OPTIMIZATION OF MICROWAVE NETWORKS BY RAZOR SEARCH

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Introduction

A new automatic optimization method called Razor Search is presented. The method, which is based on pattern search,[1] was specifically developed for the optimization by computer of networks for which the objective is to *minimize* the *maximum* deviation of some response from a desired ideal response specification. Examples falling within the scope of this paper are shown in Fig. 1.

Minimax response objectives, which can lead to *equal-ripple* optima, will in general give rise to discontinuous partial derivatives of the objective function with respect to the network parameters.[2,3] Under these circumstances otherwise efficient optimization methods -- certainly on-line manual methods -- may slow down or even fail to reach an optimum, particularly when the response hypersurface has a narrow curved valley along which the path of discontinuous derivatives lies.[3] This is probably the reason that success with the direct minimax formulation does not seem to have been previously reported. Indeed, to the authors' knowledge, the optimization of functions with discontinuous derivatives does not appear to have received any serious attention in the literature.

Essentially, the Razor Search strategy begins with a modified version of pattern search until this fails. A random point is selected automatically in the neighborhood and a second pattern search is initiated until this one fails. Using the two points where pattern search failed a new pattern in the direction of the optimum is established and a pattern search strategy resumed until it too fails. This process is repeated until any of several possible terminating criteria is satisfied. Thus, the strategy should work on problems involving narrow "razor sharp" valleys in multidimensional space.

Since the only point of interest in the network response at any given time during optimization is that point where the maximum deviation occurs (see Fig. 1), it is important to obtain this point to any desired accuracy with as few response evaluations as possible. Another direct search method called Ripple Search, which locates the extrema of multimodal functions of one variable in an efficient manner, was developed for this purpose. Unlike the usual practice of sampling, for example, a network frequency response at closely spaced fixed frequencies, the Ripple Search strategy first conducts

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a uniform search to determine the extrema and appropriate unimodal regions; subsequently, during optimization, it locates the extrema within the previously defined regions using a Fibonacci search scheme. Safeguards are built into the program to deal with continuously changing ripple patterns during optimization.

Numerical Example of the Razor Search Strategy

Consider the problem of optimizing a 2-section transmission-line transformer for a load to source impedance ratio of 10:1 over a 100% bandwidth with the section lengths fixed at their optimum values, i.e. quarter-wave at center frequency.[2,3] It has been shown that ordinary pattern search can fail to reach the optimum,[3] and that reducing the parameter increments is ineffective.

For this problem, the objective function U is the maximum magnitude of the reflection coefficient over the band of interest. The solution (Chebyshev) is available in tables[4] and is

$$Z_1 = 2.2361 \qquad Z_2 = 4.4721 \qquad \check{U} = \min(U) = 0.4286$$

Contours of U versus Z_1 and Z_2, the characteristic impedances of the two sections normalized to the source impedance, are plotted in Fig. 2. The sharp points in the contours indicate the presence of the discontinuous derivatives which arise when U jumps from one test frequency to another. In the discussion which follows, it is convenient to define

$$\underline{\phi} \equiv (\phi_1, \phi_2) \equiv (Z_1, Z_2) \ .$$

The starting point $\underline{\phi}^1 = (1.25, 4.50)$ is selected as the first base point. The objective function is evaluated at $\underline{\phi}^1$. Let its value be denoted U^1. The first exploratory move begins with ϕ_1 using a starting increment $\Delta\phi = .25$ and taking us to $\underline{\phi}^2$. Since $U^2 < U^1$ we retain $\underline{\phi}^2$ and continue exploration with ϕ_2. The next point $\underline{\phi}^3$ is rejected because $U^3 > U^2$ and ϕ_2 is incremented in the opposite direction to $\underline{\phi}^4$. $U^4 < U^2$ so $\underline{\phi}^4$ is retained in place of $\underline{\phi}^2$. The first set of exploratory moves is now complete. Since $U^4 < U^1$, $\underline{\phi}^4$ is an improvement over $\underline{\phi}^1$.

The point $\underline{\phi}^4$ becomes the second base point and in accordance with the pattern move strategy we obtain a projected point $\underline{\phi}^5$ such that $\underline{\phi}^5 - \underline{\phi}^4 = \underline{\phi}^4 - \underline{\phi}^1$. U^5 is evaluated and a second set of exploratory moves is initiated. The exploratory increment $\Delta\phi$ is set equal to $|\underline{\phi}^4 - \underline{\phi}^1|/\sqrt{k}$ where k is the dimensionality of the space; here $k = 2$. Each parameter was successfully incremented during the previous exploration therefore $\Delta\phi$ remains at .25 (otherwise $\Delta\phi$ would have been automatically reduced). Incrementing ϕ_1 in the direction previously found successful takes us to $\underline{\phi}^6$. It is found that $U^6 < U^5$ so we retain $\underline{\phi}^6$ and increment ϕ_2 in the direction previously found successful for this parameter. Point $\underline{\phi}^7$ is, however, no improvement on $\underline{\phi}^6$ so we try the opposite direction and arrive at $\underline{\phi}^8$. This point is also unsuccessful, leaving us with $\underline{\phi}^6$.

$U^6 < U^4$ therefore $\underline{\phi}^6$ becomes the third base point. We now obtain a projected point $\underline{\phi}^9$ such that $\underline{\phi}^9 - \underline{\phi}^6 = \underline{\phi}^6 - \underline{\phi}^4$ and an exploratory increment $\Delta\phi = |\underline{\phi}^6 - \underline{\phi}^4|/\sqrt{k} = \sqrt{10}/8$. Exploration around $\underline{\phi}^9$ ends at $\underline{\phi}^{12}$ with $U^{12} < U^9$. It is unsuccessful, however, since $U^{12} > U^6$.

Not wishing to abandon the pattern already established we project a point midway between $\underline{\phi}^6$ and $\underline{\phi}^9$ to $\underline{\phi}^{13}$, and reduce the exploratory increment appropriately. We finally arrive at $\underline{\phi}^{15}$ which is an improvement over $\underline{\phi}^6$. However, at this stage $\Delta\phi < \varepsilon$, the minimum allowable increment for this part of Razor Search and set at .08. The first pattern search is, therefore, terminated at $\underline{\phi}^{15}$.

A second pattern search is started from the random point $\underline{\phi}^{16}$. Eventually, we arrive at $\underline{\phi}^{36}$ where this pattern search is abandoned because $\Delta\phi < \varepsilon$, which was reduced to .04. The values U^{36} and U^{15} are compared. $U^{36} < U^{15}$ so the direction of the valley is given by $\underline{\phi}^{36} - \underline{\phi}^{15}$. Taking $\underline{\phi}^{36}$ as a base point and $\underline{\phi}^{37}$ as a projected point down the valley such that $\underline{\phi}^{37} - \underline{\phi}^{36} = \underline{\phi}^{36} - \underline{\phi}^{15}$ we continue with the pattern search strategy until $\Delta\phi < \varepsilon$. Note that $\underline{\phi}^{37} = (2.21791, 4.44943)$, i.e. at the 37th function evaluation the parameter values are within 1% and 1/2%, respectively of their optimum values.

Conclusions

In practice, Razor Search is a package of 4 subroutines called by a main program. Ripple Search is a package of 3 subprograms called by the Razor Search package, and in turn calling on a set of subprograms for evaluating the network response.

The programs have been extensively checked with multisection resistively terminated commensurate and noncommensurate transmission-line transformers for which the equal-ripple Chebyshev optima are known. Numerical results are available. They have further been used to produce new results in the broadband design of inhomogeneous waveguide transformers, for which no exact analytic synthesis theory is available. The methods described should find immediate application to the computer-aided optimization of a wide range of microwave networks, particularly where optimum broadband performance is required, but where synthesis techniques may be inappropriate or unavailable.

Acknowledgment

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References

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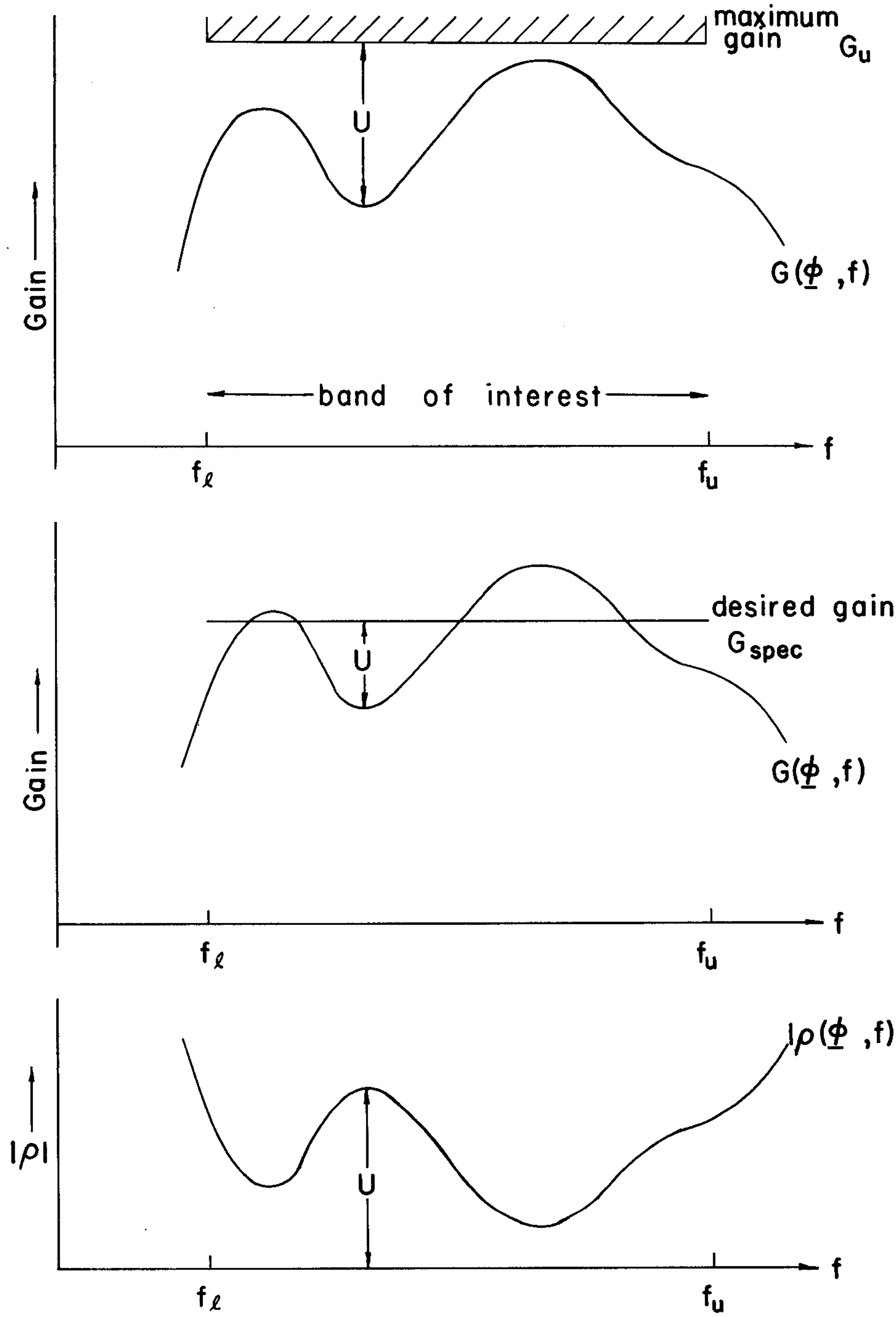

Fig. 1 Examples having minimax response objectives and falling within the scope of this paper.

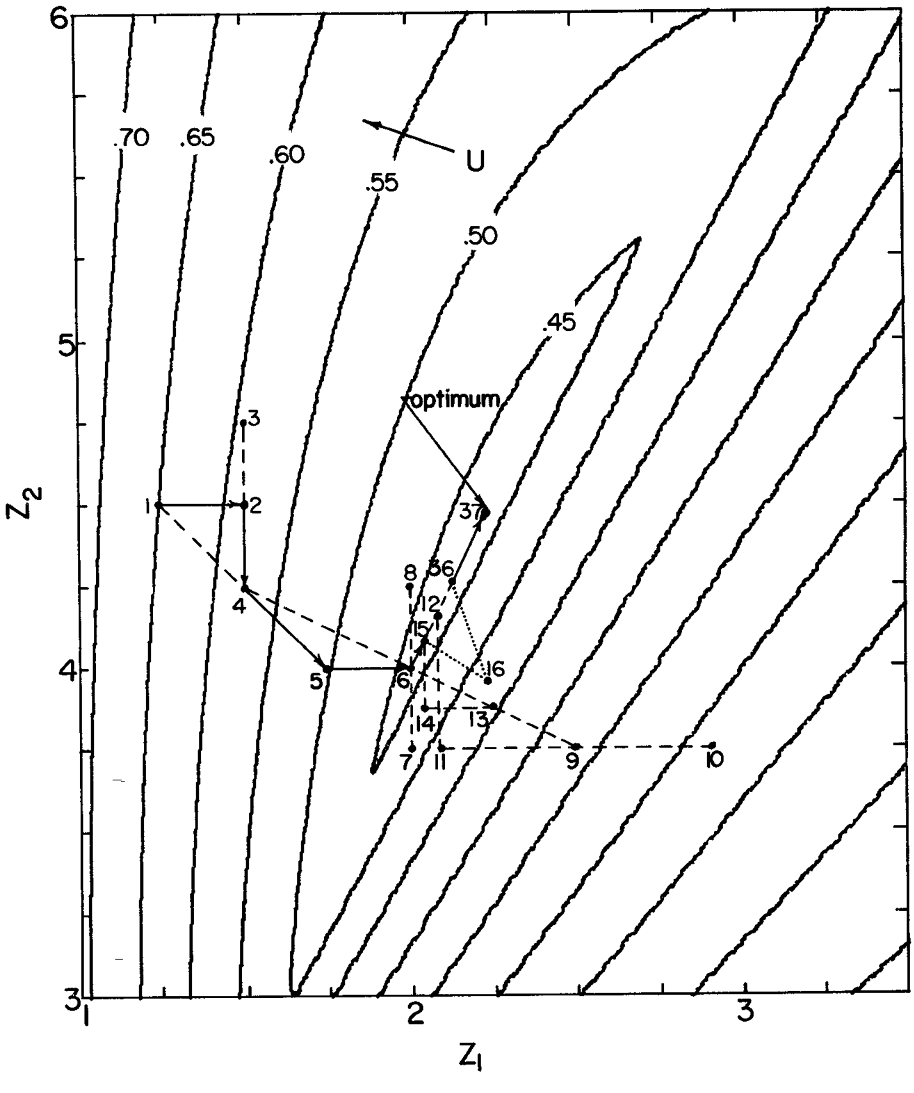

Fig. 2 A numerical example illustrating the Razor Search strategy showing how following one random move the path of discontinuous derivatives leading to the optimum is effectively located.