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Abstract

A new and elegant formulation of yield optimization is presented with applications to microwave circuits. It extends the conventional discrete Monte Carlo estimate of yield to a continuous yield probability function suitable for gradient-based optimization. The merit of the new method is demonstrated by applications to an amplifier and a nonlinear FET doubler.

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SUMMARY

Introduction

Yield optimization must be included as an integral part of the design process in order to improve first-time design success rate and to reduce manufacturing cost. Many different approaches to design centering/yield optimization have been explored. Methods which attempt to optimize the theoretical yield as an integral function are usually too complicated for practical implementation. Circuit CAD programs for microwave applications typically estimate the yield by Monte Carlo analyses.

The yield estimated by Monte Carlo analysis is a discrete function and cannot be directly optimized by a gradient-based method. The new formulation introduced in this paper extends the classical Monte Carlo concept by replacing the discrete acceptance index (outcome passed or failed) with a continuous yield probability function suitable for gradient-based optimization. The merit of the new method is demonstrated by yield optimization of a small-signal amplifier and a nonlinear FET frequency doubler. In both applications the design yield is significantly increased after optimization.

The new method is implemented in the microwave CAD/CAE system OSA90/hope™ [1]. The results compare very favorably with the one-sided ℓ_1 centering algorithm by Bandler *et al.* [2-4].

Yield Probability Function

Let ϕ denotes the vector of circuit parameters subject to tolerances and other statistical variations. We can define a generalized least pth error function $H(\phi)$ [2,5] such that $H(\phi) \leq 0$ if all the design specifications are satisfied and $H(\phi) > 0$ if some of the specifications are violated. The theoretical yield can be represented by the probability

$$P(H(\phi) \leq 0) \tag{1}$$

given the statistical distribution of ϕ.

In Monte Carlo analysis, a finite number of random outcomes are sampled from the distribution of ϕ, and the yield is estimated by the percentage of acceptable outcomes. If we define an acceptance index $I(\phi)$ such that $I(\phi) = 1$ if $H(\phi) \leq 0$ otherwise $I(\phi) = 0$, then the Monte Carlo yield estimate can be expressed as

$$Y = \sum I(\phi^k) / N \tag{2}$$

where the summation is over $k = 1, 2, \ldots N$, and N is the total number of outcomes. The estimate given by (2) is not a good candidate for gradient-based optimization due to the discrete nature of $I(\phi)$ and its discontinuity at $H(\phi) = 0$.

Our motivation is to find a substitute for $I(\phi)$ that is more suitable for gradient-based optimization. We consider a neighborhood of the outcome ϕ^k, denoted by Ω^k, and represent the yield in Ω^k by the probability

$$P^k = P(H(\phi) \leq 0 \mid \phi \in \Omega^k) \tag{3}$$

Then the overall yield can be estimated by

$$Y = \sum P^k / N \tag{4}$$

The classical Monte Carlo estimate (2) becomes a special case of (4) when Ω^k consists of a single outcome ϕ^k and consequently $P^k = I(\phi^k)$. As we extend Ω^k from a point to a *region* P^k becomes a continuous measure of the intersection of Ω^k and the acceptable region. The value of P^k varies continuously between 0 and 1.

To evaluate P^k precisely by (3), we will need to know the distribution of $H(\phi)$ in Ω^k. As an approximation, we may use the sample means of $H(\phi)$, denoted by $\overline{H}(\phi)$, to replace $H(\phi)$ in (3):

$$P^k \approx \overline{P}^k = P(\overline{H}(\phi) \leq 0 \mid \phi \in \Omega^k) \tag{5}$$

According to the *central limit theorem* [6], the frequency distribution of $\overline{H}(\phi)$ can be assumed to be normal (Gaussian). Hence $\overline{P}^k$ in (5) is the probability of a normal distribution which can be computed in a straightforward manner using, for instance, the Hastings formula [7]. $\overline{P}^k$ is a function of the mean and standard deviation of $\overline{H}(\phi)$ in Ω^k. Since Ω^k is defined as a conceptual neighborhood of ϕ^k, we use $H(\phi^k)$ as the mean of $\overline{H}(\phi)$ in Ω^k. A sketch of $\overline{P}^k$ versus $H(\phi^k)$ is shown in Fig. 1. When $H(\phi^k) >> 0$, ϕ^k is far away from the acceptable region, and therefore the yield probability $\overline{P}^k$ approaches 0. When $H(\phi^k) = 0$, ϕ^k is on the boundary of the acceptable region and $\overline{P}^k = 0.5$. As ϕ^k moves inside the acceptable region ($H(\phi^k) < 0$), $\overline{P}^k$ approaches 1. The standard deviation of $\overline{H}(\phi)$ provides a measure of the circuit performance variation in Ω^k and enters $\overline{P}^k$ as a scaling factor.

Small-Signal Amplifier

The new method is implemented in OSA90/hope [1]. We consider yield optimization of a single-stage small-signal amplifier (10MHz to 1GHz). The yield is 37.8% for the nominal design (all the yield figures are predicted by Monte Carlo analysis with 500 outcomes). The yield increases to 50% after optimization using the yield probability function method. 20 outcomes are used in the yield optimization. For comparison, we also applied the one-sided ℓ_1 centering algorithm by Bandler *et al.* [2-4]. Using the same number of outcomes in optimization the yield increases to 42.6%, and then to 45.6% after a restart. The problem is that the yield predicted by the one-sided ℓ_1 algorithm (based on 20 outcomes) is not close enough to the yield estimated by Monte Carlo analysis (based on 500 outcomes). When 100 outcomes are included in optimization the yield increases to 50%. Here, the new method demonstrates a clear advantage: no restart of optimization is needed; the predicted yield is more precise and hence a better solution (higher yield) is achieved without increasing the number of outcomes in optimization.

Nonlinear FET Doubler

A nonlinear FET frequency doubler [4] with a statistically characterized active device is also considered. The yield is 31% for the nominal design (predicted by Monte Carlo analysis with 500 outcomes). The yield increases to 74% after optimization using the new method. The histograms of the FET doubler spectral purity before and after yield optimization are shown in Fig. 2. For comparison, the yield optimized by the one-sided ℓ_1 centering algorithm [2-4] is

74.6%. In this case, the solutions and efficiency of the two methods are very similar.

Conclusions

We have introduced a new approach to yield optimization. A yield probability function has been created to replace the discrete and discontinuous acceptance index in classical Monte Carlo analysis, leading to a yield estimate suitable for gradient-based optimization. The new method has been implemented in OSA90/hope. We have demonstrated its merit through relevant circuit design applications.

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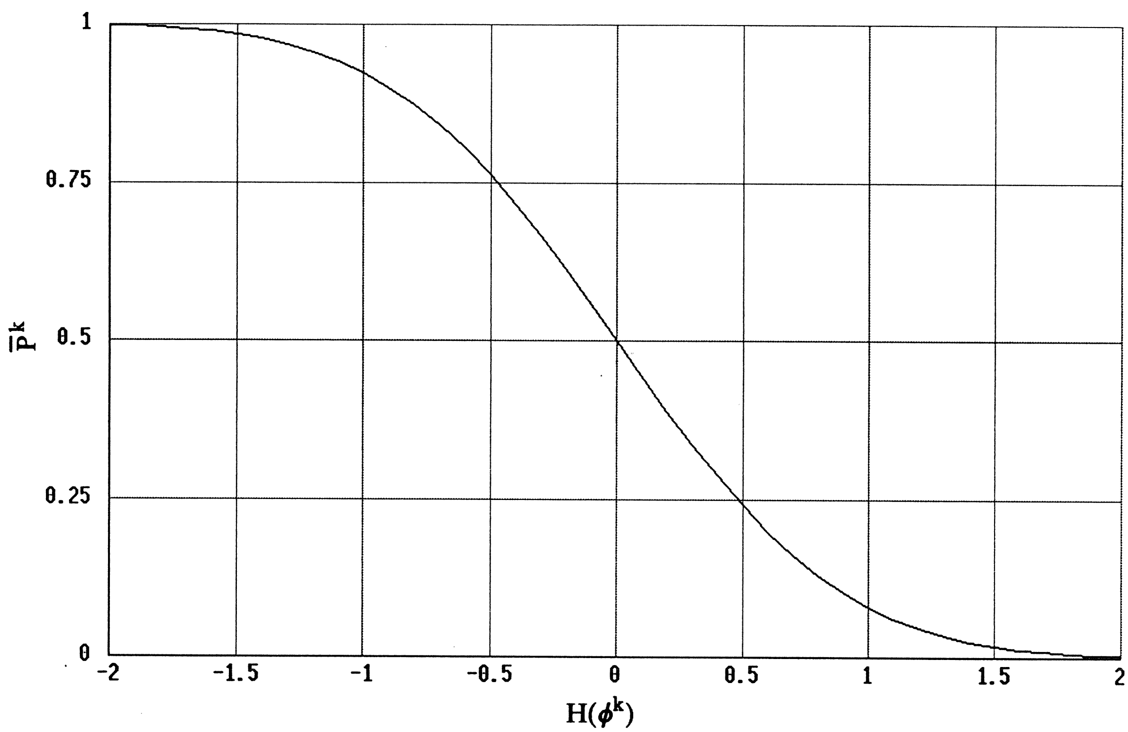
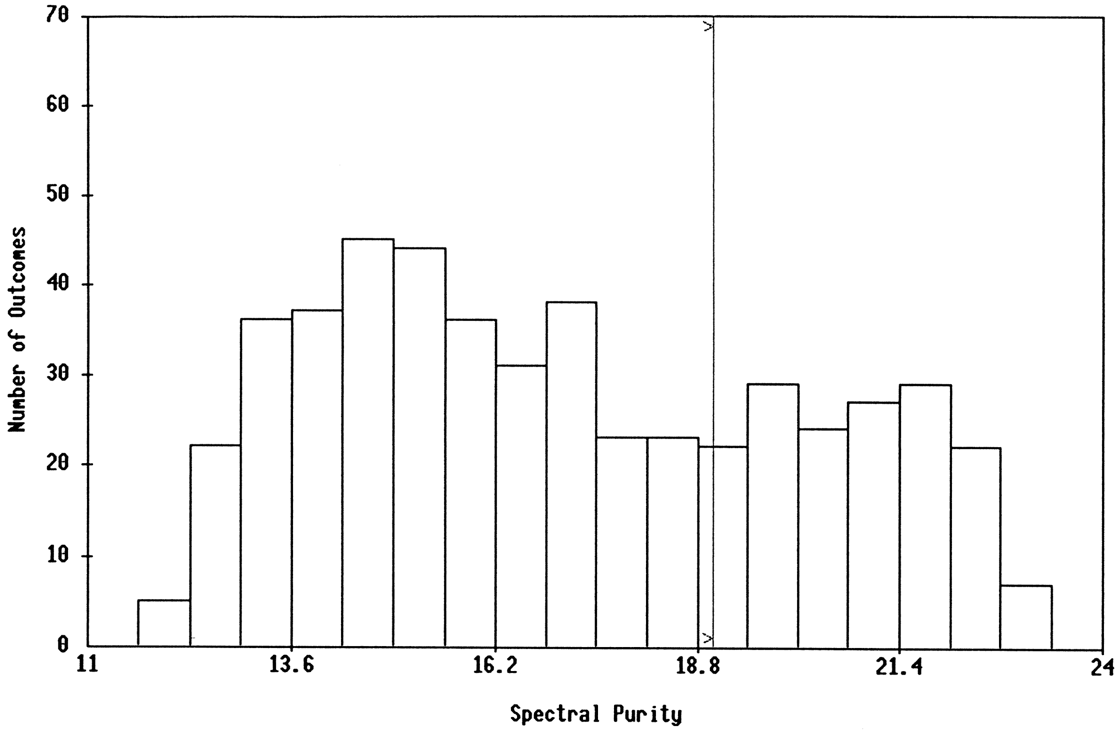
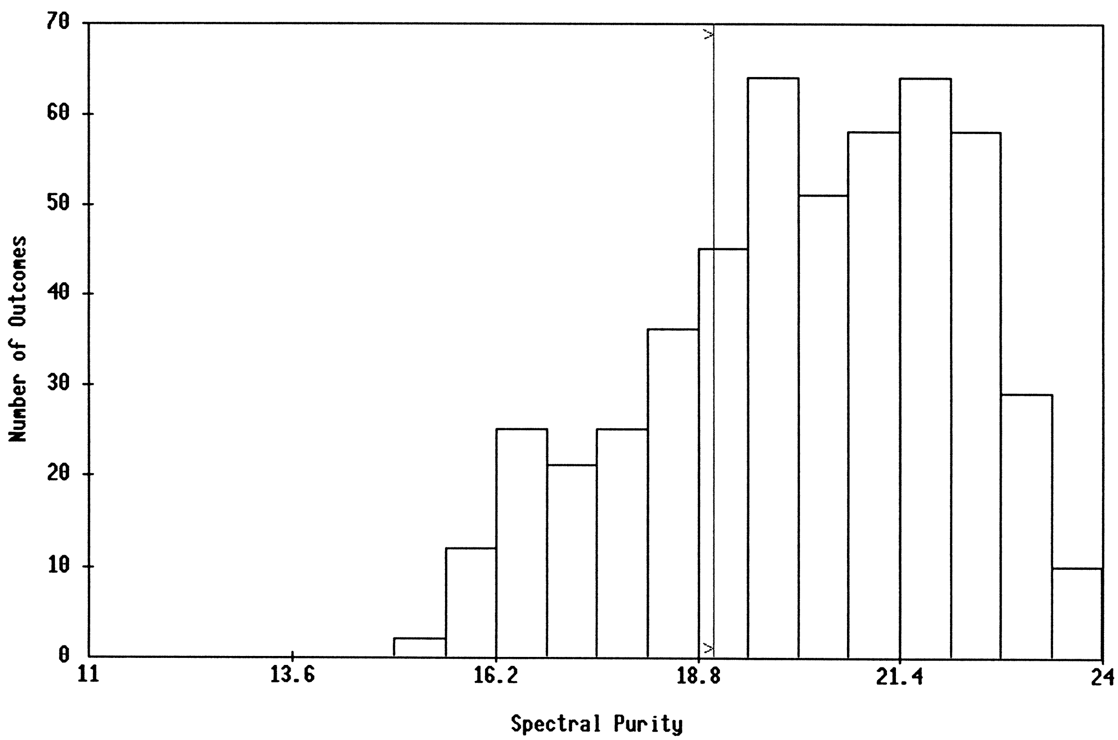


Fig. 1. Yield probability function $\overline{P}^k$ as defined by (5), assuming $H(\phi^k)$ as the mean value of $\overline{H}(\phi)$ in Ω^k and a unit standard deviation.



(a)



(b)

Fig. 2. FET doubler spectral purity: (a) before yield optimization and (b) after yield optimization.