THE ALGORITHM

1. Given $\boldsymbol{x}_f^{(1)} = \boldsymbol{x}_c^*$, $\lambda^{(1)} = 1$, $\delta^{(1)}$, α, $\boldsymbol{J}_f^{(1)} = \boldsymbol{J}_c^*$ and i=1.

2. Construct $V^{(i)}$.

3. Apply the optimization procedure

$$[\boldsymbol{B}^{(i)}, \boldsymbol{s}^{(i)}, \boldsymbol{t}^{(i)}, \sigma^{(i)}, \boldsymbol{c}^{(i)}, \gamma^{(i)}] = arg\left\{ \min_{\boldsymbol{B}, \boldsymbol{s}, \boldsymbol{t}, \sigma, \boldsymbol{c}, \gamma} \left\| \begin{bmatrix} \boldsymbol{e}_1^T & \boldsymbol{e}_2^T & \cdots & \boldsymbol{e}_N^T \end{bmatrix}^T \right\| \right\}$$

$$\boldsymbol{e}_p = \boldsymbol{R}_c(\boldsymbol{P}^{(i)}(\boldsymbol{x}_f^{(j)}, \psi_k), P_\psi^{(i)}(\boldsymbol{x}_f^{(j)}, \psi_k)) - \boldsymbol{R}_f(\boldsymbol{x}_f^{(j)}, \psi_k)\ \forall\ \boldsymbol{x}_f^{(j)} \in V^{(i)},\ \forall\, \psi_k,\ k = 1, 2, \ldots, N_\psi,\ N = N_P N_\psi$$

to obtain the mapping parameters.

4. Construct the surrogate model

$$\boldsymbol{R}_s^{(i)}(\boldsymbol{x}_f) = \lambda^{(i)} \boldsymbol{R}_m^{(i)}(\boldsymbol{x}_f) + (1 - \lambda^{(i)})(\boldsymbol{R}_f(\boldsymbol{x}_f^{(i)}) + \boldsymbol{J}_f^{(i)} \Delta \boldsymbol{x}_f),\ \lambda^{(i)} \in [0, 1]$$

where $\boldsymbol{R}_f(\boldsymbol{x}_f, \psi_j) \approx \boldsymbol{R}_m^{(i)}(\boldsymbol{x}_f, \psi_j) = \boldsymbol{R}_c(\boldsymbol{P}^{(i)}(\boldsymbol{x}_f, \psi_j), P_\psi^{(i)}(\boldsymbol{x}_f, \psi_j))$, j=1, 2, …, N_ψ

and
$$\begin{bmatrix} \boldsymbol{P}^{(i)}(\boldsymbol{x}_f, \psi_j) \\ P_\psi^{(i)}(\boldsymbol{x}_f, \psi_j) \end{bmatrix} = \begin{bmatrix} \boldsymbol{B}^{(i)} & \boldsymbol{s}^{(i)} \\ \boldsymbol{t}^{(i)T} & \sigma^{(i)} \end{bmatrix} \begin{bmatrix} \Delta \boldsymbol{x}_f \\ \psi_j \end{bmatrix} + \begin{bmatrix} \boldsymbol{c}^{(i)} \\ \gamma^{(i)} \end{bmatrix}$$

5. Obtain the suggested step $\boldsymbol{h}^{(i)}$ by solving

$$\boldsymbol{h}^{(i)} = arg\left\{ \min_{\boldsymbol{h}^{(i)}} U(\boldsymbol{R}_s^{(i)}(\boldsymbol{x}_f^{(i)} + \boldsymbol{h}^{(i)})) \right\},\ \left\| \boldsymbol{h}^{(i)} \right\| \le \delta^{(i)}$$

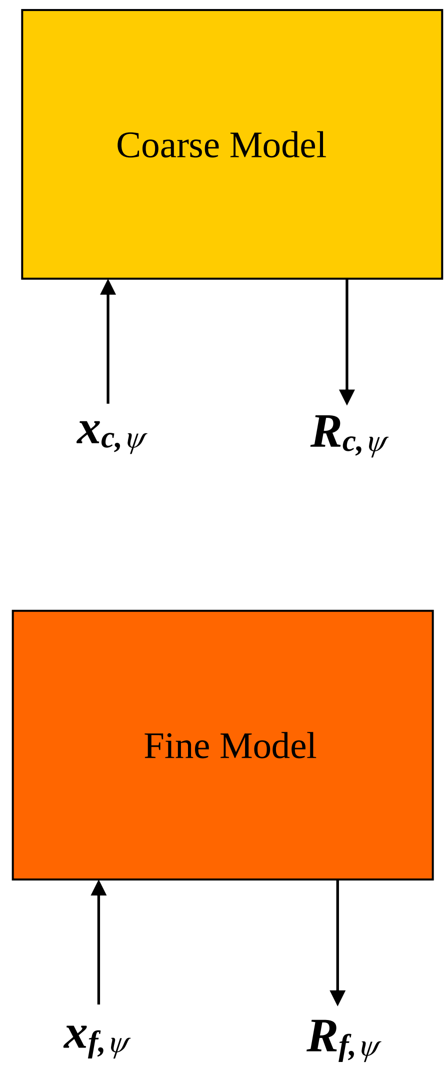
6. If $U(\boldsymbol{R}_f(\boldsymbol{x}_f^{(i)} + \boldsymbol{h}^{(i)})) < U(\boldsymbol{R}_f(\boldsymbol{x}_f^{(i)}))$, set $\boldsymbol{x}_f^{(i+1)} = \boldsymbol{x}_f^{(i)} + \boldsymbol{h}^{(i)}$ else $\boldsymbol{x}_f^{(i+1)} = \boldsymbol{x}_f^{(i)}$.

7. Update $\boldsymbol{J}_f^{(i)}$, $\delta^{(i)}$ and $\lambda^{(i)}$.

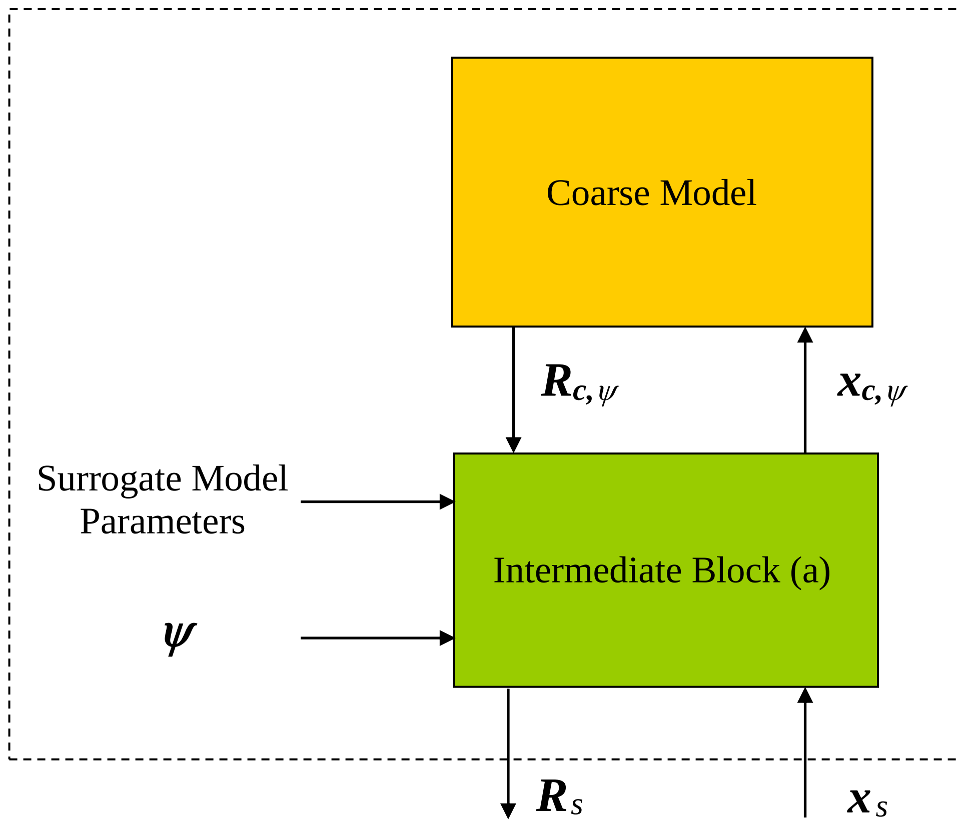
8. If the stopping criterion is satisfied stop.

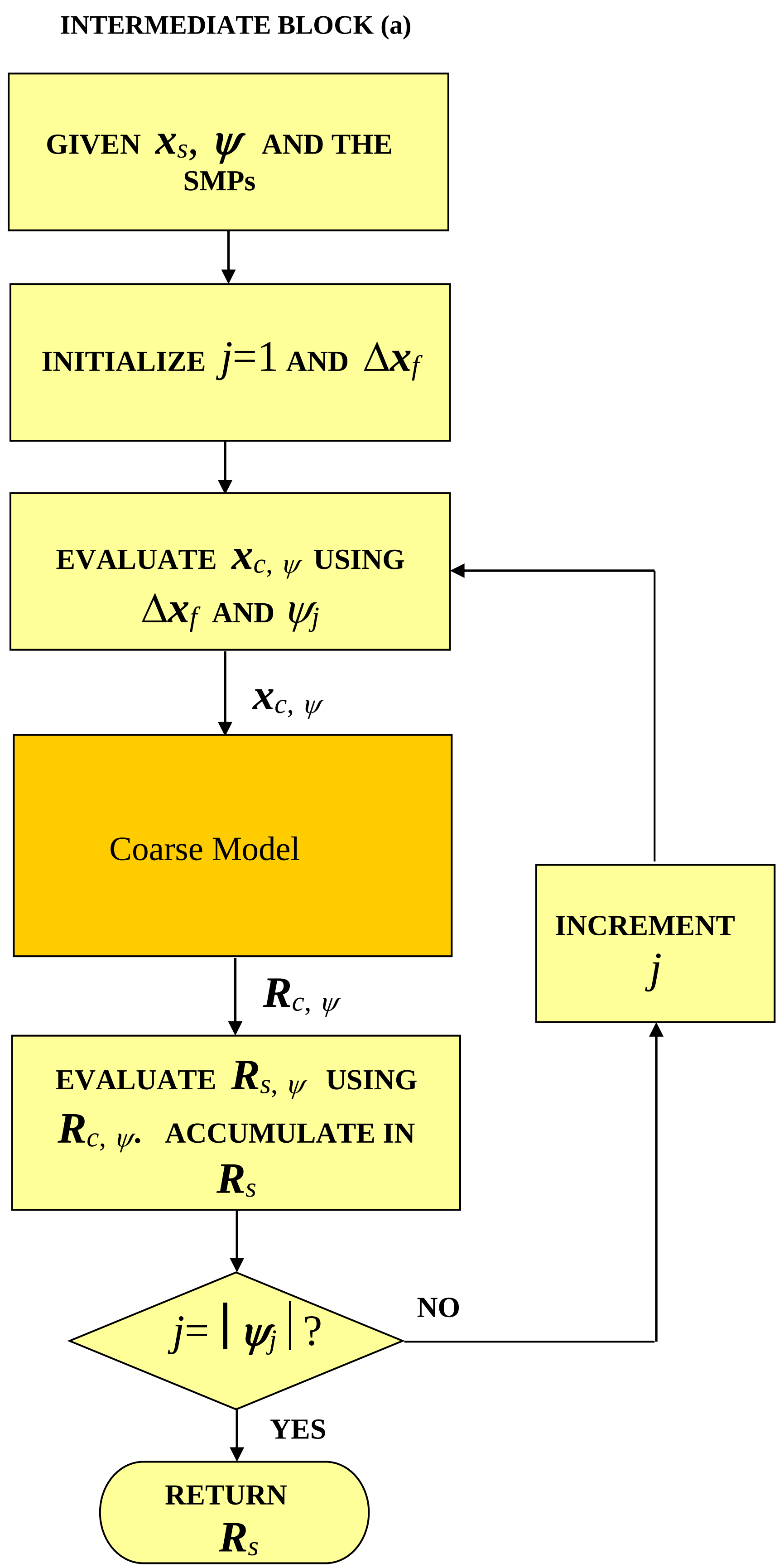
9. Set i=i+1 and go to step 2.

COARSE AND FINE MODELS

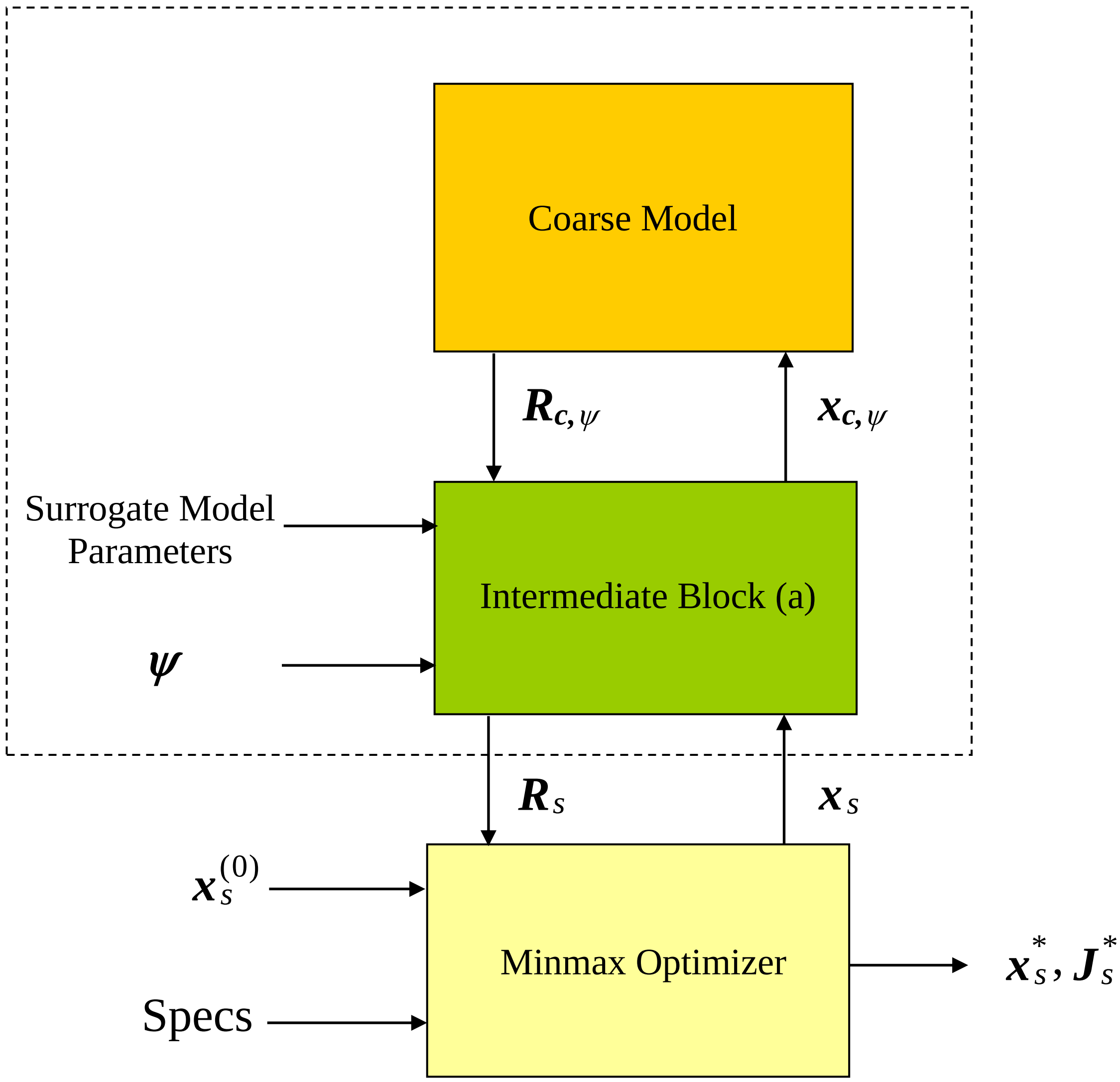


THE SURROGATE MODEL




INTERMEDIATE BLOCK (a)
GIVEN $\boldsymbol{x}_s$, ψ AND THE SMPs
INITIALIZE j=1 AND $\Delta\boldsymbol{x}_f$
EVALUATE $\boldsymbol{x}_{c,\psi}$ USING $\Delta\boldsymbol{x}_f$ AND ψ_j
$\boldsymbol{x}_{c,\psi}$
Coarse Model
$\boldsymbol{R}_{c,\psi}$
EVALUATE $\boldsymbol{R}_{s,\psi}$ USING $\boldsymbol{R}_{c,\psi}$. ACCUMULATE IN $\boldsymbol{R}_s$
$j=|\psi_j|$?
NO
INCREMENT j
YES
RETURN $\boldsymbol{R}_s$

MINMAX OPTIMIZATION OF THE SURROGATE MODEL



CASE STUDY (a)

The coarse model is given by

$$\boldsymbol{R}_c(\boldsymbol{x}_c, p) = 100p\left(x_{c2} - x_{c1}^2\right)^2 + p^2\left(1 - x_{c1}\right)^2 + 5$$

The fine model is given by

$$\boldsymbol{R}_f(\boldsymbol{x}_f, p_f) = \boldsymbol{R}_c(\boldsymbol{x}_c, p_c)$$

where

$$\begin{bmatrix} \boldsymbol{x}_c \\ p_c \end{bmatrix} = \begin{bmatrix} \boldsymbol{B} & \boldsymbol{s} \\ \boldsymbol{t} & \sigma \end{bmatrix} \begin{bmatrix} \Delta \boldsymbol{x}_f \\ p_f \end{bmatrix} + \begin{bmatrix} \boldsymbol{c} \\ \gamma \end{bmatrix},$$

and

$$\boldsymbol{B} = \begin{bmatrix} 1.1 & -0.2 \\ 0.2 & 0.9 \end{bmatrix}, \quad \boldsymbol{s} = \boldsymbol{t} = \begin{bmatrix} 0.01 \\ 0.01 \end{bmatrix}, \quad \boldsymbol{c} = \begin{bmatrix} -0.3 \\ 0.3 \end{bmatrix}, \quad \sigma = 1.1, \quad \gamma = 0.05 \quad \text{and}$$

$$\Delta \boldsymbol{x}_f = \boldsymbol{x}_f - \begin{bmatrix} 1.0 \\ 1.0 \end{bmatrix}$$

Three values of the parameter p is considered, i.e.,

$$\boldsymbol{R}_f(\boldsymbol{x}_f) = \begin{bmatrix} \boldsymbol{R}_f(\boldsymbol{x}_f, p_{f1}) \\ \boldsymbol{R}_f(\boldsymbol{x}_f, p_{f2}) \\ \boldsymbol{R}_f(\boldsymbol{x}_f, p_{f3}) \end{bmatrix} = \begin{bmatrix} \boldsymbol{R}_f(\boldsymbol{x}_f, 0.9) \\ \boldsymbol{R}_f(\boldsymbol{x}_f, 1.0) \\ \boldsymbol{R}_f(\boldsymbol{x}_f, 1.1) \end{bmatrix}$$

MATLAB FILES

```
function Rcp=Get_Coarse_Response(xcp);
xc1=xcp(1,1);
xc2=xcp(2,1);
pc=xcp(3,1);
Rc1p=(100*pc*(xc2-xc1*xc1)*(xc2-xc1*xc1))+(pc*pc*(1-xc1)*(1-
xc1))+5;
Rcp=Rc1p;
```

```
function Rs=Get_Surrogate_Response(xs);
load SMPs.mat %Load the surrogate model parameters
% These parameters are B,c,s,t,segmma,gama,Jfi,Rfi,xfi,lamda
B
c
s
t
segmma
gama
Jfi
Rfi
xfi
lamda
load P.mat %load matrix containing the values of the vector t
NP=size(P,1)
m=size(Jfi,1)
Nr=m/NP
Rs=zeros(m,1);
dx=xs-xfi
for j=1:NP
%1) The fine model point
   xs
%2)Get the jth value of the independent parameter
   Pfj=P(j,1)
%3)Get the corresponding coarse model point
   xc=B*dx+s*Pfj+c
%4)Get the corresponding jth coarse model independent parameter
   Pcj=t'*dx+segmma*Pfj+gama
%5)Construct the composed coarse model vector
   xcp=[xc
      Pcj]
%6)Get coarse model response at the transformed coarse model
%point and frequency
   Rcp=Get_Coarse_Response(xcp)
%7)Get the Jacobian of the responses corresponding to the jth
%value of P
   Jip=Jfi(((j-1)*Nr+1):(j*Nr),:)
%8)Get the fine model responses corresponding to the jth value of
%P
   Rip=Rfi(((j-1)*Nr+1):(j*Nr),1)
%9)Construct the surrogate model response at the jth value of P
   Rsp=lamda*Rcp+(1-lamda)*(Rip+Jip*dx)
%10)Accumulate this value in the vector Rs
   Rs(((j-1)*Nr+1):(j*Nr),1)=Rsp
   pause
end
```

RESULTS FOR CASE STUDY (a)

```
B =
    1.1000   -0.2000
    0.2000    0.9000

c =
   -0.3000
    0.3000

s =
    0.0100
    0.0100

t =
    0.0100
    0.0100

segmma =
    1.1000

gama =
    0.0500

Jfi =

   36.3803   37.8284
   40.0509   42.1956
   43.6871   46.6336

Rfi =
   12.0359
   13.0693
   14.1579

xfi =
     1
     1

lamda =
    0.5000

NP =
     3

m =
     3
```

RESULTS FOR CASE STUDY (a) (Cont'd)

Nr =
1

dx =
0.2000
-0.2000

First Pass

1) xs =
1.2000
0.8000

2) Pfj =
0.9000

3) xc =
-0.0310
0.1690

4) Pcj =
1.0400

5) xcp =
-0.0310
0.1690
1.0400

6) Rcp =
9.0864

7) Jip =
36.3803   37.8284

8) Rip =
12.0359

9) Rsp =
10.4163

10) Rs =
10.4163
0
0

RESULTS FOR CASE STUDY (a) (Cont'd)

Second Pass

1) xs =
1.2000
0.8000

2) Pfj =
1

3) xc =
-0.0300
0.1700

4) Pcj =
1.1500

5) xcp =
-0.0300
0.1700
1.1500

6) Rcp =
9.6914

7) Jip =
40.0509 42.1956

8) Rip =
13.0693

9) Rsp =
11.1659

10) Rs =
10.4163
11.1659
0

Third Pass
1) xs =
1.2000
0.8000

2) Pfj =
1.1000

RESULTS FOR CASE STUDY (a) (Cont'd)

3) xc =
-0.0290
0.1710

4) Pcj =
1.2600

5) xcp =
-0.0290
0.1710
1.2600

6) Rcp =
10.3292

7) Jip =
43.6871 46.6336

8) Rip =
14.1579

9) Rsp =
11.9489

10) Rs =
10.4163
11.1659
11.9489

CASE STUDY (b)

The coarse model is given by

$$\boldsymbol{R}_c(\boldsymbol{x}_c, p) = \begin{bmatrix} 100\,p\left(x_{c2} - x_{c1}^2\right)^2 + p^2\left(1 - x_{c1}\right)^2 + 5 \\ 100\,p\left(x_{c1} - 1\right)^2 + 300\,p^2\left(1 - x_{c2}\right)^2 + 10 \end{bmatrix}$$

The fine model is given by

$$\boldsymbol{R}_f(\boldsymbol{x}_f, p_f) = \boldsymbol{R}_c(\boldsymbol{x}_c, p_c)$$

where

$$\begin{bmatrix} \boldsymbol{x}_c \\ p_c \end{bmatrix} = \begin{bmatrix} \boldsymbol{B} & \boldsymbol{s} \\ \boldsymbol{t} & \sigma \end{bmatrix} \begin{bmatrix} \Delta \boldsymbol{x}_f \\ p_f \end{bmatrix} + \begin{bmatrix} \boldsymbol{c} \\ \gamma \end{bmatrix},$$

and

$$\boldsymbol{B} = \begin{bmatrix} 1.1 & -0.2 \\ 0.2 & 0.9 \end{bmatrix}, \quad \boldsymbol{s} = \boldsymbol{t} = \begin{bmatrix} 0.01 \\ 0.01 \end{bmatrix}, \quad \boldsymbol{c} = \begin{bmatrix} -0.3 \\ 0.3 \end{bmatrix}, \quad \sigma = 1.1, \quad \gamma = 0.05 \quad \text{and}$$

$$\Delta \boldsymbol{x}_f = \boldsymbol{x}_f - \begin{bmatrix} 1.0 \\ 1.0 \end{bmatrix}$$

Three values of the parameter p is considered, i.e.,

$$\boldsymbol{R}_f(\boldsymbol{x}_f) = \begin{bmatrix} \boldsymbol{R}_f(\boldsymbol{x}_f, p_{f1}) \\ \boldsymbol{R}_f(\boldsymbol{x}_f, p_{f2}) \\ \boldsymbol{R}_f(\boldsymbol{x}_f, p_{f3}) \end{bmatrix} = \begin{bmatrix} \boldsymbol{R}_f(\boldsymbol{x}_f, 0.9) \\ \boldsymbol{R}_f(\boldsymbol{x}_f, 1.0) \\ \boldsymbol{R}_f(\boldsymbol{x}_f, 1.1) \end{bmatrix}$$

MATLAB FILES

```
function Rcp=Get_Coarse_Response(xcp);
xc1=xcp(1,1);
xc2=xcp(2,1);
pc=xcp(3,1);
Rc1p=(100*pc*(xc2-xc1*xc1)*(xc2-xc1*xc1))+(pc*pc*(1-xc1)*(1-
xc1))+5;
Rc2p=(100*pc*(xc1-1)*(xc1-1))+(300*pc*pc*(1-xc2)*(1-xc2))+10;
Rcp=[ Rc1p
      Rc2p];
```

RESULTS FOR CASE STUDY (b)

B =

1.1000	**-0.2000**
0.2000	**0.9000**

c =

-0.3000
0.3000

s =

0.0100
0.0100

t =

0.0100
0.0100

segmma =

1.1000

gama =

0.0500

Jfi =

36.3803	**37.8284**
-379.0781	**-342.6384**
40.0509	**42.1956**
-429.4207	**-425.2945**
43.6871	**46.6336**
-481.6569	**-516.6460**

Rfi =

12.0359
338.2679
13.0693
390.2642
14.1579
445.4518

xfi =

1
1

lamda =

0.5000

RESULTS FOR CASE STUDY (b) (Cont'd)

NP =
3

m =
6

Nr =
2

dx =
0.2000
-0.2000

First Pass

1) xs =
1.2000
0.8000

2) Pfj =
0.9000

3) xc =
-0.0310
0.1690

4) Pcj =
1.0400

5) xcp =
-0.0310
0.1690
1.0400

6) Rcp =
9.0864
344.6212

7) Jip =
36.3803 37.8284
-379.0781 -342.6384

8) Rip =
12.0359
338.2679

RESULTS FOR CASE STUDY (b) (Cont'd)

9) Rsp =
10.4163
337.8005

10) Rs =
10.4163
337.8005
0
0
0
0

Second Pass

1) xs =
1.2000
0.8000

2) Pfj =
1

3) xc =
-0.0300
0.1700

4) Pcj =
1.1500

5) xcp =
-0.0300
0.1700
1.1500

6) Rcp =
9.6914
405.3246

7) Jip =
40.0509 42.1956
-429.4207 -425.2945

8) Rip =
13.0693
390.2642

9) Rsp =
11.1659
397.3817

10) Rs =
10.4163
337.8005
11.1659
397.3817
0
0

Third Pass

1) xs =
1.2000
0.8000

2) Pfj =
1.1000

3) xc =
-0.0290
0.1710

4) Pcj =
1.2600

5) xcp =
-0.0290
0.1710
1.2600

6) Rcp =
10.3292
470.7331

7) Jip =

43.6871	46.6336
-481.6569	-516.6460

8) Rip =
14.1579
445.4518

RESULTS FOR CASE STUDY (b) (Cont'd)

9) Rsp =

11.9489
461.5913

10) Rs =

10.4163
337.8005
11.1659
397.3817
11.9489
461.5913