THE SPACE MAPPING SUPER MODEL CONCEPT

J.W. Bandler and M.A. Ismail

SOS-98-12-R

July 1998

THE SPACE MAPPING SUPER MODEL CONCEPT

J.W. Bandler and M.A. Ismail

Simulation Optimization Systems Research Laboratory
and Department of Electrical and Computer Engineering
McMaster University, Hamilton, Canada L8S 4L7

Tel 905 525 9140 Ext. 24305
Fax 905 523 4407
Email awad@soya.sos.mcmaster.ca

Abstract This report presents a concept called Space Mapping Super Model (SMSM) to improve the accuracy of empirical models of microwave circuit elements such as microstrip open stubs, microstrip lines, coupled microstrip lines, etc. SMSM transforms the set of physical parameters of the considered microwave element to another set of parameters to be used by the empirical model of this element such that the empirical model response matches the response obtained using an electromagnetic (EM) simulator.

I. INTRODUCTION

Empirical models of microwave elements usually behave very well at low frequencies, however at higher frequencies their accuracy degrade. For example, consider a microstrip line with length L, width W, substrate height H and dielectric constant e_r equal to 50 mil, 10 mil, 15 mil and 9.8, respectively. This element was analyzed by Sonnet's ***em*** simulator [1] and by the empirical model presented by Jansen *et al*. [2] supplied by the OSA90/hope simulator [3]. The size of the grid used in Sonnet's ***em*** simulator is 1.0 mil by 1.0 mil. Fig. 1 shows the magnitude of the reflection coefficient $|\boldsymbol{S}_{11}|$ obtained by Sonnet's ***em*** simulator [1] together with $|\boldsymbol{S}_{11}|$ obtained by the empirical model in the frequency range 1 GHz to 40 GHz with a step of 3 GHz. It is clear that $|\boldsymbol{S}_{11}|$ obtained by both simulators are different at high frequencies.

This work was supported by the Natural Sciences and Engineering Research Council of Canada under Grants OGP0007239, STP0201832 and through the Micronet Network of Centers of Excellence.

In this report, we present a concept called Space Mapping Super Model (SMSM) to overcome this deficiency in empirical models. This technique transforms the physical parameters of the analyzed microwave element to another set of parameters. These transformed parameters are used by the empirical model so that the response obtained by the empirical model matches that obtained using the EM simulator in a certain frequency range. SMSM is based on a novel optimization technique called Space Mapping (SM) by Bandler *et al*. [4].

Two spaces are defined in SMSM. The first space is called the EM space and it contains the physical parameters of the microwave element to be analyzed (i.e. the length L, the width W, the substrate height H, etc.). The second space is called the linear simulator space or the empirical model space, where every element represents the transformed physical parameters to be evaluated by the SMSM. In Section II, we present the Space Mapping Super Model (SMSM) technique and the algorithm to implement it. Section III discusses the parameter extraction problem which is crucial to the performance of the SMSM technique. In Section IV, we apply the SMSM to an example and compare the results before and after using the SMSM. Finally, the report is concluded in Section V.

II. SPACE MAPPING SUPER MODEL (SMSM)

Consider a microwave element E with physical parameters represented by the vector $\boldsymbol{x}_{em}$ (for example, the parameters of a microstrip open stub are represented by the vector $x_{em} = [L \;\; W \;\; H \; \boldsymbol{e}_r]^T$). The space of $\boldsymbol{x}_{em}$ is called the EM space and is denoted by $\boldsymbol{X}_{em}$. If the element E is analyzed by an EM simulator and by an empirical model, the response $\boldsymbol{R}_{ls}(\boldsymbol{x}_{em})$ produced by the empirical model is different from the response $\boldsymbol{R}_{em}(\boldsymbol{x}_{em})$ produced by the EM simulator, particularly at high frequencies. Therefore, the aim of this work is to find a new set of parameters represented by a vector called $\boldsymbol{x}_{ls}$ such that

$$\boldsymbol{R}_{ls}(\boldsymbol{x}_{ls}) \approx \boldsymbol{R}_{em}(\boldsymbol{x}_{em}) \tag{1}$$

The space of $\boldsymbol{x}_{ls}$ is called the linear simulator space or the empirical model space and is denoted by $\boldsymbol{X}_{ls}$.

The considered problem is illustrated in Fig. 2. We have a region $\boldsymbol{Reg}_{em}$ in the EM space $\boldsymbol{X}_{em}$

and it is required to establish a mapping $\boldsymbol{P}$ from this region to a region $\boldsymbol{Reg}_{ls}$ in the $\boldsymbol{X}_{ls}$-space such that

$$\boldsymbol{x}_{ls} = \boldsymbol{P}(\boldsymbol{x}_{em}) \quad \text{and} \quad \|\boldsymbol{R}_{ls}(\boldsymbol{x}_{ls}) - \boldsymbol{R}_{em}(\boldsymbol{x}_{em})\| \le e \tag{2}$$

where $\| \ \|$ denotes a suitable norm and e is a small positive constant. The value of $\boldsymbol{x}_{ls}$ is found by solving the parameter extraction problem [4]:

$$\boldsymbol{x}_{ls} = \boldsymbol{P}(\boldsymbol{x}_{em}) = \arg \min_{\boldsymbol{x} \in \boldsymbol{X}_{ls}} \|\boldsymbol{R}_{ls}(\boldsymbol{x}) - \boldsymbol{R}_{em}(\boldsymbol{x}_{em})\| \tag{3}$$

The SMSM starts by selecting a set of base points $\boldsymbol{B}_{em}$ with m points in the region $\boldsymbol{Reg}_{em}$

$$\boldsymbol{B}_{em} = \left\{ \boldsymbol{x}_{em}^{(1)}, \boldsymbol{x}_{em}^{(2)}, \ \ldots, \boldsymbol{x}_{em}^{(m)} \right\}. \tag{4}$$

Therefore, by parameter extraction it constructs the set of points $\boldsymbol{B}_{ls}$ in the $\boldsymbol{X}_{ls}$-space

$$\boldsymbol{B}_{ls} = \left\{ \boldsymbol{x}_{ls}^{(1)}, \boldsymbol{x}_{ls}^{(2)}, \ \ldots, \boldsymbol{x}_{ls}^{(m)} \right\}. \tag{5}$$

such that

$$\boldsymbol{x}_{ls}^{(i)} = \arg \min_{x \in X_{ls}} \|\boldsymbol{R}_{ls}(\boldsymbol{x}) - \boldsymbol{R}_{em}(\boldsymbol{x}_{em}^{(i)})\|, i = 1,2,\ldots,m. \tag{6}$$

The initial mapping $\boldsymbol{P}_0$ is constructed so that it maps every element in the set $\boldsymbol{B}_{em}$ to the corresponding element in the set $\boldsymbol{B}_{ls}$. The technique updates the mapping $\boldsymbol{P}$ in an iterative manner by testing the mapping $\boldsymbol{P}_j$ established in the jth iteration on a set of testing points $\boldsymbol{C}_{em}$ (the set $\boldsymbol{C}_{em}$ has no common elements with the set $\boldsymbol{B}_{em}$). Then, it constructs using the existing mapping $\boldsymbol{P}_j$ the corresponding set $\boldsymbol{C}_{ls}$ in the $\boldsymbol{X}_{ls}$-space. That is, for every element $\boldsymbol{x}_{em} \in \boldsymbol{C}_{em}$ find the corresponding point $\boldsymbol{x}_{ls} \in \boldsymbol{C}_{ls}$ where

$$\boldsymbol{x}_{ls} = \boldsymbol{p}_j(\boldsymbol{x}_{em}) \tag{7}$$

It also finds, by parameter extraction, the corresponding points of $\boldsymbol{C}_{em}$ and put them in the set $\boldsymbol{D}_{ls}$.

The technique stops if the points in the set $\boldsymbol{C}_{ls}$ are very close to the corresponding points in the set $\boldsymbol{D}_{ls}$ within a certain accuracy. If not, it augments the set $\boldsymbol{B}_{em}$ by adding to it the elements of the set $\boldsymbol{C}_{em}$ and the set $\boldsymbol{B}_{ls}$ by adding to it the elements of $\boldsymbol{D}_{ls}$ obtained by parameter extraction . It then updates the mapping $\boldsymbol{P}_j$ to get a new mapping $\boldsymbol{P}_{j+1}$. If the number of points in $\boldsymbol{B}_{em}$ is equal to a predefined number *N* (to be set by the user) the technique terminates and considers the current mapping as the final required mapping ***P*.** The flow chart of this procedure is shown in Fig. 3.

Construction of the mapping ***P***

The mapping ***P*** is constructed in an iterative manner during the running of the SMSM algorithm. At the *j*th iteration of the algorithm, assume that the two sets of points $\boldsymbol{B}_{em}$ and $\boldsymbol{B}_{ls}$ have m_j points each. Therefore, the mapping $\boldsymbol{P}_j$ transforms every point in $\boldsymbol{B}_{em}$ to its corresponding point in $\boldsymbol{B}_{ls}$. The mapping $\boldsymbol{P}_j$ is defined as a linear combination of a predefined and fixed *t* fundamental functions

$$\hat{f}_1(\boldsymbol{x}_{em}),\ \hat{f}_2(\boldsymbol{x}_{em}),\ \hat{f}_3(\boldsymbol{x}_{em}), \ldots,\ \hat{f}_t(\boldsymbol{x}_{em}), \tag{8}$$

such that the *i*th component of $\boldsymbol{x}_{ls}$ is represented by

$$\boldsymbol{x}_{ls_i} = \sum_{s=1}^{t} \boldsymbol{a}_{is}\ \hat{f}_s(\boldsymbol{x}_{em}). \tag{9}$$

The number of fundamental functions satisfies the relation $m_j \geq t$. In matrix form, (9) is written as

$$\boldsymbol{x}_{ls} = \boldsymbol{p}_j(\boldsymbol{x}_{em}) = \boldsymbol{A}_j\ \hat{\boldsymbol{f}}(\boldsymbol{x}_{em}), \tag{10}$$

where $\boldsymbol{A}_j$ is an (*n* x *t*) matrix (*n* is the dimension of the column vector $\boldsymbol{x}_{ls}$) and $\hat{\boldsymbol{f}}(\boldsymbol{x}_{em})$ is a t-dimensional vector of fundamental functions. Consider the mapping $\boldsymbol{P}_j$ for all points in $\boldsymbol{B}_{em}$ and $\boldsymbol{B}_{ls}$ and expand (10) to get

$$\left[\boldsymbol{x}_{ls}^{(1)}\ \boldsymbol{x}_{ls}^{(2)} \ldots \boldsymbol{x}_{ls}^{(m_j)}\right] = \boldsymbol{A}_j\left[\hat{\boldsymbol{f}}(\boldsymbol{x}_{em}^{(1)})\ \hat{\boldsymbol{f}}(\boldsymbol{x}_{em}^{(2)}) \ldots \hat{\boldsymbol{f}}(\boldsymbol{x}_{em}^{(m_j)})\right]. \tag{11}$$

The matrix of constant coefficients $\boldsymbol{A}_j$ can be evaluated using the least–squares method. Using the notations:

$$\boldsymbol{V} = \left[\boldsymbol{x}_{ls}^{(1)}\ \boldsymbol{x}_{ls}^{(2)} \ldots \boldsymbol{x}_{ls}^{(m_j)}\right]^T, \quad \boldsymbol{S} = \left[\hat{\boldsymbol{f}}(\boldsymbol{x}_{em}^{(1)})\ \hat{\boldsymbol{f}}(\boldsymbol{x}_{em}^{(2)}) \ldots \hat{\boldsymbol{f}}(\boldsymbol{x}_{em}^{(m_j)})\right]^T, \tag{12}$$

(11) can be written as

$$\boldsymbol{V}^T = \boldsymbol{A}_j \ \boldsymbol{S}^T . \tag{13}$$

By taking the transpose of both sides we get

$$\boldsymbol{S} \ \boldsymbol{A}_j^T = \boldsymbol{V} , \tag{14}$$

which is an overdetermined system of linear equations. The least–squares solution of (14) is given by

$$\boldsymbol{A}_j^T = (\boldsymbol{S}^T \boldsymbol{S})^{-1} \boldsymbol{S}^T \boldsymbol{V} \tag{15}$$

In this report, we are using linear mapping. Accordingly, the vector of fundamental functions in (10) is given by

$$\hat{\boldsymbol{f}}(\boldsymbol{x}_{em}) = \begin{bmatrix} 1 & x_{em_1} & x_{em_2} & \dots & x_{em_n} \end{bmatrix}^T . \tag{16}$$

Consequently, the matrix $\boldsymbol{S}$ in (15) is given by

$$\boldsymbol{S} = \begin{bmatrix} 1 & 1 & \dots & 1 \\ \boldsymbol{x}_{em}^{(1)} & \boldsymbol{x}_{em}^{(2)} & \dots & \boldsymbol{x}_{em}^{(m_j)} \end{bmatrix}^T . \tag{17}$$

The mapping $\boldsymbol{P}_j$ is fully determined by (10) and (15).

The sets $\boldsymbol{B}_{em}$ and $\boldsymbol{C}_{em}$

The way the two sets $\boldsymbol{B}_{em}$ and $\boldsymbol{C}_{em}$ are constructed is crucial to the performance of the SMSM technique. When constructing these sets, one should compromise between the number of points they contain and the amount of information these points have (i.e., the distribution of these points in the region $\boldsymbol{Reg}_{em}$). Of course, using a very large number of points in $\boldsymbol{B}_{em}$ and $\boldsymbol{C}_{em}$ improves the accuracy of the mapping but it is time extensive.

If the vector $\boldsymbol{x}_{em}$ is a two–dimensional vector, then the region $\boldsymbol{Reg}_{em}$ is a two–dimensional box with dimensions d_1 and d_2. The set of base points $\boldsymbol{B}_{em}$ contains the four vertices of the box and the point at the centre of the box as shown in Fig. 4(a). At the *j*th iteration, the set of test points $\boldsymbol{C}_{em}$ contains the vertices of the box in Fig. 4(b) whose dimensions are $(d_1 - 2j\, h_1)$ and $(d_2 - 2j\, h_2)$ where h_1 and h_2 are two predefined increments. During the running time of the algorithm, the maximum number of the sets of test points is given by

$$M = \min\left\{ \left\lfloor \frac{d_1 - h_1}{2h_1} \right\rfloor, \left\lfloor \frac{d_2 - h_2}{2h_2} \right\rfloor \right\}, \tag{18}$$

where $\lfloor x \rfloor$ is the largest integer less than or equal to *x*. The maximum number of points that can be used during the SMSM algorithm is $N = 5 + 4 * M$.

The way of selecting the sets $\boldsymbol{B}_{em}$ and $\boldsymbol{C}_{em}$ in two dimensions can be easily generalized in the n-dimension case, where the region $\boldsymbol{Reg}_{em}$ is an n-dimensional box with side lengths $d_1, d_2, ..., d_n$. The number of the sets of test points is given by

$$M = \min_i \left\{ \left\lfloor \frac{d_i - h_i}{2h_i} \right\rfloor \right\}, \text{where } i = 1, 2, ..., n \tag{18}$$

and the maximum number of points that can be used during the SMSM algorithm is

$$N = 1 + 2^n (1 + M) . \tag{19}$$

The SMSM Algorithm

The SMSM can be implemented through the following steps.

Step 1 Determine the maximum number of points *N* in the region $\boldsymbol{Reg}_{em}$ using (19).

Step 2 Construct the set $\boldsymbol{B}_{em}$.

Step 3 Construct by parameter extraction the set of points $\boldsymbol{B}_{ls}$ and Initialize *j*=0.

Step 4 Establish the mapping $\boldsymbol{P}_j$ by evaluating the matrix $\boldsymbol{A}_j^T = (\boldsymbol{S}^T \boldsymbol{S})^{-1} \boldsymbol{S}^T \boldsymbol{V}$ where the matrix $\boldsymbol{V}$ is constructed using the points in $\boldsymbol{B}_{ls}$ according to (12) and the matrix $\boldsymbol{S}$ is constructed using the points in $\boldsymbol{B}_{em}$ according to (17).

Step 5 If the total number of elements in the set $\boldsymbol{B}_{em}$ equals *N*, then stop and consider the current mapping $\boldsymbol{P}_j$ as the final mapping ***P*.**

Step 6 Construct the set of test points $\boldsymbol{C}_{em}$ and find the corresponding elements by using the current mapping $\boldsymbol{P}_j$.

Step 7 If the mapped elements found in step 6 are very close to the corresponding elements of $\boldsymbol{C}_{em}$ obtained by parameter extraction, stop and set $\boldsymbol{P} = \boldsymbol{P}_j$. Else, augment the set $\boldsymbol{B}_{em}$ by adding to it the elements of $\boldsymbol{C}_{em}$ and augment the set $\boldsymbol{B}_{ls}$ by adding to it the points obtained by parameter extraction corresponding to the points in $\boldsymbol{C}_{em}$ and set $j = j+1$ and go to step 4.

III. PARAMETER EXTRACTION

Parameter extraction process is very important to the SMSM technique. In each iteration of the SMSM we have to perform parameter extraction more than once as presented in Section II. Recall that we have a microwave element E with physical parameters represented by the vector $\boldsymbol{x}_{em}$, and we want to find transformed parameters represented by the vector $\boldsymbol{x}_{ls}$ such that (1) is satisfied. The parameter extraction process involves the solution of the optimization problem (3). The norm mentioned in (3) can be the ℓ_1–norm, ℓ_2–norm or Huber–norm [6]. In this report the Huber–norm was chosen since it is robust against gross errors and flexible with respect to small errors [6].

IV. EXAMPLE

Consider two model of the microstrip line. The first model is a transmission line model using two parameters, the characteristic impedance Z_0 and the line length, see Fig. 4. The second model is the empirical model of microstrip line presented by Jansen [2]. The empirical model is considered as the EM model while transmission line model is considered as the linear simulator model. The parameters of the microstrip empirical model are $40\,\text{mil} \le L \le 65\,\text{mil}$, $10\,\text{mil} \le W \le 35\,\text{mil}$, the substrate height H=15 mil and $\boldsymbol{e}_r = 9.8$. Therefore, the region $\boldsymbol{Reg}_{em}$ and the vector $\boldsymbol{x}_{em}$ in the EM space are given by

$$\boldsymbol{Reg}_{em} \cong \{(W, L) | 40\text{mil} \le L \le 65\text{mil and} 10\text{mil} \le W \le 35\text{mil}\}$$
$$\boldsymbol{x}_{em} = [W, L]^T$$

The vector $\boldsymbol{x}_{ls}$ to be used in the transmission line model is given by

$$\boldsymbol{x}_{ls} = [W_1, L_1]^{\boldsymbol{T}} .$$

Therefore, the parameters used in the transmission line are the length L_1 and the characteristic impedance Z_0 (which is computed using the quasi-static model in [7] for the microstrip transmission line in terms of the width W_1, the substrate height H and the relative dielectric constant e_r).

The SMSM algorithm was applied to this problem in order to match the response obtained by the transmission line model with the response obtained by the empirical model implemented in OSA90/hope [3]. The frequency range used is from 40 GHz to 50GHz with a step of 0.5 GHz. The algorithm terminated after two iterations (13 parameter extraction were needed) and produced the mapping $\boldsymbol{P}$ which is represented by

$$\boldsymbol{x}_{ls} = \boldsymbol{p}(\boldsymbol{x}_{em}) = \boldsymbol{A}\begin{bmatrix}1 \\ \boldsymbol{x}_{em}\end{bmatrix} \text{and } \boldsymbol{A} = \begin{bmatrix} 0.4042 & 0.9074 & -0.0409 \\ -0.7487 & 0.0290 & 1.0960 \end{bmatrix},$$

or in other form

$$\begin{aligned} W_1 &= 0.4042 + 0.9074W - 0.0409L \\ L_1 &= -0.7487 + 0.0290W + 1.0960L \end{aligned}$$

To show the benefit of the SMSM algorithm we performed yield analysis supplied by OSA90/hope [3] in the region $\boldsymbol{Reg}_{em}$. We generated 50 uniformly distributed random points inside $\boldsymbol{Reg}_{em}$. The specifications used to obtain the yield are the difference between the real part or the imaginary part of the scattering parameters obtained by the two models is less than (0.04). Before using the SMSM technique the yield is found to be zero and after using it the yield became 90%. Figs. 6, 7, 8 and 9 show the difference of the real part and the imaginary part of the scattering parameters obtained by the empirical model and by the transmission line model. The circuit and the MATLAB [5] files required to implement the SMSM technique for this problem are given in Appendix A. While the circuit file (accepted by OSA90/hope [3]) to perform yield analysis is given in Appendix B.

V. CONCLUSIONS

In this report we presented the Space Mapping Super Model (SMSM) technique to enhance the accuracy of microwave empirical models. For a microwave element, the technique establishes a mapping to transform the parameters of this element to another set of parameters. These transformed parameters are used by the fast but less accurate empirical model of this element to match its response with that obtained by an accurate but time extensive EM simulator. The proposed technique was tested in an example and gave good results.

APPENDIX A

```
!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!
!!This file performs parameter extraction to match the response of the transmission
!!line model with the response of the microstrip empirical model
!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!
Model
    !!!!!!!!!!!! parameters values and frequency range
    F_MIN=40ghz;
    F_MAX=50ghz;
    F_STEP=0.5ghz;
    N=13;  ! the number of points we perform parameter extraction up on
    k=1;   ! just an index

   L[N]=[40mil 40 mil 65mil 65 mil 52.5mil 45mil 45 mil 60mil 60 mil
           50mil 50 mil 55mil 55 mil];
   L1[N]=[?43.3562mil? ?43.9348mil? ?71.4879mil? ?70.4699mil? ?57.565mil?
             ?49.0647mil? ?49.2502mil? ?65.9403mil? ?65.4628mil? ?54.7716mil?
             ?54.8464mil? ?60.3621mil? ?60.2439mil?];

W[N]=[10mil 35mil 35 mil 10mil 22.5mil 15mil 30mil 30 mil 15mil
           20mil 25mil 25 mil 20mil];

 W1[N]=[?7.82576mil? ?30.8619mil? ?29.4513mil? ?7.68279mil? ?18.2689mil?
             ?12.0043mil? ?26.8623mil? ?24.9827mil? ?11.7219mil? ?15.9467mil?
             ?20.6309mil? ?20.5213mil? ?16.0013mil?];
H=15mil;
EPSR=9.8;
!!!!!!! The empirical model for the microstrip!!!!!!!!!!
   MSUB EPSR=EPSR H=H;
   MSL 1 2  W=W[k] L=L[k];
   PORTS 1 0 2 0;
   !!!!!!!!!!!!!!!!!!!!!!!The TL model!!!!!!!!!!!!!!!!!!
   !! compute the effective dielectric constant and the characteristic impedance
   !! using the quasistatic analysis
   !! that is transform W1, H1, EPSR1 to Z0 and the effective dielectric constant
   EPSR_E=( EPSR+1)/2+(EPSR-1)/(2 *sqrt(1+12*H/W1[k]));

   Z0=if((W1[k]/H)<1)
      (60/sqrt(EPSR_E) * log(8*H/W1[k]+W1[k]/(4*H)))
       else
      (120*pi/(sqrt(EPSR_E)*(W1[k]/H+1.393+0.667*log(W1[k]/H+1.444))));

   TRL 3  4 Z=Z0 L=L1[k] K=EPSR_E F=FREQ;

   PORTS 3 0 4 0;
   CIRCUIT;

   !! This is for displaying purposes only
   D_RS11=abs(RS11-RS33);
   D_RS12=abs(RS12-RS34);
   D_IS11=abs(IS11-IS33);
   D_IS12=abs(IS12-IS34);
```

```
 end
 Sweep
      AC:
         k: from 1 to N step=1
         FREQ: from F_MIN to F_MAX step=F_STEP
         D_PS D_MS D_RS D_IS
     end
 Spec
   AC:
      k: from 1 to N step=1
      FREQ: from F_MIN to F_MAX step=F_STEP
  !!! the goals of the parameter extraction
  RS11=RS33
  RS12=RS34
  IS11=IS33
  IS12=IS34;
 end
 control
 optimizer = HUBER;
end
!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!! End of file !!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!

% %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% This Matlab file is to establish the mapping P in the Space Mapping Super Model Algorithm
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

function  AJ = P_Linear;
% where AJ is the same matrix mentioned in the report
% n is the dimension of xem and xls, assuming it is the same
Ns= 4;  %this is the number of points in the set of testing points Cem
N=2;   % the maximum number of iterations
[xls1, xls2, xem1, xem2]=get_data;  % get data is the function the extracted parameters values
% compute the matrix V
V1=xls1(1:Ns+1);  % the first vector of V
V2=xls2(1:Ns+1);  % the second vector of V
V=[V1 V2];
%compute the matrix S
S1=xem1(1:Ns+1);  % the first vector of  S
S2=xem2(1:Ns+1);  % the second vector of S
U(1:Ns+1,1)=1;
S=[U S1 S2];
% construct the initial mapping P0
epslon=0.03; % the maximum error
AJ=(inv(S.'*S)*S.'*V).'
for i=1:N
            AJ=(inv(S.'*S)*S.'*V).';
            Cem1=xem1((2+i*Ns): (1+(i+1)*Ns));
            Cem2=xem2((2+i*Ns): (1+(i+1)*Ns));
            U1(1:Ns,1)=1;
            Cem=[U1 Cem1 Cem2];
            Dls = Cem *AJ.';
            Cls1=xls1((2+i*Ns): (1+(i+1)*Ns));
            Cls2=xls2((2+i*Ns): (1+(i+1)*Ns));
            Cls=[Cls1 Cls2];
            Error=max(max(abs((Dls-Cls)./Cls)))
```

```
 if(Error<= epslon)
      break;
     end
   V1=[V1 ; Cls1];
   V2=[V2 ; Cls2];
   V=[V1 V2];
   S1=[S1;Cem1];
   S2=[S2;Cem2];
   U(1: (1+(i+1)*Ns),1)=1;
   S=[U S1 S2];
   AJ=(inv(S.'*S)*S.'*V).'

end
%%%%%%%%%%%%%%%%%%%% End of the file %%%%%%%%%%%%%%%%%%%%%%

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% This function returns the values of the set of base points Bem and the set of test points Cem
%obtained by parameter extraction
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
function [Xls1, Xls2, Xem1, Xem2]=get_data;

Xls1=[11.0158   38.3204  38.4779  10.9578  24.087
         16.4918  32.8907  32.9397  16.4927
         21.8889   27.3376  27.3844  21.9269 ];

Xls2=[39.3525  39.0836  63.6283    63.7337  50.6077
         43.9241   43.7737  58.4877     58.551
         48.6866  48.6413  53.5232   53.5666 ];
Xem1=[10; 35; 35; 10; 22;
           15; 30; 30; 15;
           20; 25; 25; 20];
Xem2=[40; 40; 65; 65; 52;
           45; 45; 60; 60;
          50; 50; 55; 55];
```

APPENDIX B

```
!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!
!! In this file we do montcarlo analysis to evaluate the Space Mapping Super Model used to
!! match the transmission line model response with the microstrip empirical model response
!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!
Model
   !!!!!!! this is for montcarlo analysis!!!!!!!!!!!!!!!!!!!!!!!!!!!!
   !! define L as a uniform random variable between 40 mil and 65 mil
   L_: 40 {Uniform TOL=25 LOW=0.0 HIGH=1.0};
   !! define W as a uniform random variable between 10 mil and 35 mil
   W_: 10 {Uniform TOL=25 LOW=0.0 HIGH=1.0};
   !!!!!!!!!!!!!!!!!!!!!!!!!! circuit definition!!!!!!!!!
   !! define the parameters values and the frequency range
    F_MIN=40ghz;
    F_MAX=50ghz;
    F_STEP=0.5ghz;

    L=L_*1mil;
    W=W_*1mil;
    H=15mil;
    EPSR=9.8;
    ! Without SMSM
    !W1=W;
   !L1=L;
   !!( L1,W1)=P(L,W) and it should be replaced by L1=L and W1=W if the SMSM is not used
   !W1=( 0.4042 + 0.9074 * W_ - 0.0409 *L_) *1mil;
   !L1=(-0.7487 + 0.0290 * W_ + 1.0960 *L_) * 1mil;
!!!!!!!!!!!!!!!! The empirical model for the microstrip TL !!!!!!!!!!
   MSUB EPSR=EPSR H=H;
   MSL 1 2  W=W L=L;
   PORTS 1 0 2 0;
   !!!!!!!!!!!!!!!!!!!!!!The TL physical model!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!
   EPSR_E=( EPSR+1)/2+(EPSR-1)/(2 *sqrt(1+12*H/W1));  !!! compute the effective dielectric constant
   Z0=if((W1/H)<1)
      (60/sqrt(EPSR_E) * log(8*H/W1+W1/(4*H)))
       else
      (120*pi/(sqrt(EPSR_E)*(W1/H+1.393+0.667*log(W1/H+1.444))));
   TRL 3  4 Z=Z0 L=L1 K=EPSR_E F=FREQ;
   PORTS 3 0 4 0;
CIRCUIT;
!! define the goal as the differences between the corresponding responses
D_PS11_1=abs(PS11-PS33);
D_RS11=abs(RS11-RS33);
D_RS12=abs(RS12-RS34);
D_IS11=abs(IS11-IS33);
D_IS12=abs(IS12-IS34);
D_RS[1,2]=[D_RS11  D_RS12];
D_IS[1,2]=[D_IS11  D_IS12];
end
Sweep
   AC: FREQ: from F_MIN to F_MAX step =F_STEP
   D_PS D_MS D_RS D_IS

end
```

```
MonteCarlo

 AC:  FREQ: from F_MIN to F_MAX step =F_STEP
N_Outcomes=500
D_RS11<0.04
D_RS12<0.04
D_IS11<0.04
D_IS12<0.04;

end
```

REFERENCES

[1] ***em***™, Sonnet Software, Inc., 1020 Seventh North Street, Suite 210, Liverpool, NY13088.

[2] M. Kirschning and R. Jansen, "Accurate model for effective dielectric constant of microstrip with validity up to millimeter-wave frequencies," *Electronics Letters*, vol. 18, 1982, pp. 272–273.

[3] *OSA90/hope*™, Optimization Systems Associate Inc., Dundas, Ontario, Canada, 1997.

[4] J.W. Bandler, R.M. Biernacki, S.H. Chen, P.A. Grobelny and R.H. Hemmers, "Space mapping technique for electromagnetic optimization," *IEEE Trans. Microwave Theory Tech*., vol. 42, 1994, pp. 2536–2544.

[5] *MATLAB*, The MathWorks, Inc., Simon & Schuster/A Viacom Company Upper Saddle River, New Jersey 07458, 1996.

[6] J.W. Bandler, S.H. Chen, R.M. Biernacki, Li Gao, K. Madsen and H. Yu, "Huber optimization of circuits: a robust approach," *IEEE Trans. Microwave Theory Tech*., vol. 41, 1993, pp. 2279–2287.

[7] D.M. Pozar, *Microwave Engineering*, New York: Addison-Wesley, 1990.

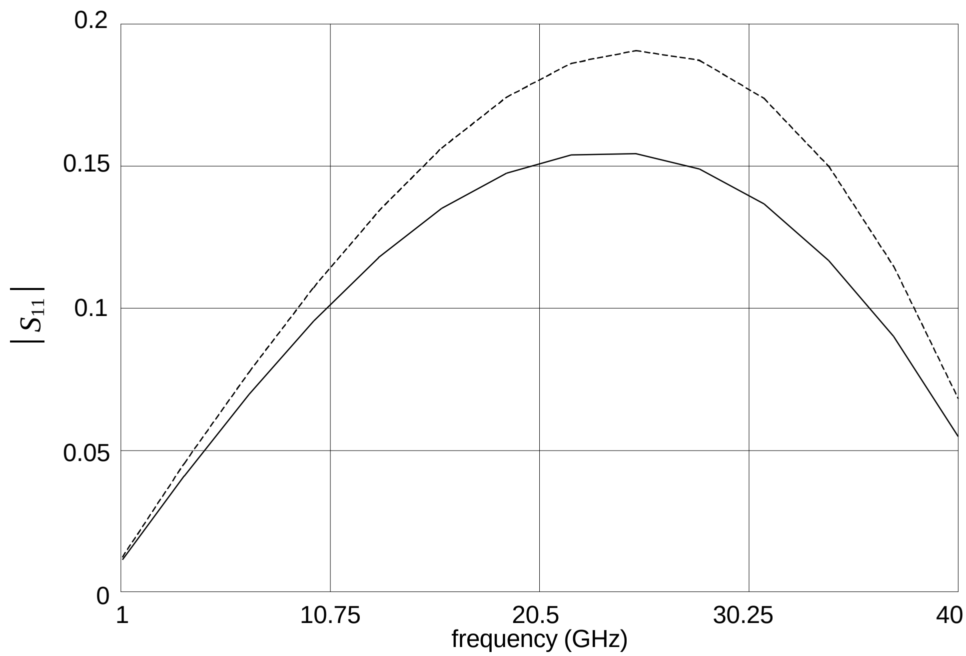


Fig. 1. The magnitude of the reflection coefficient $|S_{11}|$ of a microstrip analyzed by using the Sonnet's ***em*** simulator [1] (—) and by using the Jansen's empirical model [2](---).

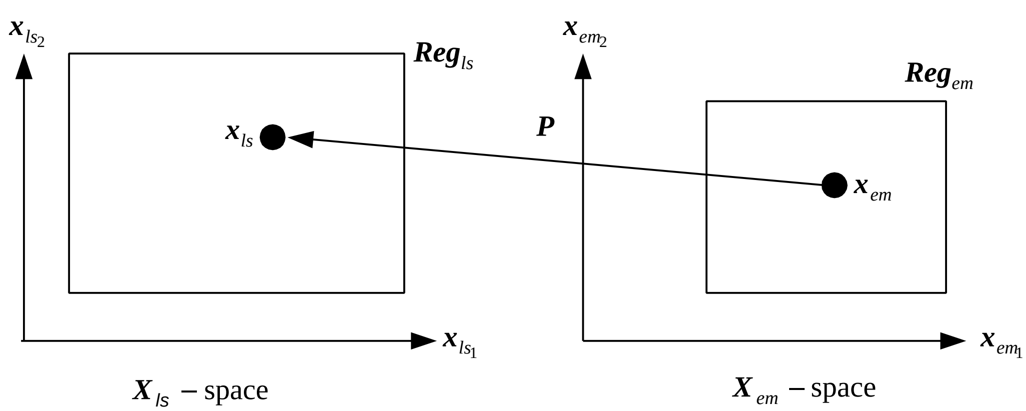


Fig. 2. The mapping $\boldsymbol{P}$ which transforms any element in $\boldsymbol{Reg}_{em}$ to the corresponding element in $\boldsymbol{Reg}_{ls}$ such that (2) is satisfied.

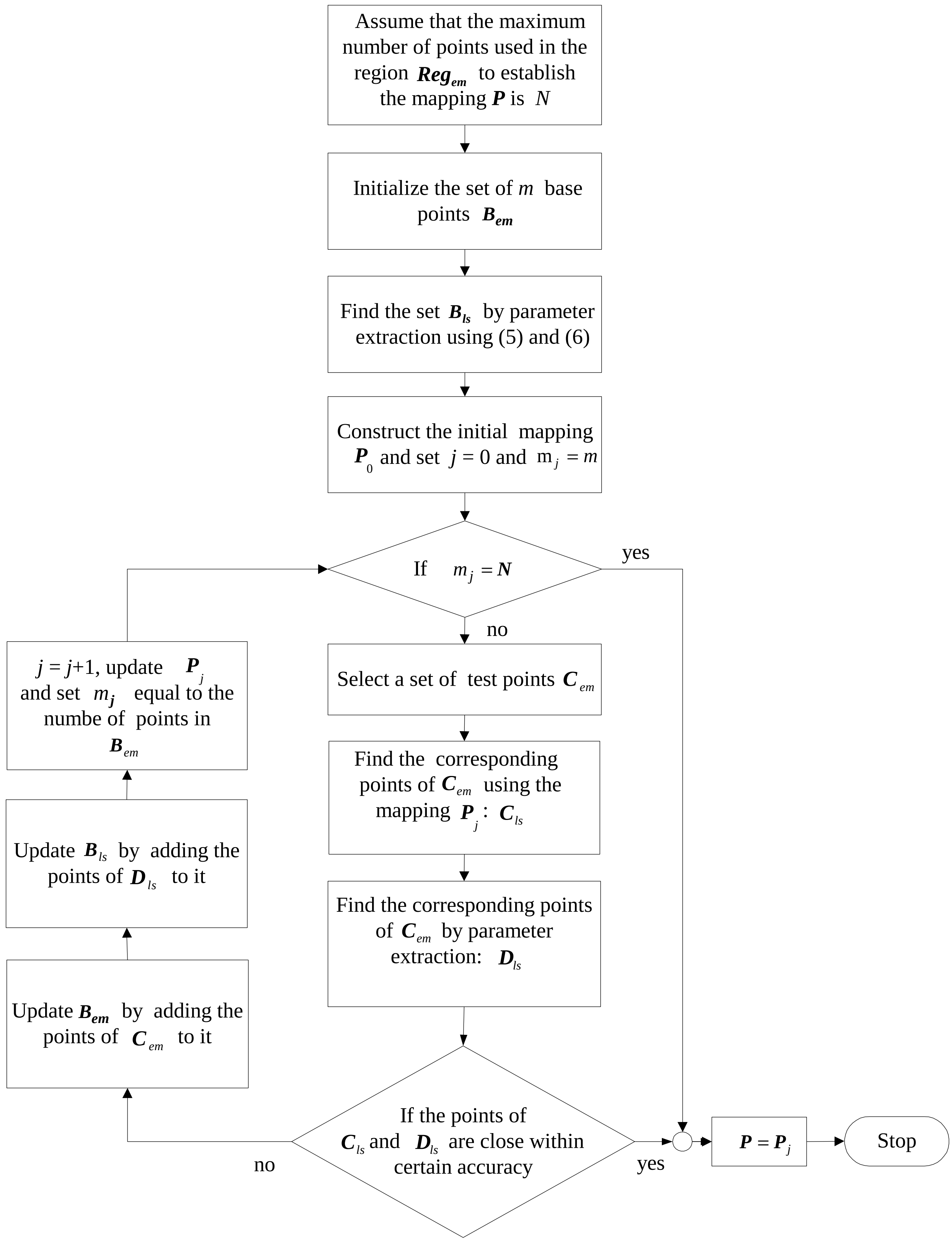
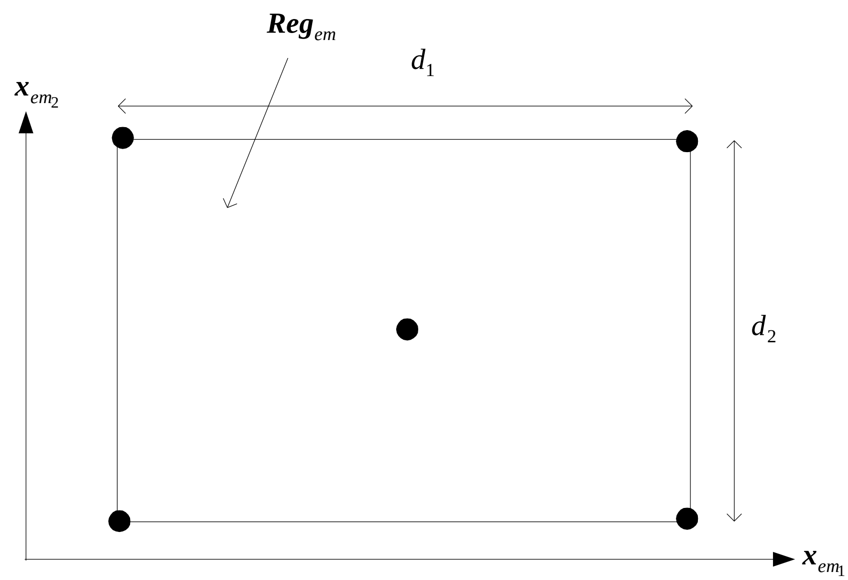
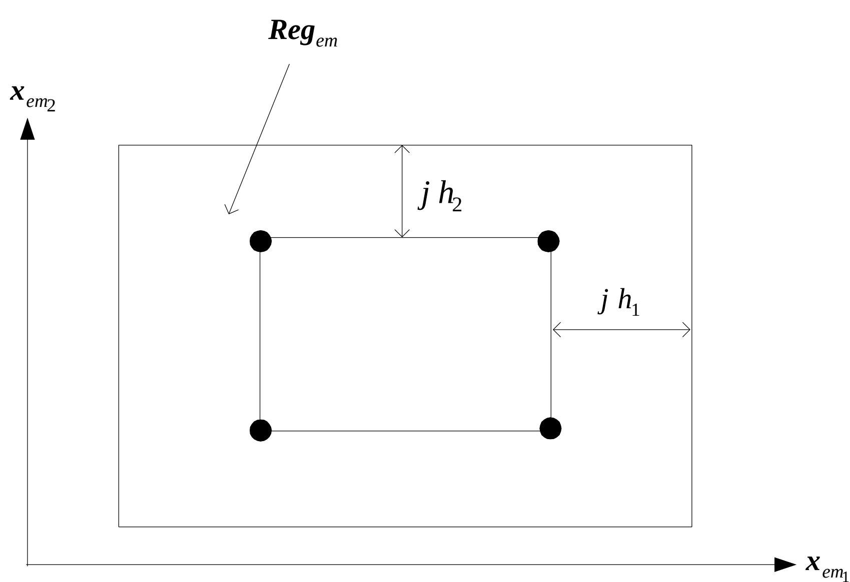


Fig. 3. The flow chart of the SMSM technique.



(a)



(b)

Fig. 4 For two–dimensional region $\boldsymbol{Reg}_{em}$: (a) the points in the set $\boldsymbol{B}_{em}$; (b) the points in the set $\boldsymbol{C}_{em}$.

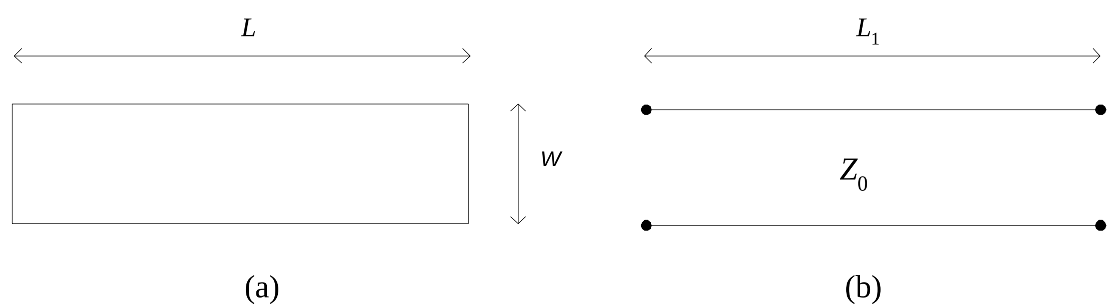
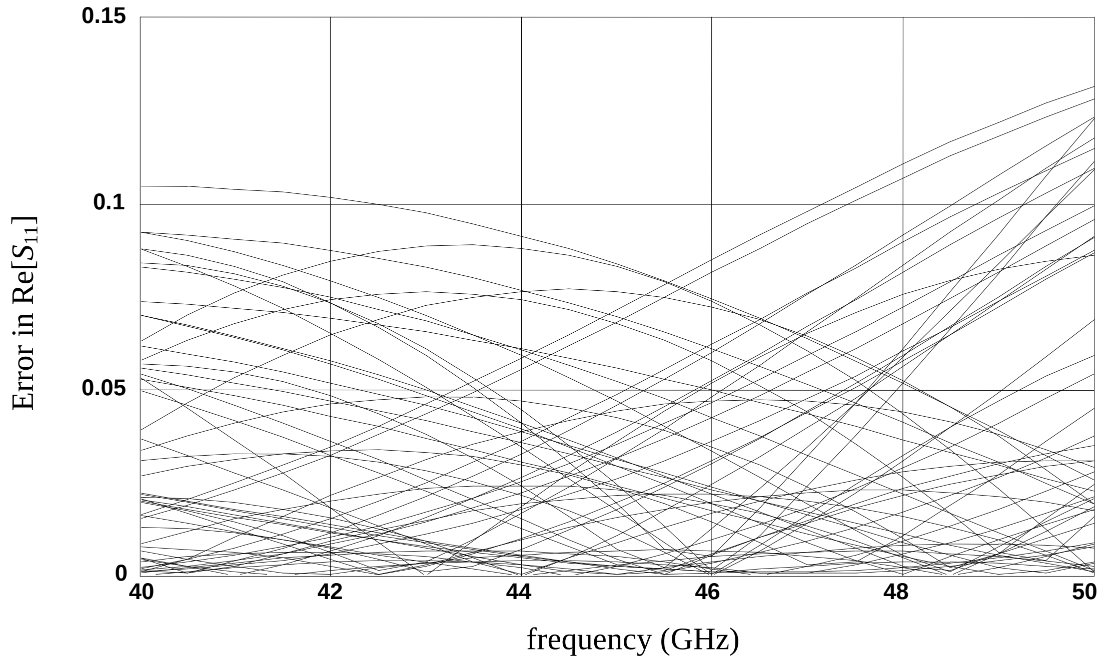
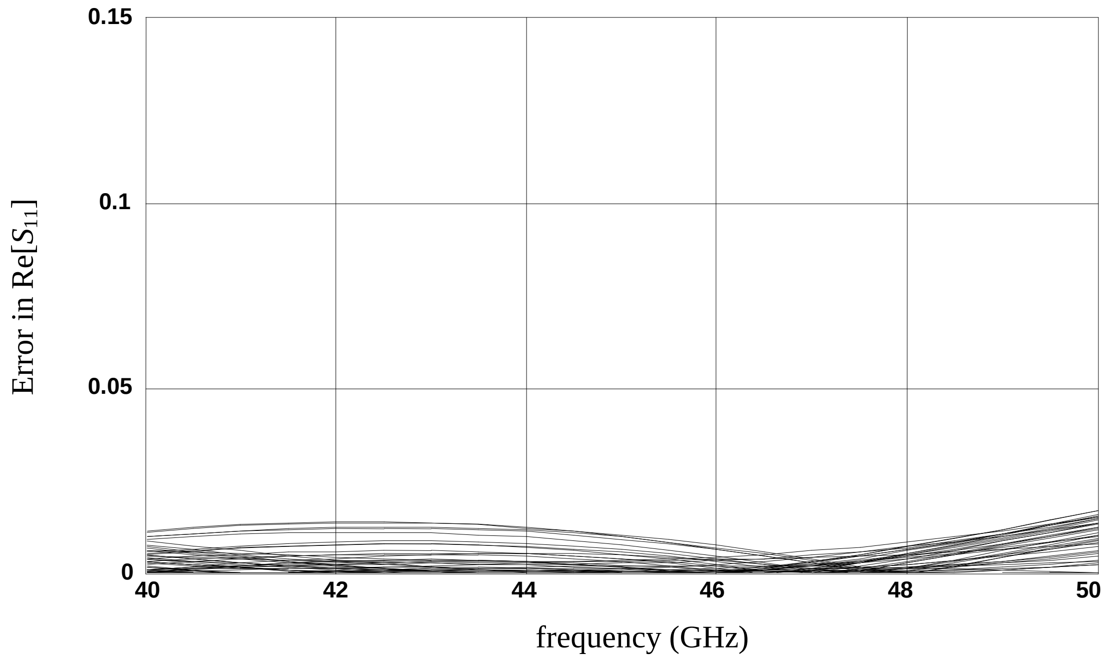


Fig . 5. (a) microstrip line and the corresponding transmission line (b).

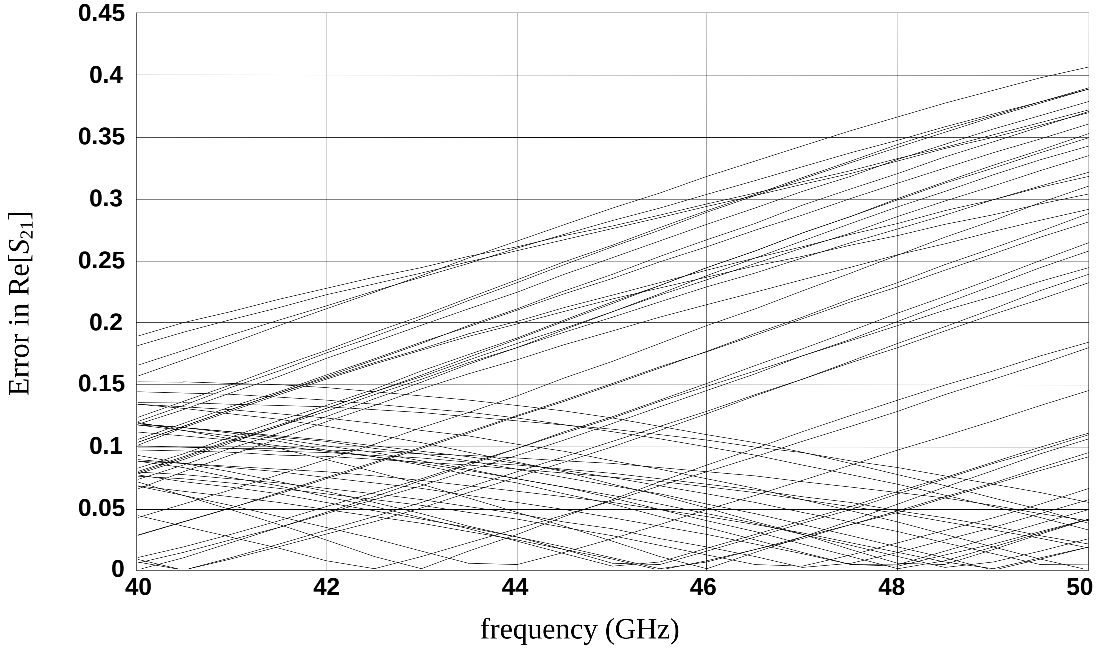


(a)

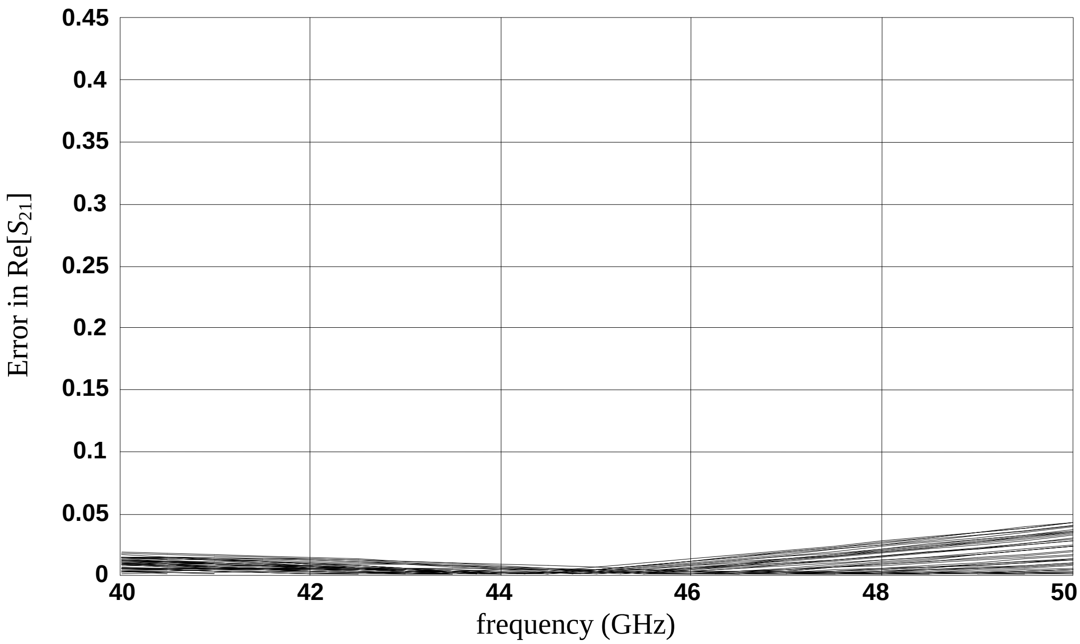


(b)

Fig. 6. The difference between the real part of S_{11} obtained by the empirical model of the microstrip line and by the transmission line model before applying the SMSM technique (a) and after applying it (b).

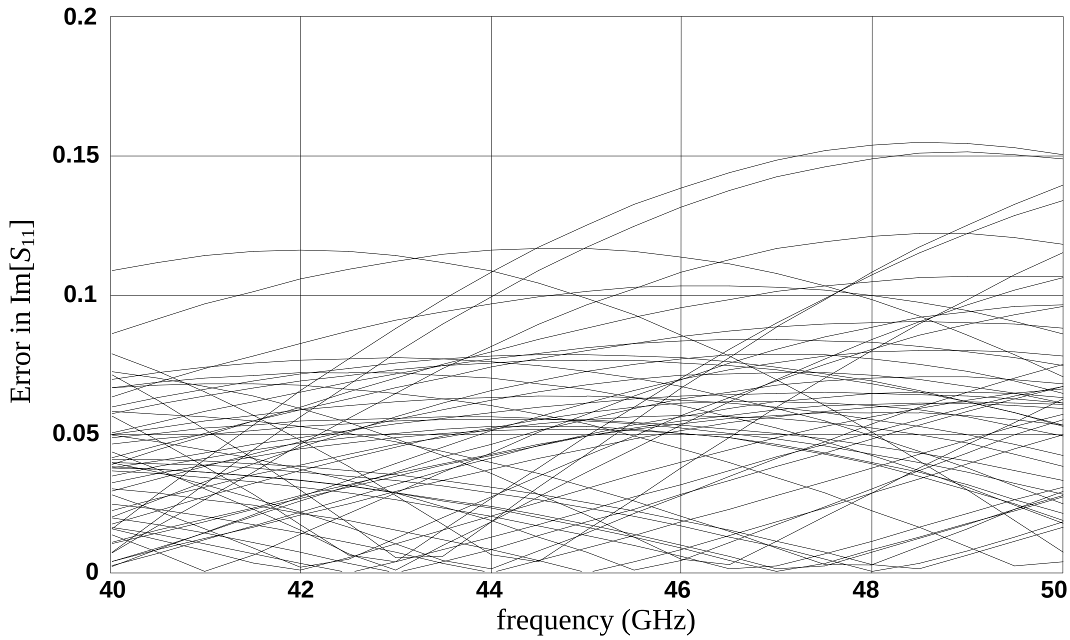


(a)

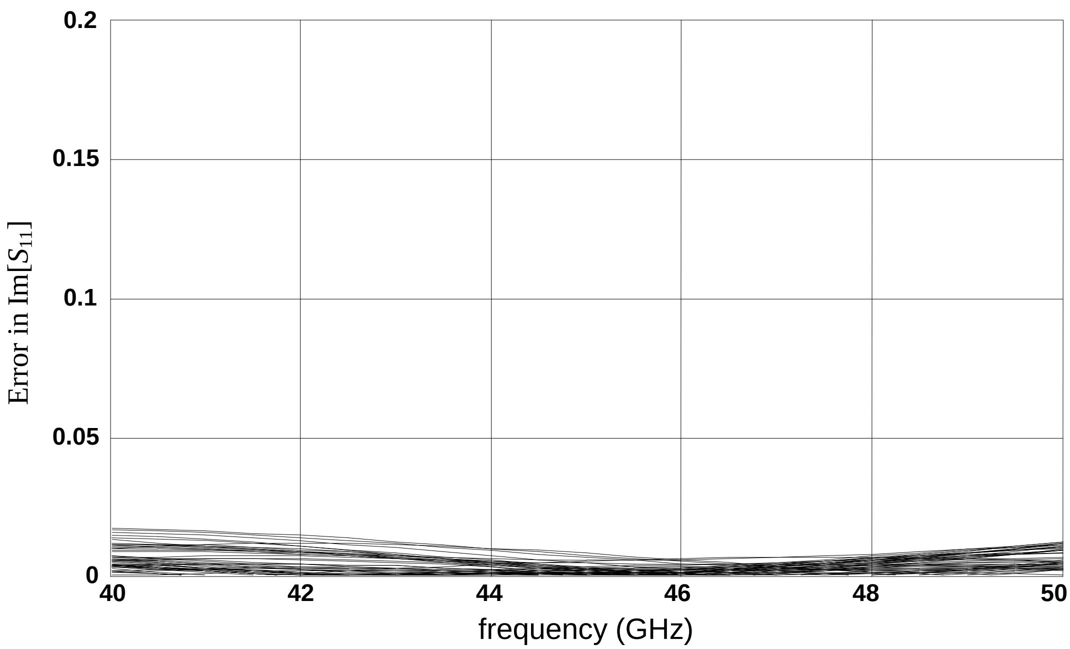


(b)

Fig. 7. The difference between the real part of S_{21} obtained by the empirical model of the microstrip line and by the transmission line model before applying the SMSM technique (a) and after applying it (b).

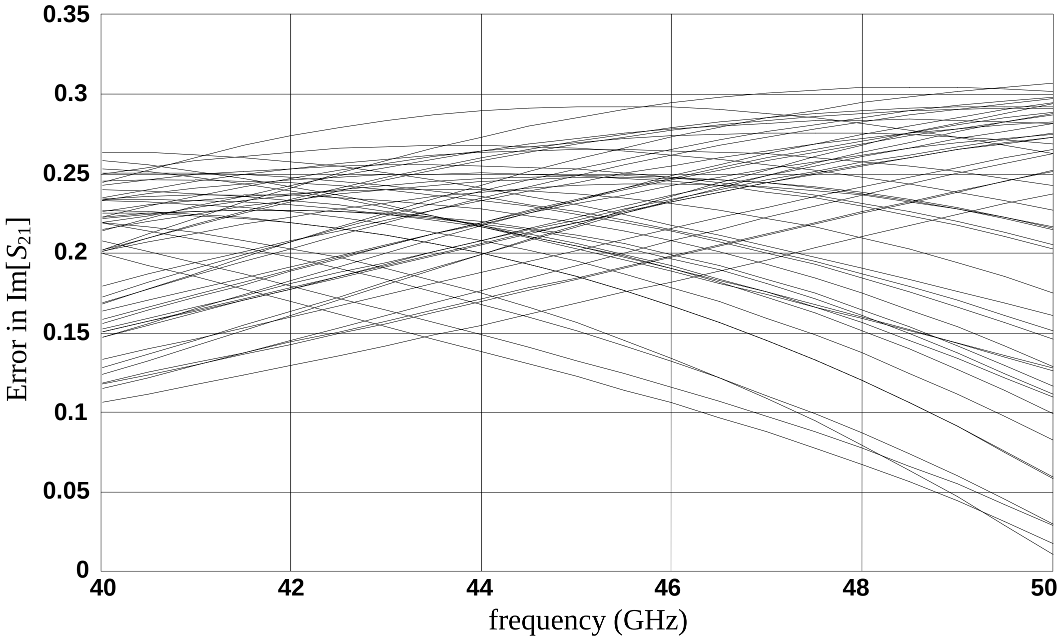


(a)

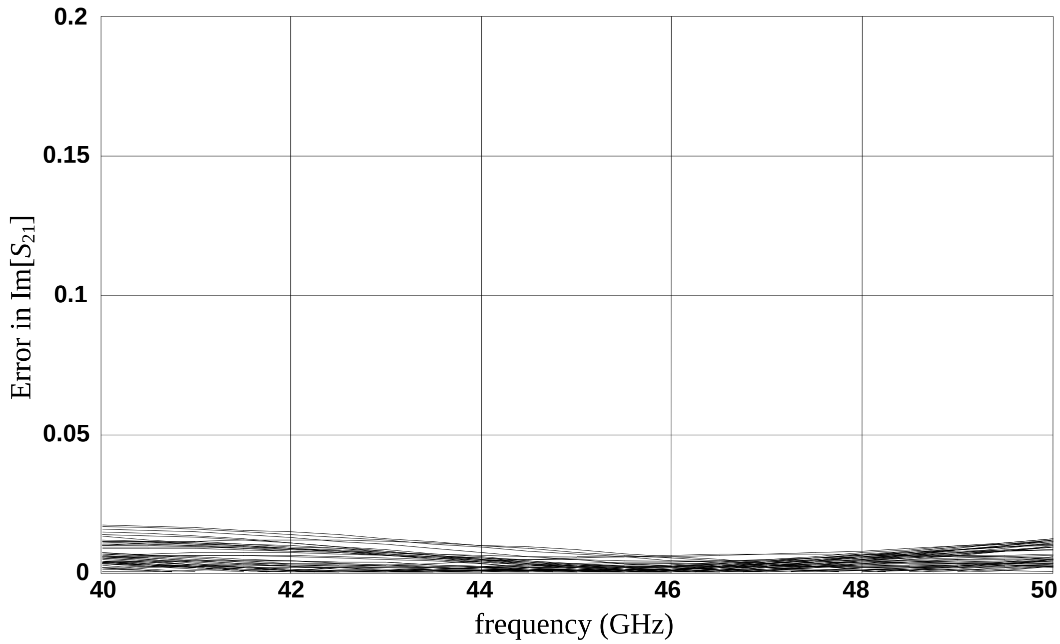


(b)

Fig. 8. The difference between the imaginary part of S_{11} obtained by the empirical model of the microstrip line and by the transmission line model before applying the SMSM technique (a) and after applying it (b).



(a)



(b)

Fig. 9. The difference between the imaginary part of S_{21} obtained by the empirical model of the microstrip line and by the transmission line model before applying the SMSM technique (a) and after applying it (b).