

# BROADBAND PHYSICS-BASED MODELING OF MICROWAVE PASSIVE DEVICES THROUGH FREQUENCY MAPPING

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## ABSTRACT

We present a new methodology to develop physics-based models for passive components. We coherently integrate full-wave EM simulations, artificial neural networks, multivariable rational functions, dimensional analysis and frequency mapping. We consider frequency-independent and frequency-dependent models. Various examples include a microstrip right angle bend and a CPW short-circuit stub.

## INTRODUCTION

We present a new computer-aided modeling methodology to develop physics-based empirical models (“coarse” models) for microwave passive components. We consider frequency-independent empirical models (FIEM) and frequency-dependent empirical models (FDEM). In the FDEM we use the frequency mapping approach [1] to introduce frequency dependency into the model elements. We also exploit the odd property of the frequency mapping, that is the transformed or “coarse” model frequency must be an odd function of frequency. ANNs or MRFs [2] are used to approximate the model elements as well as the frequency mapping. MRFs enable us to transform a simple FDEM to an equivalent FIEM. This transformation can be expedited by impedance synthesis [3] as we will see in the examples. Dimensional analysis [4,5]

This work was supported in part by the Natural Sciences and Engineering Research Council of Canada (NSERC) under Grants OGP0007239, STP0201832 and through the Micronet Network of Centres of Excellence.

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reduces the number of input parameters to the ANN or the MRF. We illustrate the process through various examples, including a microstrip right angle bend and a CPW short-circuit stub.

## FREQUENCY-INDEPENDENT EMPIRICAL MODELS (FIEM)

Consider a microwave component modeled by a fine model (typically a suitable full-wave EM simulator) and a coarse, equivalent circuit (empirical) model. We assume that the model topology is known but empirical formulas are to be determined. Let  $\mathbf{x}_f$  be an  $n$ -dimensional vector representing the parameters of the component,  $\mathbf{R}_f$  is a vector representing the fine model responses, e.g., the scattering parameters,  $\omega$  is the frequency and  $\mathbf{R}_c$  is a vector representing the coarse model responses. The development of the FIEM is carried out by evaluating an  $l$ -dimensional vector  $\mathbf{y}$  which represents the empirical formulas. Applying dimensional analysis  $\mathbf{y}$  becomes a function of an  $n_r$ -dimensional vector  $\mathbf{x}_r$  ( $n_r < n$ ), which we call the reduced input parameter vector. Through ANNs or MRFs [2] we approximate  $\mathbf{y}$  in a region of parameters and frequency as

$$\mathbf{y} \approx \mathbf{Q}(\mathbf{x}_r, \mathbf{w}) \quad (1)$$

where  $\mathbf{w}$  is the set of parameters of the ANN or the MRF. This set  $\mathbf{w}$  is evaluated by solving

$$\min_{\mathbf{w}} \left\| [\mathbf{e}_{11}^T \cdots \mathbf{e}_{1M}^T \mathbf{e}_{21}^T \cdots \mathbf{e}_{2M}^T \cdots \mathbf{e}_{N1}^T \cdots \mathbf{e}_{NM}^T]^T \right\| \quad (2)$$

where  $\| \cdot \|$  is a suitable norm,  $N$  is the total number of training points,  $M$  is the number of frequency points per frequency sweep and  $\mathbf{e}_{ij}$  is an error vector given by

$$\mathbf{e}_{ij} = \mathbf{R}_f(\mathbf{x}_{f_i}, \omega_j) - \mathbf{R}_c(\mathbf{Q}(\mathbf{x}_{r_i}, \mathbf{w}), \omega_j) \quad (3)$$

Training points are selected according to the Central Composite Design (CCD) [6]. More points are added if necessary.

### FREQUENCY-DEPENDENT EMPIRICAL MODELS (FDEM)

Two approaches can be used to introduce frequency dependency to the elements of the FDEM. One approach is to make the reduced vector  $\mathbf{x}_r$  and hence  $\mathbf{y}$  depend on frequency as well as other physical parameters. The second approach exploits the frequency mapping concept [1], where we simulate the coarse model at a different frequency from the fine model. We call this frequency the coarse model frequency  $\omega_c$ . Frequency transformations (mappings) have roots in classical filter design, for example, low-pass to band-pass transformations [7]. Dimensional analysis is also applied to determine the dependency of  $\omega_c$  on  $\omega$  as well as the physical parameters. Both  $\mathbf{y}$  and  $\omega_c$  can be approximated by an ANN or an MRF as

$$\mathbf{y} \approx \mathcal{Q}(\mathbf{x}_r, \mathbf{w}_1), \quad \omega_c \approx \Omega(\mathbf{x}_r, \omega, \mathbf{w}_2) \quad (4)$$

where  $\mathbf{w}_1$  and  $\mathbf{w}_2$  are the parameters of the ANN or MRF. These parameters are evaluated by solving an optimization problem similar to (2).

### PROPERTIES OF THE FREQUENCY MAPPING

Simulating the coarse model at a different frequency from that of the fine model implicitly introduces frequency dependency to the coarse model. For example, if the device is lossless the model contains only lossless lumped-elements (inductors and capacitors). In this case, an FDEM simulated at  $\omega_c$  and with a circuit element vector  $\mathbf{y}$  is equivalent to an FDEM simulated at  $\omega$  and circuit element vector  $\mathbf{y}_1 = (\omega_c/\omega)\mathbf{y}$ . Furthermore,  $\omega_c$  should be an odd function of  $\omega$ . This results from the even and odd properties [7] of an arbitrary frequency-dependent impedance  $Z(\omega)$ . For example, if an inductor  $L$  is simulated at frequency  $\omega_c$  the equivalent impedance  $Z_L = j\omega_c L$  is purely imaginary, hence  $Z_L$  and consequently  $\omega_c$  should be an odd function of  $\omega$ . (The odd

property is also preserved for low-pass filter to high- or band-pass transformations [7]). Exploiting this property with dimensional analysis further reduces the number of ANN or MRF parameters approximating  $\omega_c$ .

### TRANSFORMATION OF FDEMS INTO FIEMS

The advantage of using MRFs to approximate the frequency mapping is that we can transform the FDEM into an equivalent FIEM. This transformation involves one-port impedance synthesis, which states that the impedance we want to realize should be a rational function. For example, the frequency mapping used in two examples presented here (microstrip right angle bend and microstrip via) takes the form

$$\omega_c = \omega \frac{f_1 - \omega^2 f_2}{f_3 - \omega^2 f_4} \quad (6)$$

where  $f_1, f_2, f_3, f_4$  are polynomials of the device physical parameters. We believe that (6) may be useful for other devices such as microstrip mitered bends, microstrip step junctions, etc.

To display the results in a compact way we define the error in the scattering parameter  $S_{ij}$  as the modulus of the difference between the scattering parameter  $S_{ij}^f$  computed by the fine model and the scattering parameter  $S_{ij}^c$  computed by the coarse model

$$\text{error in } S_{ij} = |S_{ij}^f - S_{ij}^c| \quad (7)$$

where  $i = 1, 2, \dots, M$  and  $j = 1, 2, \dots, M$  ( $M$  is the number of ports of the microwave device).

### MICROSTRIP RIGHT ANGLE BEND

Here, we develop an FIEM and an FDEM for a microstrip right angle bend. The fine model is analyzed by Sonnet's *em* [8] and the coarse model consists of the LC circuit [9]. The vector of input parameters  $\mathbf{x}_f = [W/H \ \epsilon_r]^T$  and the vector  $\mathbf{y} = [L/H \ C/H]^T$ . Applying dimensional analysis [4,5], we can show that  $\mathbf{y}$  is given by  $L/H = \mu_0 f(W/H)$ ,  $C/H = \epsilon_0 f(W/H, \epsilon_r)$  (8)

Therefore,  $\mathbf{y}$  is a function of  $\mathbf{x}_r = [W/H \ \varepsilon_r]^T$ . We first develop an FIEM in the frequency range [1, 11] GHz. The region of interest is  $0.2 < W/H < 6$  and  $2 < \varepsilon_r < 11$ . The substrate height  $H$  is chosen in the range [5, 30] mil. We use a three-layer perceptron ANN to approximate  $\mathbf{y}$ . The training points are chosen according to the Central Composite Design (CCD) [6] in addition to 4 more points (total 13 training points). The parameters of the ANNs are obtained by the Huber optimizer in OSA90/hope [10]. Fig. 1 shows the error in the scattering parameter  $S_{11}$  at 16 test points in the region of interest for the FIEM. These results are comparable with those of the Jansen model [11] at the same test points.

The results obtained by the FIEM and by the Jansen empirical model [11] over the range [1, 31] GHz are shown in Figs. 2 (a) and (b), respectively. It is clear that neither the FIEM nor the empirical model in [11] are accurate at high frequencies. Therefore, we develop an FDEM, where  $\omega_c$  is a function of  $\omega$  and the other parameters. Applying dimensional analysis and using the odd property of  $\omega_c$  we get

$$\omega_c = \omega \ \gamma(W/H, \varepsilon_r, (\omega H/c)^2) \quad (9)$$

where  $c$  is the speed of light and  $\gamma$  is an unknown function to be approximated. We use MRFs to approximate  $\mathbf{y}$  as well as  $\gamma$ . Fig. 3 shows the error in the scattering parameters  $S_{11}$  at 16 test points in the region of interest for the FDEM.

We transform the FDEM into an equivalent FIEM as follows. The frequency  $\omega_c$  is given by (6), hence the impedances associated with  $L$  and  $C$  are given by

$$\begin{aligned} Z_L &= j\omega_c L = j \omega L \frac{f_1 - \omega^2 f_2}{f_3 - \omega^2 f_4} \\ Z_C &= \frac{1}{j\omega_c C} = \frac{1}{j\omega} \frac{f_3 - \omega^2 f_4}{f_1 - \omega^2 f_2} \end{aligned} \quad (10)$$

which can be realized by network synthesis [3], in our case by the first Foster realization.

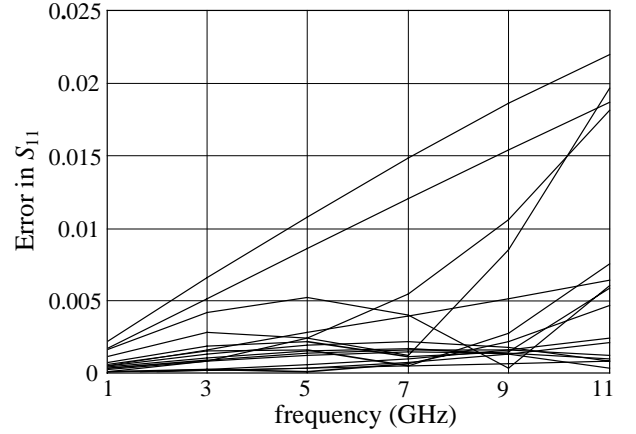
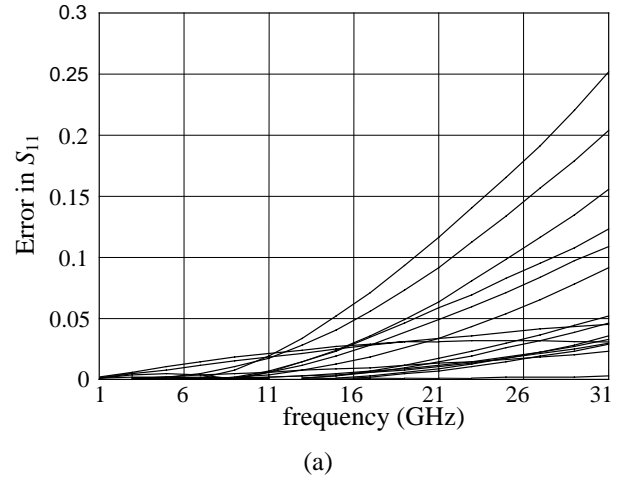
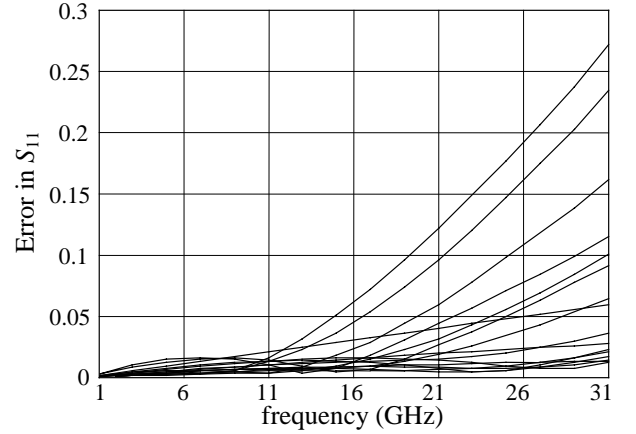


Fig. 1. The error in  $S_{11}$  of the microstrip right angle bend FIEM with respect to  $em^{TM}$  at the test points.



(a)



(b)

Fig. 2. The error in  $S_{11}$  of the microstrip right angle bend with respect to  $em^{TM}$  at the test points: (a) the FIEM in [1, 31] GHz; (b) the empirical model from [11].

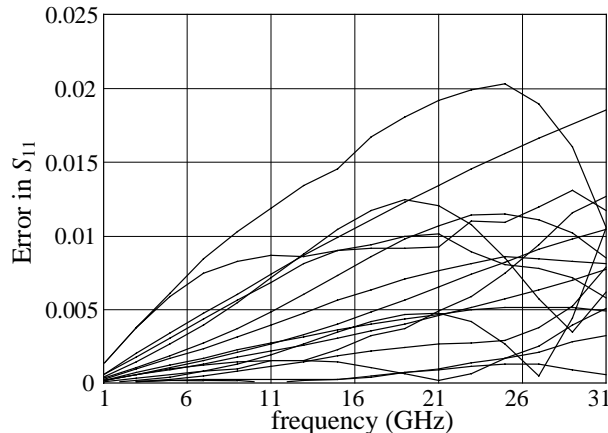


Fig. 3. The error in  $S_{11}$  of the microstrip right angle bend FDEM with respect to  $em^{TM}$  at the test points.

### COPLANAR WAVEGUIDE (CPW) SHORT-CIRCUIT STUB

In this example, we develop an FIEM for the CPW short-circuit stub. The reference plane is taken at the stub end. The fine model is analyzed by Momentum [12] and the coarse model consists of an inductance  $L$  to ground. The input parameter vector  $\mathbf{x}_f = [W \ G \ H]^T$ . The vector  $\mathbf{y}$  contains only one element, namely

$$L/H = \mu_0 f(W/H, G/W) \quad (13)$$

Therefore,  $\mathbf{y}$  is a function of  $\mathbf{x}_r = [W/H \ G/W]^T$ . We develop an FIEM in the range [1, 25] GHz. The region of interest is  $0.2 < W/H < 2$  and  $0.2 < G/W < 2$ . The substrate used is GaAs ( $\epsilon_r=12.9$ ) with height  $H$  in the range [100, 635]  $\mu\text{m}$ . We use a three-layer perceptron ANN to approximate  $L/H$ . The training points are chosen according to the Central Composite Design (CCD) [6] in addition to 4 more points (total 13 training points).

The worst case % errors in  $S_{11}$  of the FIEM of the CPW short-circuit stub is 2%. We observe good results of the FIEM in the range [1, 40] GHz (worst case % error of 4%).

### CONCLUSIONS

We present a unified computer-aided modeling methodology for developing broadband models of microwave passive components. Two types of model are considered: FIEMs and FDEMs.

FDEMs can be transformed to equivalent FIEMs if we use an MRF to approximate the frequency mapping. This is important since the FIEMs are readily implementable in conventional circuit simulators. We applied our modeling methodology to develop broadband empirical models for several microwave components.

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