DESIGN AND SIMULATION OF INTERDIGITAL FILTERS

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Abstract

A 5th order interdigital C-band filter is designed. Traditional synthesis approach used for this design is reviewed. The procedure is enhanced by a proposed new approach to find a suitable placement of the input and output tapped lines and the lengths of the input and output resonators. Some equations and conditions are obtained for the design of the T-junctions. The electromagnetic simulator ***em***™ driven by *OSA90/hope*™ through the *Empipe*™ interface are used to simulate the filter. It is demonstrated that the procedure gives reasonably good design.

I. INTRODUCTION

Since its early emergence [1], interdigital filters have attracted significant attention. Such structures have the advantages of compact size and easy applications for both narrow and wide band filtering. It was first synthesized by Matthaei [2] using traditional synthesis techniques. Then, a number of papers (e.g. [3-6]) were published advancing theory and improving design accuracy. Among them, the papers by Wenzel [4,5] and Getsinger [6] are quite impressive and helpful from a practical point of view. A new structure emerged in the 1970s in which tapped lines were introduced at the input and output resonators [7,8]. Such a structure offers both space and cost saving over conventional filters because the first and the last end sections can be eliminated. Another advantage is that the tapped structure can be realized for very weak couplings when the traditional structure becomes impractical. Unfortunately, exact design of tapped line filters is not simple, and the

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available techniques are not directly applicable to microstrip configurations. Rather recently, Lo and Tzuang [9] analyzed this type of structure by a 3D full-wave spectral domain method for the suspended stripline case, and excellent results were reported. Swanson recently reported [10] a combination of 2D and 3D simulations using ***em***™ [11], *OSA90/hope*™ and *Empipe*™ [12]. He obtained a very close agreement with measurements.

The tapped line structure is very important because of its critical position in popular classes of filters including interdigital, combline and hairpin configurations. For exact design, a 3D full-wave approach would be the first choice. For practical reasons, however, we may be forced to use some approximate techniques to avoid excessive computer time. In this report, we propose a new such design procedure for tapped structures.

II. TRADITIONAL SYNTHESIS

Fig. 1 shows the structure of a traditional interdigital filter. Here, the microstrip sections 1, 2, ..., n function as n quarter-wavelength resonators with width w_i, i=1, 2, ..., n, the two end strips with widths w_0 and w_{n+1} work as input and output impedance transformers, and $s_{k,k+1}$ is the distance from resonator k to resonator k+1. The synthesis procedure can be summarized as [2]

$$\begin{aligned}
\theta_1 &= \frac{\pi}{2}\frac{\omega_1}{\omega_0} = \frac{\pi}{2}\left(1-\frac{\Delta\omega}{2}\right) \\
\frac{J_{01}}{Y_A} &= \frac{1}{\sqrt{g_0 g_1 \omega_1'}}\,, \quad \frac{J_{n,n+1}}{Y_A} = \frac{1}{\sqrt{g_n g_{n+1} \omega_1'}} \\
\left.\frac{J_{k,k+1}}{Y_A}\right|_{k=1,\,2,\,\ldots,\,n-1} &= \frac{1}{\omega_1' \sqrt{g_k g_{k+1}}}
\end{aligned} \tag{1}$$

where, ω_1 and ω_2 are the passband cutoff frequencies, $\omega_0 = (\omega_1+\omega_2)/2$, $\Delta\omega = (\omega_2-\omega_1)/\omega_0$, ω_1' is the cutoff frequency of the lowpass prototype, $g_0, g_1, ..., g_n, g_{n+1}$ are the normalized values of inductances and capacitances for the lowpass prototype, Y_A is the characteristic admittance of the input or output line, and $J_{k,k+1}$, k=0, 1, ..., n, are the coefficients of the admittance inverters.

Some intermediate parameters can be expressed as

$$M_1 = Y_A(\frac{J_{01}}{Y_A}\sqrt{h}+1)\ ,\quad M_n = Y_A(\frac{J_{n,n+1}}{Y_A}\sqrt{h}+1)$$
$$N_{k,k+1}|_{k=1,\,2,\,...,\,n-1} = \sqrt{(\frac{J_{k,k+1}}{Y_A})^2+\frac{\tan^2\theta_1}{4}} \tag{2}$$

where the parameter h is unknown.

Generally, to get a good Q, the impedance of each resonator should be around 70 ohms [2]. This leads to

$$\frac{2C_{k-1,k}}{\varepsilon}+\frac{C_k}{\varepsilon}+\frac{2C_{k,k+1}}{\varepsilon}|_{k=\frac{n}{2}\ for\ even\ n,\ \frac{n+1}{2}\ for\ odd\ n} \approx 5.4 \tag{3}$$

where C_k/ε and $C_{k,k+1}/\varepsilon$ are normalized self and mutual capacitances per unit length and can be expressed as [2]

$$\frac{C_0}{\varepsilon} = \frac{376.7}{\sqrt{\varepsilon_r}}[2Y_A-M_1],\quad \frac{C_{n+1}}{\varepsilon} = \frac{376.7}{\sqrt{\varepsilon_r}}[2Y_A-M_n]$$
$$\frac{C_1}{\varepsilon} = \frac{376.7}{\sqrt{\varepsilon_r}}[Y_A-M_1+hY_A(\frac{\tan\theta_1}{2}+(\frac{J_{01}}{Y_A})^2+N_{12}-\frac{J_{12}}{Y_A})]$$
$$\frac{C_k}{\varepsilon}|_{k=2,\,3,\,...,\,n-1} = \frac{376.7}{\sqrt{\varepsilon_r}}\ hY_A(N_{k-1,k}+N_{k,k+1}-\frac{J_{k-1,k}}{Y_A}-\frac{J_{k,k+1}}{Y_A}) \tag{4}$$
$$\frac{C_n}{\varepsilon} = \frac{376.7}{\sqrt{\varepsilon_r}}[Y_A-M_n+hY_A(\frac{\tan\theta_1}{2}+(\frac{J_{n,n+1}}{Y_A})^2+N_{n-1,n}-\frac{J_{n-1,n}}{Y_A})]$$

$$\frac{C_{01}}{\varepsilon} = \frac{376.7}{\sqrt{\varepsilon_r}}[M_1 - Y_A], \quad \frac{C_{n,n+1}}{\varepsilon} = \frac{376.7}{\sqrt{\varepsilon_r}}[M_n - Y_A]$$

$$\frac{C_{k,k+1}}{\varepsilon}\Big|_{k=1,\,2,\,\ldots,\,n-1} = \frac{376.7}{\sqrt{\varepsilon_r}} h Y_A\left(\frac{J_{k,k+1}}{Y_A}\right) \tag{5}$$

All the dimensions of the interdigital filter can be determined from these self and mutual capacitances.

First, we choose b, the height of the cavity, and t/b, the relative thickness of the strips (Fig. 2). We often select $t/b = 0$ for very thin conductor strips. Then, from the plots provided in Figs. 3-5 in [6] and Fig. 2 in [13], the relative gap $s_{k,k+1}/b$ can be obtained. In those figures,

$$\frac{(\Delta C)_{k,k+1}}{\varepsilon} = \frac{C_{k,k+1}}{\varepsilon}$$

At the same time, we can get the fringing capacitances $(C_{fe}')_{k,k+1}/\varepsilon$ and C_f'/ε from which the strip widths can be determined [6]

$$\frac{w_k}{b} = \frac{1}{2}\left(1-\frac{t}{b}\right)\left[\frac{1}{2}\left(\frac{C_k}{\varepsilon}\right) - \frac{(C_{fe}')_{k-1,k}}{\varepsilon} - \frac{(C_{fe}')_{k,k+1}}{\varepsilon}\right], \qquad k=1,\,2,\,\ldots,\,n$$

$$\frac{w_0}{b} = \frac{1}{2}\left(1-\frac{t}{b}\right)\left[\frac{1}{2}\left(\frac{C_0}{\varepsilon}\right) - \frac{C_f'}{\varepsilon} - \frac{(C_{fe}')_{01}}{\varepsilon}\right] \tag{6}$$

If $\frac{w_k}{b} < 0.35\left(1-\frac{t}{b}\right)$, the width correction is given by

$$\frac{w_k'}{b} = \frac{0.07\left(1-\frac{t}{b}\right) + \frac{w_k}{b}}{1.2}$$

Usually, this routine is for filters of narrow to moderate bandwidth. Interested readers can refer to [2] for similar formulas for the wide bandwidth case.

For CAD, closed form expressions can be obtained by an interpolation technique, and the computer can find the corresponding values automatically.

It must be pointed out that this procedure often leads to a narrower bandwidth compared with the required one. It is suggested [2] that the filter fractional bandwidth $\Delta\omega$ be specified to be 6 or

7 percent larger than is actually desired, but the actual bandwidth $\Delta\omega$ is used to determine the order of the filter. It should be noted that the preceding synthesis is for the case where the dielectric is filled homogenously in the whole structure of the filter, while planar interdigital filters often involve two or more dielectric layers. Therefore, the transformation shown in Fig. 2 is needed, where the first-order approximation formula can be found in [14,15], and dispersion must be considered.

More accurate design can be obtained from the coupling coefficients obtained directly for the microstrip technology [16,17]. In this case, the capacitances and impedances of the even and odd modes of the line can be calculated directly for the microstrip line and coupled line. The resonances and coupling can then be determined. Finally, various dimensions can be synthesized from these parameters. This approach avoids the loss of accuracy during the transformation. The design procedure for unshielded microstrip is the same unless radiation loss needs to be considered.

III. MICROSTRIP TAPPED LINE INTERDIGITAL FILTER

In the previous section, the input and output bars (or strips) function only as impedance transformers not as resonators. For even more compact designs these two end transformers can be eliminated by introducing two tapped lines [7-9] on the first and the last resonators directly, as shown in Fig. 3. In this case, the impedance transformations are accomplished by the input and output T-junctions, which themselves also act as resonators. The design of the inner resonators follows the same procedure as in the previous section, but the design of the input and output T-junctions is rather different.

Suppose the generator is matched. According to [8] the Q value of the resonator and the tapped position l are related by

$$\frac{Q_{si}}{R/Z_0} = \frac{\pi}{4\sin^2\dfrac{\pi l}{2L}} \tag{7}$$

where Q_{si} is the loaded Q of the first or last resonator, R and Z_0 are the impedances of the generator and the internal resonator, respectively. Knowing the Q_{si} value and the resonator length L, the tapped place l can be decided. It is still not clear on how to choose the Q_{si} to get the optimum specified filter response. In a practical design, the length of the end resonator is often different from a quarter wavelength, as illustrated in Fig. 4. Furthermore, this length is critical in such a filter.

IV. A NEW APPROACH FOR DESIGN OF TAPPED STRUCTURES

We usually make the resonators resonate at the center frequency of the passband in the bandpass filter design. The principle of the interdigital filter design is the same. The end strip functions as a resonator for the tapped case, therefore, it should resonate at the given center frequency. Fig. 4 shows the input structure (T-junction) of an interdigital filter. In this figure, L and $L' = L + \Delta L$ are the lengths of the internal and the end resonators, Z and Z_0 are the characteristic impedances of the input line and the resonator, the input line is tapped at a distance l from the short-circuit end of the resonator.

Suppose the generator is matched and the coupling of resonators 1 and 2 is weak enough compared with that of the input line. The equivalent circuit of the structure shown in Fig. 4 can be drawn as in Fig. 5. Looking towards the right at the BB' reference plane, we have

$$\bar{z}_B = \frac{\bar{z}\, j\tan\beta l}{\bar{z} + j\tan\beta l}$$

where $\bar{z} = Z/Z_0$, $\bar{z}_B = Z_B/Z_0$, and β is the propagation constant of the line. The input impedance at reference plane AA' can be written as

$$\bar{z}_A = \frac{\bar{z}_B + j\tan\beta(L'-l)}{1 + j\bar{z}_B\tan\beta(L'-l)}$$

To make it resonate at f_0 with optimum Q, we might conclude that the impedance and the resistance at AA' should be maximum. These relations result in

$$\frac{\partial|\bar{z}_A|}{\partial L'}\bigg|_{f=f_0} = 0 \qquad\qquad (8)$$
$$\frac{\partial Re(\bar{z}_A)}{\partial l}\bigg|_{f=f_0} = 0$$

To solve these two equations a 2D root-finding procedure can be employed. We suggest that the domains to be explored are $[L, 1.15L]$ for L', and $[0, 0.2L]$ for l. We can then obtain the input and output positions and the first and last resonator lengths quickly.

V. NUMERICAL RESULTS

To illustrate the procedures described in the preceding sections, we designed and simulated a C-band filter. For this designed filter, the specifications can be written as

Passband cutoff:	$f_1 = 4.9$ GHz, $f_2 = 5.3$ GHz
Passband ripple:	$r = 0.1$ dB
Isolation bandwidth:	$BWI = 1$ GHz
Isolation:	$DBI = 35$ dB

The order of the filter can be found to be 5 by [18]

$$N = int\left[\frac{DBI+RL+6}{20\log(GR+\sqrt{GR^2-1})}+0.999\right]$$

where

$$GR = \frac{BWI}{f_2-f_1}, \quad RL = 10\log(1-10^{-r/10})$$

We choose a 15 mil thick alumina substrate with $\varepsilon_r = 9.8$. The characteristic impedance of each resonator is selected to be about 56 ohms. The reason for this value is that the strip width of 70 ohms alumina is relatively narrow, which normally increases the loss and decreases the Q value, therefore, a lower impedance line is more practical. The width of each strip and the effective dielectric constant are determined [15] to be 10.5 mil and 6.82, respectively. Following the well known relation

$$L = \lambda_g/4 = \lambda_0/4\sqrt{\varepsilon_{eff}}$$

where λ_g and λ_0 are the wavelength of the microstrip line and free-space, respectively, and ε_{eff} is the effective dielectric constant, the resonator length L can be found to be 221.7 mil. Now, (1)-(5) are used to calculate the mutual capacitances of the coupling structure and the various distances of the different resonators can then be determined by the plots provided in Figs. 3-5 in [6] and Fig. 2 in [13]. Finally, the remaining two unknowns L' and l are found by solving (8). The synthesized dimensions are as follows

$$L = 221.7 \text{ mil}, L' = 227.5 \text{ mil}, l = 40 \text{ mil},$$

$$w_0 = 10.5 \text{ mil}, w_{in} = 10.5 \text{ mil}, w_{out} = 10.5 \text{ mil},$$

$$s_{12} = s_{45} = 27.0 \text{ mil}$$

$$s_{23} = s_{34} = 31.95 \text{ mil}$$

The structure of this filter is shown in Fig. 6.

The synthesized interdigital filter was then simulated by ***em***™ [11] driven by *OSA90/hope*™ through *Empipe*™ [12]. In the simulation, the dimensions are rounded to the nearest on-grid values as

$$L = 220.5 \text{ mil}, L' = 227.5 \text{ mil}, l = 40 \text{ mil},$$

$$w_0 = 10.0 \text{ mil}, w_{in} = 10.5 \text{ mil}, w_{out} = 10.5 \text{ mil},$$

$$s_{12} = s_{45} = 27.5 \text{ mil}$$

$$s_{23} = s_{34} = 32.5 \text{ mil}$$

The reason for adopting these dimensions is a compromise between accuracy and computer time. The cell size was set as 2.5 mil by 3.5 mil. The simulated results are shown in Fig. 7 and Fig. 8. From these two figures, it can be seen that excellent results are obtained. We see very good bandwidth and center frequency, perfect isolation and good skirt shape. One problem is that the passband return loss is not good enough. Possible reasons for this include: (1) we did not use the exact synthesized dimensions; (2) we did not use an exact solution for the design of the input T-junction; (3) we did not optimize the structure, including input and output line width and transitions. We can, however, say that the task of synthesis has been done. Certainly, the response might be further improved by optimization.

VI. CONCLUSIONS

A technique for synthesis of interdigital filters has been reviewed. The technique has been enhanced to determine the parameters of the structures, specially the location of the tapped T-junctions and the lengths of the end resonators. Combining this with the conventional synthesis technique, a complete approach is established. A 5th order C-band interdigital filter was synthesized using this approach. Then, electromagnetic simulation tools (***em***™, *OSA90/hope*™ and *Empipe*™) were used to validate the design. An excellent response was obtained. It shows that this technique leads to very accurate bandwidth and center frequency. The passband attenuation is not as small as desired, it is, however, less than 1.5 dB. Nevertheless, the shape of the filter response is excellent. It can be concluded that the procedure reported here can be developed as a synthesis routine for engineering applications.

In the future, a promising method to solve the input and output junctions with more accurate results and reasonable computer time could be the 3D planar waveguide model.

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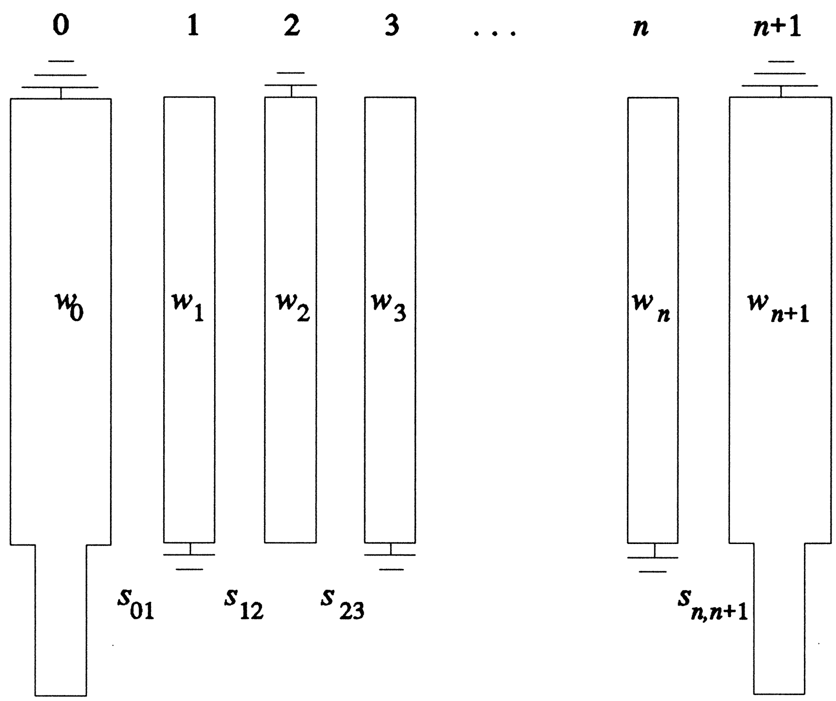

Fig. 1. Basic configuration of an interdigital filter.

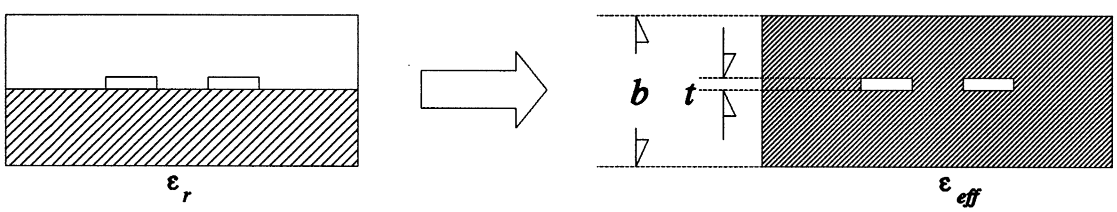

Fig. 2. The equivalent relation from inhomogeneous coupled microstrip lines to homogeneous coupled microstrip lines.

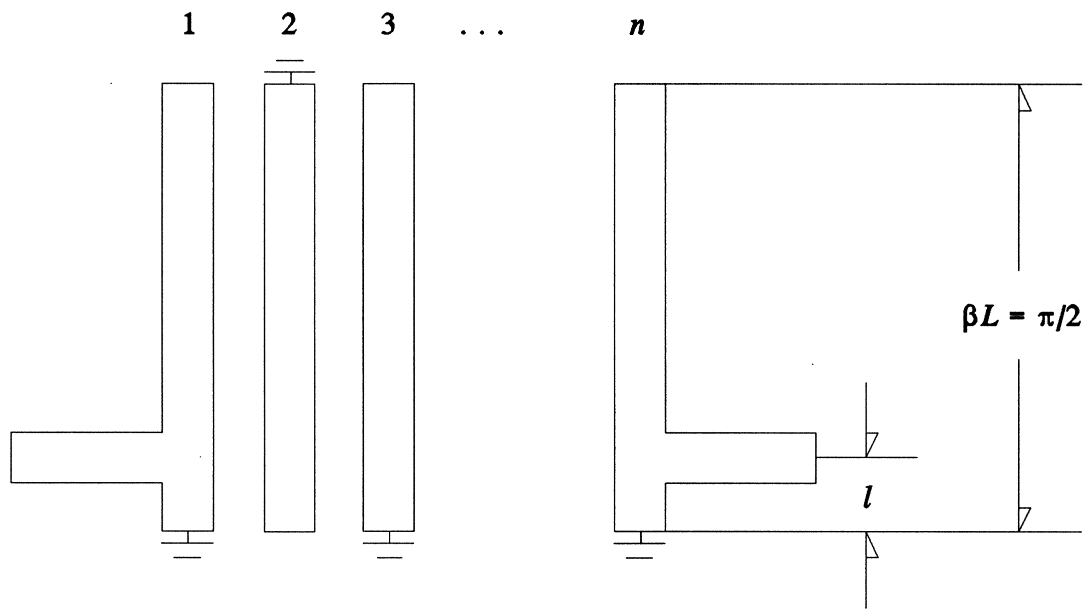

Fig. 3. Tapped line interdigital filter of order n. The electrical length of each resonator is $\pi/2$ and the T-junctions act as transformers.

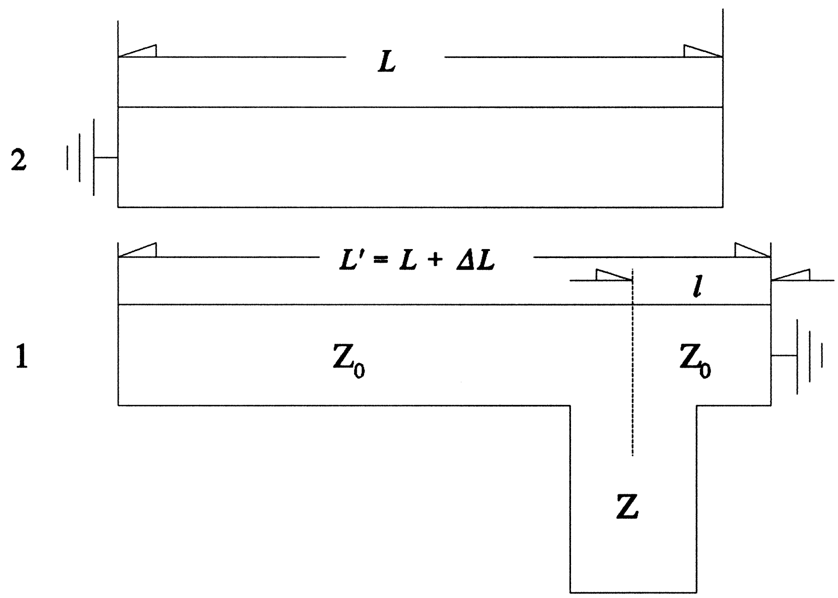

Fig. 4. The input T-junction of the interdigital filter with different input resonator length from those of the inner resonators.

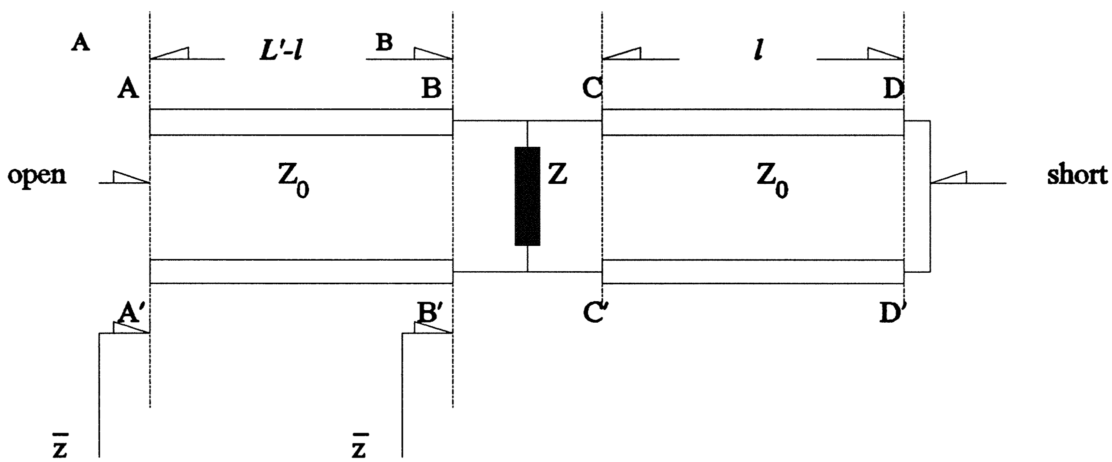

Fig. 5. Equivalent circuit of the input T-junction and resonator with matched generator. Suppose that the coupling between the first and second resonators is weak enough compared with the coupling of the generator.

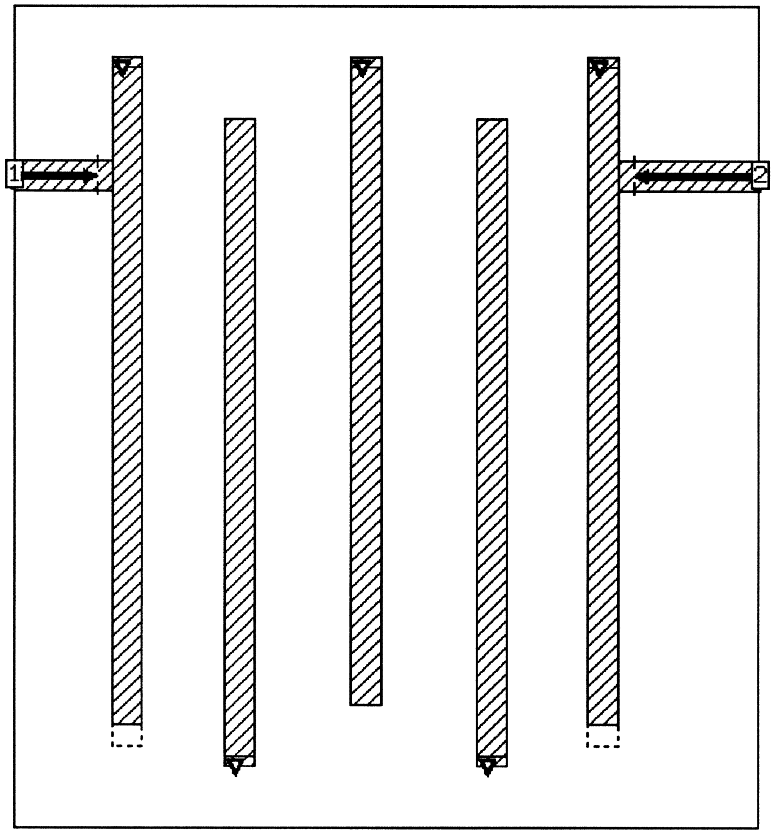

Fig. 6. The structure of the 5-pole interdigital filter with idealized shorted-circuits using ***xgeom***.

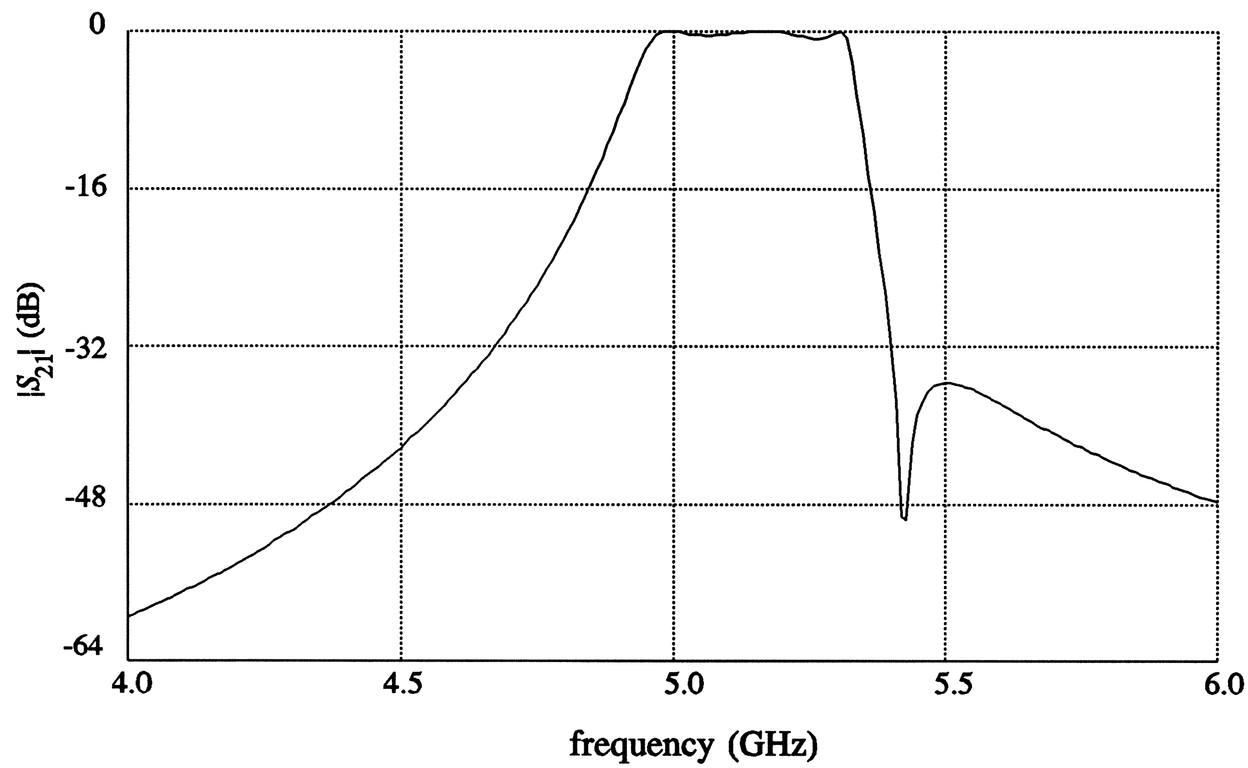

Fig. 7. The attenuation response $|S_{21}|$ of the designed filter without consideration of losses.

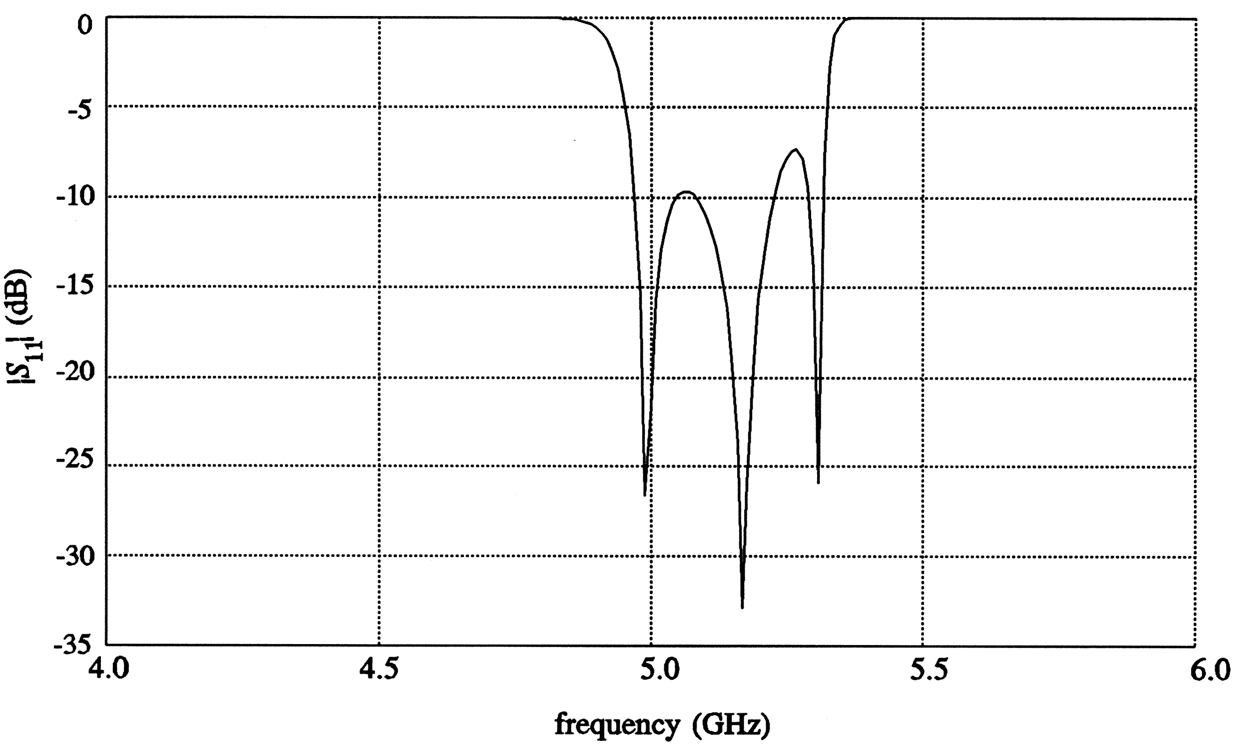

Fig. 8. The return loss response $|S_{11}|$ of the designed filter without consideration of losses.