GRADIENT QUADRATIC APPROXIMATION SCHEME FOR YIELD-DRIVEN DESIGN

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Outline

introduction

quadratic approximation

approximation of responses and gradients

filter design

amplifier design

conclusions

Yield-driven Design

yield-driven design is an indispensable tool to

- improve competitiveness
- ensure first-time-success design
- reduce production costs

computationally intensive and inefficient in general-purpose CAD systems

The Motivation of Our Work

reduce the number of time-consuming actual circuit simulations

utilize available gradient information

simultaneously approximate both circuit responses and gradients to improve accuracy

Quadratic Model

$$q(\boldsymbol{x}) = a_0 + \sum_{i=1}^{n} a_i (x_i - r_i) + \sum_{i,j=1, i \leq j}^{n} a_{ij} (x_i - r_i)(x_j - r_j)$$

where $\boldsymbol{r} = \begin{bmatrix} r_1 & r_2 & . & . & . & r_n \end{bmatrix}^T$ is a given reference point

$$\begin{bmatrix} Q_{11} & Q_{12} \\ Q_{21} & Q_{22} \end{bmatrix} \begin{bmatrix} \boldsymbol{a} \\ \boldsymbol{v} \end{bmatrix} = \begin{bmatrix} \boldsymbol{f}_1 \\ \boldsymbol{f}_2 \end{bmatrix}$$

$\boldsymbol{f}_1$, $\boldsymbol{f}_2$: function values evaluated at m base points

$\boldsymbol{a}$, $\boldsymbol{v}$: coefficients of the quadratic model

Maximally Flat Interpolation

applying the least-squares constraint to the second order term coefficients $\boldsymbol{v}$ leads to the unique solution

$$\boldsymbol{v} = C^T(CC^T)^{-1}\boldsymbol{e}$$

$$C = Q_{22} - Q_{21}Q_{11}^{-1}Q_{12}$$

$$\boldsymbol{e} = \boldsymbol{f}_2 - Q_{21}Q_{11}^{-1}\boldsymbol{f}_1$$

$$\boldsymbol{a} = Q_{11}^{-1}\boldsymbol{f}_1 - Q_{11}^{-1}Q_{12}\boldsymbol{v}$$

Fixed Base Point Pattern

m base points

where $n + 1 < m < 2n + 1$

$\boldsymbol{x}^1 = \boldsymbol{r}$

$$\boldsymbol{x}^{i+1} = \boldsymbol{r} + \left[0 \ldots 0 \; \beta_i \; 0 \ldots 0\right]^T, \quad i = 1, 2, \ldots, n$$

$$\boldsymbol{x}^{n+1+i} = \boldsymbol{r} + \left[0 \ldots 0 \; \gamma_i \; 0 \ldots 0\right]^T, \quad i = 1, 2, \ldots, m-(n+1)$$

$\beta_i \neq \gamma_i$: perturbations

Coefficient Calculation

$$a_{ii} = \frac{1}{\gamma_i - \beta_i}\left[\frac{f(x^{n+1+i}) - f(x^1)}{\gamma_i} - \frac{f(x^{i+1}) - f(x^1)}{\beta_i}\right], \quad i = 1, 2, \ldots, m-(n-1)$$

$$a_{ii} = 0, \; i = m-n, \ldots, n, \qquad a_{ij} = 0, \; i \neq j, \; i, j = 1, 2, \ldots, n$$

$$a_0 = f(x^1), \qquad a_i = \frac{[f(x^{i+1}) - f(x^1)]}{\beta_i} - \beta_i a_{ii}, \quad i = 1, 2, \ldots, n$$

Type of Variables

x_{DS}: designable variables with statistics

x_D: designable variables without statistics

x_S: non-designable variables with statistics

Approximation to Responses and Gradients

R_i: responses, $i = 1, 2, \ldots, k$

∇R_i: gradients, defined as

$$\nabla R_i = \left[\left[\frac{\partial R_i}{\partial x_{DS}^0} \right]^T \left[\frac{\partial R_i}{\partial x_D^0} \right]^T \right]^T$$

$x = \begin{bmatrix} x_{DS} \\ x_S \end{bmatrix}$: variables for the quadratic models

$k \times (1 + n_{DS} + n_D)$: the number of quadratic models

Statistical Outcomes

$\begin{bmatrix} x_{DS}^i \\ x_S^i \end{bmatrix}$: statistical outcomes, generated as

$$\begin{bmatrix} x_{DS}^i \\ x_S^i \end{bmatrix} = \begin{bmatrix} x_{DS}^0 \\ x_S^0 \end{bmatrix} + \begin{bmatrix} \Delta x_{DS}^i \\ \Delta x_S^i \end{bmatrix}, \quad i = 1, 2, \ldots, N$$

N: number of outcomes

x^0: nominal values

Implementation Environment

OSA90

gradient information available through the *FAST* technique

the one-sided ℓ_1 centering approach

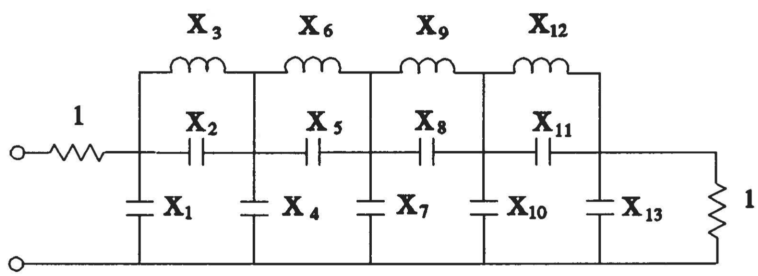
Implementation of Quadratic Approximation

use the same set of $2(n_{DS} + n_S) + 1$ base points for both responses and gradients

quadratic models are rebuilt for all responses and gradients at each iteration of optimization

use the models to approximate all responses and their gradients for all outcomes

13-Element Low-Pass Filter



13-Element Low-Pass Filter

Specifications

insertion loss < 0.4dB at 21 angular frequencies from 0.25 to 1
insertion loss > 49dB at 7 frequencies from 1.05 to 1.115

Design Parameters

13 elements, normal distribution with 0.5% standard deviation

Starting Point

the minimax solution with an estimated yield of 33.4%

13-Element Low-Pass Filter

Calculation Details

two optimizations: with and without modeling

both solutions are after one phase of yield optimization

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normal distribution with $\sigma = 0.5\%$ is assumed for all parameters

100 outcomes used in optimization

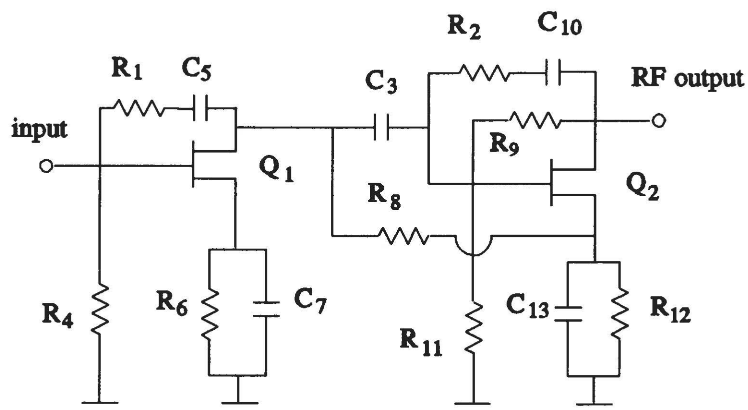
1000 outcomes used in yield estimation

YIELD OPTIMIZATION OF THE LC 13-ELEMENT FILTER WITH AND WITHOUT GRADIENT QUADRATIC APPROXIMATIONS

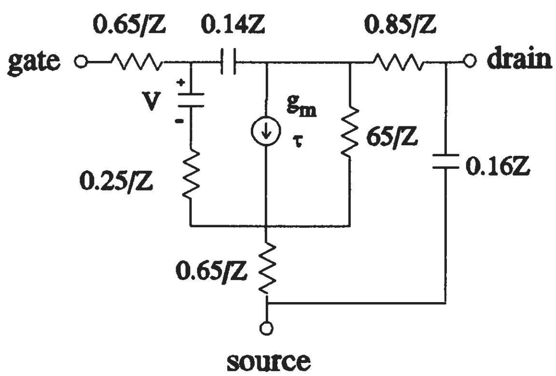
Parameter	Initial	Solution†	Solution††
x_1	0.2088	0.2145	0.2205
x_2	0.03594	0.03642	0.03929
x_3	0.1822	0.1800	0.1775
x_4	0.2340	0.2347	0.2266
x_5	0.2424	0.2426	0.2556
x_6	0.08776	0.08702	0.08426
x_7	0.1333	0.1290	0.1234
x_8	0.3549	0.3535	0.3551
x_9	0.06477	0.06496	0.06481
x_{10}	0.1674	0.1625	0.1561
x_{11}	0.1422	0.1435	0.1498
x_{12}	0.1140	0.1120	0.1098
x_{13}	0.1433	0.1414	0.1303
Yield Estimate	33.4%	75.6%	80.7%
CPU		7min.	30min.

† The solution using quadratic modeling
†† The solution using exact simulation

Two-Stage GaAs MMIC Feedback Amplifier



Small-signal FET model



Two-Stage GaAs MMIC Feedback Amplifier

Specifications

small-signal gain of 8dB±1dB
VSWR at the input port of less than 2
VSWR at the output port less than 2.2

Optimization Parameters

R_1, R_2 and C_3

Starting Point

the minimax solution with an estimated yield 32.1%

PARAMETER VALUES AND TOLERANCES
FOR THE MMIC AMPLIFIER

Element Parameter	Mean Value	Standard Deviation
$Z(\mu m)$	300	3%
$R_4(\Omega)$	400	0%
$C_5(pF)$	4	2%
$R_6(\Omega)$	20	2%
$C_7(pF)$	10	2%
$R_8(\Omega)$	145	2%
$R_9(\Omega)$	2200	0%
$C_{10}(pF)$	4	2%
$R_{11}(\Omega)$	6000	0%
$R_{12}(\Omega)$	500	2%
$C_{13}(pF)$	10	2%

Z is the gate width of the FETs

Two-Stage GaAs MMIC Feedback Amplifier

Calculation Details

two optimizations: with and without modeling

both solutions are after one phase of yield optimization

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100 outcomes used in optimization

1000 outcomes used in yield estimation

YIELD OPTIMIZATION OF THE MMIC AMPLIFIER WITH AND WITHOUT GRADIENT QUADRATIC APPROXIMATIONS

Parameter	Initial	Solution†	Solution††
R_1	201.02	207.63	207.73
R_2	504.82	627.94	630.53
C_3	5.3501	2.7742	2.7563
Yield Estimate	32.1%	77.8%	77.3%
CPU		9min.	39min.

† The solution using quadratic modeling
†† The solution using exact simulation

Conclusions

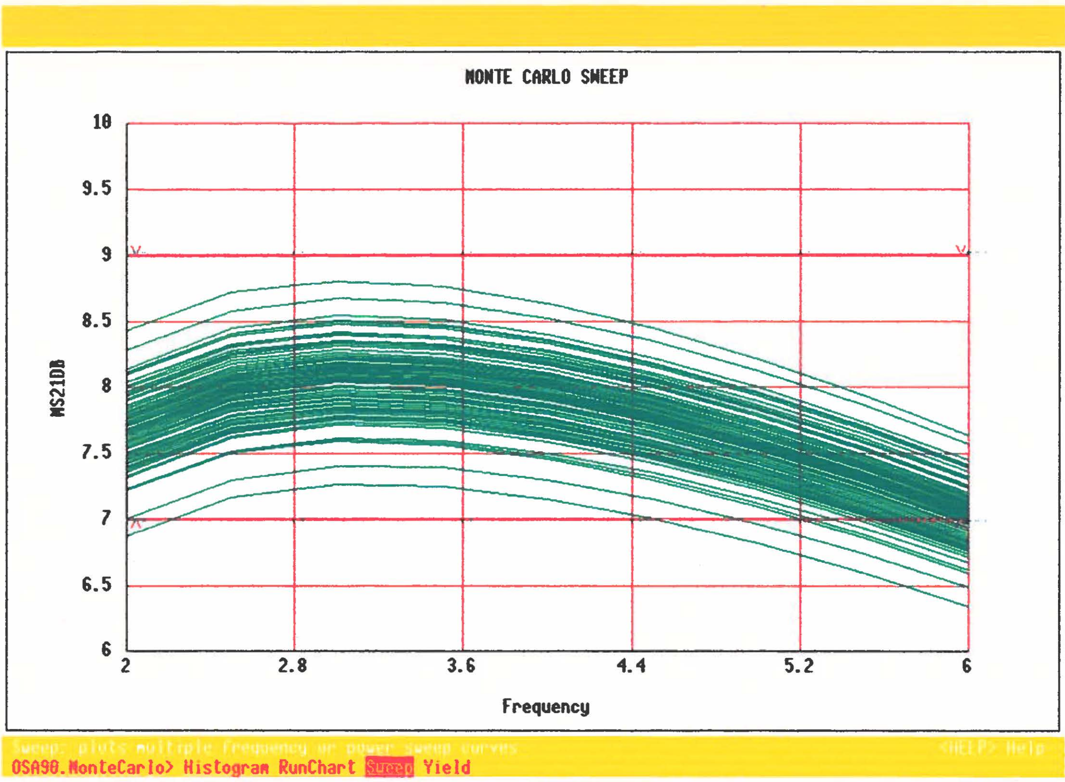
a highly efficient quadratic approximation technique is applied simultaneously to responses and their gradients

our approach is especially suitable for gradient-based yield-driven design

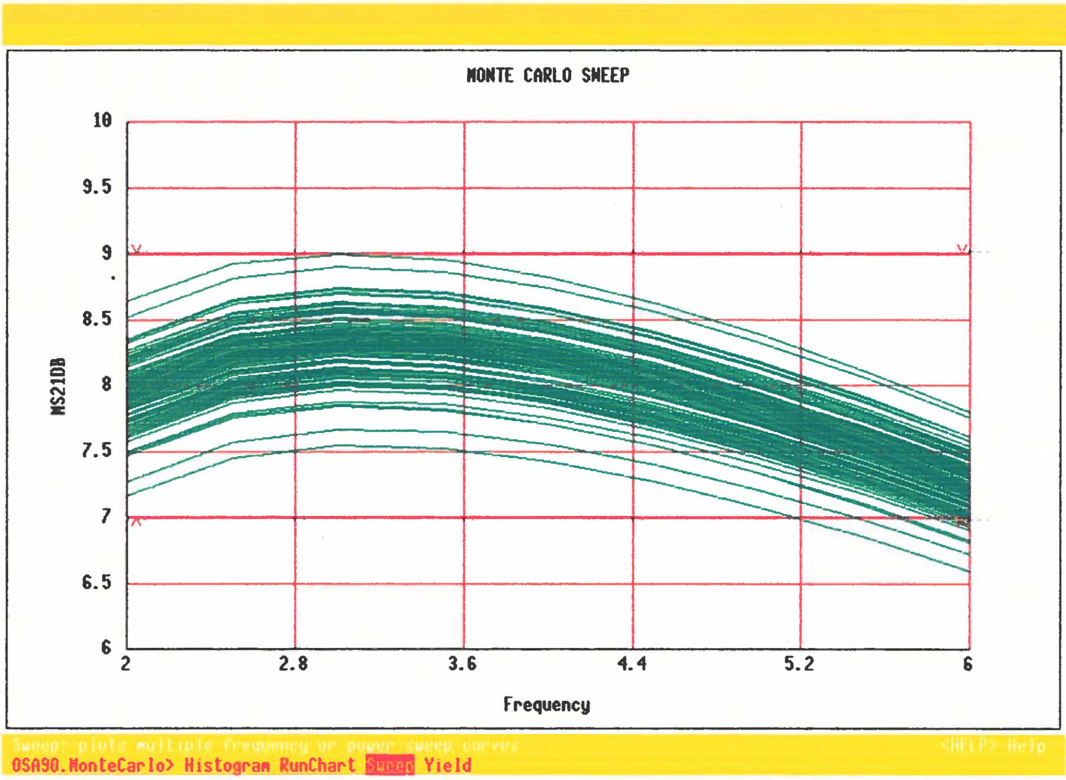
a low-pass filter and an MMIC amplifier design illustrate the merits

suitable for other applications where a large number of expensive simulations are required

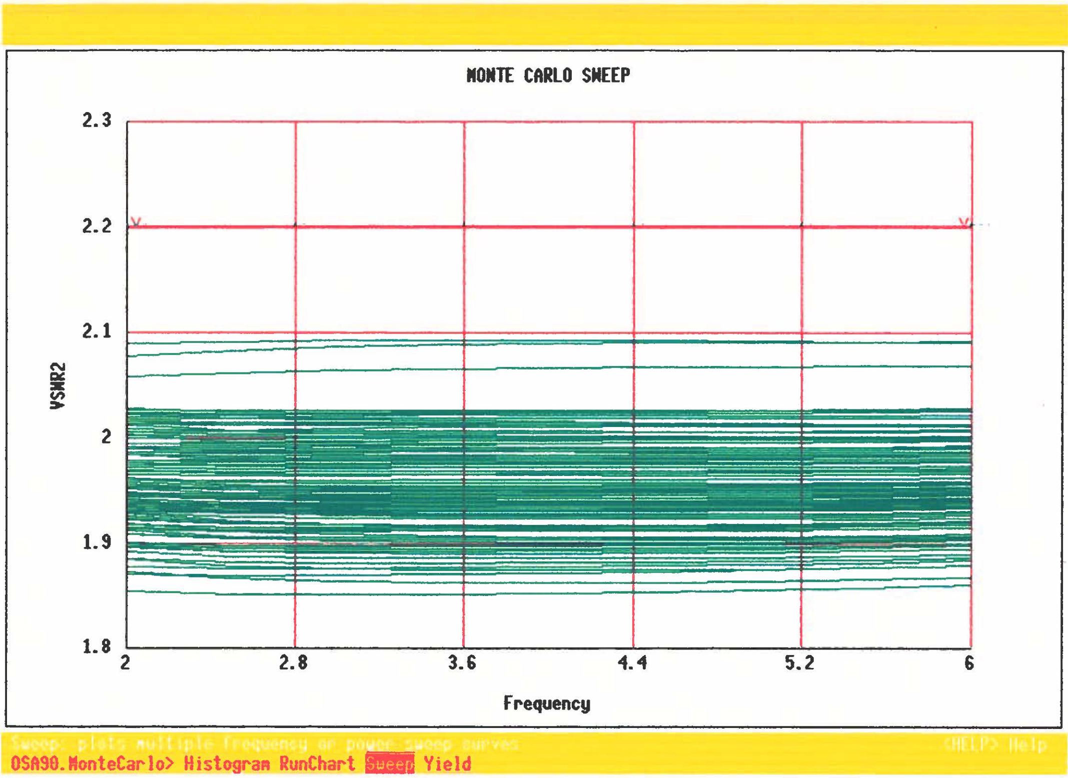
Small-signal Gains of 100 Outcomes before Yield Optimization



Small-signal Gains of 100 Outcomes after Yield Optimization



VSWR at the Output Port of 100 Outcomes before Yield Optimization



VSWR at the Output Port of 100 Outcomes after Yield Optimization