OPTIMIZATION OF CIRCUITS

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SUMMARY

A number of recently proposed and implemented objective function formulations for circuit optimization are reviewed. The emphasis is on formulations which can allow explicit and implicit constraints on the circuit parameters and responses to be taken into account. The formulations considered include the ones used by Bandler and Macdonald; Waren, Lasdon and Suchman; Ishizaki and Watanabe; and Temes and Zai. They can all be used in the computer-aided optimization of circuits for which the objective is to try to *minimize* the *maximum* deviation of some response from a desired ideal response specification.

INTRODUCTION

In the author's experience from numerous discussions with both students and practicing engineers, a difficulty often encountered in the automatic optimization of circuits by computer is the selection of a suitable objective function. Weighted least squares types of objective function seem to be rather popular, possibly because least squares approximation techniques are more widely documented and because they are relatively easy to implement. For many circuit optimization problems least squares solutions may not provide the most desirable responses. A *minimax* solution, i.e., one in which the maximum deviation of the response from an ideal response specification has been minimized, might be preferred.

In this context, a number of recently proposed and implemented objective function formulations are presented and discussed. The emphasis is on formulations which can allow explicit and implicit constraints on the circuit parameters and responses to be taken into account. The formulations considered include the ones used by Bandler and Macdonald [1]; Waren, Lasdon and Suchman [2]; Ishizaki and Watanabe [3]; and Temes and Zai [4].

DIRECT MINIMAX FORMULATION

An ideal objective for network optimization is

minimize U

where

$$U = U(\underline{\phi},\psi) = \max_{[\psi_\ell,\psi_u]} [w_u(F-S_u), - w_\ell(F-S_\ell)] \tag{1}$$

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and where

$F = F(\underline{\phi},\psi)$ is the response function
$\underline{\phi}$ represents the network parameters
ψ is an independent variable, e.g. frequency or time
$S_u = S_u(\psi)$ is a desired upper response specification
$S_\ell = S_\ell(\psi)$ is a desired lower response specification
$w_u = w_u(\psi)$ is a weighting factor for S_u
$w_\ell = w_\ell(\psi)$ is a weighting factor for S_ℓ
ψ_u is the upper bound on ψ
ψ_ℓ is the lower bound on ψ.

This formulation is illustrated by Fig. 1. Fig. 1(a) shows a response function satisfying arbitrary specifications; Fig. 1(b) shows a response function failing to satisfy a bandpass filter specification; Fig. 1(c) shows a response function just satisfying a possible amplifier specification. F will often be expressible as a continuous function of $\underline{\phi}$ and ψ. But $S_\ell(\psi)$, $S_u(\psi)$, $w_\ell(\psi)$ and $w_u(\psi)$ are likely to be discontinuous.

The following restrictions are imposed:

$$S_u \geq S_\ell \tag{2}$$

$$w_u > 0 \tag{3}$$

$$w_\ell > 0 \,. \tag{4}$$

Under these conditions $w_u(F-S_u)$ and $-w_\ell(F-S_\ell)$ are both positive when the specifications are not met; they are zero when the specifications are just met; and they are negative when the specifications are exceeded. The objective is, therefore, to minimize the maximum (weighted) amount by which the network response fails to meet the specifications; or to maximize the minimum amount by which the network response exceeds the specifications. Note the special case when

$$S_u = S_\ell = S \tag{5}$$

and

$$w_u = w_\ell = w \tag{6}$$

which reduces (1) to

$$U = \max_{[\psi_\ell,\psi_u]} [|w(F-S)|] \,. \tag{7}$$

This form may be recognized as the more conventional Chebyshev type of objective.

The direct minimax formulation, the optimum of which represents the best possible attempt at satisfying the design specifications within the constraints of the particular problem, appears to have received very little attention in the literature on circuit optimization. This is chiefly due to the fact that discontinuous derivatives are generated in the response hypersurface when the maximum deviation jumps abruptly from one point on the ψ-axis to another, and that multidimensional optimization methods which deal effectively with such problems are rather scarce [5]. In spite of these difficulties, some success with objectives in the form of (7) has been reported [1]. But it is felt that considerable research into methods for dealing with objectives in the form of (1) remains to be done.

FORMULATION IN TERMS OF INEQUALITY CONSTRAINTS

A less direct formulation than the previous one, but one which seems to have provided considerable success, is the formulation in terms of inequality constraints on the network response [2,3]. The problem is

minimize U

subject to

$$U \geq w_{ui}(F_i(\phi) - S_{ui}) \qquad i \in I_u \tag{8}$$

$$U \geq -w_{\ell i}(F_i(\phi) - S_{\ell i}) \qquad i \in I_\ell \tag{9}$$

and other constraints, e.g., upper and lower bounds on the parameters, where U is now an *additional independent variable* and where the subscript i refers to quantities (already defined) evaluated at discrete values of ψ which form the set $\{\psi_i\}$ in the interval $[\psi_\ell, \psi_u]$. The index sets I_u and I_ℓ, which are not necessarily disjoint, contain those values of i which refer to the upper and lower specifications, respectively. Thus, in the case of Fig. 1(a), the index set I_u and I_ℓ could be identical. For Fig. 1(b), the set I_u would refer to the passband and the set I_ℓ to the stopbands. In Fig. 1(c), there might be an intersection between I_u and I_ℓ.

At a minimum at least one of the constraints (8) or (9) must be an equality otherwise U could be further reduced without any violation of the constraints. If $\breve{U} < 0$ (where $\breve{U}$ is the minimum value of U) then the minimum amount by which the network response exceeds the specifications has been maximized. If $\breve{U} > 0$ then the maximum amount by which the network response fails to meet the specifications has been minimized. It is clear that both this and the previous formulations have ultimately similar objectives. Indeed, if the sets I_u and I_ℓ are infinite then the optimum solutions given by both formulations may be identical. Not surprisingly such a problem may be described as one which has an infinite number of constraints.

A special case again arises when

$$S_{ui} = S_{\ell i} = S_i \tag{10}$$

$$w_{ui} = w_{\ell i} = w_i \tag{11}$$

$$I_u = I_\ell = I \tag{12}$$

which reduces (8) and (9) to

$$U \geq w_i(F_i(\phi) - S_i) \tag{13}$$

$$U \geq -w_i(F_i(\phi) - S_i) \qquad i \in I \tag{14}$$

The formulation in terms of inequality constraints is equivalent to a nonlinear programming problem. Two methods of solution have so far been proposed. One of these, described by Waren, Lasdon and Suchman [2], involves the transformation of the constrained objective into a penalized unconstrained objective. A sequence of unconstrained minimizations follows, preferably using an efficient quadratically convergent method, the minimum of which ultimately approaches the constrained minimum $\breve{U}$. The other method, described by Ishizaki and Watanabe [3], reduces the nonlinear programming problem at a particular stage to a linear programming problem by linearization of $F_i(\phi)$. The sequence of linear programming problems are each solved by the simplex method.

WEIGHTING FACTORS

Essentially, the task of weighting factors is to emphasize or deemphasize various parts of the response to suit the designer's requirements. For example, if one of the factors w_u or w_ℓ is unity and the other very much greater than unity then if the specifications are not satisfied, a great deal of effort will be devoted to forcing the response associated with the large weighting factor to meeting the specifications at the expense of the rest of the response. Once the specifications are satisfied, then effort is quickly switched to the rest of the response while the response associated with the large weighting factor is virtually left alone. In this way, once certain parts of the network response reach acceptable levels they are effectively maintained at those levels while further effort is spent on improving other parts.

LEAST PTH APPROXIMATION

A frequently employed class of objective functions may be written in the generalized form

$$U = U(\phi, \psi) = \sum_{i=1}^{n} |w_i(F_i(\phi) - S_i)|^p = \sum_{i=1}^{n} |e_i(\phi)|^p \tag{15}$$

where ψ represents the sample points and where the subscript i refers to quantities evaluated at the sample point ψ_i. Thus, the objective is essentially to minimize the sum of the magnitudes raised to some power p of the weighted deviations $e_i(\phi)$ of the response from a desired response over a set of sample points $\{\psi_i\}$. p may, in the present case, by any positive integer.

The sample points are commonly spaced uniformly along the ψ-axis in the interval $[\psi_\ell,\psi_u]$. If the objective is effectively to minimize the area under a curve then sufficient sample points must be used to ensure that (15) is a good approximation to the area. However, it should be remembered that function evaluations are often by far the most time consuming parts of an optimization process. So the number of sample points should be carefully chosen for the particular problem under consideration. These arguments apply, of course, to any formulation which involves sampling.

With p = 1, (15) represents the area under the deviation magnitude curve if sufficient sample points are used. With p = 2 we have a least squares type of formulation. Obviously, the higher the value of p the more emphasis will be given to those deviations which are largest. So if the requirement is to concentrate more on minimizing the maximum deviation a sufficiently large value of p must be chosen. The basis of such a formulation is the fact that

$$\max_{[\psi_\ell,\psi_u]}[|e(\underline{\phi},\psi)|] = \lim_{p\to\infty}\left[\frac{1}{\psi_u - \psi_\ell}\int_{\psi_\ell}^{\psi_u}|e(\underline{\phi},\psi)|^p \, d\psi\right]^{\frac{1}{p}} \quad (16)$$

when $|e(\underline{\phi},\psi)|$ is defined in the interval $[\psi_\ell,\psi_u]$. In terms of a sampled response deviation the corresponding statement is

$$\max_i[|e_i(\underline{\phi})|] = \lim_{p\to\infty}[\sum_i|e_i(\underline{\phi})|^p]^{\frac{1}{p}} . \quad (17)$$

In practice, values of p from 4 to 10 may provide an adequate approximation for engineering purposes to the ideal objective. A good choice of the weighting factors w_i will also assist in emphasizing or deemphasizing parts of the response deviation. It may also be found advantageous to switch objective functions, number of sample points or weighting factors after any complete optimization if the optimum is unsatisfactory. For example, one may optimize with the weighting factors set to unity and with p = 2. If the maximum deviation is larger than desired, one could select appropriate scale factors and/or a higher value of p and try again from the previous 'optimum'.

The contribution by Temes and Zai [4], who have employed a least pth formulation, is their extension of the familiar least squares *method* of approximation to a more general least pth method.

COMBINED OBJECTIVES

The objective function can consist of several objectives. Indeed, the form of (1) and (15) suggest such a possibility. For example, we could have a linear combination

$$U = \alpha_1 U_1 + \alpha_2 U_2 + \dots \quad (18)$$

where U_1, U_2, ... could take the form of (15). For an amplifier a compromise might have to be reached between gain and noise figure; another example is the problem of approximating the input resistance and reactance of a model to experimental data. The factors α_1, α_2, ... would then be given values commensurate with the importance of U_1, U_2, ..., respectively.

CONCLUSIONS

A brief review of some recently proposed objective function formulations for computer-aided circuit optimization has been presented. The formulations aim at reducing the maximum deviation of the circuit response from ideal response specifications, and can, therefore, result in more desirable optimum responses.

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