AGGRESSIVE SPACE MAPPING WITH MULTIPLE POINT PARAMETER EXTRACTION: A STEP BY STEP ILLUSTRATION USING TWO ROSENBROCK FUNCTIONS

J.W. Bandler and J.E. Rayas-Sánchez

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McMaster University, Hamilton, Canada L8S 4L7

presented at

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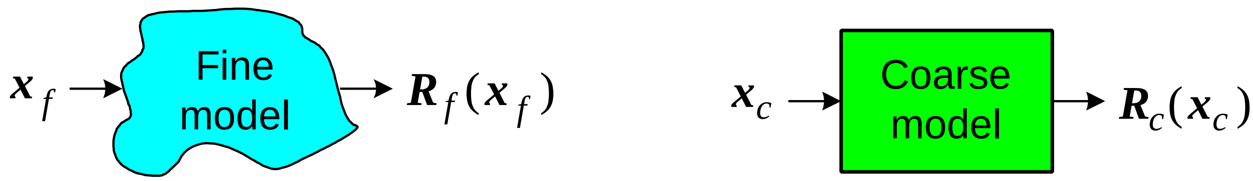
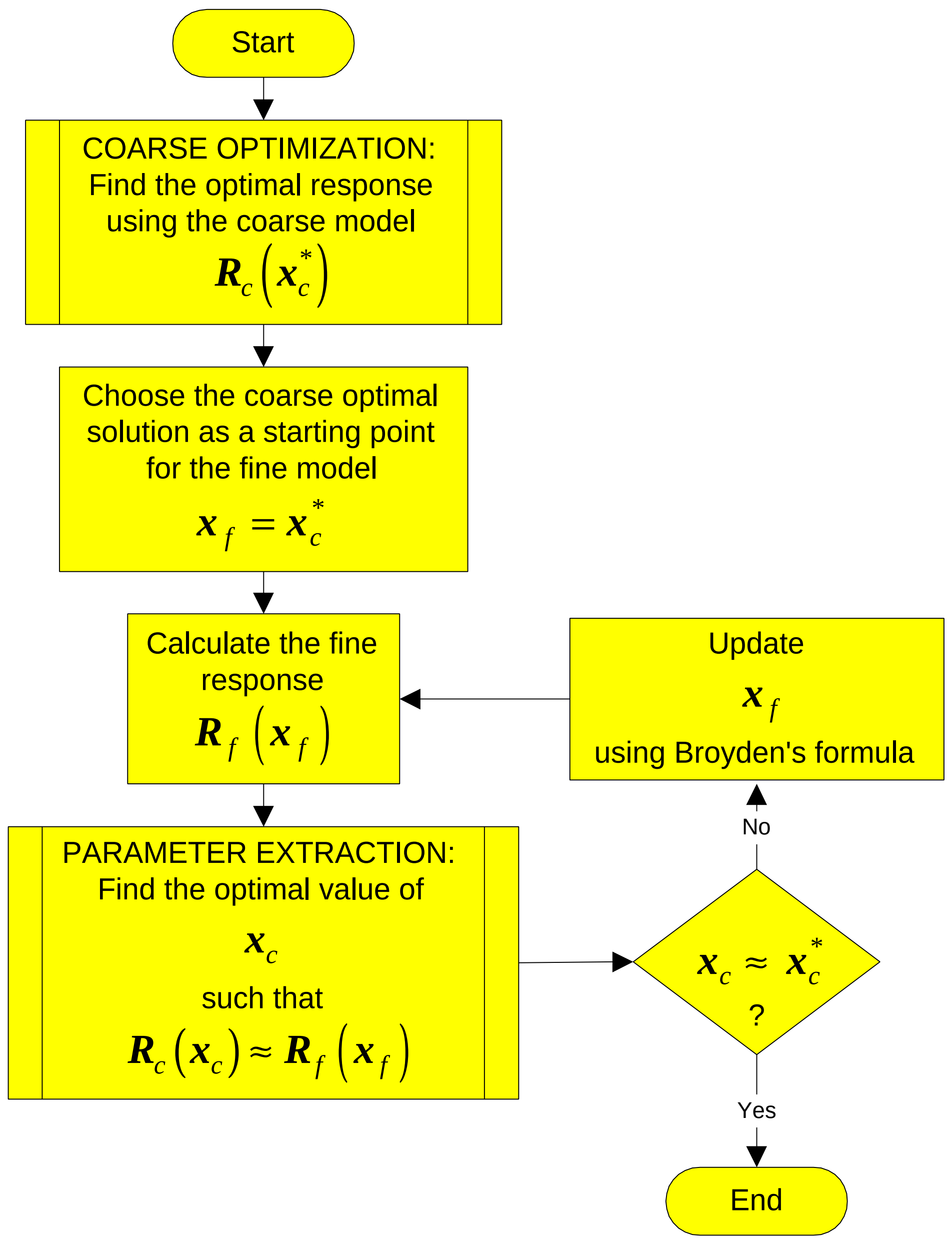
Aggressive Space Mapping (ASM) Concept





ASM Algorithm (Bandler *et al.*, 1995)

Step 0. Initialize $\boldsymbol{x}_f^{(1)} = \boldsymbol{x}_c^*$, $\boldsymbol{B}^{(1)} = \boldsymbol{1}$, $j = 1$.

Step 1. Evaluate $\boldsymbol{R}_f(\boldsymbol{x}_f^{(1)})$.

Step 2. Extract $\boldsymbol{x}_c^{(1)}$ such that $\boldsymbol{R}_c(\boldsymbol{x}_c^{(1)}) \approx \boldsymbol{R}_f(\boldsymbol{x}_f^{(1)})$.

Step 3. Evaluate $\boldsymbol{f}^{(1)} = \boldsymbol{x}_c^{(1)} - \boldsymbol{x}_c^*$. Stop if $\left\| \boldsymbol{f}^{(1)} \right\| \leq \eta$.

Step 4. Solve $\boldsymbol{B}^{(j)} \boldsymbol{h}^{(j)} = -\boldsymbol{f}^{(j)}$ for $\boldsymbol{h}^{(j)}$.

Step 5. Set $\boldsymbol{x}_f^{(j+1)} = \boldsymbol{x}_f^{(j)} + \boldsymbol{h}^{(j)}$.

Step 6. Evaluate $\boldsymbol{R}_f(\boldsymbol{x}_f^{(j+1)})$.

Step 7. Extract $\boldsymbol{x}_c^{(j+1)}$ such that $\boldsymbol{R}_c(\boldsymbol{x}_c^{(j+1)}) \approx \boldsymbol{R}_f(\boldsymbol{x}_f^{(j+1)})$.

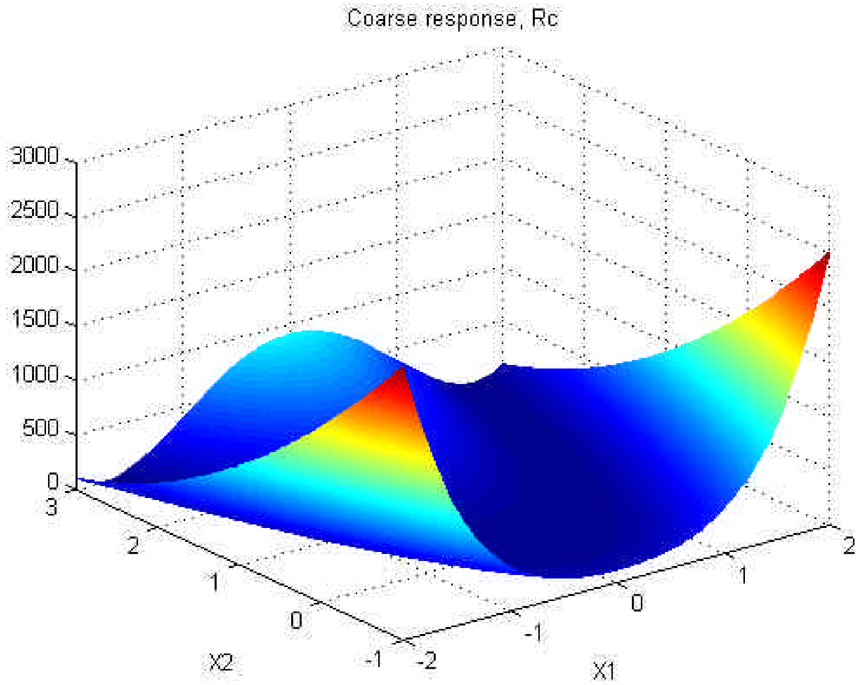
Step 8. Evaluate $\boldsymbol{f}^{(j+1)} = \boldsymbol{x}_c^{(j+1)} - \boldsymbol{x}_c^*$. Stop if $\left\| \boldsymbol{f}^{(j+1)} \right\| \leq \eta$.

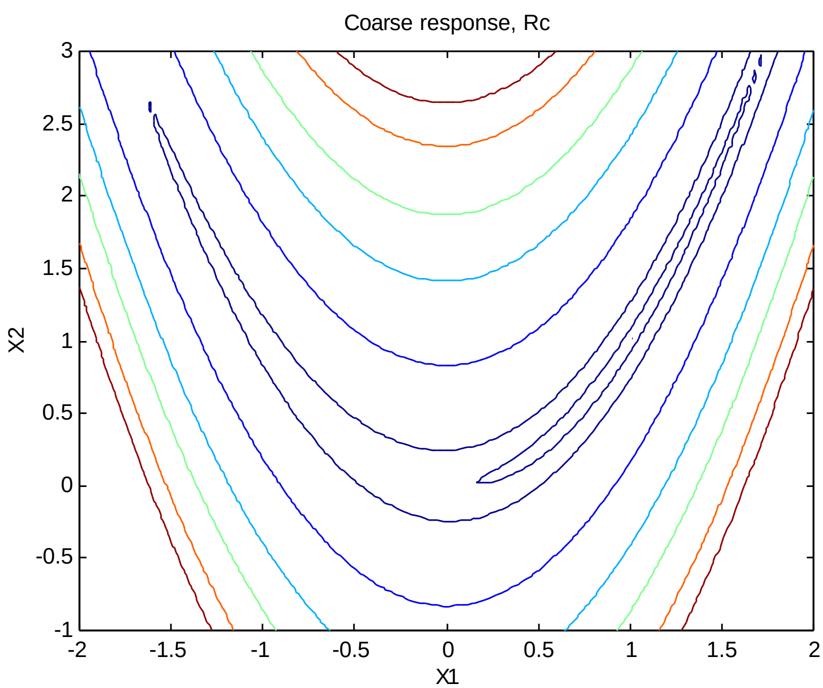
Step 9. Update $\boldsymbol{B}^{(j+1)} = \boldsymbol{B}^{(j)} + \dfrac{\boldsymbol{f}^{(j+1)} \boldsymbol{h}^{(j)T}}{\boldsymbol{h}^{(j)T} \boldsymbol{h}^{(j)}}$.

Step 10. Set $j = j + 1$; go to *Step* 4.

Coarse Model: Rosenbrock Function

$$R_c(\boldsymbol{x}) = 100(x_2 - x_1^2)^2 + (1 - x_1)^2 \text{ , where } \boldsymbol{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$



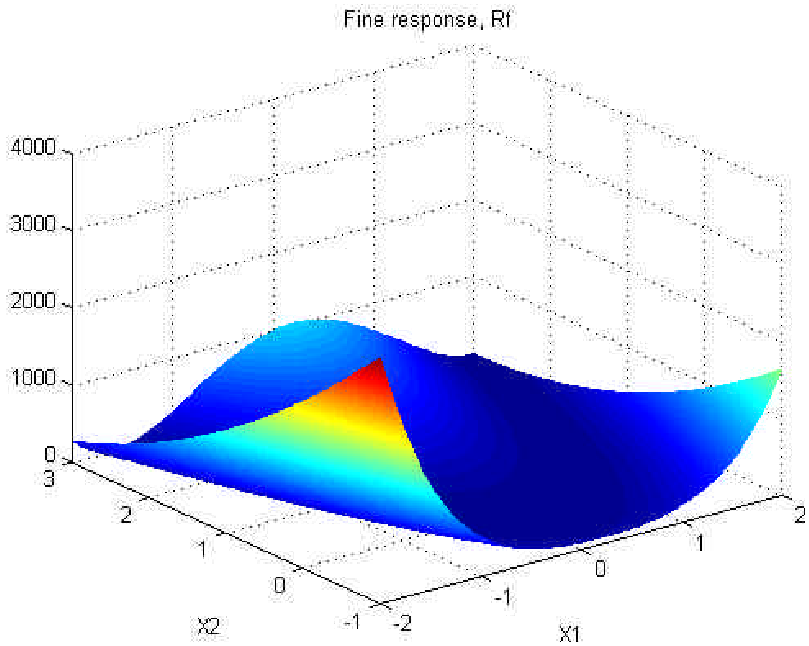


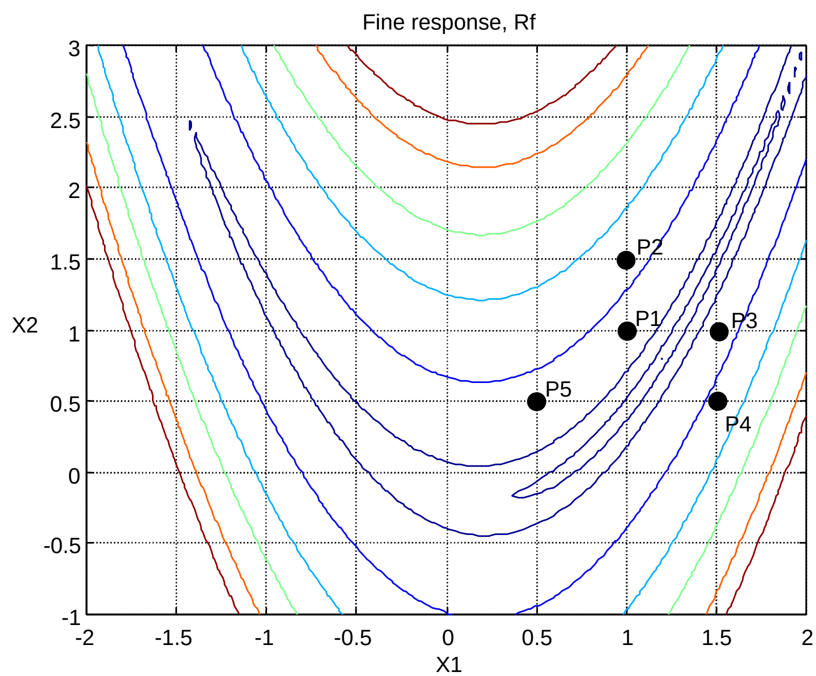
$$\boldsymbol{x}_c^* = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$R_c^* = R_c(\boldsymbol{x}_c^*) = 0$$

Fine Model: Perturbed Rosenbrock Function

$$R_f(\boldsymbol{x}) = 100(u_2 - u_1^2)^2 + (1-u_1)^2 \text{ where } \boldsymbol{u} = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \boldsymbol{x} + \begin{bmatrix} -0.2 \\ +0.2 \end{bmatrix}$$





Multiple Point Parameter Extraction

Instead of minimizing $\left\| R_c\,(\boldsymbol{x}) - R_f\,(\boldsymbol{x}_c^*) \right\|_p$ we will minimize a parameter extraction objective function considering five matching points

$$\left\| R_c\,(\boldsymbol{x} + \boldsymbol{B}^{(j)}?\ \boldsymbol{x}_i) - R_f\,(\boldsymbol{x}_c^* + ?\ \boldsymbol{x}_i) \right\|_p \quad , \quad \text{where}$$

$$\Delta\boldsymbol{x}_1 = \begin{bmatrix} 0 \\ 0 \end{bmatrix},\ \Delta\boldsymbol{x}_2 = \begin{bmatrix} 0 \\ 0.5 \end{bmatrix},\ \Delta\boldsymbol{x}_3 = \begin{bmatrix} 0.5 \\ 0 \end{bmatrix},\ \Delta\boldsymbol{x}_4 = \begin{bmatrix} 0.5 \\ -0.5 \end{bmatrix},\ \Delta\boldsymbol{x}_5 = \begin{bmatrix} -0.5 \\ -0.5 \end{bmatrix}$$

Since the matrix $\boldsymbol{B}^{(j)} = \boldsymbol{1}$ for the first parameter extraction optimization, the corresponding five error functions E_i are given by

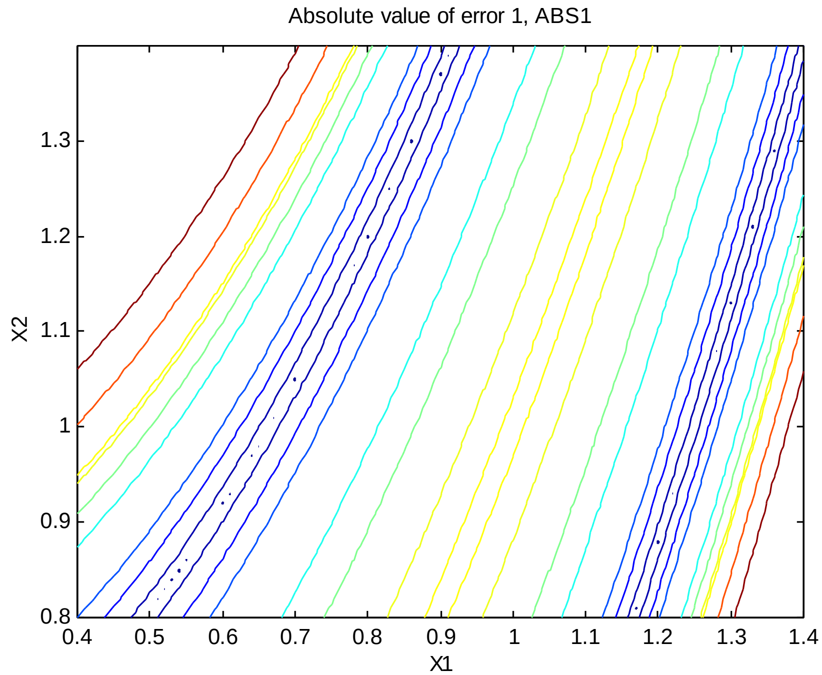
$$E_i = R_c\,(\boldsymbol{x} + ?\ \boldsymbol{x}_i) - R_f\,(\boldsymbol{x}_c^* + ?\ \boldsymbol{x}_i)$$

Considering only l_1 and l_2 norms, the corresponding objective functions are

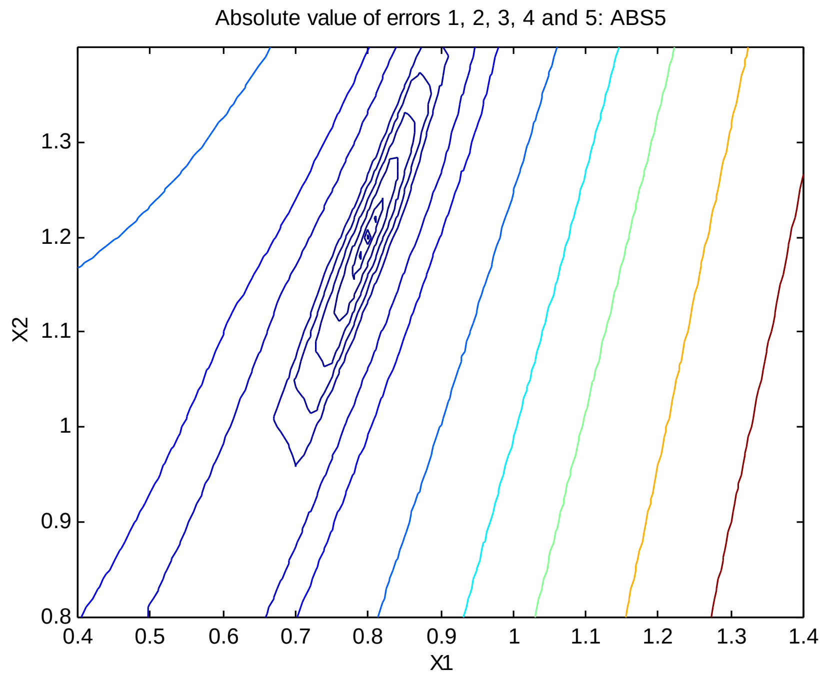
$$ABS_n = |E_1| + |E_2| + \ldots + |E_n|$$

$$SQR_n = (E_1)^2 + (E_2)^2 + \ldots + (E_n)^2$$

l_1 Objective Function for the Parameter Extraction Problem

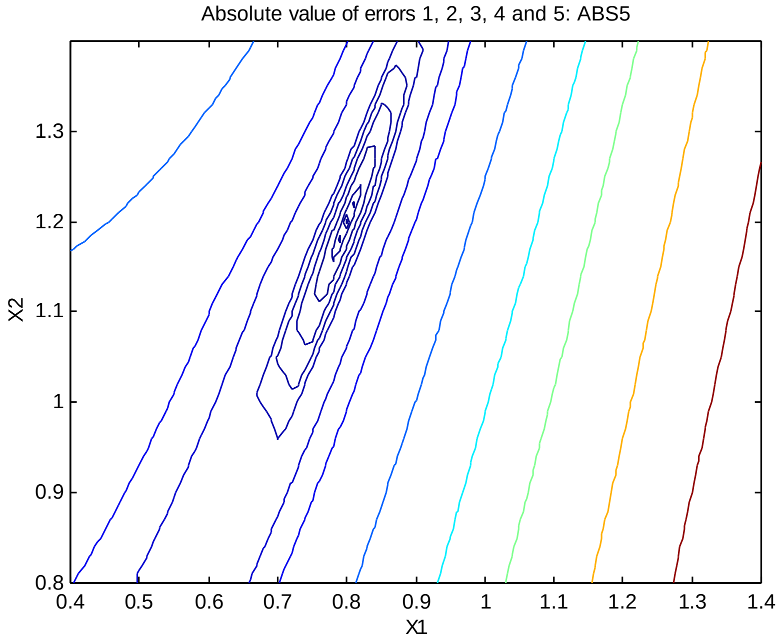


(a) single point parameter extraction

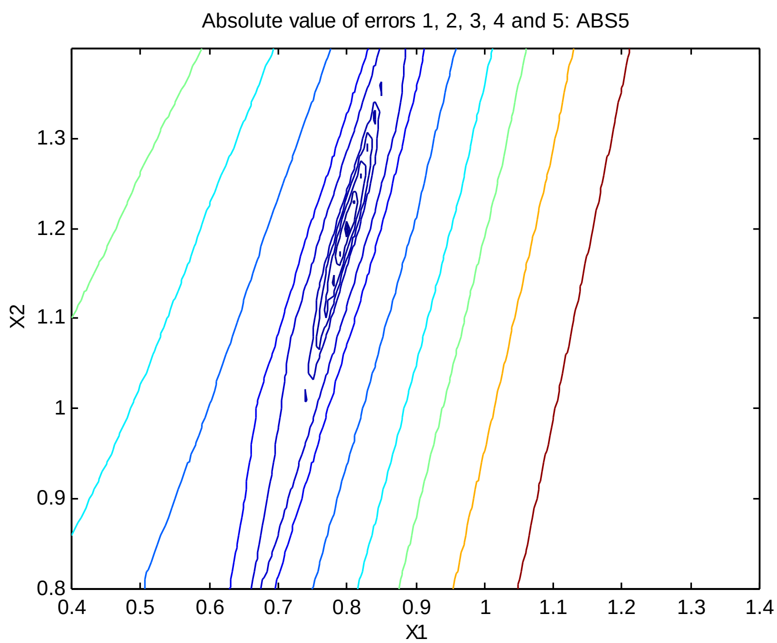


(b) multiple point parameter extraction (4 additional points)

l_1 Objective Function for 5-point Parameter Extraction

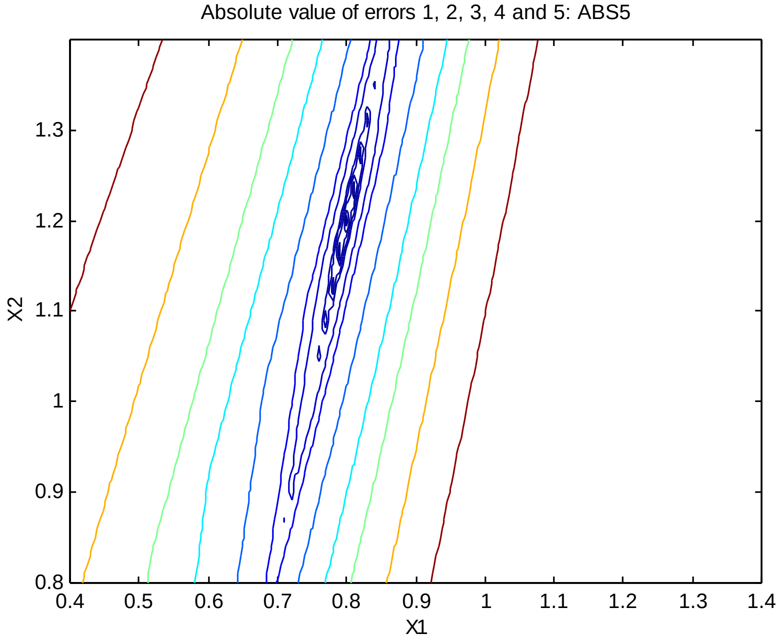


$$\Delta \boldsymbol{x}_1 = \begin{bmatrix} 0 \\ 0 \end{bmatrix},\ \Delta \boldsymbol{x}_2 = \begin{bmatrix} 0 \\ 0.5 \end{bmatrix},\ \Delta \boldsymbol{x}_3 = \begin{bmatrix} 0.5 \\ 0 \end{bmatrix},\ \Delta \boldsymbol{x}_4 = \begin{bmatrix} 0.5 \\ -0.5 \end{bmatrix},\ \Delta \boldsymbol{x}_5 = \begin{bmatrix} -0.5 \\ -0.5 \end{bmatrix}$$

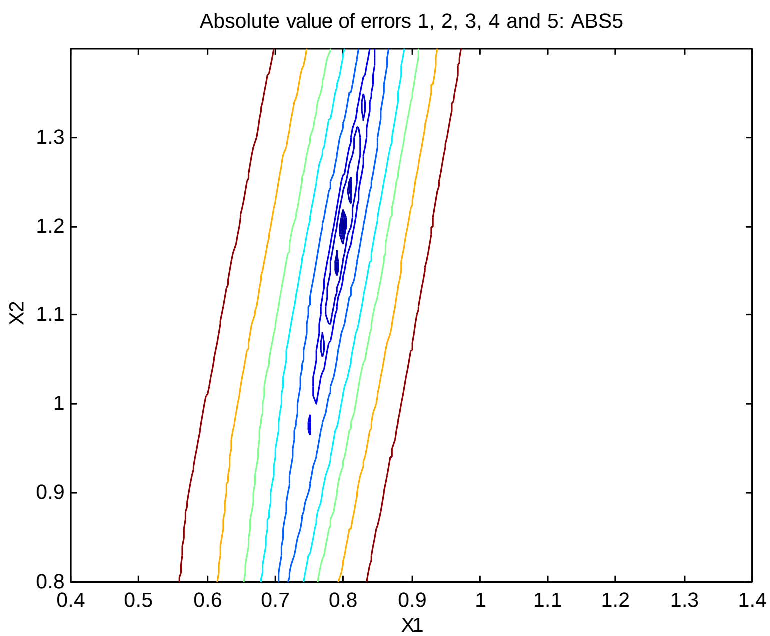


$$\Delta \boldsymbol{x}_1 = \begin{bmatrix} 0 \\ 0 \end{bmatrix},\ \Delta \boldsymbol{x}_2 = \begin{bmatrix} 0 \\ 0.8 \end{bmatrix},\ \Delta \boldsymbol{x}_3 = \begin{bmatrix} 0.8 \\ 0 \end{bmatrix},\ \Delta \boldsymbol{x}_4 = \begin{bmatrix} 0.8 \\ -0.8 \end{bmatrix},\ \Delta \boldsymbol{x}_5 = \begin{bmatrix} -0.8 \\ -0.8 \end{bmatrix}$$

l_1 Objective Function for 5-point Parameter Extraction

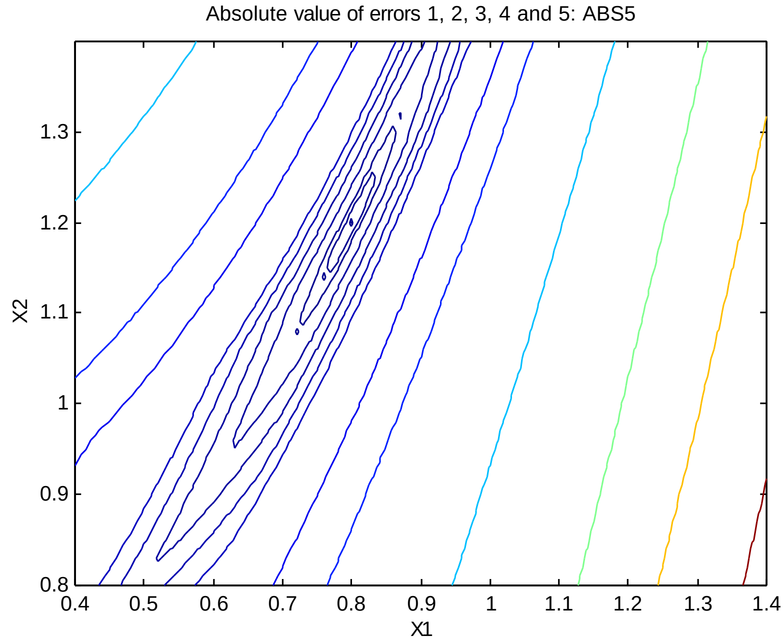


$$\Delta \boldsymbol{x}_1 = \begin{bmatrix} 0 \\ 0 \end{bmatrix},\ \Delta \boldsymbol{x}_2 = \begin{bmatrix} 0 \\ 1.1 \end{bmatrix},\ \Delta \boldsymbol{x}_3 = \begin{bmatrix} 1.1 \\ 0 \end{bmatrix},\ \Delta \boldsymbol{x}_4 = \begin{bmatrix} 1.1 \\ -1.1 \end{bmatrix},\ \Delta \boldsymbol{x}_5 = \begin{bmatrix} -1.1 \\ -1.1 \end{bmatrix}$$

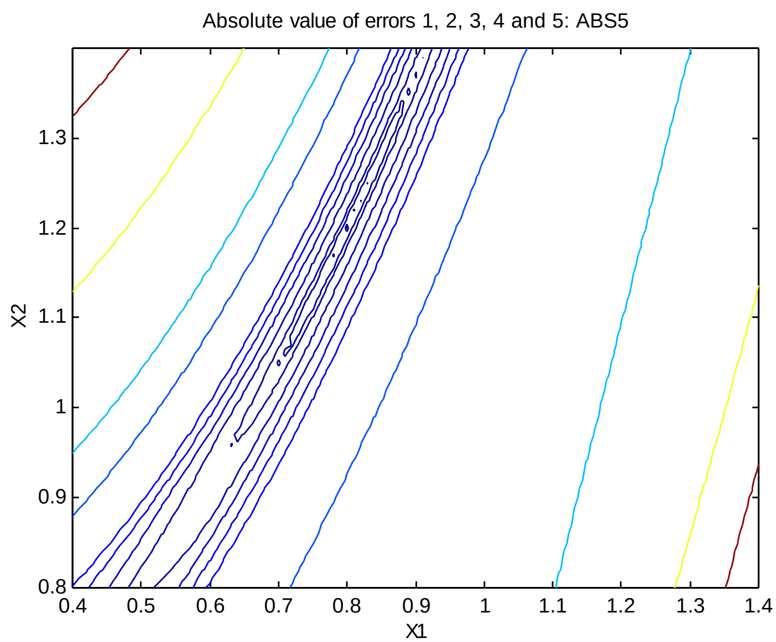


$$\Delta \boldsymbol{x}_1 = \begin{bmatrix} 0 \\ 0 \end{bmatrix},\ \Delta \boldsymbol{x}_2 = \begin{bmatrix} 0 \\ 1.5 \end{bmatrix},\ \Delta \boldsymbol{x}_3 = \begin{bmatrix} 1.5 \\ 0 \end{bmatrix},\ \Delta \boldsymbol{x}_4 = \begin{bmatrix} 1.5 \\ -1.5 \end{bmatrix},\ \Delta \boldsymbol{x}_5 = \begin{bmatrix} -1.5 \\ -1.5 \end{bmatrix}$$

l_1 Objective Function for 5-point Parameter Extraction

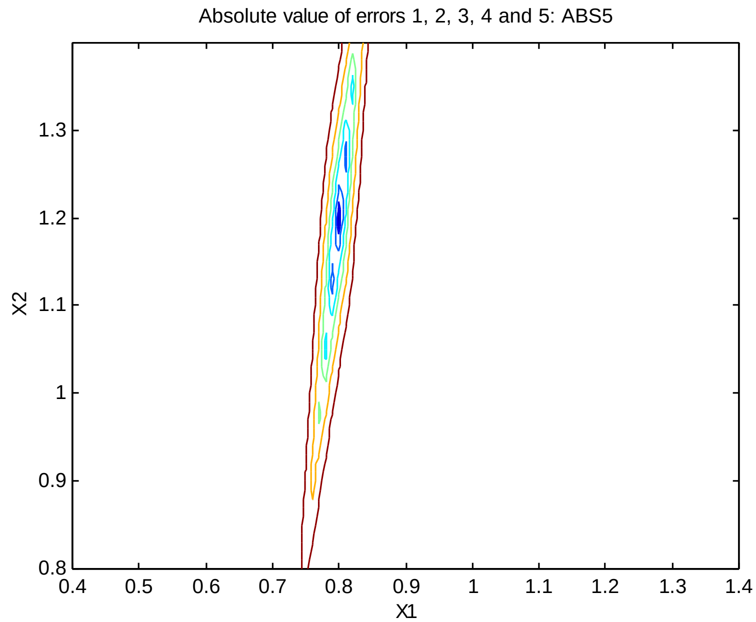


$$\Delta \boldsymbol{x}_1 = \begin{bmatrix} 0 \\ 0 \end{bmatrix},\ \Delta \boldsymbol{x}_2 = \begin{bmatrix} 0 \\ 0.3 \end{bmatrix},\ \Delta \boldsymbol{x}_3 = \begin{bmatrix} 0.3 \\ 0 \end{bmatrix},\ \Delta \boldsymbol{x}_4 = \begin{bmatrix} 0.3 \\ -0.3 \end{bmatrix},\ \Delta \boldsymbol{x}_5 = \begin{bmatrix} -0.3 \\ -0.3 \end{bmatrix}$$

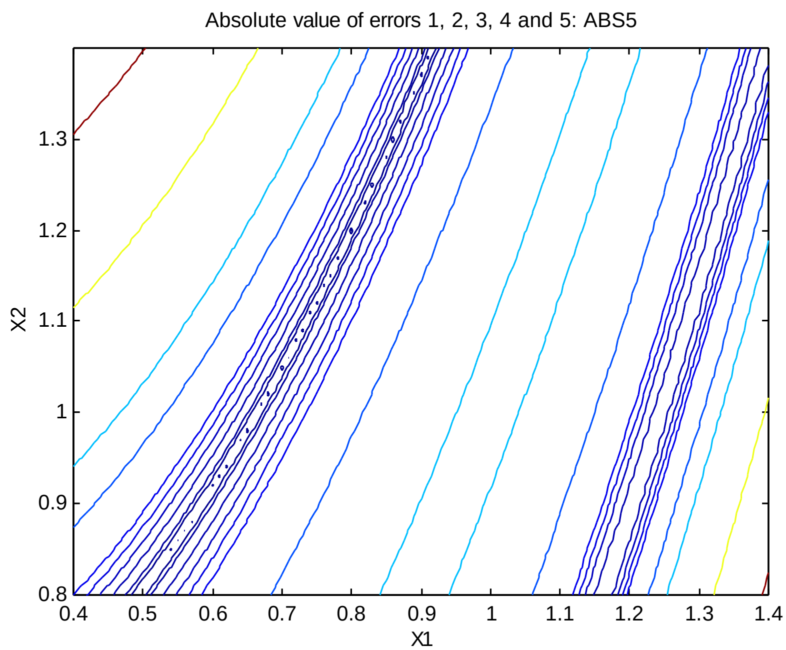


$$\Delta \boldsymbol{x}_1 = \begin{bmatrix} 0 \\ 0 \end{bmatrix},\ \Delta \boldsymbol{x}_2 = \begin{bmatrix} 0 \\ 0.1 \end{bmatrix},\ \Delta \boldsymbol{x}_3 = \begin{bmatrix} 0.1 \\ 0 \end{bmatrix},\ \Delta \boldsymbol{x}_4 = \begin{bmatrix} 0.1 \\ -0.1 \end{bmatrix},\ \Delta \boldsymbol{x}_5 = \begin{bmatrix} -0.1 \\ -0.1 \end{bmatrix}$$

l_1 Objective Function for 5-point Parameter Extraction

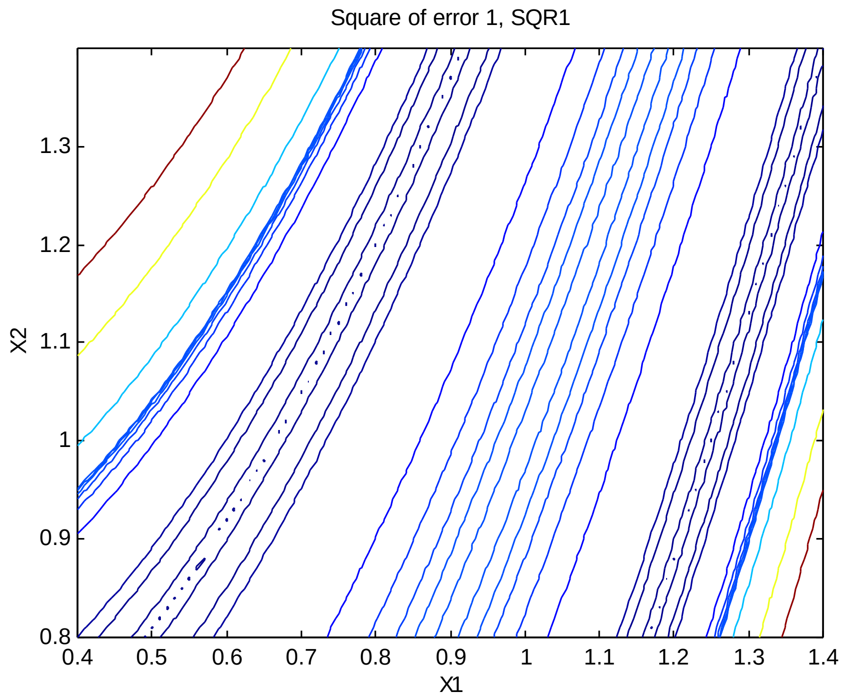


$$\Delta \boldsymbol{x}_1 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \ \Delta \boldsymbol{x}_2 = \begin{bmatrix} 0 \\ 3 \end{bmatrix}, \ \Delta \boldsymbol{x}_3 = \begin{bmatrix} 3 \\ 0 \end{bmatrix}, \ \Delta \boldsymbol{x}_4 = \begin{bmatrix} 3 \\ -3 \end{bmatrix}, \ \Delta \boldsymbol{x}_5 = \begin{bmatrix} -3 \\ -3 \end{bmatrix}$$

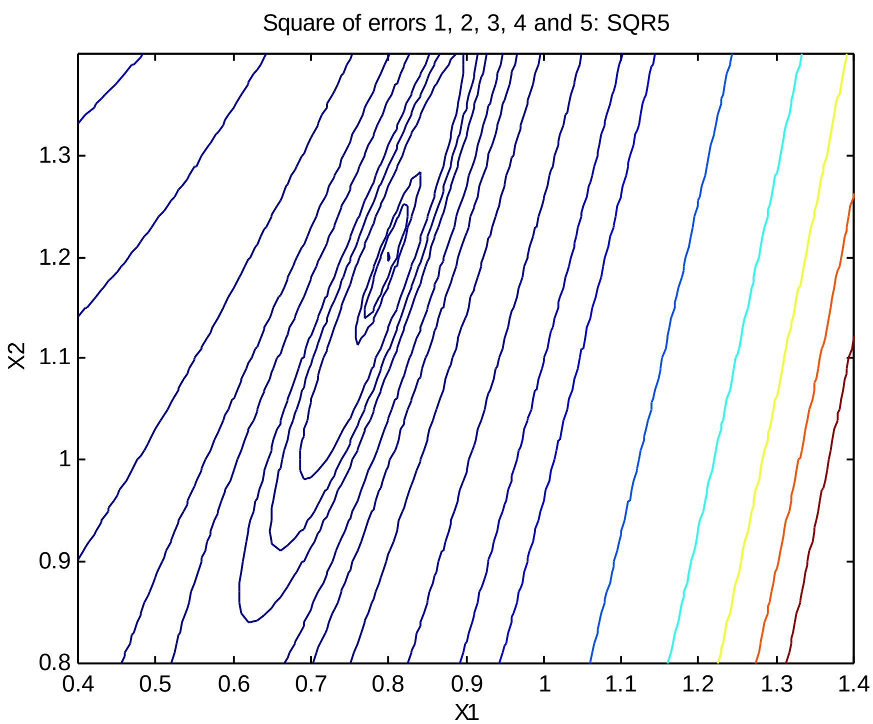


$$\Delta \boldsymbol{x}_1 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \Delta \boldsymbol{x}_2 = \begin{bmatrix} 0 \\ 0.01 \end{bmatrix}, \Delta \boldsymbol{x}_3 = \begin{bmatrix} 0.01 \\ 0 \end{bmatrix}, \Delta \boldsymbol{x}_4 = \begin{bmatrix} 0.01 \\ -0.01 \end{bmatrix}, \Delta \boldsymbol{x}_5 = \begin{bmatrix} -0.01 \\ -0.01 \end{bmatrix}$$

l_2 Objective Function for the Parameter Extraction Problem

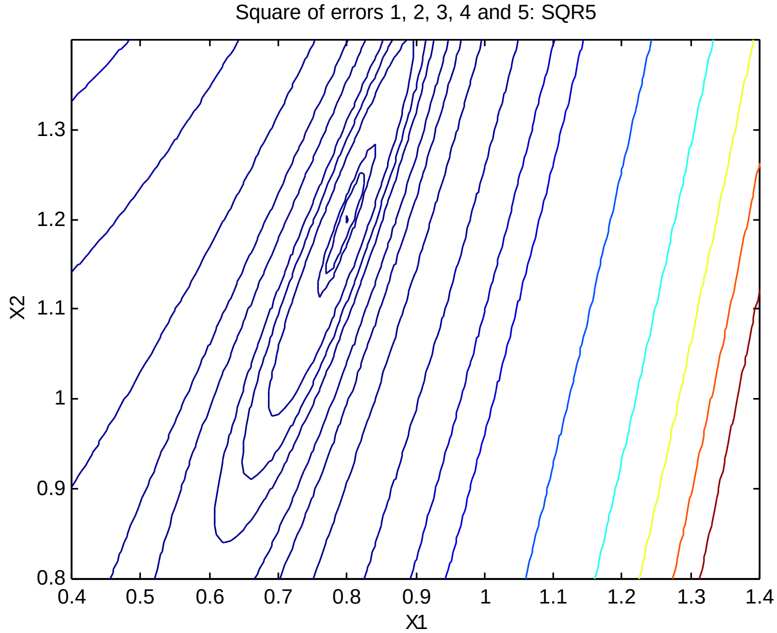


(a) single point parameter extraction

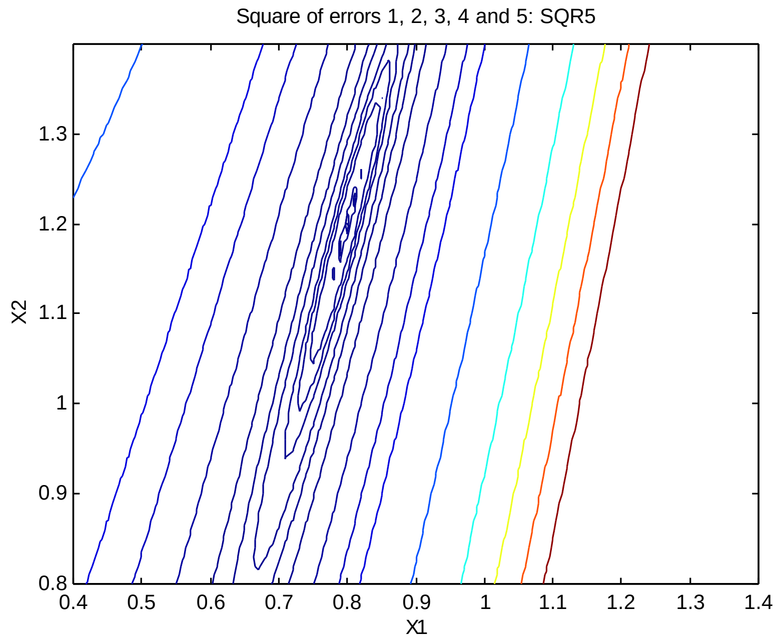


(b) multiple point parameter extraction (with 4 additional points)

l_2 Objective Function for 5-point Parameter Extraction

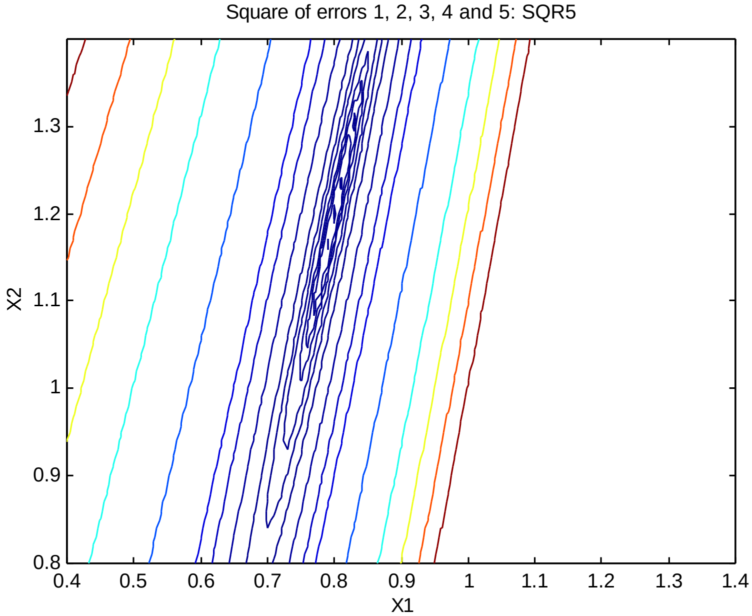


$$\Delta \boldsymbol{x}_1 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \ \Delta \boldsymbol{x}_2 = \begin{bmatrix} 0 \\ 0.5 \end{bmatrix}, \ \Delta \boldsymbol{x}_3 = \begin{bmatrix} 0.5 \\ 0 \end{bmatrix}, \ \Delta \boldsymbol{x}_4 = \begin{bmatrix} 0.5 \\ -0.5 \end{bmatrix}, \ \Delta \boldsymbol{x}_5 = \begin{bmatrix} -0.5 \\ -0.5 \end{bmatrix}$$

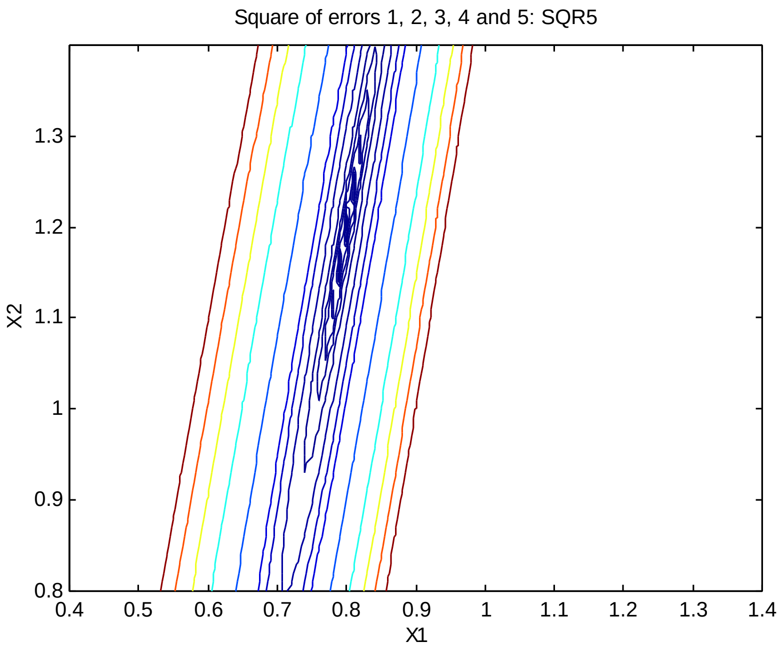


$$\Delta \boldsymbol{x}_1 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \ \Delta \boldsymbol{x}_2 = \begin{bmatrix} 0 \\ 0.8 \end{bmatrix}, \ \Delta \boldsymbol{x}_3 = \begin{bmatrix} 0.8 \\ 0 \end{bmatrix}, \ \Delta \boldsymbol{x}_4 = \begin{bmatrix} 0.8 \\ -0.8 \end{bmatrix}, \ \Delta \boldsymbol{x}_5 = \begin{bmatrix} -0.8 \\ -0.8 \end{bmatrix}$$

l_2 Objective Function for 5-point Parameter Extraction

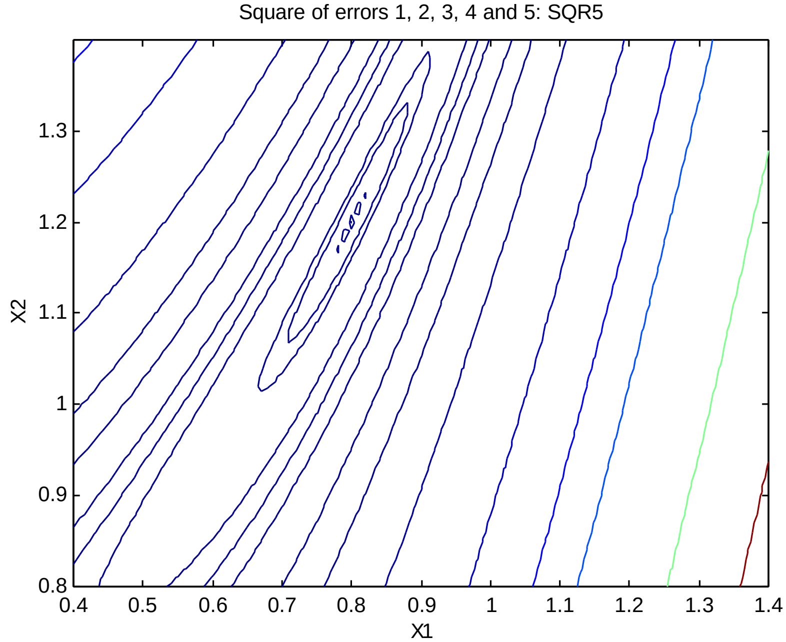


$$\Delta \boldsymbol{x}_1 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \; \Delta \boldsymbol{x}_2 = \begin{bmatrix} 0 \\ 1.1 \end{bmatrix}, \; \Delta \boldsymbol{x}_3 = \begin{bmatrix} 1.1 \\ 0 \end{bmatrix}, \; \Delta \boldsymbol{x}_4 = \begin{bmatrix} 1.1 \\ -1.1 \end{bmatrix}, \; \Delta \boldsymbol{x}_5 = \begin{bmatrix} -1.1 \\ -1.1 \end{bmatrix}$$

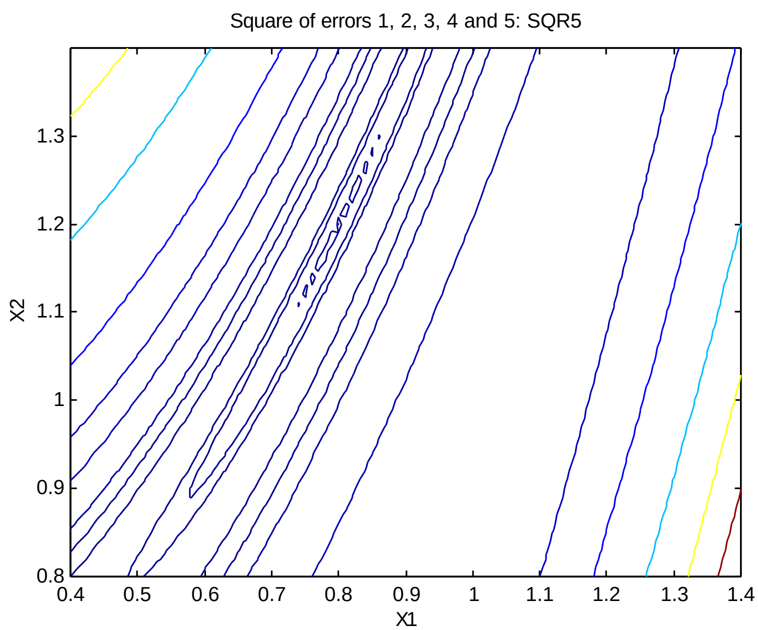


$$\Delta \boldsymbol{x}_1 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \; \Delta \boldsymbol{x}_2 = \begin{bmatrix} 0 \\ 1.5 \end{bmatrix}, \; \Delta \boldsymbol{x}_3 = \begin{bmatrix} 1.5 \\ 0 \end{bmatrix}, \; \Delta \boldsymbol{x}_4 = \begin{bmatrix} 1.5 \\ -1.5 \end{bmatrix}, \; \Delta \boldsymbol{x}_5 = \begin{bmatrix} -1.5 \\ -1.5 \end{bmatrix}$$

l_2 Objective Function for 5-point Parameter Extraction

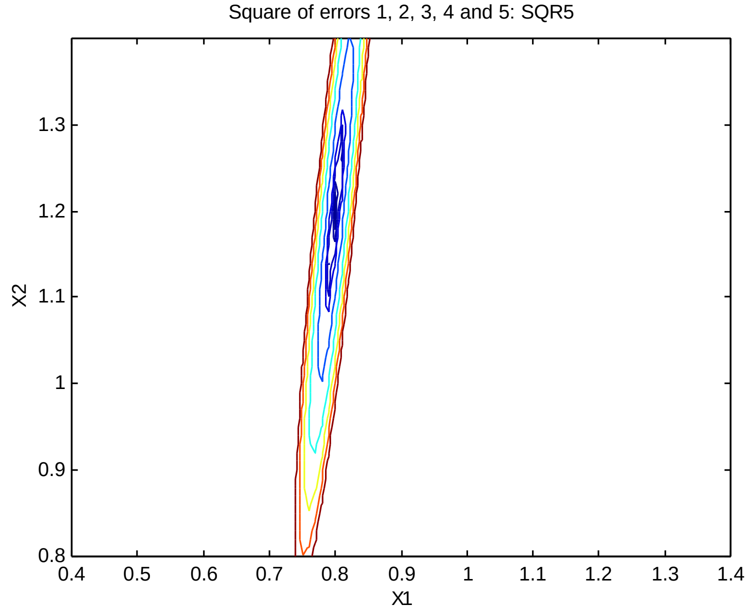


$$\Delta \boldsymbol{x}_1 = \begin{bmatrix} 0 \\ 0 \end{bmatrix},\ \Delta \boldsymbol{x}_2 = \begin{bmatrix} 0 \\ 0.3 \end{bmatrix},\ \Delta \boldsymbol{x}_3 = \begin{bmatrix} 0.3 \\ 0 \end{bmatrix},\ \Delta \boldsymbol{x}_4 = \begin{bmatrix} 0.3 \\ -0.3 \end{bmatrix},\ \Delta \boldsymbol{x}_5 = \begin{bmatrix} -0.3 \\ -0.3 \end{bmatrix}$$

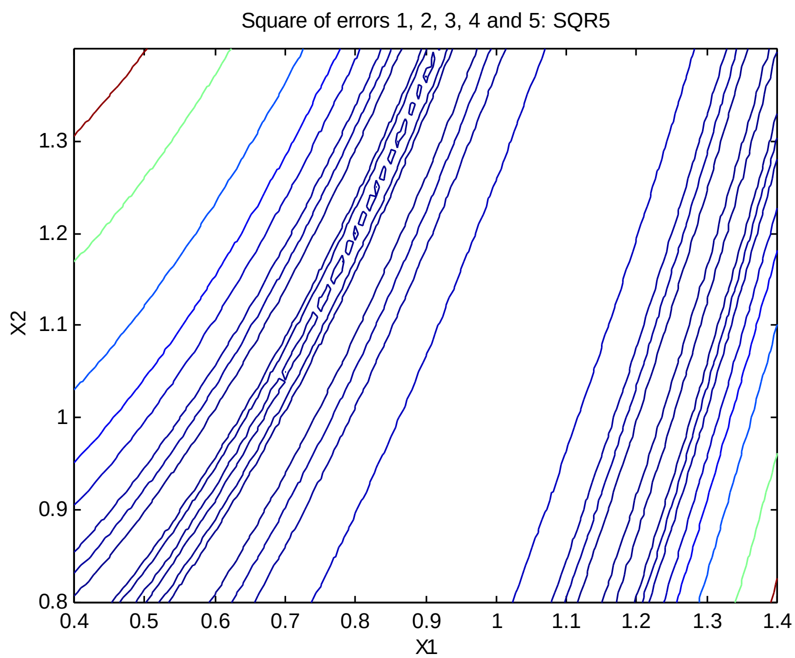


$$\Delta \boldsymbol{x}_1 = \begin{bmatrix} 0 \\ 0 \end{bmatrix},\ \Delta \boldsymbol{x}_2 = \begin{bmatrix} 0 \\ 0.1 \end{bmatrix},\ \Delta \boldsymbol{x}_3 = \begin{bmatrix} 0.1 \\ 0 \end{bmatrix},\ \Delta \boldsymbol{x}_4 = \begin{bmatrix} 0.1 \\ -0.1 \end{bmatrix},\ \Delta \boldsymbol{x}_5 = \begin{bmatrix} -0.1 \\ -0.1 \end{bmatrix}$$

l_2 Objective Function for 5-point Parameter Extraction



$$\Delta \boldsymbol{x}_1 = \begin{bmatrix} 0 \\ 0 \end{bmatrix},\ \Delta \boldsymbol{x}_2 = \begin{bmatrix} 0 \\ 3 \end{bmatrix},\ \Delta \boldsymbol{x}_3 = \begin{bmatrix} 3 \\ 0 \end{bmatrix},\ \Delta \boldsymbol{x}_4 = \begin{bmatrix} 3 \\ -3 \end{bmatrix},\ \Delta \boldsymbol{x}_5 = \begin{bmatrix} -3 \\ -3 \end{bmatrix}$$



$$\Delta \boldsymbol{x}_1 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \Delta \boldsymbol{x}_2 = \begin{bmatrix} 0 \\ 0.01 \end{bmatrix},\ \Delta \boldsymbol{x}_3 = \begin{bmatrix} 0.01 \\ 0 \end{bmatrix},\ \Delta \boldsymbol{x}_4 = \begin{bmatrix} 0.01 \\ -0.01 \end{bmatrix},\ \Delta \boldsymbol{x}_5 = \begin{bmatrix} -0.01 \\ -0.01 \end{bmatrix}$$

It can be visualized from the plots of ABS_5 and SQR_5 that the solution for the parameter extraction problem is $\boldsymbol{x} = \begin{bmatrix} 0.8 \\ 1.2 \end{bmatrix}$. It can also be verified that

$$R_c\left(\begin{bmatrix} 0.8 \\ 1.2 \end{bmatrix} + ?\ \boldsymbol{x}_1\right) = R_f\,(\boldsymbol{x}_c^* + ?\ \boldsymbol{x}_1) = 31.4,$$

$$R_c\left(\begin{bmatrix} 0.8 \\ 1.2 \end{bmatrix} + ?\ \boldsymbol{x}_2\right) = R_f\,(\boldsymbol{x}_c^* + ?\ \boldsymbol{x}_2) = 112.4,$$

$$R_c\left(\begin{bmatrix} 0.8 \\ 1.2 \end{bmatrix} + ?\ \boldsymbol{x}_3\right) = R_f\,(\boldsymbol{x}_c^* + ?\ \boldsymbol{x}_3) = 24.1\ ,$$

$$R_c\left(\begin{bmatrix} 0.8 \\ 1.2 \end{bmatrix} + ?\ \boldsymbol{x}_4\right) = R_f\,(\boldsymbol{x}_c^* + ?\ \boldsymbol{x}_4) = 98.1,\ \text{and}$$

$$R_c\left(\begin{bmatrix} 0.8 \\ 1.2 \end{bmatrix} + ?\ \boldsymbol{x}_5\right) = R_f\,(\boldsymbol{x}_c^* + ?\ \boldsymbol{x}_5) = 37.7$$

Space Mapped Solution

Step 0. $\boldsymbol{x}_f^{(1)} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \ \boldsymbol{B}^{(1)} = \boldsymbol{1}, \ j = 1$

Step 1. $R_f(\boldsymbol{x}_f^{(1)}) = 31.4$

Step 2. When $\boldsymbol{x}_c^{(1)} = \begin{bmatrix} 0.8 \\ 1.2 \end{bmatrix}, \ R_c(\boldsymbol{x}_c^{(1)}) = R_f(\boldsymbol{x}_f^{(1)})$

Step 3. $\boldsymbol{f}^{(1)} = \begin{bmatrix} 0.8 \\ 1.2 \end{bmatrix} - \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} -0.2 \\ 0.2 \end{bmatrix}$

Step 4. Since $\boldsymbol{B}^{(1)} = \boldsymbol{1}, \ \boldsymbol{h}^{(1)} = \begin{bmatrix} 0.2 \\ -0.2 \end{bmatrix}$

Step 5. Set $\boldsymbol{x}_f^{(2)} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} + \begin{bmatrix} 0.2 \\ -0.2 \end{bmatrix} = \begin{bmatrix} 1.2 \\ 0.8 \end{bmatrix}$

Step 6. $R_f(\boldsymbol{x}_f^{(2)}) = 0$

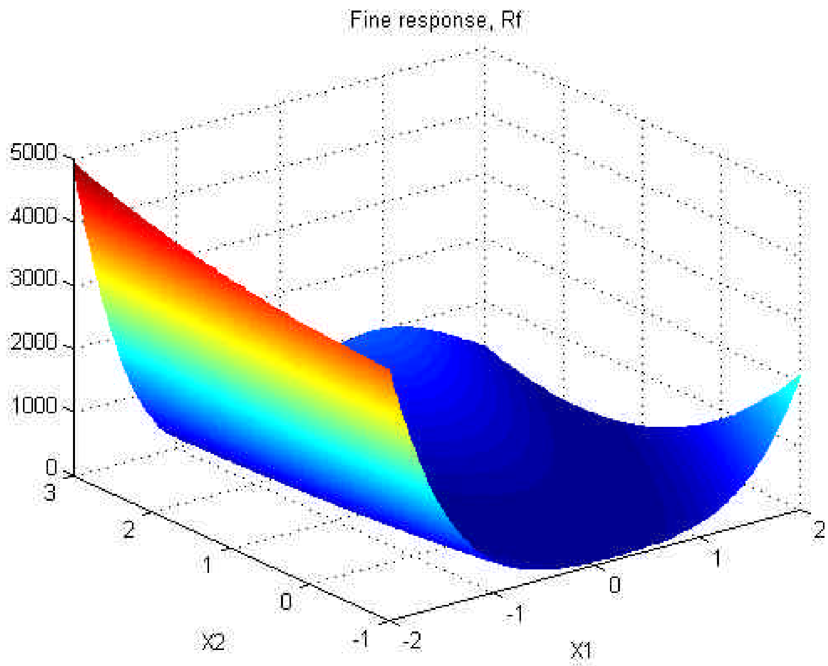
Step 7. $\boldsymbol{x}_c^{(2)} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$, because we know $R_c(\boldsymbol{x}_c^{(2)}) = 0$

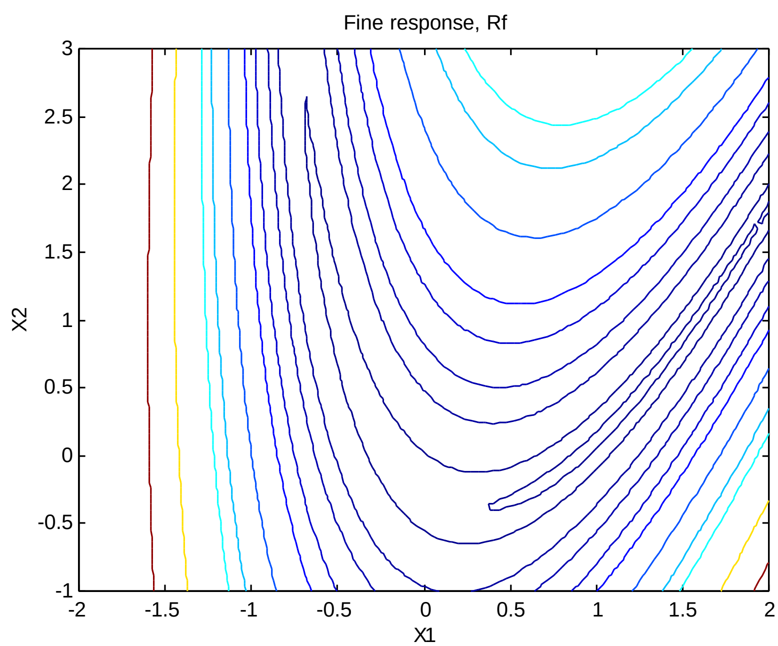
Step 8. Since $\boldsymbol{f}^{(2)} = \boldsymbol{x}_c^{(2)} - \boldsymbol{x}_c^{*} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$, then $\bar{\boldsymbol{x}}_f = \boldsymbol{x}_f^{(2)} = \begin{bmatrix} 1.2 \\ 0.8 \end{bmatrix}$

2nd Fine Model: Perturbed Rosenbrock Function

$$R_f\ (\boldsymbol{x}) \ = \ 100\,(u_2 - u_1^2)^2 + (1-u_1)^2 \quad \text{where}$$

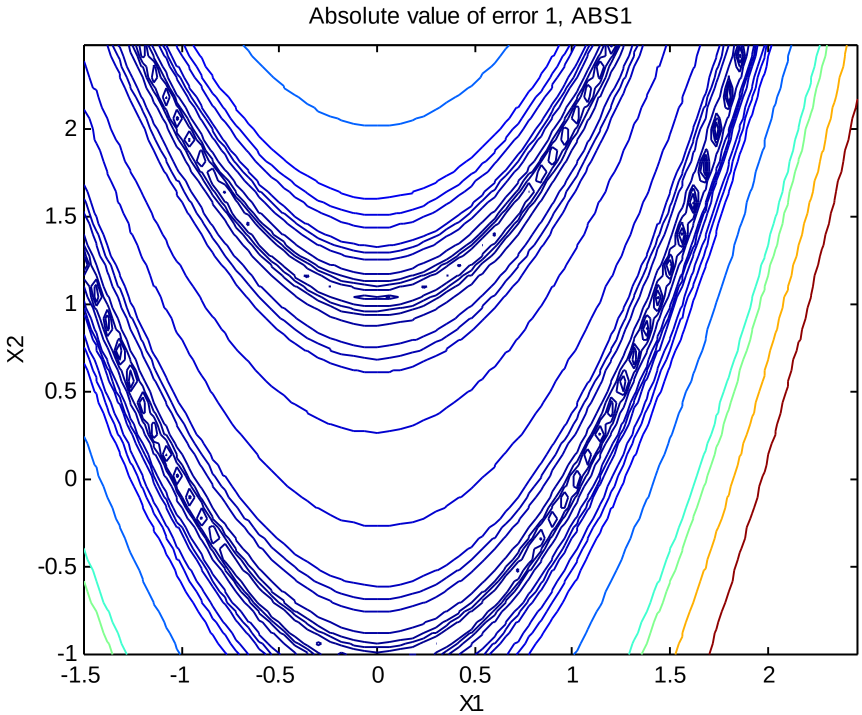
$$\boldsymbol{u} = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} 1.1 & -0.2 \\ 0.2 & 0.9 \end{bmatrix} \boldsymbol{x} + \begin{bmatrix} -0.3 \\ 0.3 \end{bmatrix}$$



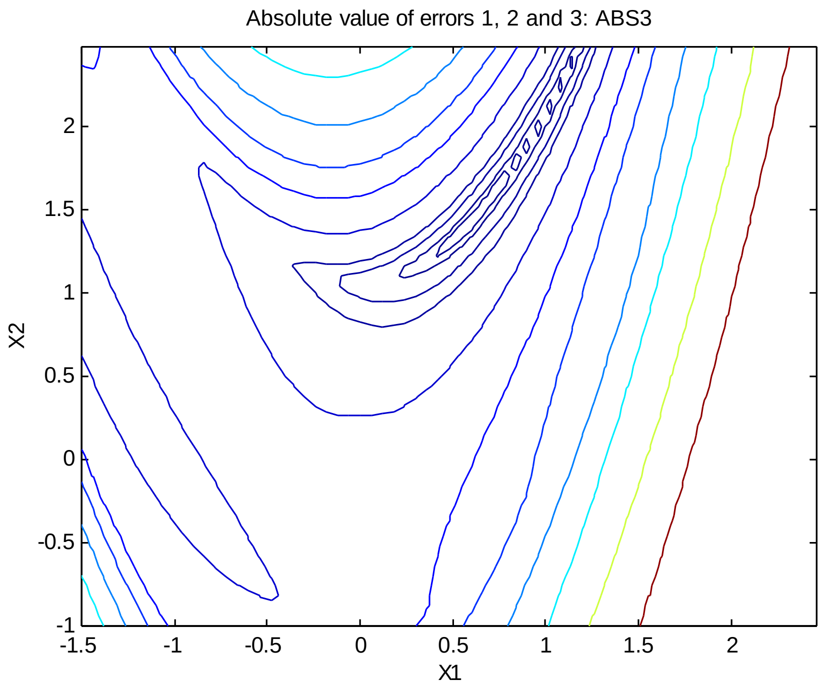


$$R_f\,(\boldsymbol{x}_c^*) = 108.3$$

First l_1 Parameter Extraction



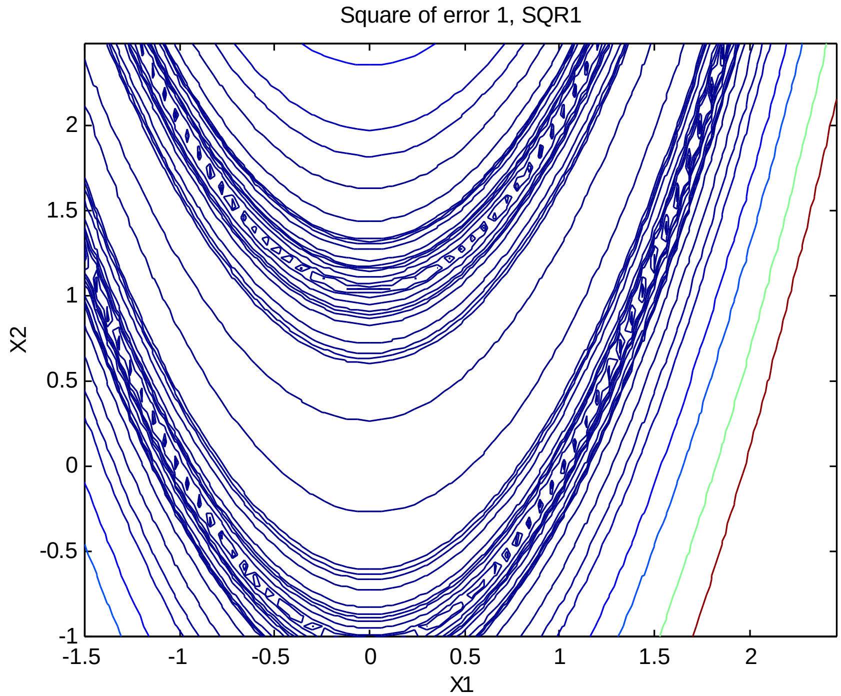
(a) single point parameter extraction



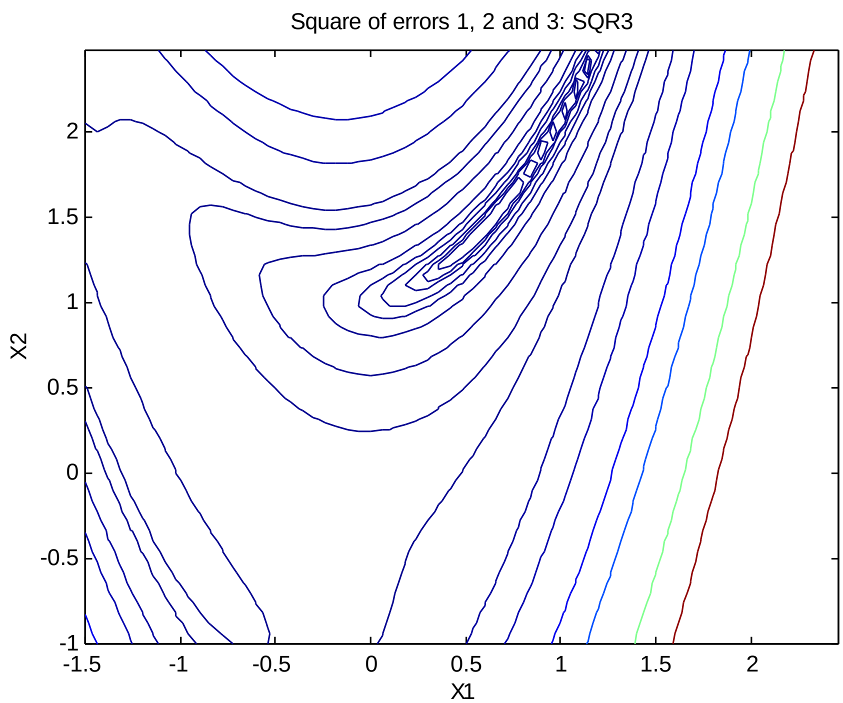
$$x_c^{(1)} = \begin{bmatrix} 0.6 \\ 1.4 \end{bmatrix}$$

(b) multiple point parameter extraction (2 additional points)

First l_2 Parameter Extraction



(b) single point parameter extraction



$$\boldsymbol{x}_c^{(1)} = \begin{bmatrix} 0.6 \\ 1.4 \end{bmatrix}$$

(b) multiple point parameter extraction (with 2 additional points)

Space Mapped Solution

Step 0. $\boldsymbol{x}_f^{(1)} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \ \boldsymbol{B}^{(1)} = \boldsymbol{1}, \ j = 1$

Step 1. $R_f(\boldsymbol{x}_f^{(1)}) = 108.32$

Step 2. When $\boldsymbol{x}_c^{(1)} = \begin{bmatrix} 0.6 \\ 1.4 \end{bmatrix}, \ R_c(\boldsymbol{x}_c^{(1)}) = R_f(\boldsymbol{x}_f^{(1)})$

Step 3. $\boldsymbol{f}^{(1)} = \begin{bmatrix} 0.6 \\ 1.4 \end{bmatrix} - \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} -0.4 \\ 0.4 \end{bmatrix}$

Step 4. Since $\boldsymbol{B}^{(1)} = \boldsymbol{1}, \ \boldsymbol{h}^{(1)} = \begin{bmatrix} 0.4 \\ -0.4 \end{bmatrix}$

Step 5. Set $\boldsymbol{x}_f^{(2)} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} + \begin{bmatrix} 0.4 \\ -0.4 \end{bmatrix} = \begin{bmatrix} 1.4 \\ 0.6 \end{bmatrix}$

Step 6. $R_f(\boldsymbol{x}_f^{(2)}) = 1.8207$

Step 7. When $\boldsymbol{x}_c^{(2)} = \begin{bmatrix} 1.12 \\ 1.12 \end{bmatrix}, \ R_c(\boldsymbol{x}_c^{(2)}) = R_f(\boldsymbol{x}_f^{(2)})$

Step 8. $\boldsymbol{f}^{(2)} = \begin{bmatrix} 1.12 \\ 1.12 \end{bmatrix} - \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0.12 \\ 0.12 \end{bmatrix}$

Step 9. $\boldsymbol{B}^{(2)} = \boldsymbol{B}^{(1)} + \dfrac{\boldsymbol{f}^{(2)}\boldsymbol{h}^{(1)^T}}{\boldsymbol{h}^{(1)^T}\boldsymbol{h}^{(1)}} = \begin{bmatrix} 1.15 & -0.15 \\ 0.15 & 0.85 \end{bmatrix}$

Step 4b. $\boldsymbol{h}^{(2)} = -\boldsymbol{B}^{(2)^{-1}}\boldsymbol{f}^{(2)} = \begin{bmatrix} -0.12 \\ -0.12 \end{bmatrix}$

Step 5b. Set $\boldsymbol{x}_f^{(3)} = \begin{bmatrix} 1.4 \\ 0.6 \end{bmatrix} + \begin{bmatrix} -0.12 \\ -0.12 \end{bmatrix} = \begin{bmatrix} 1.28 \\ 0.48 \end{bmatrix}$

Step 6b. $R_f(\boldsymbol{x}_f^{(3)}) = 0.1308$

Step 7b. When $\boldsymbol{x}_c^{(3)} = \begin{bmatrix} 1.012 \\ 0.988 \end{bmatrix}$, $R_c(\boldsymbol{x}_c^{(3)}) = R_f(\boldsymbol{x}_f^{(3)})$

Step 8b. $\boldsymbol{f}^{(3)} = \begin{bmatrix} 1.012 \\ 0.988 \end{bmatrix} - \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0.012 \\ -0.012 \end{bmatrix}$

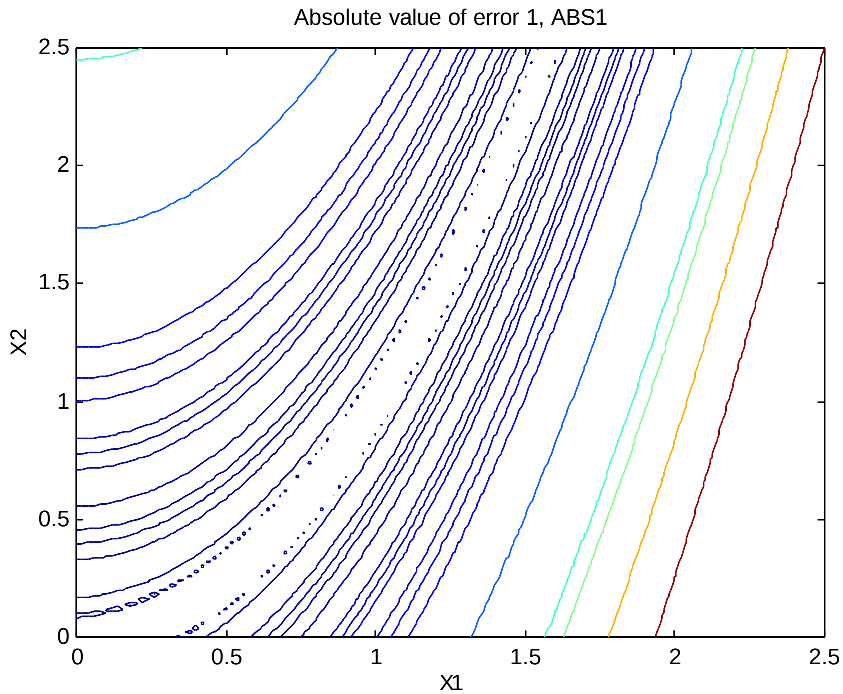
Step 9b. $\boldsymbol{B}^{(3)} = \boldsymbol{B}^{(2)} + \dfrac{\boldsymbol{f}^{(3)}\boldsymbol{h}^{(2)^T}}{\boldsymbol{h}^{(2)^T}\boldsymbol{h}^{(2)}} = \begin{bmatrix} 1.1 & -0.2 \\ 0.2 & 0.9 \end{bmatrix}$

Step 4c. $\boldsymbol{h}^{(3)} = -\boldsymbol{B}^{(3)^{-1}}\boldsymbol{f}^{(3)} = \begin{bmatrix} -0.0082 \\ 0.0151 \end{bmatrix}$

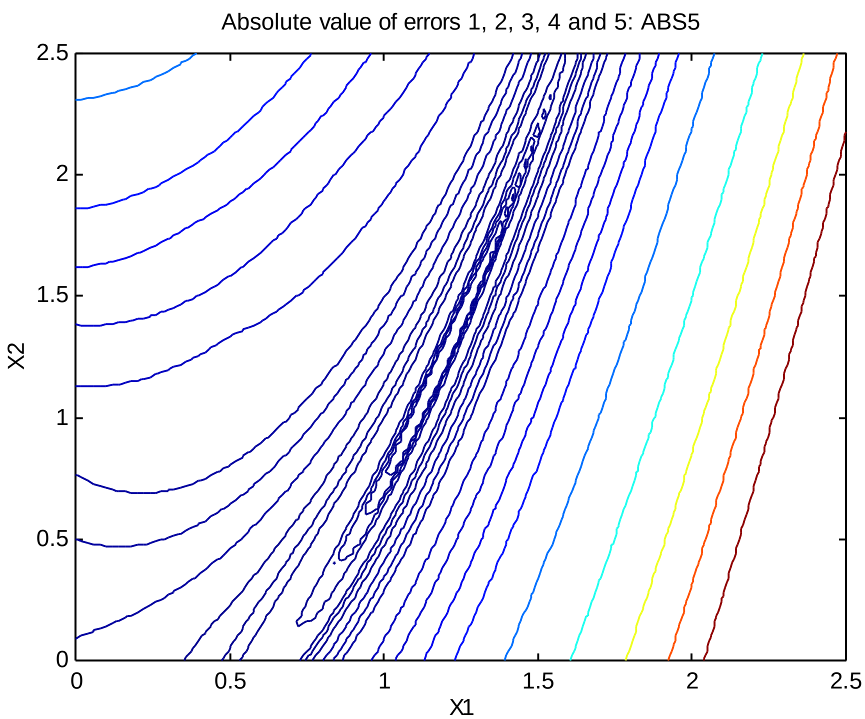
Step 5c. Set $\boldsymbol{x}_f^{(4)} = \begin{bmatrix} 1.28 \\ 0.48 \end{bmatrix} + \begin{bmatrix} -0.0082 \\ 0.0151 \end{bmatrix} = \begin{bmatrix} 1.2718 \\ 0.4951 \end{bmatrix}$

Step 6c. $R_f(\boldsymbol{x}_f^{(4)}) = 0$, then $\bar{\boldsymbol{x}}_f = \boldsymbol{x}_f^{(4)}$ and the algorithm ends

Second l_1 Parameter Extraction



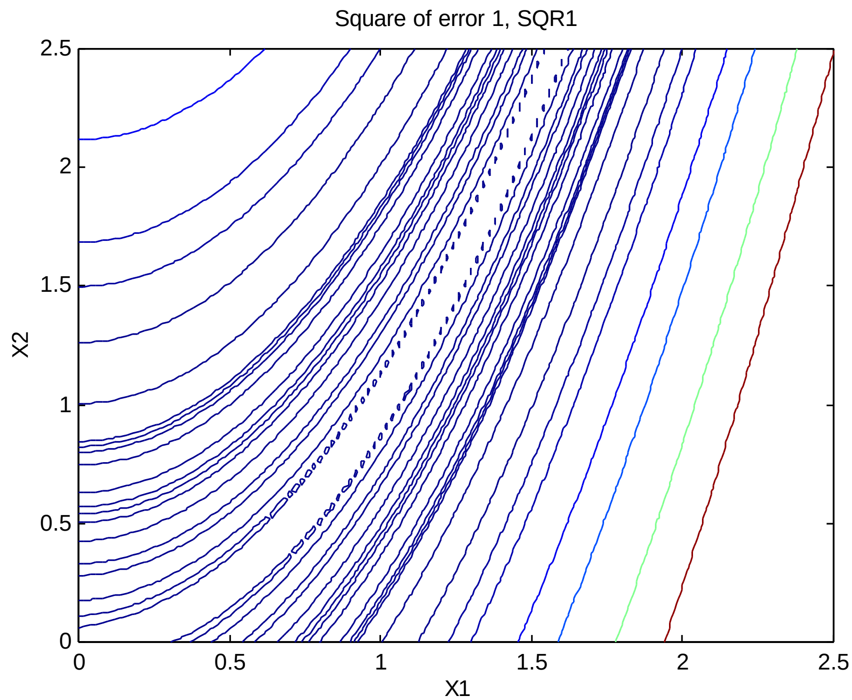
(a) single point parameter extraction



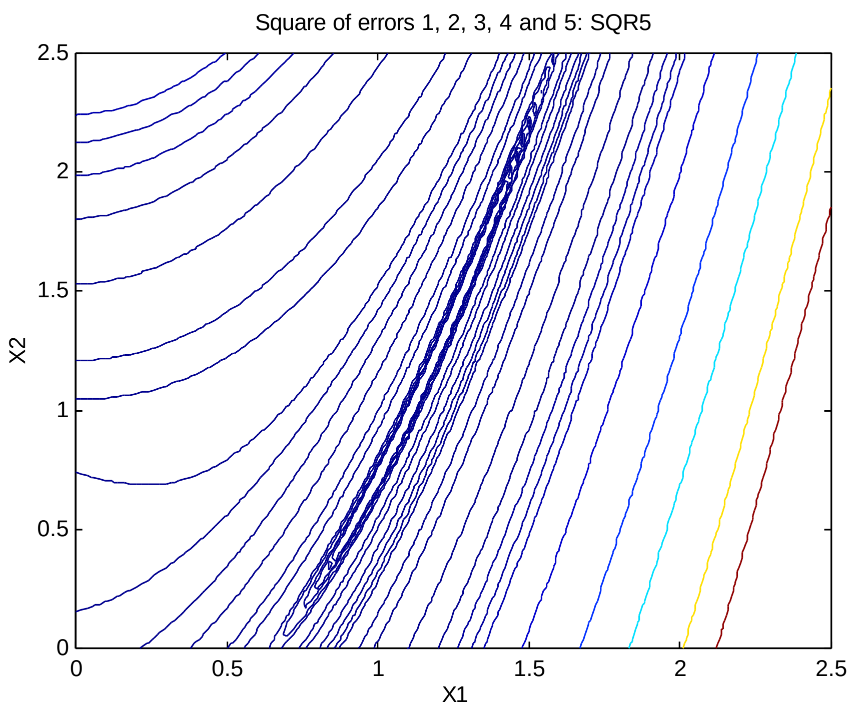
$$x_c^{(2)} = \begin{bmatrix} 1.12 \\ 1.12 \end{bmatrix}$$

(b) multiple point parameter extraction (4 additional points)

Second l_2 Parameter Extraction



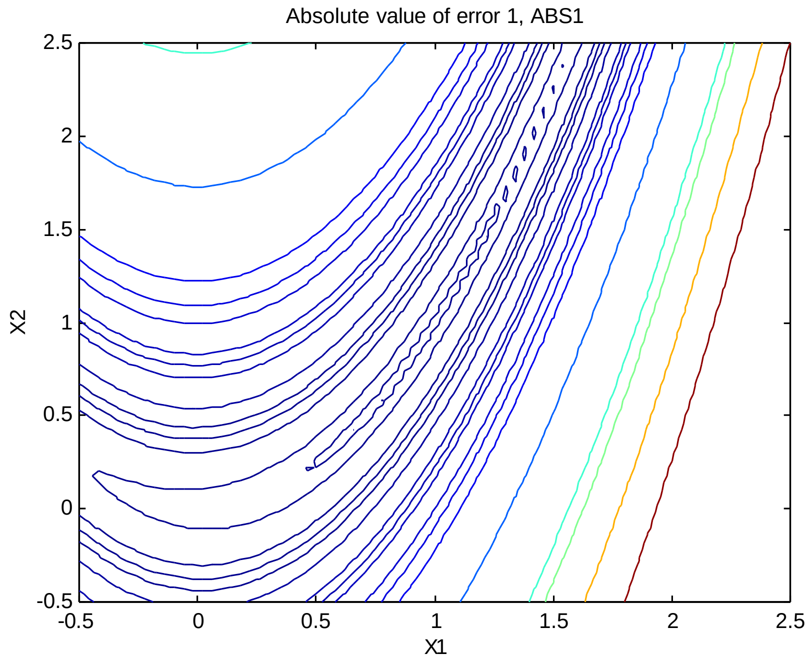
(b) single point parameter extraction



$$\boldsymbol{x}_c^{(2)} = \begin{bmatrix} 1.12 \\ 1.12 \end{bmatrix}$$

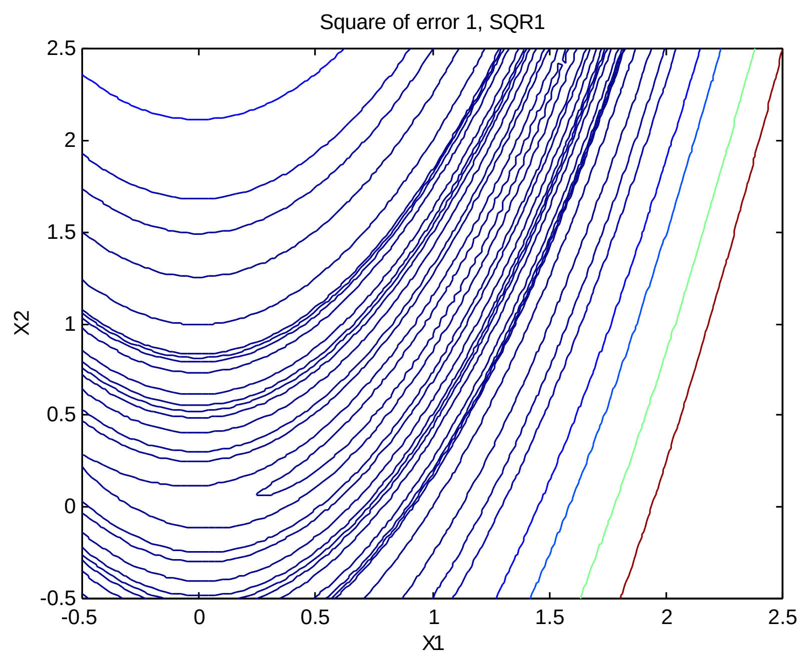
(b) multiple point parameter extraction (with 4 additional points)

Third Parameter Extraction



$$\boldsymbol{x}_c^{(3)} = \begin{bmatrix} 1.02 \\ 0.98 \end{bmatrix}$$

(a) single point l_1 parameter extraction



$$\boldsymbol{x}_c^{(3)} = \begin{bmatrix} 1.02 \\ 0.98 \end{bmatrix}$$

(b) single point l_2 parameter extraction

Conclusions

the Aggressive Space Mapping (ASM) algorithm with multiple point parameter extraction was illustrated making use of two analytical Rosenbrock functions

the improvement in the uniqueness of the parameter extraction due to multiple point matching was graphically illustrated

the sizes and directions of the $\Delta \boldsymbol{x}_i$ used for multiple point parameter extraction affects the degree of improvement in the uniqueness of the parameter extraction solution

the ASM algorithm was executed in a step by step fashion, and the corresponding space mapped solution was accomplished after only one SM iteration for the case of shifting in the input parameters, and after three SM iterations for the case of shifting, scaling and rotation of the input parameters

the mapping calculated by the ASM algorithm is equal to the original linear mapping between both parameter spaces