ : \\ Y'_{rb} \end{bmatrix} V' = I' .$$

It is straightforward from KCL that

$$\begin{bmatrix} \Sigma Y'_{1j} \\ Y'_{11} \\ : \\ \Sigma Y'_{2j} \\ Y'_{21} \\ : \\ \Sigma Y'_{mj} \\ Y'_{m1} \\ : \\ Y'_{r1} \\ : \\ Y'_{rb} \end{bmatrix} V' = \begin{bmatrix} 0 \\ I_{11} \\ : \\ 0 \\ I_{21} \\ : \\ 0 \\ I_{m1} \\ : \\ 0 \\ : \\ 0 \end{bmatrix}$$

or

$$Y''\ V'' = I'' \tag{4}$$

where $\Sigma Y'_{ij}$ is the summation of all the rows corresponding to N_i, and $V'' = V'$.

Then, the multiport matrix can be obtained by the following steps.

Step 1 Select a ground node to reduce Y″ to a full rank matrix. If *m0* is selected to be the ground node, i.e., $V_{m0}=0$, we delete the row and column in Y″ corresponding to *m0*.

Step 2 Similar to the Gauss elimination method, eliminate columns in Y″ corresponding to *i0*, i=1, .., m-1, and *rj*, j=1, .., b.

Step 3 Delete rows and columns in Y″ corresponding to *i0*, i=1, .., m-1, and *rj*, j=1, .., b.

The final form of the multiport matrix equation reads

$$Y_{MP} \begin{bmatrix} V_{11}-V_{10} \\ V_{12}-V_{10} \\ \vdots \\ V_{21}-V_{20} \\ V_{22}-V_{20} \\ \vdots \\ V_{m1}-V_{m0} \\ V_{m2}-V_{m0} \\ \vdots \end{bmatrix} = \begin{bmatrix} I_{11} \\ I_{12} \\ \vdots \\ I_{21} \\ I_{22} \\ \vdots \\ I_{m1} \\ I_{m2} \\ \vdots \end{bmatrix} \tag{5}$$

where Y_{MP} stands for the multiport matrix suitable for harmonic balance simulation.

III. EXAMPLE

To illustrate the approach presented in the previous section, let us consider a simple circuit example, which has one current excitation and one two-terminal nonlinear element, as shown in Fig. 2.

Step 1 The indefinite nodal matrix is

$$Y = \begin{bmatrix} Y_4+Y_5 & 0 & -Y_5 & 0 & -Y_4 \\ 0 & Y_1 & 0 & -Y_1 & 0 \\ -Y_5 & 0 & Y_3+Y_5 & 0 & -Y_3 \\ 0 & -Y_1 & 0 & Y_1+Y_2 & -Y_2 \\ -Y_4 & 0 & -Y_3 & -Y_2 & Y_2+Y_3+Y_4 \end{bmatrix}$$

Step 2

$$Y' = \begin{bmatrix} Y_4+Y_5 & 0 & -Y_5 & 0 & -Y_4 \\ Y_1 & Y_1 & -Y_1 & -Y_1 & 0 \\ -Y_5 & 0 & Y_3+Y_5 & 0 & -Y_3 \\ -Y_1 & -Y_1 & Y_1+Y_2 & Y_1+Y_2 & -Y_2 \\ -Y_4 & 0 & -Y_2-Y_3 & -Y_2 & Y_2+Y_3+Y_4 \end{bmatrix}$$

Step 3

$$Y'' = \begin{bmatrix} Y_1+Y_4+Y_5 & Y_1 & -Y_1-Y_5 & -Y_1 & -Y_4 \\ Y_1 & Y_1 & -Y_1 & -Y_1 & 0 \\ -Y_1-Y_5 & -Y_1 & Y_1+Y_2+Y_3+Y_5 & Y_1+Y_2 & -Y_2-Y_3 \\ -Y_1 & -Y_1 & Y_1+Y_2 & Y_1+Y_2 & -Y_2 \\ -Y_4 & 0 & -Y_2-Y_3 & -Y_2 & Y_2+Y_3+Y_4 \end{bmatrix}$$

Step 4 Select *10* as the ground, therefore delete the row and column corresponding to node *10*,

$$\begin{bmatrix} Y_1 & -Y_1 & -Y_1 & 0 \\ -Y_1 & Y_1+Y_2+Y_3+Y_5 & Y_1+Y_2 & -Y_2-Y_3 \\ -Y_1 & Y_1+Y_2 & Y_1+Y_2 & -Y_2 \\ 0 & -Y_2-Y_3 & -Y_2 & Y_2+Y_3+Y_4 \end{bmatrix}$$

Step 5 Eliminate the column corresponding to node *20*

$$\begin{bmatrix} Y_1(Y_2+Y_3+Y_5)/\Delta_1 & 0 & -Y_1(Y_3+Y_5)/\Delta_1 & -Y_1(Y_2+Y_3)/\Delta_1 \\ -Y_1 & Y_1+Y_2+Y_3+Y_5 & Y_1+Y_2 & -Y_2-Y_3 \\ -Y_1(Y_3+Y_5)/\Delta_1 & 0 & (Y_1+Y_2)(Y_3+Y_5)/\Delta_1 & (Y_1Y_3-Y_2Y_5)/\Delta_1 \\ -Y_1(Y_2+Y_3)/\Delta_1 & 0 & (Y_1Y_3-Y_2Y_5)/\Delta_1 & (Y_2+Y_3)(Y_1+Y_5)/\Delta_1+Y_4 \end{bmatrix}$$

where $\Delta_1 = Y_1+Y_2+Y_3+Y_5$, and the column corresponding to *r1*

$$\begin{bmatrix} y_{11} & 0 & y_{12} & 0 \\ -Y_1 & Y_1+Y_2+Y_3+Y_5 & Y_1+Y_2 & -Y_2-Y_3 \\ y_{11} & 0 & y_{22} & 0 \\ -Y_1(Y_2+Y_3)/\Delta_1 & 0 & (Y_1Y_3-Y_2Y_5)/\Delta_1 & (Y_2+Y_3)(Y_1+Y_5)/\Delta_1+Y_4 \end{bmatrix}$$

where

$$y_{11} = Y_1(Y_2+Y_3+Y_5)/\Delta_1 - [Y_1(Y_2+Y_3)/\Delta_1]^2/\Delta_2$$

$$y_{12} = -Y_1(Y_3+Y_5)/\Delta_1 + [(Y_1Y_3-Y_2Y_5)/\Delta_1][Y_1(Y_2+Y_3)/\Delta_1]/\Delta_2$$

$$y_{21} = y_{12}$$

$$y_{22} = (Y_1+Y_2)(Y_3+Y_5)/\Delta_1 - [(Y_1Y_3-Y_2Y_5)/\Delta_1]^2/\Delta_2$$

$$\Delta_2 = (Y_2+Y_3)(Y_1+Y_5)/\Delta_1+Y_4 \ .$$

Step 6 The multiport matrix equation is

$$\begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} V_{11}-V_{10} \\ V_{21}-V_{20} \end{bmatrix} = \begin{bmatrix} I_{11} \\ I_{21} \end{bmatrix}$$

where $V_{10} = 0$.

IV. DISCUSSION

We present a simple approach to construct a multiport matrix for nonlinear circuit simulation by the harmonic balance technique. The approach is highly efficient, for it only needs certain matrix row and column additions, and b+m-1 matrix column eliminations, where the order of the matrix is b + (total number of terminals between the linear circuit and m external ports). The approach is quite simple and therefore easy to program. This approach was been successfully implemented in a large-signal parameter extraction program [3] which exploits harmonic balance as the nonlinear circuit simulation method.

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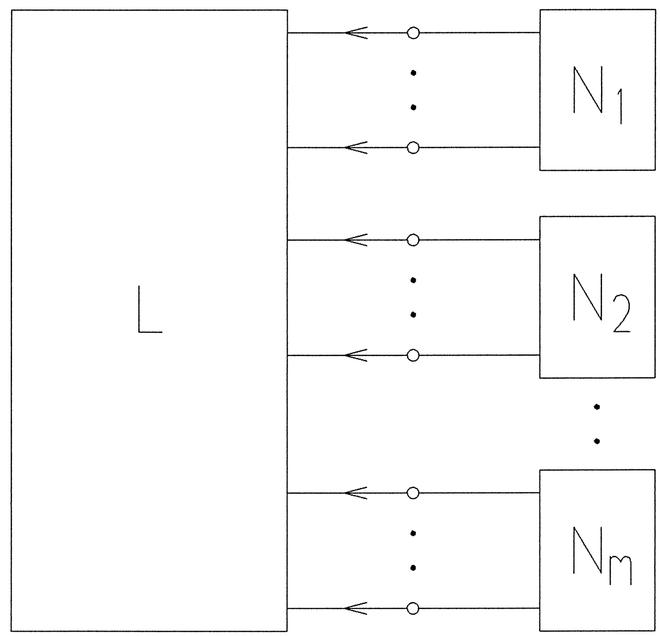
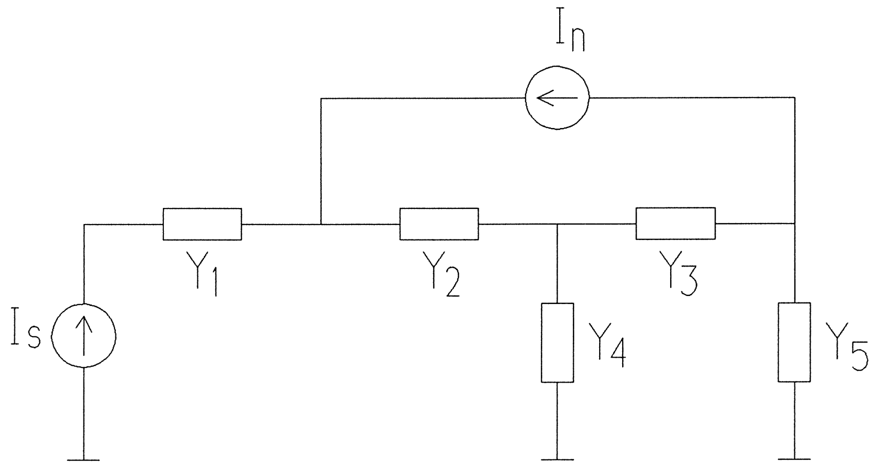
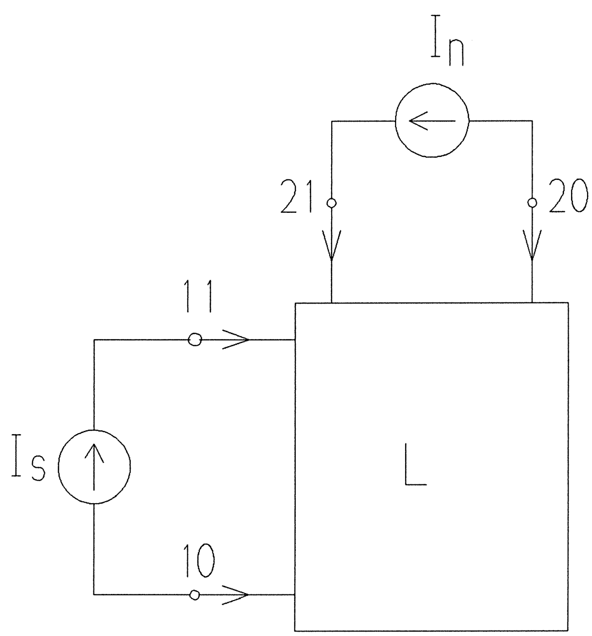


Fig. 1. Diagram illustrating the separation between linear and nonlinear parts of a nonlinear circuit.



(a)



(b)

Fig. 2. Circuit example discussed in Section III. (a) Circuit diagram where I_s and I_n correspond to a circuit excitation and a nonlinear element in the circuit, respectively. (b) Diagram after separating the linear and nonlinear part.