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*Abstract*

Powerful multilevel multidimensional quadratic modeling has been developed for efficient yield-driven design. This approach makes it possible, for the first time, to perform direct yield optimization of circuits with components simulated by an electromagnetic simulator. Efficiency and accuracy of our technique are demonstrated by yield optimization of a three-stage microstrip transformer and a small-signal amplifier.

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**SUMMARY**

*Introduction*

A new multilevel multidimensional modeling technique is presented for effective and efficient yield-driven design. This approach makes it possible, for the first time, to perform yield optimization of circuits with microstrip structures simulated by an electromagnetic (EM) simulator.

Yield optimization is now recognized as effective, not only for massively manufactured circuits but also to ensure first-pass success in any design where the prototype development is lengthy and expensive. Complexity of calculations involved in yield optimization requires special numerical techniques, e.g. [1-4]. In this paper we extend our previously published [2,4], highly efficient quadratic approximation technique to multilevel modeling. It is particularly suitable for circuits containing complex subcircuits or components whose simulation requires significant computational effort.

With the increasing availability of EM simulators [5-7] it is very tempting to include them into performance-driven and even yield-driven circuit optimization. However, direct utilization of EM simulation for yield optimization might seem to be computationally prohibitive. By constructing local quadratic models for each component simulated by an EM simulator we effectively overcome the computational burden of repeated EM simulations, which would otherwise be invoked for many statistical circuit outcomes throughout all yield optimization iterations.

We show that when the proposed multilevel quadratic modeling technique is used together with expensive, but more accurate simulations at the component level, the results are more reliable than those obtained from traditional empirical component simulations.

Efficiency and accuracy of our technique are demonstrated by yield optimization of a three stage microstrip transformer and a small-signal amplifier. Optimization was performed within the OSA90/hope™ [8] simulation and optimization environment.

*Yield Optimization*

We define yield to be the ratio

$$N_{pass} \,/\, N_t \tag{1}$$

where $N_{pass}$ is the number of circuit outcomes meeting the design specifications and $N_t$ is the total number of circuit outcomes. If a simulator is involved in yield estimation or yield optimization the outcomes are generated by a random number generator according to a statistical distribution of circuit parameters. For yield optimization we use the one-sided $\ell_1$ objective function [1,9]

$$U(\boldsymbol{\phi}^0) = \sum \alpha_i \, v(\boldsymbol{\phi}^i) \tag{2}$$

where $\boldsymbol{\phi}^0$ is the vector of nominal parameter values, $\boldsymbol{\phi}^i$ is the vector of parameter values for the $i$th outcome, $\alpha_i$ are suitably chosen positive multipliers and $v(\boldsymbol{\phi}^i)$ is the generalized $\ell_p$ function combining all errors (deviations between design specifications and the $i$th outcome responses).

*Efficient Quadratic Modeling*

The *Q-model* to approximate a generic response $f(\boldsymbol{x})$, i.e., any response or gradient function at any level (see Fig.1) for which we want to build and utilize the model, is a multidimensional quadratic polynomial of the form

$$q(\boldsymbol{x}) = a_0 + \sum a_i(x_i - r_i) + \sum \; a_{ij}(x_i - r_i)(x_j - r_j) \tag{3}$$

where $\boldsymbol{x} = [x_1 \; x_2 \; . \; . \; . \; x_n]^T$ is the vector of generic parameters in terms of which the response is defined, and $\boldsymbol{r} = [r_1 \; r_2 \; . \; . \; . \; r_n]^T$ is a chosen reference point in the parameter space.

To build the model we use $n+1 < m \leq 2n+1$ base points at which the function $f(\boldsymbol{x})$ is evaluated using actual circuit, subcircuit or component simulations. The reference point $\boldsymbol{r}$ is selected as the first base point $\boldsymbol{x}^1$. The remaining $m-1$ base points are selected by perturbing one variable at a time around $\boldsymbol{r}$

$$\boldsymbol{x}^{i+1} = \boldsymbol{r} + [0 \; . \; . \; . \; 0 \; \beta_i \; 0 \; . \; . \; . \; 0]^T, \quad i = 1, 2, ..., n \tag{4}$$

$$\boldsymbol{x}^{n+1+i} = \boldsymbol{r} + [0 \; . \; . \; . \; 0 \; -\beta_i \; 0 \; . \; . \; . \; 0]^T, \quad i = 1, 2, ..., m-(n+1) \tag{5}$$

where $\beta_i$ is a predetermined perturbation. If a variable is perturbed twice the second perturbation is located symmetrically w.r.t. $\boldsymbol{r}$. In the full paper we will show that the maximally flat quadratic interpolation [3], i.e., such that the second-order term coefficients in (3) are minimized in the leastsquares sense, leads to the following, simple formula for $q(\boldsymbol{x})$

$$q(\boldsymbol{x}) = f(\boldsymbol{r}) + \sum \{[f(\boldsymbol{x}^{i+1}) - f(\boldsymbol{x}^{n+1+i}) + (f(\boldsymbol{x}^{i+1})+f(\boldsymbol{x}^{n+1+i})-2f(\boldsymbol{r}))(x_i-r_i)/\beta_i](x_i-r_i)/(2\beta_i)\} \quad (6)$$

The gradient of the model response can also be easily evaluated as

$$\partial q(\boldsymbol{x})/\partial x_i = [(f(\boldsymbol{x}^{i+1})-f(\boldsymbol{x}^{n+1+i}))/2 + (f(\boldsymbol{x}^{i+1})+f(\boldsymbol{x}^{n+1+i})-2f(\boldsymbol{r}))(x_i-r_i)/\beta_i]/\beta_i \quad (7)$$

Efficiency of the approach is unsurpassed and the computational effort increases only linearly with the number of variables $n$.

*Multilevel Modeling*

Multilevel simulation and modeling is depicted in Fig. 1. A Q-model can be established and, subsequently utilized, at any level for some or all subcircuits and components. The models are built from the results of exact simulations of the corresponding subcircuit, component, or the overall circuit. Once the model is established, it is used in place of the corresponding simulator. For particular problems many Q-models may exist changing the path of calculations as indicated by different links in Fig. 1.

The circuit, subcircuit, or component parameters can be, in general, categorized as: designable $\boldsymbol{x}_D$, statistical $\boldsymbol{x}_S$, or discrete $\boldsymbol{x}_G$. All other parameters are fixed. The vector $\boldsymbol{x}$ of model variables may contain different combinations of $\boldsymbol{x}_D$, $\boldsymbol{x}_S$ and $\boldsymbol{x}_G$, depending on the capabilities of the corresponding simulator. For example, as proposed in [4] the Q-models at the circuit level can be built for both response and gradient functions in terms of $\boldsymbol{x}_S$ only if the exact simulator returns gradient information.

The importance of bringing the discrete parameters $\boldsymbol{x}_G$ into the Q-model is illustrated in Fig. 2. The discrete parameters are such that simulation can be performed at discrete values located on the grid. For example, this is applicable in numerical EM simulation. Normally, the reference vector $\boldsymbol{r}$ is taken as the nominal point $\boldsymbol{x}^0$. This is likely to be off-the-grid. Similarly, the other base points $\boldsymbol{x}^{i+1}$ and $\boldsymbol{x}^{n+1+i}$ are likely to be off-the-grid. Then, local interpolation involving several simulations on the grid in the vicinity of each of the base points must be performed. In order to avoid these excessive simulations those base points are modified to snap to the grid. Significant computational savings can be achieved.

*Yield Optimization of a 3-Section Microstrip Transformer*

The 3-section 3:1 microstrip impedance transformer is shown in Fig. 3. The source and load impedances are 50 and 150 ohms, respectively. The design specification is set for input reflection coefficient

$$|S_{11}| \leq 0.12, \quad \text{for } 5 \text{ GHz} < f < 15 \text{ GHz}$$

The error functions for yield optimization are calculated for frequencies from 5 GHz to 15 GHz with a 0.5 GHz step.

We start yield optimization from the solution of a nominal design with $W_1$, $W_2$, $W_3$, $L_1$, $L_2$ and $L_3$ as variables. Normal distribution with 2% standard deviations were assumed for $W_1$, $W_2$ and $W_3$ and 1% standard deviations for $L_1$, $L_2$ and $L_3$. The component level Q-model is established for the transformer structure at the nominal point using the simulator ***em*** [7]. The perturbation size of the model is chosen to cover the statistical outcome region.

Two experiments were conducted to demonstrate multilevel quadratic modeling: (1) yield optimization using a single Q-model, i.e., a Q-model was employed only at the component level, and (2) yield optimization using two-level Q-models, i.e., Q-models were employed at both the component and the circuit response levels. 50 statistical outcomes were used for yield optimization. The solutions in both cases are almost identical: yield (estimated by 250 outcomes) is increased from 42% to 81%. However, the CPU time was reduced by two thirds when multilevel Q-models were used. Fig. 4(a) illustrates the Monte Carlo sweep before optimization and Fig. 4(b) shows the corresponding sweep after yield optimization using one-level modeling.

We also investigated yield optimization with only $W_1$, $W_2$ and $W_3$ as variables. $L_1$, $L_2$ and $L_3$ are fixed (without tolerances). Yield is increased from 66% to 77% after optimization.

*Yield Optimization of a Small-Signal Amplifier*

The specification for the single-stage 6-18 GHz small-signal amplifier shown in Fig. 5, is

$$7 \text{ dB} \leq |S_{21}| \leq 8 \text{ dB}, \quad \text{for } 6 \text{ GHz} \leq f \leq 18 \text{ GHz}.$$

The statistics of the small-signal FET model were extracted from measurement data [10]. The gate and drain microstrip T-junctions and the feedback microstrip line are built on a 10 mil thick substrate with the relative dielectric constant 9.9. We first performed a nominal design optimization using analytical/empirical microstrip components. The variables were $W_1$, $W_2$, $L_1$, $L_2$ for the gate T-junction and $W_3$, $W_4$, $L_3$, $L_4$ for the drain T-junction as shown in Fig. 5.

We assume 0.5 mil tolerance and uniform distribution for the geometrical parameters of the microstrip components. 92.8% yield was estimated using Monte Carlo simulation with 250 outcomes. To obtain a more accurate estimate, we used component level Q-models built from ***em*** [7] simulations for the microstrip components. Monte Carlo simulation with 250 outcomes indicated a drop to 68% yield. Figs. 6(a) and 6(b) show the Monte Carlo sweeps obtained using analytical/empirical and EM quadratic microstrip models, respectively. Close similarities can be observed for lower frequencies, while discrepancies become larger for higher frequencies.

Utilizing the component level quadratic Q-models built from ***em*** simulations, we further performed yield optimization of this amplifier. Yield estimated by 250 Monte Carlo simulations was increased to 82%. The corresponding Monte Carlo sweep diagram is shown in Fig. 7. Yield and the values of the optimization variables before and after optimization are given in Table I.

*Conclusions*

We have presented a new multilevel quadratic modeling technique suitable for effective and efficient yield-driven design optimization. This approach is particularly useful for circuits containing complex subcircuits or components whose simulation requires significant computational effort. The efficiency of this technique allowed us to perform yield-driven design of circuits containing microstrip structures accurately simulated by ***em*** [7]. Our approach, illustrated by yield optimization of a three-stage microstrip transformer and a small-signal amplifier, significantly extends the microwave CAD applicability of yield optimization techniques.

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[10] Measurement data provided by Plessey Research Caswell Ltd., Caswell, Towcester, Northamptonshire, England NN12 8EQ, 1990.

TABLE I
YIELD OPTIMIZATION OF THE AMPLIFIER

| Variable | Nominal design | Centered design |
|---|---|---|
| $W_1$(mil) | 17.45 | 19.03 |
| $L_1$(mil) | 35.54 | 34.54 |
| $W_2$(mil) | 9.01 | 8.581 |
| $L_2$(mil) | 30.97 | 32.03 |
| $W_3$(mil) | 8.562 | 6.97 |
| $L_3$(mil) | 4.668 | 6.03 |
| $W_4$(mil) | 3.926 | 3.628 |
| $L_4$(mil) | 9.902 | 11.01 |
| Yield (250 outcomes) | 68% | 82% |

50 outcomes were used in yield optimization

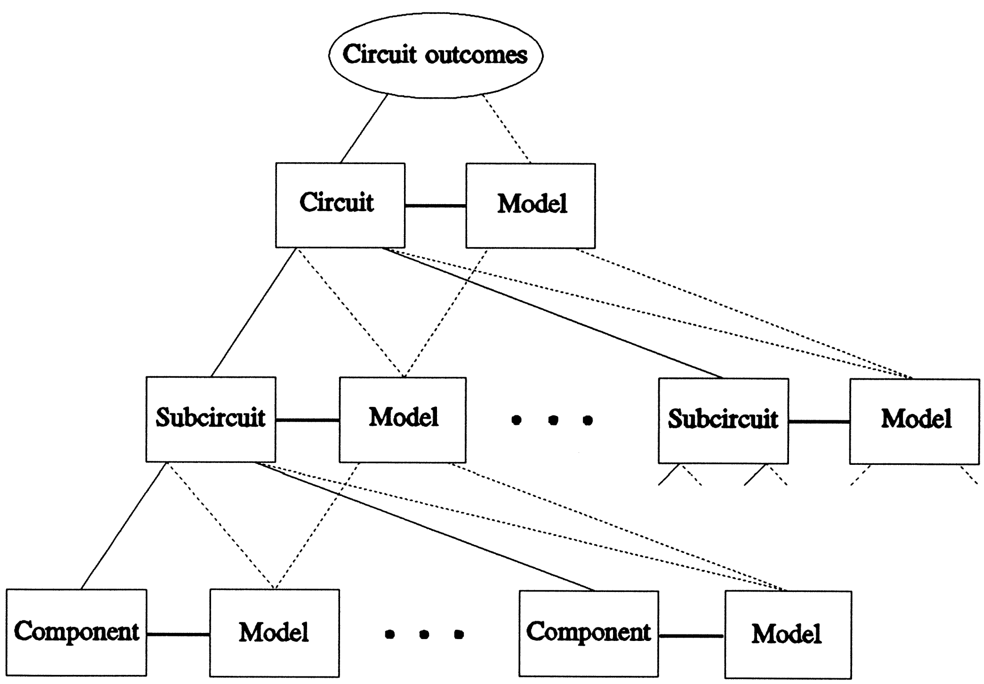



Fig. 1 Schematic diagram illustrating multilevel modeling for yield-driven optimization.

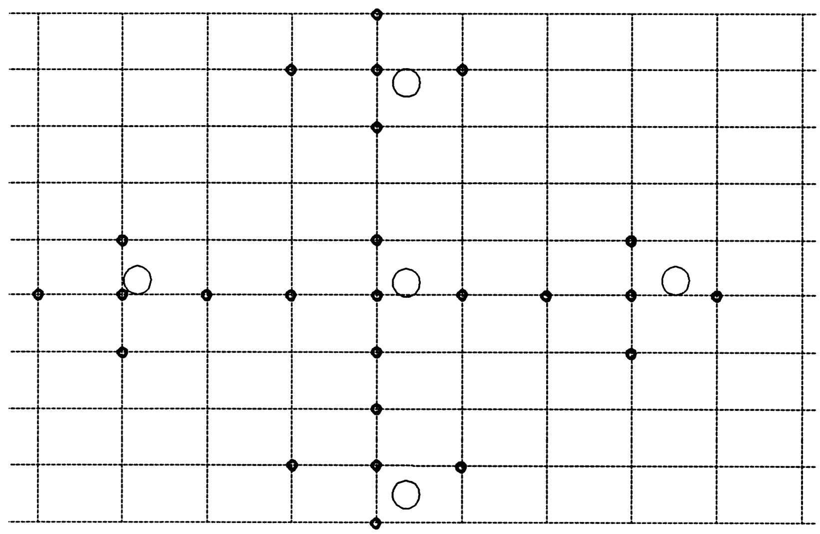


Fig. 2 Illustration of base points and discrete points. The large circles represent possible location of base points w.r.t. a grid. The solid dots indicate discrete simulation points on the grid. If the base points are snapped to the grid, the number of simulations can be significantly reduced.

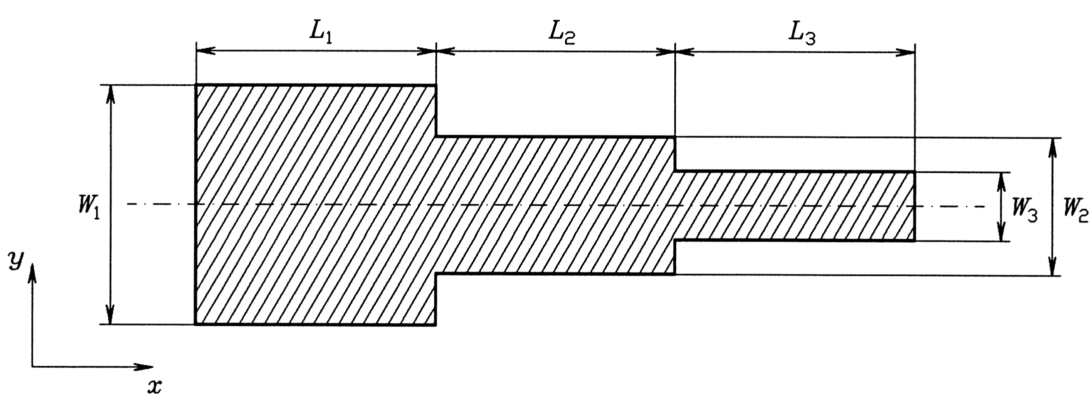
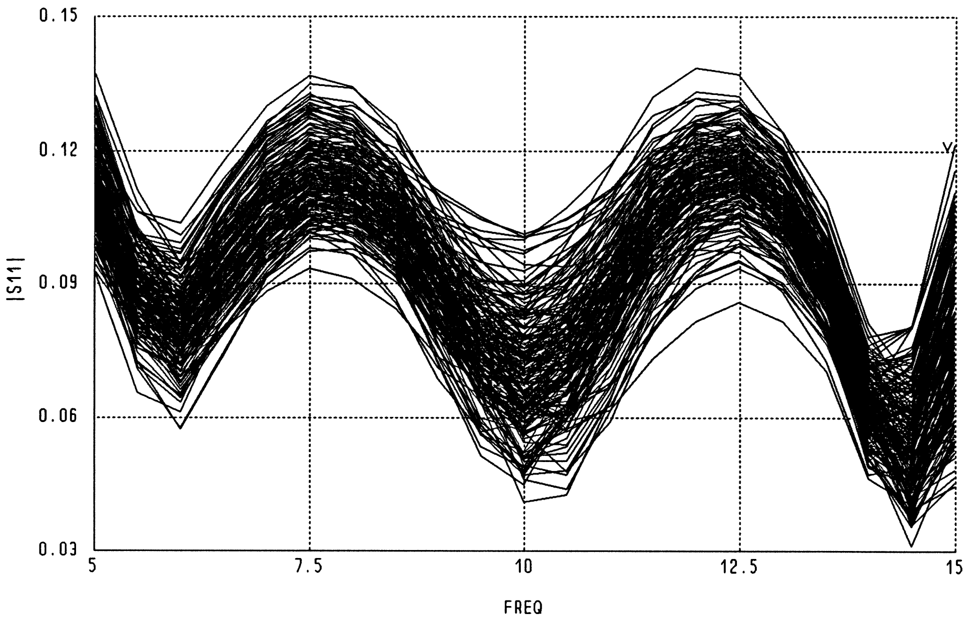
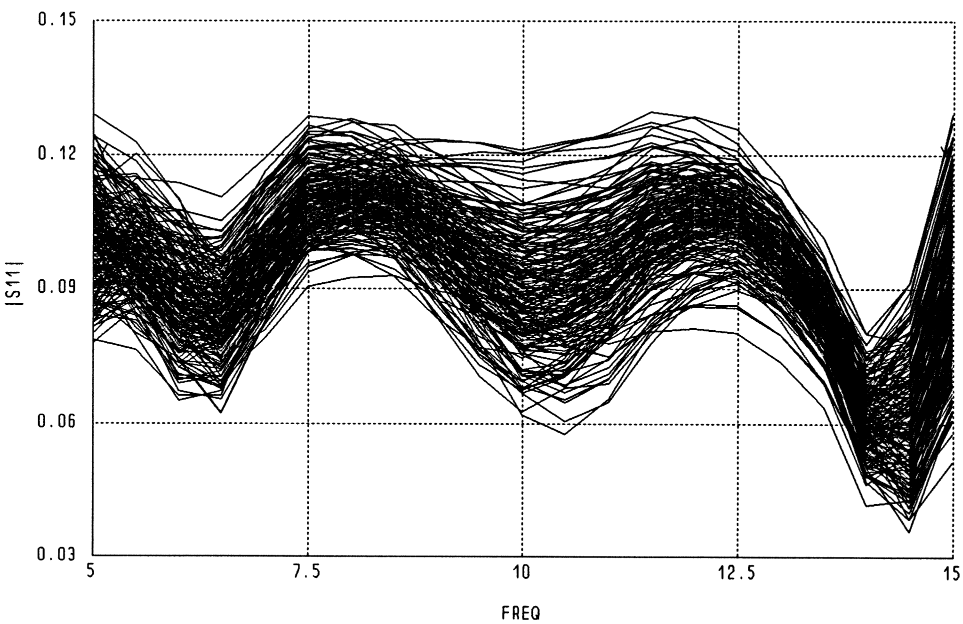



Fig. 3 3-section 3:1 microstrip impedance transformer. The thickness and dielectric constant of the substrate are 0.635mm and 9.7, respectively.




(a)




(b)

Fig. 4 Reflection coefficient of the 3-section microstrip impedance transformer vs. frequency for 250 statistical outcomes: (a) before yield optimization, and (b) after yield optimization. Yield is increased from 42% to 81% after optimization using component level Q-model.

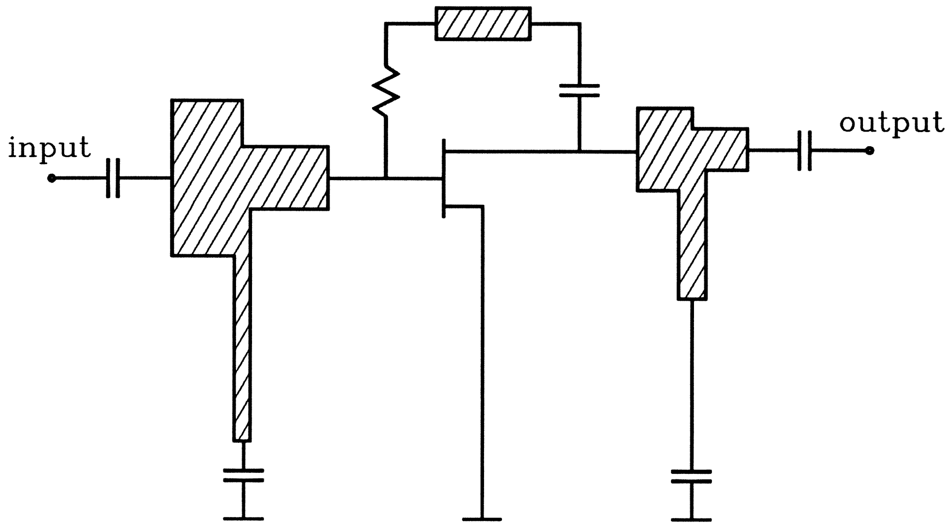
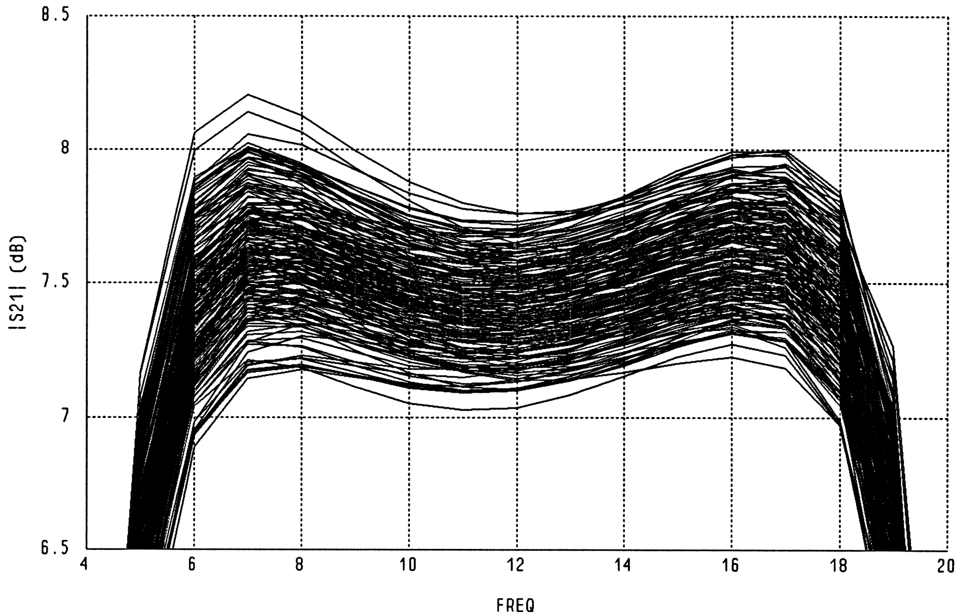
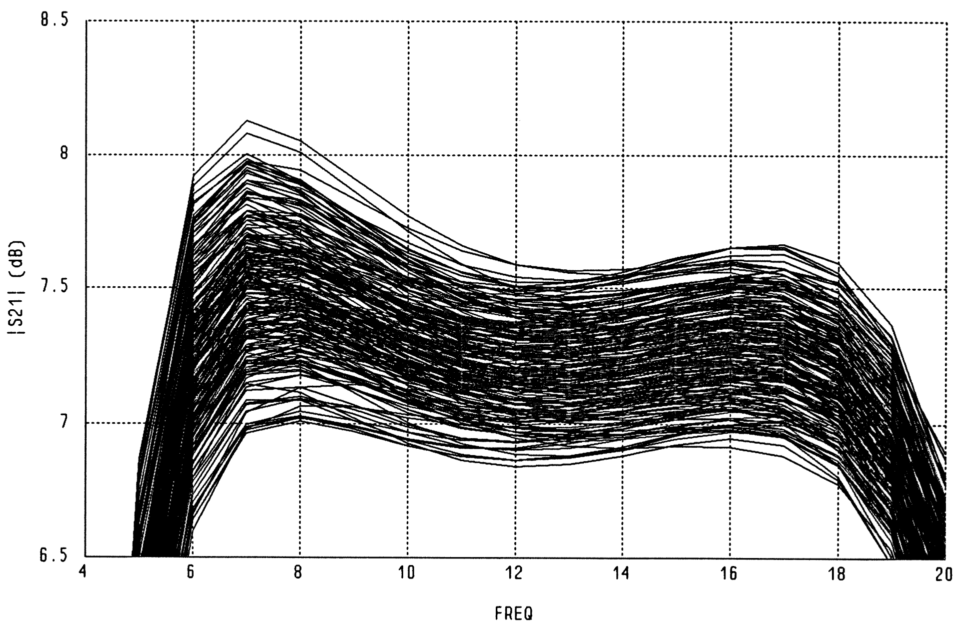



Fig. 5 Circuit diagram of the 6-18 GHz small-signal amplifier. We use *em* [7] to model the two T-junctions and the microstrip line.




(a)




(b)

Fig. 6 $|S_{21}|$ of the small-signal amplifier for 250 statistical outcomes at the nominal design: (a) using analytical/empirical microstrip component models, and (b) using *em* [7].

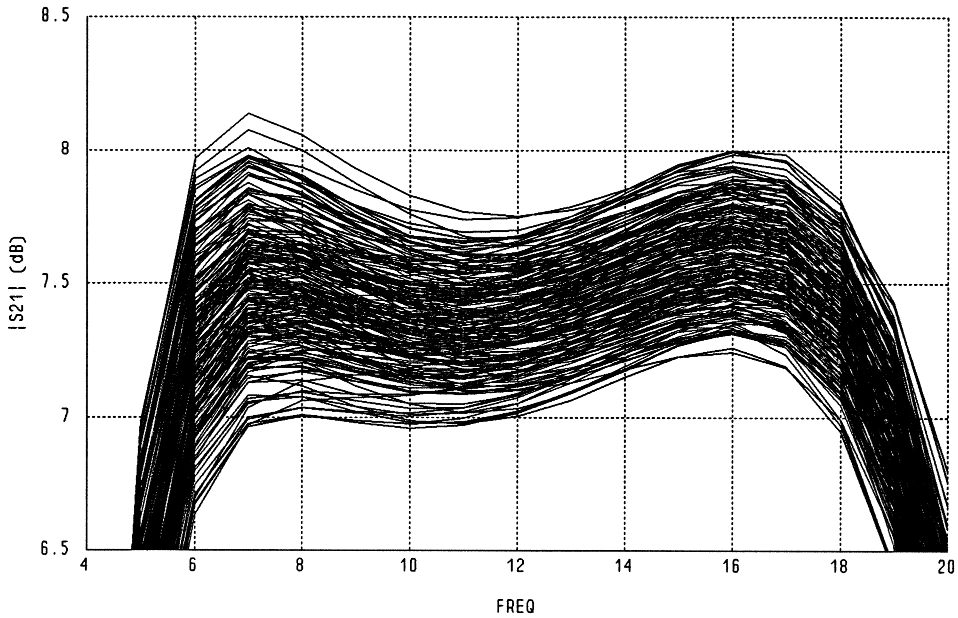



Fig. 7 $|S_{21}|$ of the small-signal amplifier for 250 statistical outcomes after yield optimization. Yield is increased from 68% to 82%.