

**PERFORMANCE AND YIELD OPTIMIZATION  
OF ELECTRONIC DEVICES AND CIRCUITS**

**J.W. Bandler**

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ELECTRONIC DEVICES AND CIRCUITS**

J.W. Bandler

Simulation Optimization Systems Research Laboratory  
and Department of Electrical and Computer Engineering  
McMaster University, Hamilton, Canada L8S 4L7





## **Introduction**

recent work in performance and yield optimization (design centering) of electronic devices and circuits

yield-driven design is effective for massively manufactured circuits and for first-pass success when prototype development is lengthy and expensive

formulations are usually nonlinear but we use algorithms employing trust regions and linear programming

minimax and  $\ell_1$  formulations implemented in the user-friendly optimization environment OSA90/hope

direct utilization of electromagnetic simulators

results on minimax microwave filter design

multidimensional response surface modeling

one-sided  $\ell_1$  algorithm for yield optimization

robust circuit optimization using Huber functions: device statistical modeling, microwave multiplexer optimization, analog fault location and data fitting

one-sided Huber function for large-scale optimization



## Yield Optimization

the problem of yield optimization can be formulated as

$$\underset{\phi^0}{\text{maximize}} \left\{ Y(\phi^0) = \int_{R^n} I_a(\phi) f_{\phi}(\phi^0, \phi) d\phi \right\}$$

where

- $\phi^0$             nominal circuit parameters
- $\phi$              actual circuit outcome parameters
- $Y(\phi^0)$        design yield
- $f_{\phi}(\phi^0, \phi)$  probability density function of  $\phi$  around  $\phi^0$
- $I_a(\phi) = \begin{cases} 1 & \text{if } \phi \in A \\ 0 & \text{if } \phi \notin A \end{cases}$
- $A$              acceptability region

in practice, the integral is approximated using  $K$  Monte Carlo circuit outcomes  $\phi^i$  and yield is estimated by

$$Y(\phi^0) \approx \frac{1}{K} \left( \sum_{i=1}^K I_a(\phi^i) \right)$$

the outcomes  $\phi^i$  are generated by a random number generator according to  $f_{\phi}(\phi^0, \phi)$



## Error and Objective Functions

to estimate yield we create a set of multi-circuit error functions  $e(\phi^1), e(\phi^2), \dots, e(\phi^K)$

the error functions  $e(\phi^i)$  are derived from the circuit responses  $R_j$  and lower specifications ( $S_l$ ) and upper specifications ( $S_u$ ) as

$$e_j(\phi^i) = R_j(\phi^i) - S_{uj} \quad \text{or} \quad e_j(\phi^i) = S_{lj} - R_j(\phi^i)$$

for yield optimization we use the one-sided  $\ell_1$  objective function

$$U(\phi^0) = \sum_{i \in J} \alpha_i v(\phi^i)$$

where

$$J = \{i \mid v(\phi^i) > 0\}$$

$\alpha_i$  suitably chosen positive multipliers

$v(\phi^i)$  generalized  $\ell_1$  function

consequently,  $U(\phi^0)$  becomes an approximation to the percentage of outcomes violating design specifications and minimization of  $U(\phi^0)$  leads to yield improvement



## Efficient Q-Modeling - Concept (Biernacki et al., 1989)

the *Q-model* to approximate a generic response  $f(x)$  is a multidimensional quadratic polynomial of the form

$$q(x) = a_0 + \sum_{i=1}^n a_i (x_i - r_i) + \sum_{\substack{i=1 \\ j \geq i}}^n a_{ij} (x_i - r_i) (x_j - r_j)$$

where

$$\begin{aligned} x &= [x_1 \ x_2 \ \dots \ x_n]^T && \text{vector of generic circuit parameters} \\ r &= [r_1 \ r_2 \ \dots \ r_n]^T && \text{chosen reference point} \end{aligned}$$

to build the model we use  $n+1 < m \leq 2n+1$  base points at which the function  $f(x)$  is evaluated

the reference point  $r$  is selected as the first base point  $x^1$

the remaining  $m-1$  base points are selected by perturbing one variable at a time around  $r$  with a predetermined perturbation  $\beta_i$

$$x^{i+1} = r + [0 \ \dots \ 0 \ \beta_i \ 0 \ \dots \ 0]^T, \quad i = 1, 2, \dots, n$$

$$x^{n+1+i} = r + [0 \ \dots \ 0 \ -\beta_i \ 0 \ \dots \ 0]^T, \quad i = 1, 2, \dots, m-(n+1)$$



## Efficient Q-Modeling - Formulas

applying the Maximally Flat Quadratic Interpolation (MFQI) technique to the set of base points yields

$$\begin{aligned} q(x) = & f(\mathbf{r}) + \sum_{i=1}^{m-(n+1)} \left\{ [f(\mathbf{x}^{i+1}) - f(\mathbf{x}^{n+1+i}) + (f(\mathbf{x}^{i+1}) \right. \\ & \left. + f(\mathbf{x}^{n+1+i}) - 2f(\mathbf{r}))(x_i - r_i)/\beta_i](x_i - r_i)/(2\beta_i) \right\} \\ & + \sum_{i=m-n}^n \left\{ [f(\mathbf{x}^{i+1}) - f(\mathbf{r})](x_i - r_i)/\beta_i \right\} \end{aligned}$$

to apply a gradient-based optimizer we need the gradient of  $q(x)$

$$\begin{aligned} \partial q(x)/\partial x_i = & [(f(\mathbf{x}^{i+1}) - f(\mathbf{x}^{n+1+i}))/2 + (f(\mathbf{x}^{i+1}) + f(\mathbf{x}^{n+1+i}) \\ & - 2f(\mathbf{r}))(x_i - r_i)/\beta_i]/\beta_i, \quad i = 1, \dots, m-(n+1) \end{aligned}$$

and, if  $m < 2n + 1$ ,

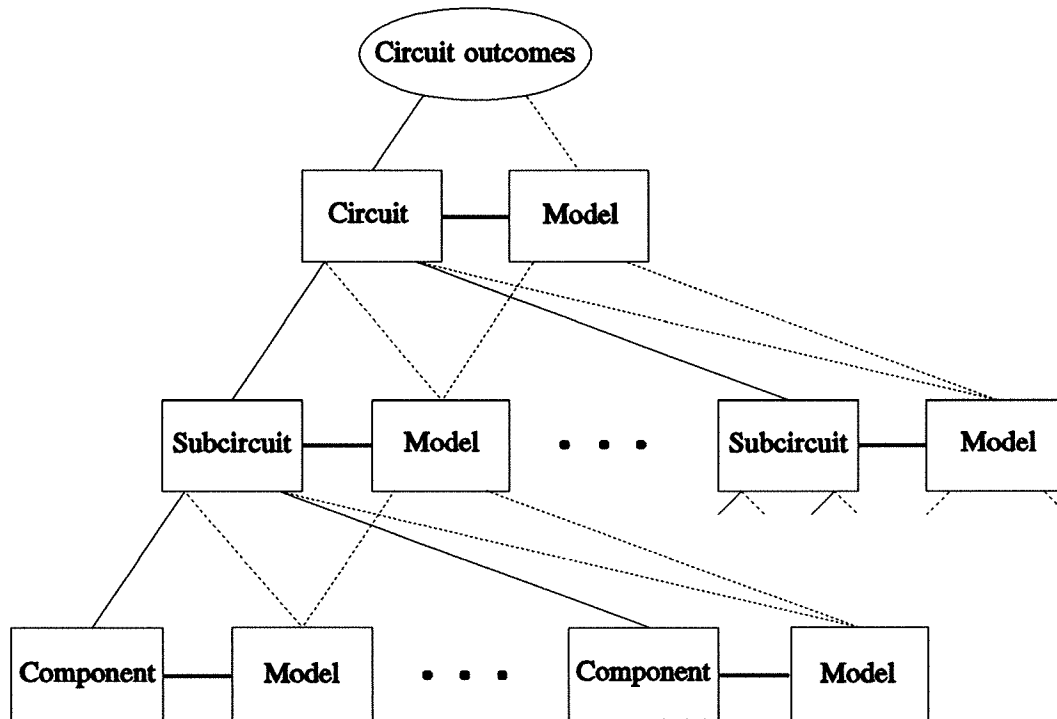
$$\partial q(x)/\partial x_i = [f(\mathbf{x}^{i+1}) - f(\mathbf{r})]/\beta_i, \quad i = m-n, \dots, n$$

the simplicity of these formulas results in unsurpassed efficiency





## Multilevel Modeling



a Q-model can be established at any level for some or all subcircuits and components

the models are built from the results of exact simulations of the corresponding component, subcircuit, or the overall circuit

once the Q-model is established, it is used in place of the corresponding simulator

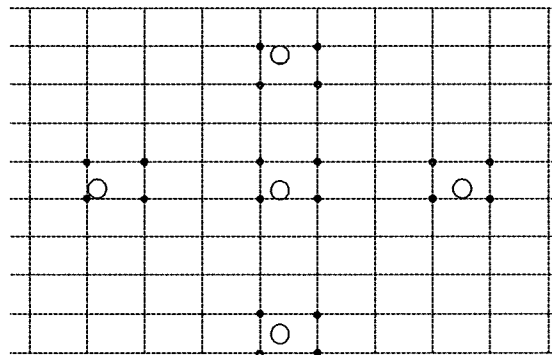
many Q-models may exist changing the path of calculations as indicated by different links in the figure



## **Model Variables**

the vector  $x$  of circuit, subcircuit or component model variables may contain different combinations of designable  $x_D$ , statistical  $x_S$ , or discrete  $x_G$  parameters

the discrete parameters  $x_G$  are those for which simulation can only be performed at discrete values located on the grid as in EM simulation



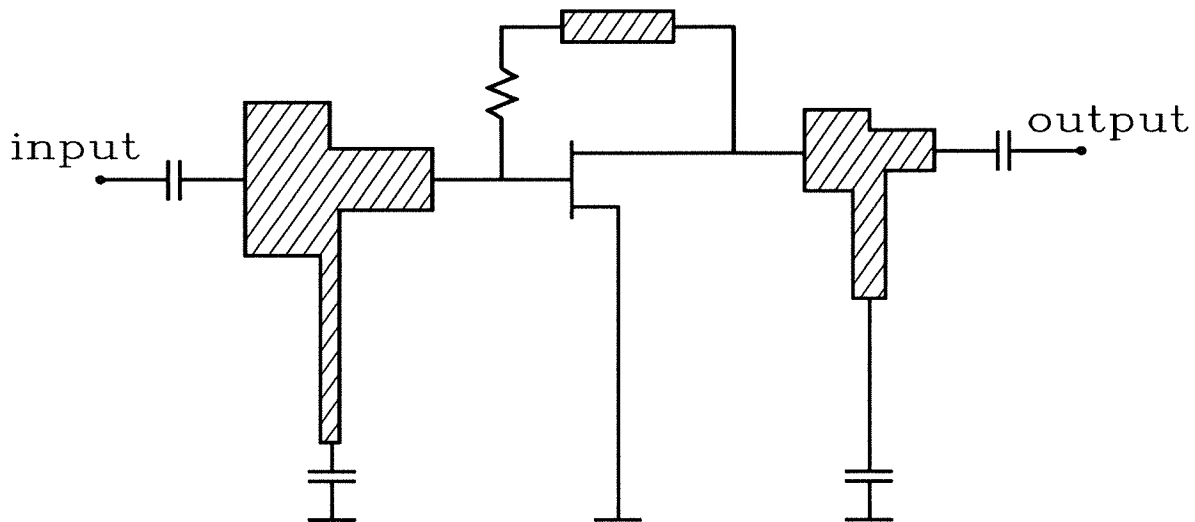
the reference vector and other base points are likely to be off-the-grid

local interpolation involving several simulations on the grid in the vicinity of each of the base points must then be performed

in order to avoid excessive simulations the base points are modified to snap to the grid



## Optimization of a Small-Signal Amplifier



the specifications for yield optimization of the amplifier are

$$7 \text{ dB} \leq |S_{21}| \leq 8 \text{ dB} \quad \text{for} \quad 6 \text{ GHz} < f < 18 \text{ GHz}$$

the gate and drain circuit microstrip T-junctions and the feedback microstrip line are built on a 10 mil thick substrate with relative dielectric constant 9.9

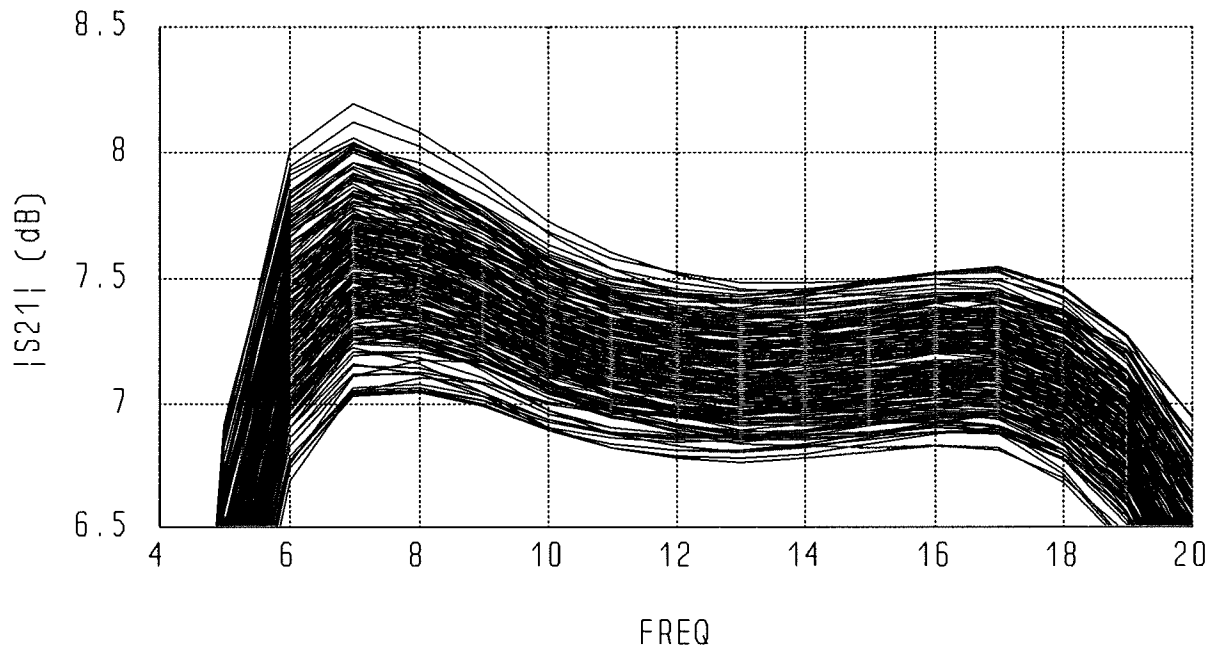
the microstrip components of the amplifier are simulated using component level Q-models built from EM simulations

we used *em*<sup>TM</sup> from *Sonnet Software* for EM simulations



## **Small-Signal Amplifier Yield Before Optimization**

the starting point for yield optimization was obtained by nominal minimax optimization using analytical/empirical microstrip component models



## **Monte Carlo simulation**

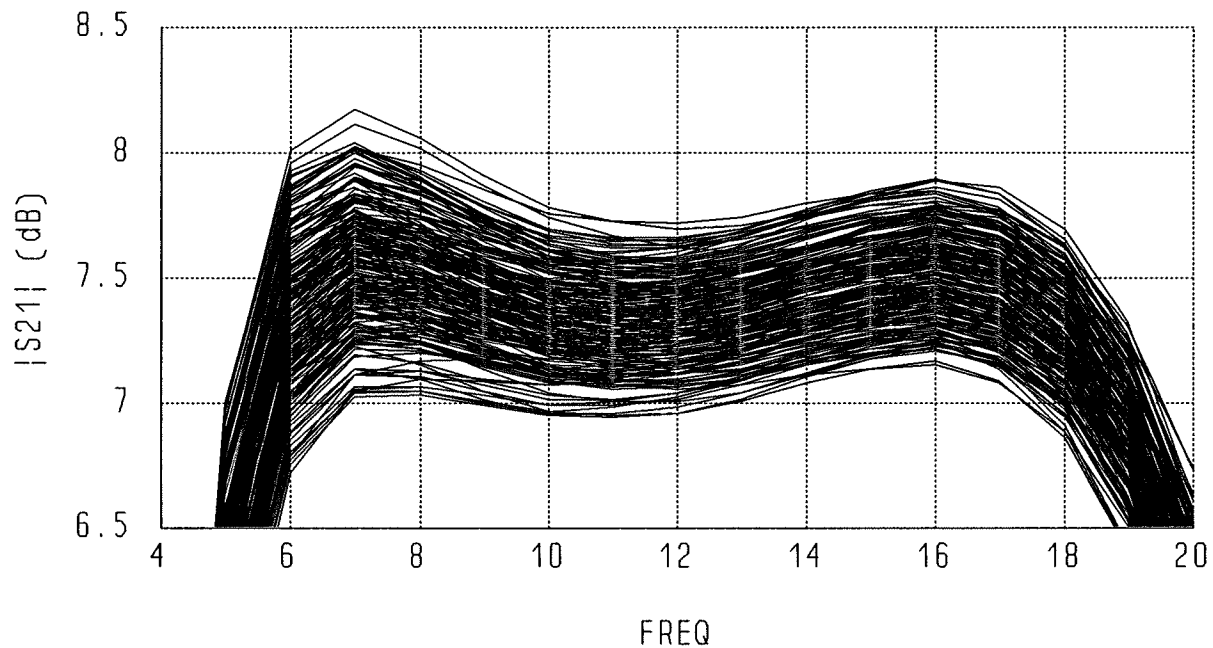
**250 outcomes**

**55% yield**



## Small-Signal Amplifier Yield After Optimization

the component level Q-models were used in yield optimization



yield estimated by 250 Monte Carlo simulations increased to 82%

optimization was performed by OSA90/hope<sup>TM</sup> with Empipe<sup>TM</sup> driving *em*<sup>TM</sup>



## Optimization Results

### MICROSTRIP PARAMETERS OF THE AMPLIFIER

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Parameters	Nominal design	Centered design
$W_{g1}$	17.45	19.0
$L_{g1}$	35.54	34.53
$W_{g2}$	9.01	8.611
$L_{g2}$	30.97	32.0
$W_{g3}$	3.0*	3.0*
$L_{g3}$	107.0*	107.0*
$W_{d1}$	8.562	7.0
$L_{d1}$	4.668	6.0
$W_{d2}$	3.926	3.628
$L_{d2}$	9.902	11.0
$W_{d3}$	3.5*	3.5*
$L_{d3}$	50.0*	50.0*
$W$	2.0*	2.0*
$L$	10.0*	10.0*

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Yield (250 outcomes)	55%	82%
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\* Parameters not optimized.

Dimensions of the parameters are in mils. 50 outcomes were used for yield optimization. 0.5 mil tolerance and uniform distribution were assumed for all the parameters.

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## **Minimax Design Optimization**

$$\underset{\phi}{\text{minimize}} \{ \max_j (e_j(\phi)) \}$$

where

$\phi$  the vector of optimization variables

$R_j(\phi)$   $j=1,2,..$  - the circuit responses ( $S$  parameters, return loss, insertion loss, etc.)

$S_{uj}, S_{lj}$  upper/lower specification on  $R_j(\phi)$

the individual errors  $e_j(\phi)$  are of the form

$$e_j(\phi) = R_j(\phi) - S_{uj}$$

or

$$e_j(\phi) = S_{lj} - R_j(\phi)$$

negative/positive error value indicates that the corresponding specification is satisfied/violated

effective minimax optimization requires a dedicated optimizer and accurate gradients of individual errors w.r.t. the optimization variables  $\phi$



## Geometrical Parameters

the vector  $\psi$  of all geometrical parameters (structure lengths, widths, spacings, etc.) can be written as

$$\psi = [\psi_{opt}^T(\phi) \quad \psi_{fix}^T]^T$$

where

$\psi_{opt}(\phi)$  designable geometrical parameters which are either directly the optimization variables or are functions of the optimization variables  $\phi$

$\psi_{fix}$  fixed geometrical parameters

each component of  $\psi$  belongs to one of the three physical orientations ( $x$ ,  $y$ , or  $z$ ) and the vector  $\psi$  can be rearranged as

$$\psi = [\psi^{xT} \quad \psi^{yT} \quad \psi^{zT}]^T$$

numerical EM simulation is performed for discretized values of geometrical parameters  $\psi$

the discretization matrix  $\delta$  is defined by the grid sizes  $\Delta x_i$ ,  $\Delta y_i$  and  $\Delta z_i$  as

$$\delta = \text{diag}\{\delta_i\} = \text{diag}\{\Delta x_1, \Delta x_2, \dots, \Delta y_1, \Delta y_2, \dots, \Delta z_1, \Delta z_2, \dots\}$$





## **Interpolation Base**

if the point is off-the-grid we use interpolation to determine each response  $R(\psi)$

the interpolation base  $B$  is defined as a set of grid points in the space of geometrical parameters

$$B = \{\psi^c\} \cup \{\psi \mid \psi = \psi^c + S \delta \eta, \eta \in B^\eta\}$$

where

$\psi^c$             the centre base point

$B^\eta$             the relative interpolation base (a set of selected integer vectors)

$S$             the symmetry matrix accounting for double grid size increments for parameters whose dimensions are modified by extending or contracting both ends simultaneously

while the centre point may move during optimization the relative interpolation base is fixed

the interpolation base is used as the set of *base points*  $\psi^c$  and  $\psi^{bj}$  at which EM simulation is invoked to evaluate the corresponding responses



## Geometrical Interpolation

the relative deviation  $\theta$  of  $\psi$  from the centre base point  $\psi^c$  is defined by the equation

$$\psi = \psi^c + \delta\theta$$

the interpolating function is devised such that it passes through the exact response values at the base points and can be evaluated as

$$R(\psi) = R_{EM}(\psi^c) + f^T(\delta\theta) F^{-1}(S\delta, B^\eta) \Delta R_{EM}(B)$$

where

$f(\delta\theta)$  the vector of fundamental interpolating functions

$\Delta R_{EM}(B)$  the vector of response deviations at the base points:

$$\Delta R_{EM}(\psi^{bj}) = R_{EM}(\psi^{bj}) - R_{EM}(\psi^c)$$

the matrix  $F^{-1}(S\delta, B^\eta)$  depends only on the selection of the fundamental interpolating functions and the relative interpolation base  $B^\eta$  and can be determined prior to all calculations



## **Gradient Estimation**

facilitates the use of an efficient and robust dedicated gradient minimax optimizer

we need to provide the gradients of the error functions

$$\nabla_{\phi} R_j(\phi) = \nabla_{\phi} \psi^T(\phi) \nabla_{\psi} R(\psi)$$

the first factor is readily available

the second factor must be determined using EM simulations; it is most appropriate from the optimizer's point of view to provide the gradient of the interpolating function, i.e., the function that is actually returned to the optimizer

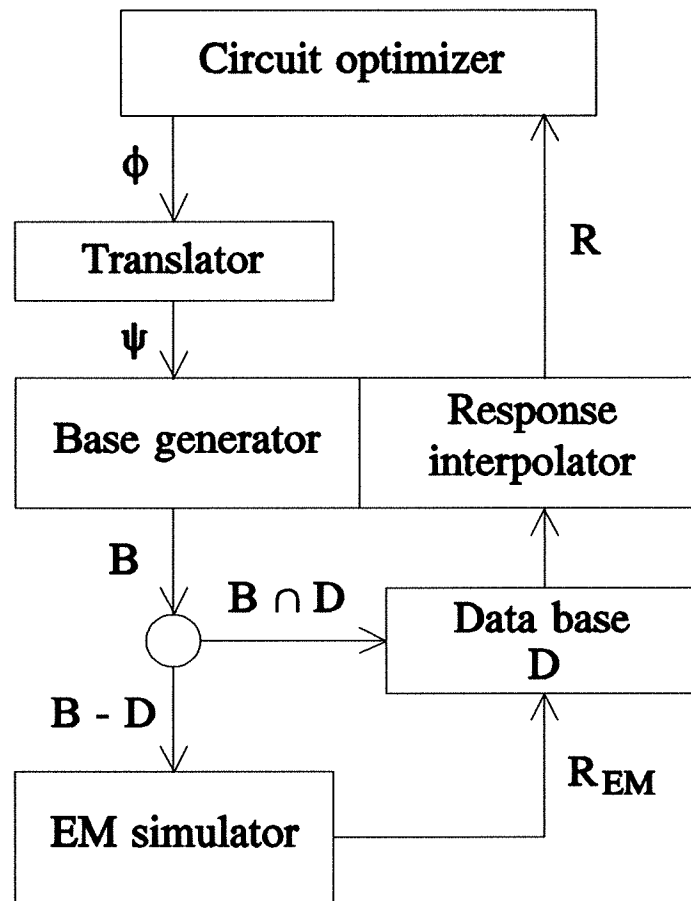
$$\nabla_{\psi} R(\psi) = \nabla_{\delta\theta} f^T(\delta\theta) F^{-1}(S\delta, B^{\eta}) R_{EM}(\psi^c)$$

this gives accurate gradient information for the optimizer in a simple, straightforward and efficient manner

note that  $F^{-1}(S\delta, B^{\eta})$  and  $R_{EM}(\psi^c)$  are already available from response interpolation

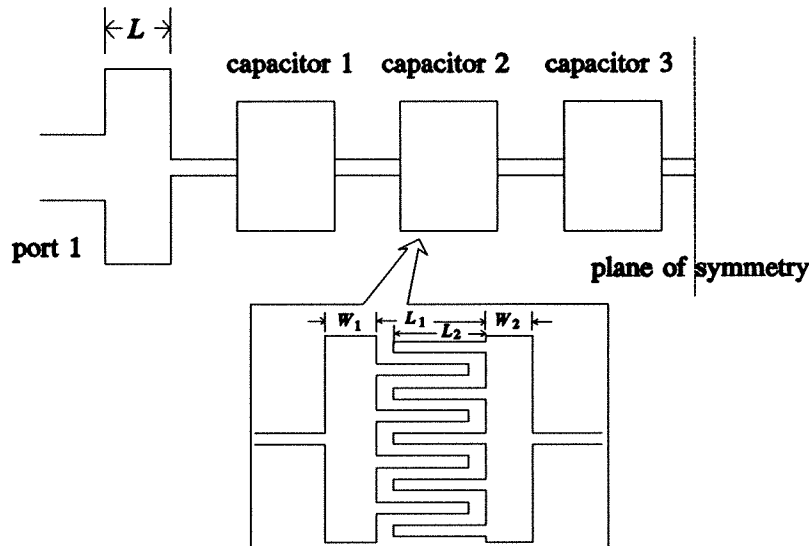


## **Interconnection Between a Circuit Optimizer and a Numerical EM Simulator**





## 26-40 GHz Interdigital Microstrip Bandpass Filter



utilizes thin microstrip lines and interdigital capacitors to realize inductances and capacitances of a synthesized lumped ladder circuit

the original microstrip design was determined by matching the lumped prototype at the center frequency using *em*<sup>TM</sup>

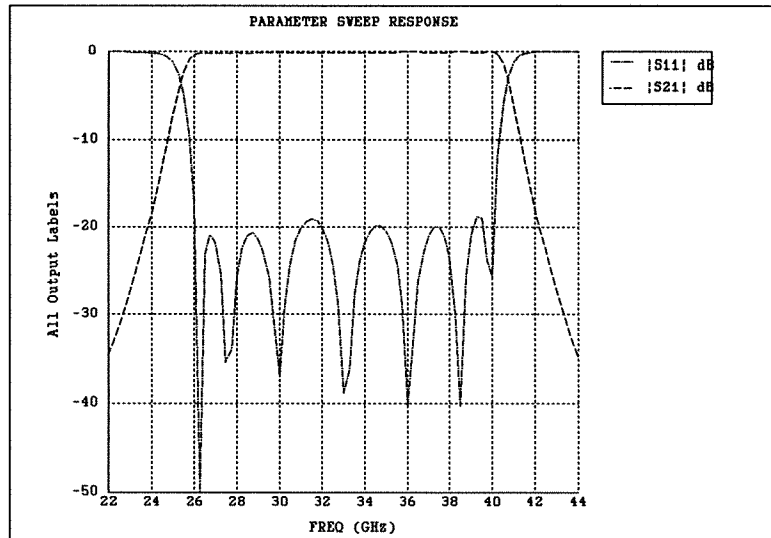
when the filter was simulated by *em*<sup>TM</sup> in the whole frequency range the results exhibited significant discrepancies w.r.t. the prototype

it necessitated manual adjustment and made a satisfactory design very difficult to achieve



## **Simulation of the 26-40 GHz Interdigital Capacitor Filter After Optimization**

filter response after optimization



a typical minimax equal-ripple response of the filter was achieved after a series of consecutive optimizations with different subsets of optimization variables and frequency points

the resulting geometrical dimensions were finally rounded to 0.1 mil resolution



## Huber Functions

$$\rho_k(f) = \begin{cases} f^2/2 & \text{if } |f| \leq k \\ k|f| - k^2/2 & \text{if } |f| > k \end{cases}$$

where  $k$  is a positive constant

the Huber function  $\rho_k$  is a hybrid of the least-squares ( $\ell_2$ ) (when  $|f| \leq k$ ) and the  $\ell_1$  (when  $|f| > k$ )

if  $k$  is sufficiently large, the Huber function becomes the  $\ell_2$  function; if  $k$  approaches zero,  $\rho_k$  approaches the  $\ell_1$  function

## Huber Optimization

$$\underset{x}{\text{minimize}} \quad F(x) \triangleq \sum_{j=1}^m \rho_k(f_j(x))$$

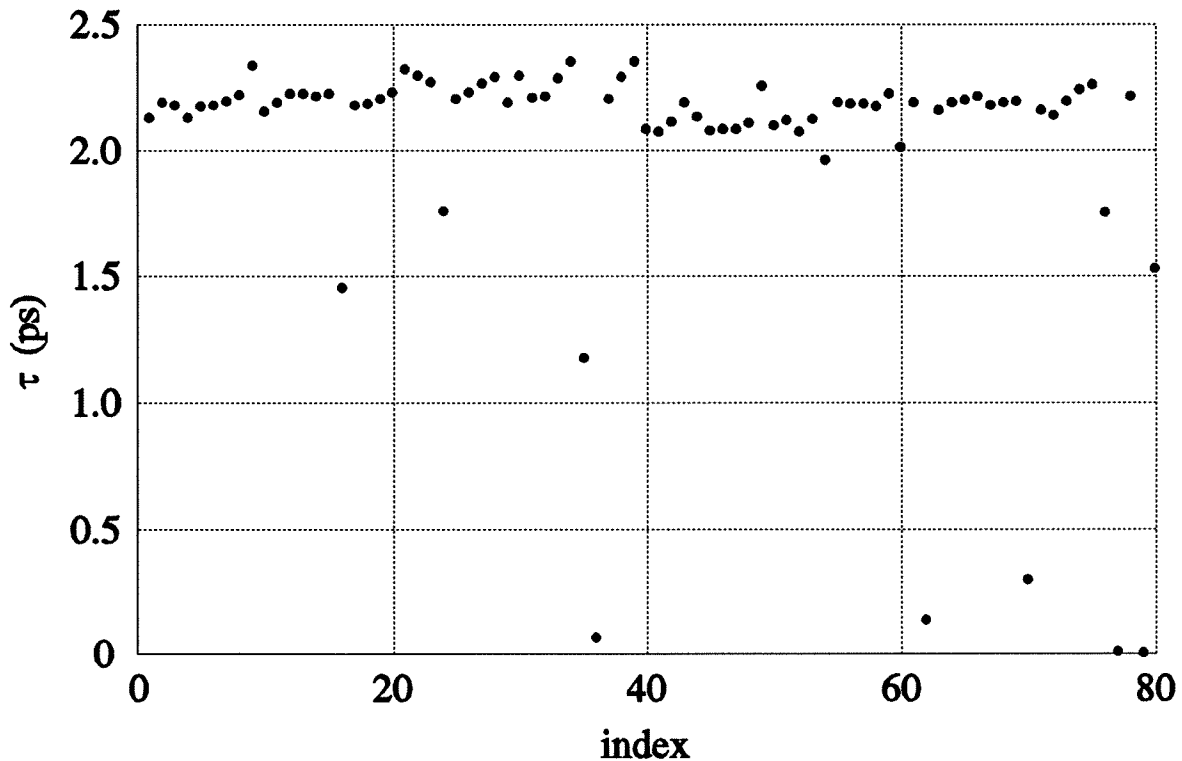
where

$x = [x_1 \ x_2 \ \dots \ x_n]^T$  is the set of variables

$f_j \ j = 1, 2, \dots, m$ , are the error functions



## Data Containing Wild Points



run chart of the extracted FET time-delay  $\tau$

a few abnormal values in the data due to faulty devices  
and/or gross measurement errors

manual deletion has been adopted to exclude abnormal data  
points in earlier work using  $\ell_2$  estimator





## **Analog Diagnosis Using Huber Optimization**

penalty function approach

$$\underset{\mathbf{x}}{\text{minimize}} \quad \sum_{j=1}^{n+K} \rho_k(f_j(\mathbf{x}))$$

where

$$f_i(\mathbf{x}) = \Delta x_i / x_i^0, \quad i = 1, 2, \dots, n$$

$$f_{n+i}(\mathbf{x}) = \beta_i (V_i^c - V_i^m), \quad i = 1, 2, \dots, K$$

$\beta_i$                                       appropriate multipliers for the  
penalty terms

$\mathbf{x} \equiv [x_1 \ x_2 \ \dots \ x_n]^T$  vector of circuit parameters

$\mathbf{x}^0$                                       nominal values

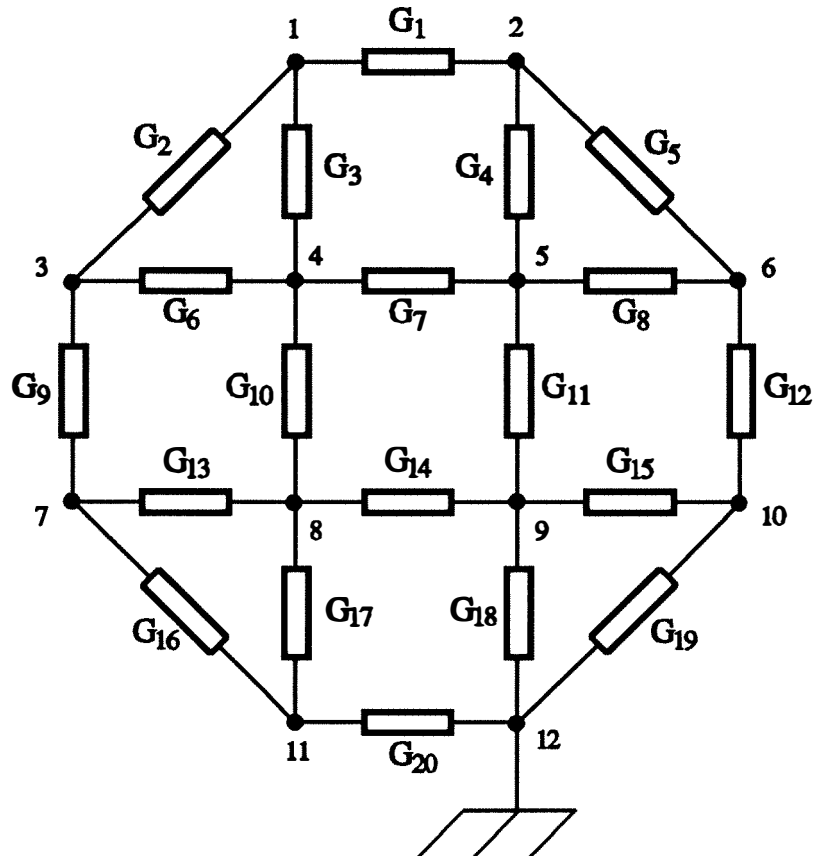
$\Delta \mathbf{x} \equiv \mathbf{x} - \mathbf{x}^0$                       deviations from the nominal

$V_1^m, \dots, V_K^m$                        $K$  measurements

$V_1^c, \dots, V_K^c$                       calculated circuit responses



## Analog Diagnosis Using Huber Optimization



resistive mesh circuit

only external nodes are available for excitation and measurements



## Huber Solution of Analog Diagnosis Problem

### FAULT LOCATION OF THE RESISTIVE MESH CIRCUIT

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Element	Nominal Value	Actual Value	Percentage Deviation		
			Actual	$\ell_1$	Huber
$G_1$	1.0	0.98	-2.0	0.00	-0.11
$G_2$	1.0	0.50	-50.0*	-48.89	-47.28
$G_3$	1.0	1.04	4.0	0.00	-2.46
$G_4$	1.0	0.97	-3.0	0.00	-1.18
$G_5$	1.0	0.95	-5.0	-2.70	-3.16
$G_6$	1.0	0.99	-1.0	0.00	-0.06
$G_7$	1.0	1.02	2.0	0.00	-0.19
$G_8$	1.0	1.05	5.0	0.00	-0.41
$G_9$	1.0	1.02	2.0	2.41	3.75
$G_{10}$	1.0	0.98	-2.0	0.00	0.39
$G_{11}$	1.0	1.04	4.0	0.00	-0.37
$G_{12}$	1.0	1.01	1.0	2.73	1.32
$G_{13}$	1.0	0.99	-1.0	0.00	-0.26
$G_{14}$	1.0	0.98	-2.0	0.00	-0.50
$G_{15}$	1.0	1.02	2.0	0.00	-0.05
$G_{16}$	1.0	0.96	-4.0	-3.36	-2.67
$G_{17}$	1.0	1.02	2.0	0.00	-0.61
$G_{18}$	1.0	0.50	-50.0*	-50.09	-47.33
$G_{19}$	1.0	0.98	-2.0	-1.41	-3.81
$G_{20}$	1.0	0.96	-4.0	-4.40	-4.72

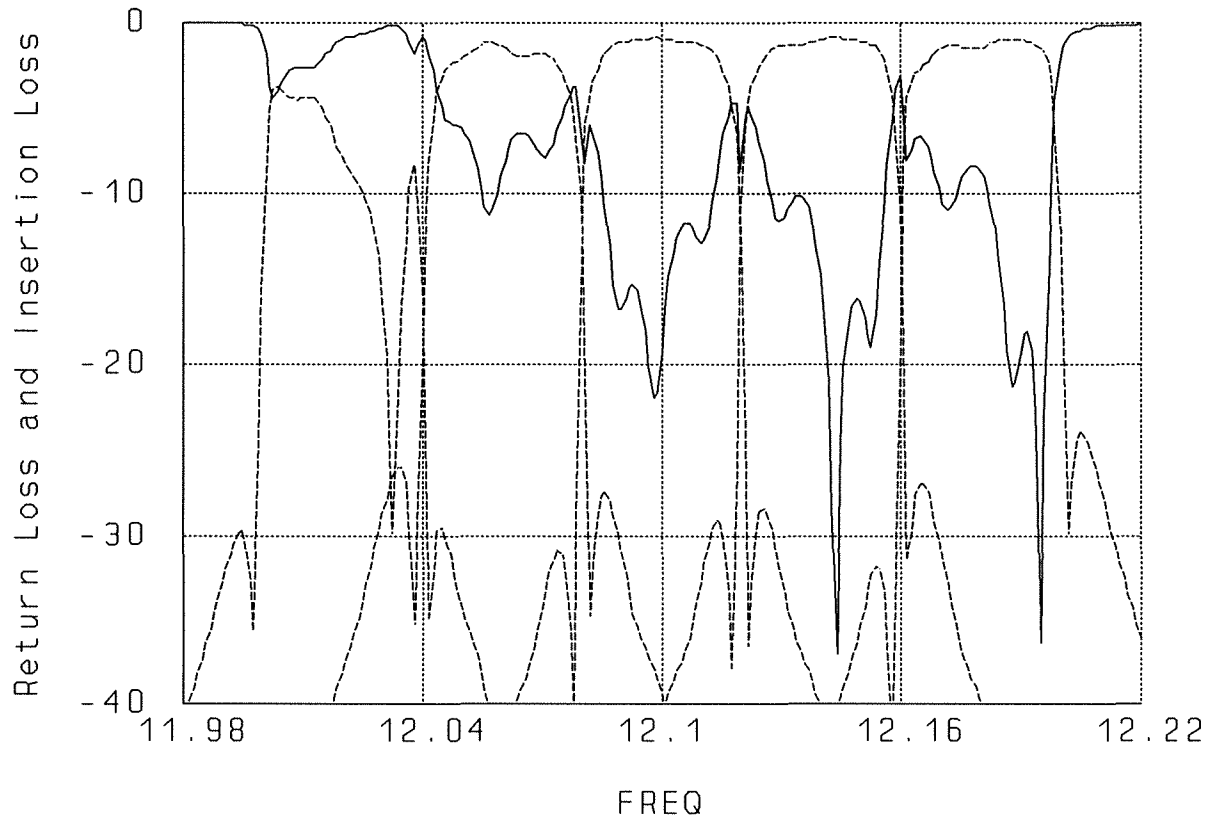
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\* Faults

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## **Five-channel 12 GHz Waveguide Manifold Multiplexer**



responses before optimization

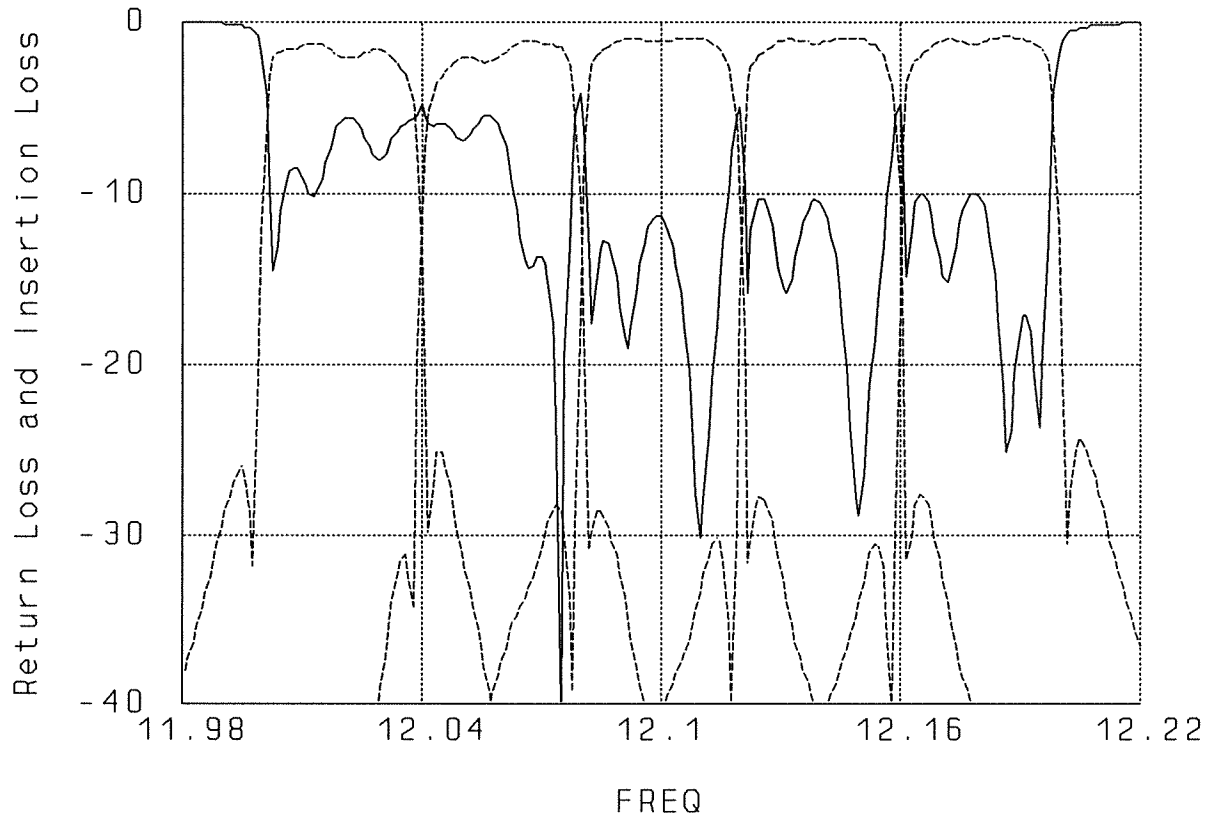
(——) common port return loss

(-----) individual channel insertion losses

75 optimizable variables



## **Partial Minimax Optimization of the 5-Channel Multiplexer**



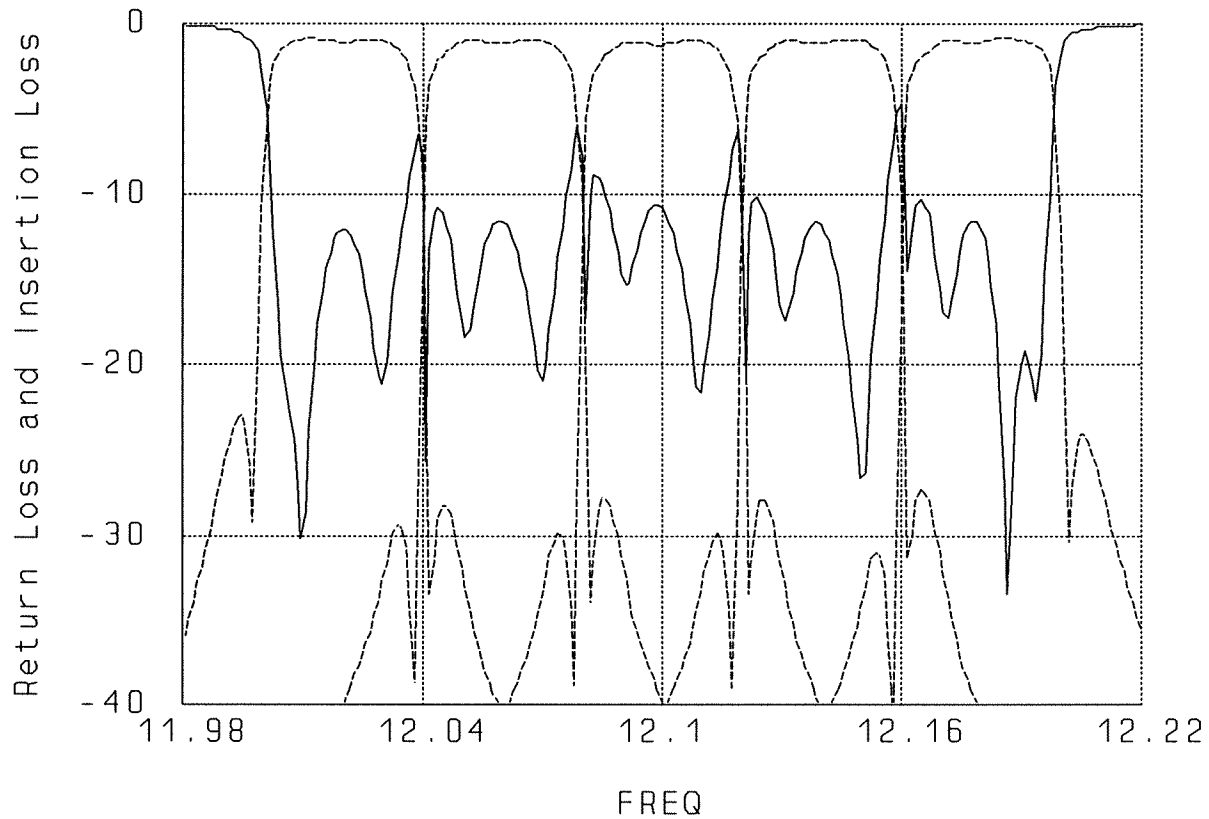
**multiplexer responses after minimax optimization with 10 variables: spacings and channel input transformer ratios**

**worst-case errors cannot be further reduced with only 10 variables by minimax**

**hardly improved upon the starting point**



## **Partial Huber Optimization of the 5-Channel Multiplexer**



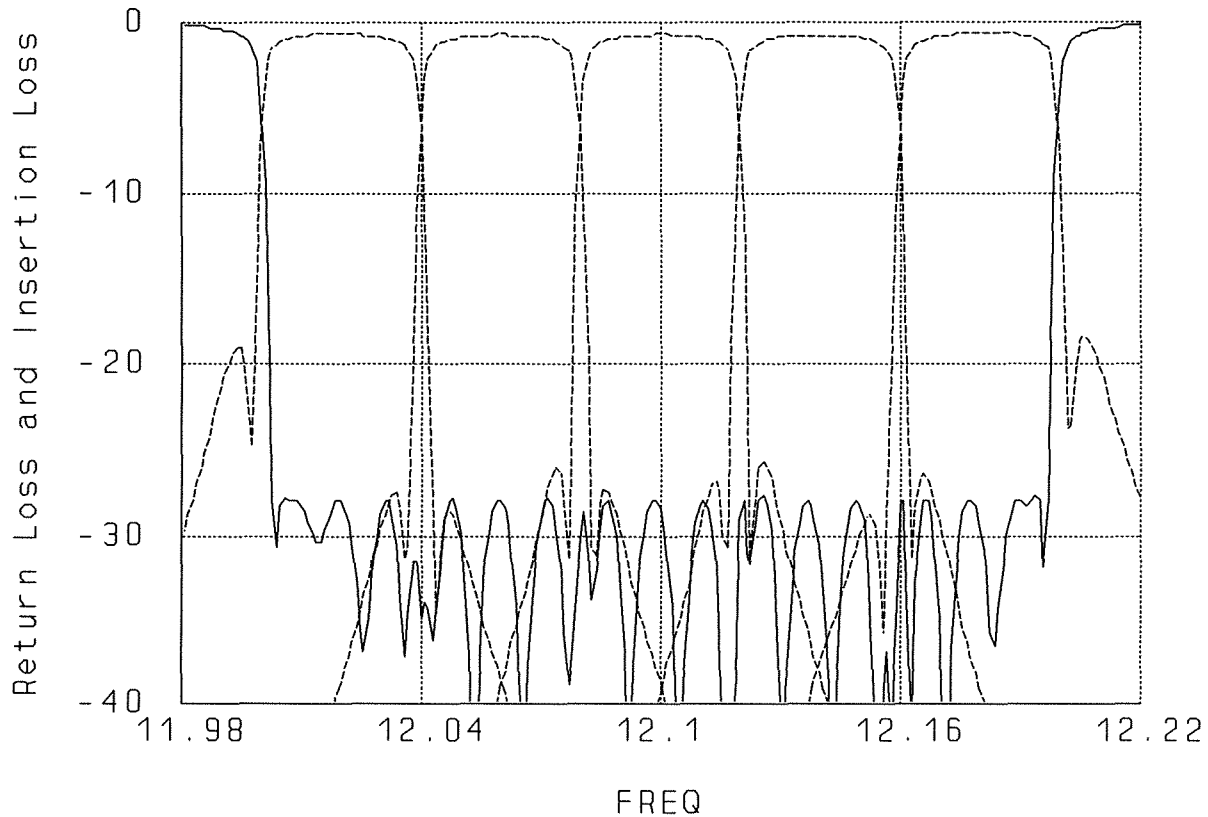
**multiplexer responses after one-sided Huber optimization  
with 10 variables**

**significantly better than minimax solution with the same 10  
variables**

**provides a good starting point for full-scale minimax  
optimization**



## **Full-scale Minimax Optimization of the 5-Channel Multiplexer**



**multiplexer responses after minimax optimization with the full set of 75 variables**