TABLE I
COMPARISON OF VARIOUS GRAPHICAL I/O SYSTEMS FOR CIRCUIT ANALYSIS

	AEDNET	CIRCAL	GINA	OLCA	2250 ECAP
Graphics Language	AED	AED	CALLIGRAPH	GRIN	GPACK
Computer	Project MAC	Project MAC	7094	PDP-5	360 Mod 40
Input Medium	Keyboard	Keyboard	Light pen	Light pen	Keyboard
Alphanumeric Keyboard	yes	yes	no	yes	yes
Analysis Program	AEDNET	CIRCAL	CALAHAN POTTLE } projected CIRCUS }	HYBRID	ECAP

Finally, it should be said that only OLCA and GINA have hard copy provisions, and only GINA has a built-in movie-making function.

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Cascaded Noncommensurate Transmission-Line Networks as Optimization Problems

Interest is growing both in computer-aided design and optimization of electrical networks generally, and in the synthesis of noncommensurate transmission-line networks in particular. Multivariable networks of the latter kind are, to date, still optimized in the real frequency domain [1]–[3]. To obtain equal-ripple responses directly involves the minimization of functions of several variables, the functions being characterized by discontinuous partial derivatives. Our experience indicates that virtually all available automatic optimization methods can be expected to fail, in general, to reach even a local optimum for such situations.

This correspondence presents and discusses the relevant results of a study of the optimization of cascaded noncommensurate transmission lines acting as transformers between resistive terminations. The discussion is illustrated by contour plots of those parts of the response hypersurface generated by varying two parameters of a typical network, the remaining parameters being fixed. The behavior of some direct search strategies [1]–[8] on these contours, particularly pattern search [2], [4]–[6], is discussed.

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An earlier publication [2] found that the optima for the problem of minimizing the maximum input reflection coefficeint over specified bandwidths for networks of the type shown Fig. 1 (a) turn out to be the known quarter-wave Chebyshev designs [9]. A formal proof of this does not yet appear to have been reported, however.

The present correspondence will restrict itself to the two-section transformer (four variables) shown in Fig. 1(b). As shown, it has a load-to-source impedance ratio R of 10:1 and requires optimum performance over a 100 percent bandwidth. Formally, the problem is to minimize

$$U = \max_i |\rho(\phi, f_i)| \qquad i = 1, 2, \cdots, n \quad (1)$$

where

$$\phi = \begin{bmatrix} l_1 \\ Z_1 \\ l_2 \\ Z_2 \end{bmatrix} \quad (2)$$

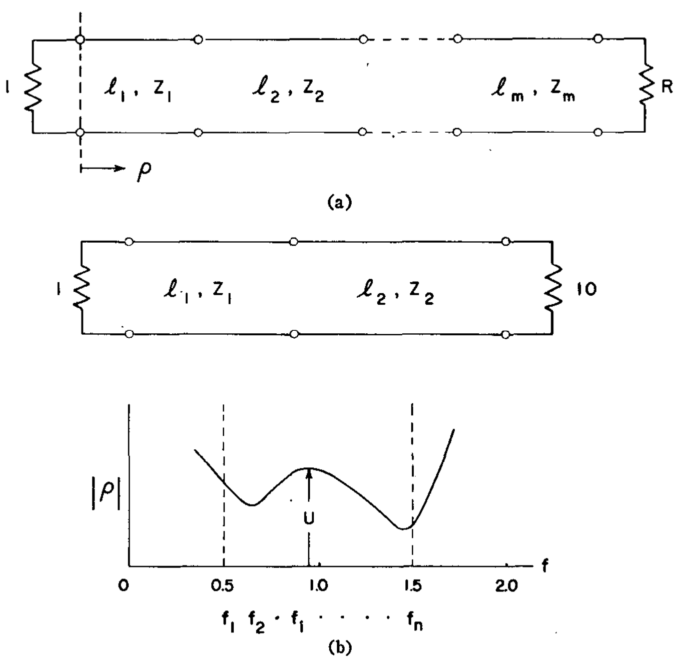


Fig. 1. Examples of multivariable cascaded transmission-line networks. (a) m-section resistively terminated noncommensurate transformer. (b) Two-section 10:1 transformer for optimum performance over 100 percent bandwidth.

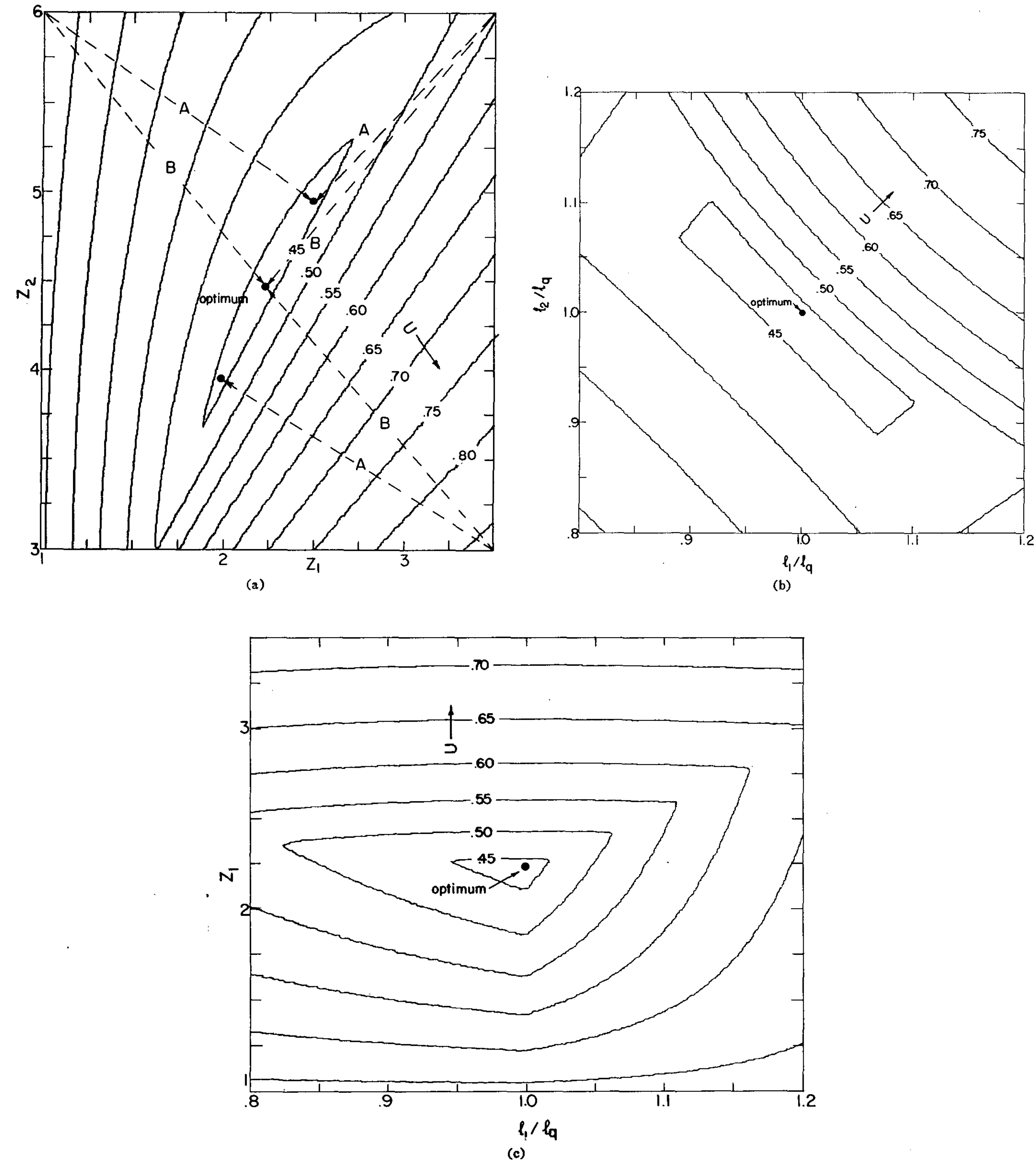


Fig. 2. (a) Contours of U when $l_1 = l_2 = l_q$. (b) Contours of U when $Z_1 = 2.2361$ and $Z_2 = 4.4721$. (c) Contours of U when $l_2 = l_q$ and $Z_2 = 4.4721$.

subject to

$$\left.\begin{aligned} l_k &\geq 0 \\ Z_k &> 0 \end{aligned}\right\} \qquad k = 1, 2 \qquad \begin{aligned} &(3) \\ &(4) \end{aligned}$$

where $f_l = 0.5$ and $f_h = 1.5$ (normalized to the center frequency f_0).

The solution is [9]

$$U_{\min} = 0.4286$$
$$l_1 = l_2 = l_q$$
$$Z_1 = 2.2361 \qquad Z_2 = 4.4721$$

where

$$l_q = c/4\,f_0.$$

Any two parameters can be fixed and the others varied to produce contours of U. Perhaps the most interesting for our purpose are the three cases shown in Fig. 2. In Fig. 2(a), the lengths are fixed at their optimum values and the impedances are varied; in Fig. 2(b), the impedances are kept fixed at their optimum values and the lengths are varied; and in Fig. 2(c), the parameters of the second section are held at their optimum values while those of the first section are varied. The sharp points in the contours indicate the presence of the discontinuous partial derivatives of U. The discontinuities occur, of course, when U jumps from one test frequency to another.

The pattern search strategy[1] was started in each corner of the three diagrams in Fig. 2. Two runs per corner were made, one with parameter increments (for the exploratory moves) of one division on the Z scales and/or 2/3 of a division on the l/l_q scales, the other with double these increments. The increments were then reduced as necessary to keep the pattern search going in an effort to reach the optimum, either until the increments were less than 10^{-5} or the number of evaluations of U exceeded 1000.

The only situation that presented difficulties was that of Fig. 2(a). For point (1, 3), the optimization process terminated outside the bounded area. Convergence from the other corners onto points very close to the known optimum resulted when parameter increments of one division ($\Delta Z = 0.25$) were used. The corresponding paths are labeled B.[2] For $\Delta Z = 0.5$ (paths labeled A), the search failed to reach the optimum.

Similar difficulties also manifested themselves in the more general four-variable optimization described by (1)–(4) and in other multivariable examples investigated [2].

A brief explanation of the failure is in order. Fig. 3 reproduces the contours of Fig. 2(a). It shows how pattern search[1] using $\Delta Z = 0.25$ and starting at a fails at f after 16 function evaluations. Thereafter, the parameter increments would be reduced and the process restarted at f. But it is clear that, unless a fortuitous choice of increments is struck, pattern search will repeatedly fail no matter how small the increments are made, and eventually terminate very close to f.

In those cases when the optimum was reached, between 72 and 280 function evaluations were required taking from 1/4 to 1 second on the IBM 360/65. No attempt to minimize the number of function evaluations was made.

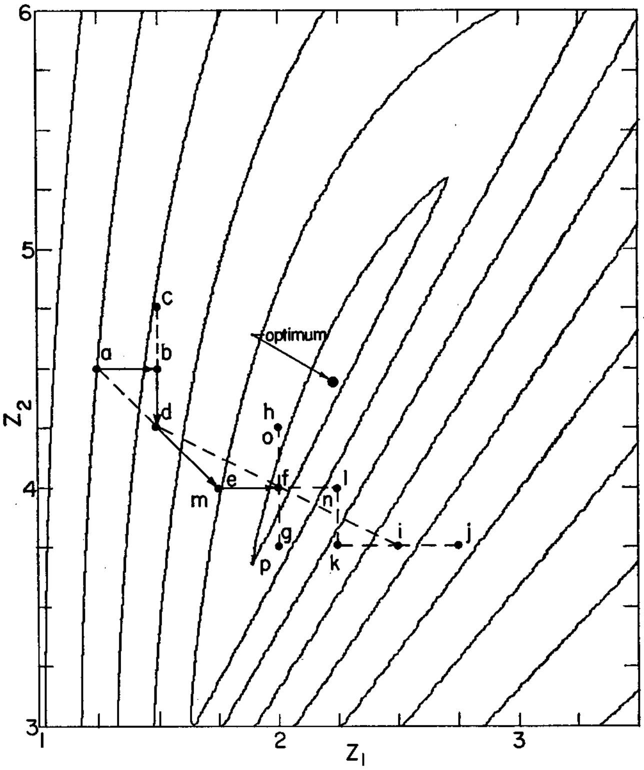
Although all the contours in Fig. 2 have discontinuous derivatives, only in Fig. 2(a) do the contours lie entirely in one quadrant. Once the search stops at a point of discontinuous derivatives, exploration parallel to the coordinate axes will not yield any improvement. Wilde [10] and Wilde and Beightler [11] have discussed similar situations when methods that are usually good at following narrow valleys can fail. Powell's minimization method [7], [8], which does not restrict itself to exploring along the coordinate axes, was also programmed, but failed in the same way as pattern search. O'Hagan's method [1], which relies on orthogonal exploratory moves in directions oblique to the coordinate axes chosen at random, might eventually find the direction of the valley because of its reliance on randomness.

Little research appears to have been done to overcome this problem as contours involving discontinuous derivatives seem to be unpopular among exponents of optimization. It is usual to reformulate the problem by specifying an objective function that provides a nearly equal-ripple response (see, for example, Temes and Calahan [12]).

It is felt, however, that optimization in the frequency domain is of sufficient importance to warrant a deeper investigation of methods for handling equal-ripple response criteria directly. Even if the synthesis problem for multivariable transmission-line networks is solved, the relatively narrow range of practical characteristic impedance values (say 15 to 150 ohms), might still make optimization in the frequency domain more attractive. The example presented here might be found useful for testing new optimization methods [13].[3]

[1] More recent modifications [5], [6] of the pattern search strategy of Hooke and Jeeves [4] were incorporated into the computer program.

[2] The dashed lines labeled A and B in Fig. 2(a) are not, of course, the actual paths taken.



Function Evaluation	Description of Move
a	initial base (or starting) point
b c d	exploratory moves from a
e	pattern move in direction ad
f g h	exploratory moves from e
i	pattern move in direction df
j k l	exploratory moves from i
m n o p	exploratory moves from f

Fig. 3. The behavior of pattern search for the case $l_1 = l_2 = l_q$ starting from a typical point when $\Delta Z = 0.25$.

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[3] As described in a more recent paper [13], the authors have followed up this work by developing a direct search method based on pattern search which performs reliably on the problems presented in this correspondence.

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Transient Responses of Conventional Distributed Amplifiers

In a recent correspondence [1], the transient responses of a distributed amplifier using bridged-T filter sections have been presented. In [2], we have discussed the effects on the gain and transient responses of a distributed amplifier by inserting an extra section into the plateline of the amplifier. In this correspondence, we shall supplement the previous two works by presenting the transient responses of a conventional distributed amplifier. The theoretical results will confirm Sarma's experimental claim [3] that the staggering of the lines of the amplifier will improve the transient responses.

To obtain the transient response, it is necessary to start with the voltage gain function of the amplifier. The voltage gain function $A(s)$ is derived in [4], and is given by

$$A(s) = \frac{A_0}{n(1 + x_g'^2)^{1/2}} \cdot \frac{\sinh \frac{1}{2}n(\theta_g - \theta_p)}{\sinh \frac{1}{2}(\theta_g - \theta_p)} \cdot \exp\left\{-\tfrac{1}{2}n(\theta_g + \theta_p)\right\}, \tag{1}$$

where $A_0 = -\frac{1}{2}ng_m R_{0p}$, the zero-frequency voltage gain; g_m is the transconductance of the valves employed; n is the number of valves used in an amplifier stage; R_{0p} is the nominal image impedance of the plateline; ω_{0p} and ω_{0g} are the plateline and gridline cutoff frequencies, respectively; $x_p' = s/\omega_{0p}$ and $x_g' = s/\omega_{0g}$; and $\theta_p = 2 \sinh^{-1} x_p'$ and $\theta_g = 2 \sinh^{-1} x_g'$.

In order to obtain the Laplace inverse transform of (1), it is simpler to express the hyperbolic functions in terms of the exponential functions. Since

$$\frac{\sinh \frac{1}{2}n(\theta_g - \theta_p)}{\sinh \frac{1}{2}(\theta_g - \theta_p)} \cdot \exp\left\{-\tfrac{1}{2}(\theta_g + \theta_p)\right\} = \sum_{k=1}^{n} \exp\left\{-(k - \tfrac{1}{2})\theta_g - (n - k + \tfrac{1}{2})\theta_p\right\},$$

and $\exp \frac{1}{2}\theta_p = x_p' + (1 + x_p'^2)^{1/2}$ and $\exp \frac{1}{2}\theta_g = x_g' + (1 + x_g'^2)^{1/2}$, the voltage gain function $A(s)$ becomes

$$A(s) = (A_0/n) \sum_{k=1}^{n} (1 + x_p'^2)^{-1/2}\{x_g' + (1 + x_g'^2)^{1/2}\}^{1-2k} \cdot \{x_p' + (1 + x_p'^2)^{1/2}\}^{2(k-n)-1}. \tag{2}$$

In order to observe the effects of the staggering of the lines on the transient responses of the amplifier, it is necessary that (2) be normalized with respect to the same quantity. Thus, let $P = \omega_{0g}/\omega_{0p}$. The term P is called the *staggering factor* of the amplifier. Since the plateline usually has a higher cutoff frequency than that of the gridline, the value of P is ranged from 0 to 1. Using the following

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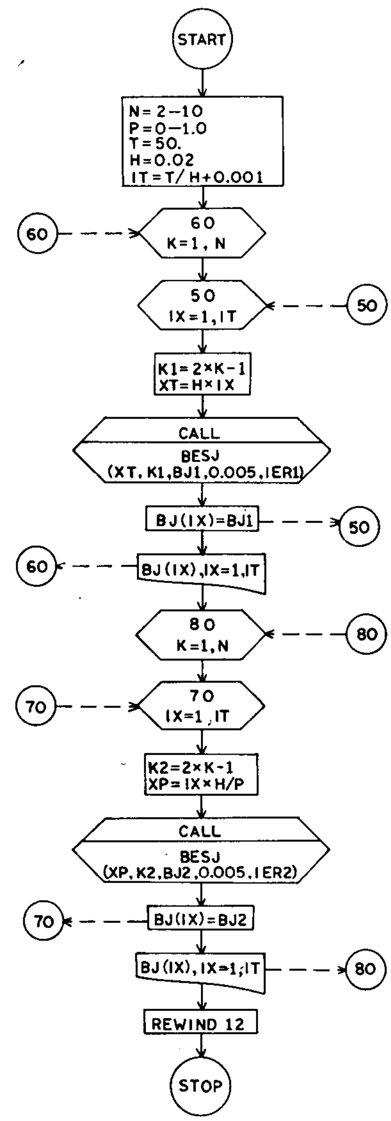

Fig. 1. Flow chart used to generate the values of the Bessel functions.

Laplace inverse transform indentities,

$$(1 + s^2)^{-1/2}\{s + (1 + s^2)^{1/2}\}^{-k} = J_k(t)$$
$$\{s + (a^2 + s^2)^{1/2}\}^{-k} = (k/a^k)J_k(at)/t, \tag{3}$$

where $J_k(t)$ is the standard symbol for a Bessel function, in conjunction with the fact that $P = x_p'/x_g'$, the output voltage $e_{np}^*(\omega_{0g}t)$ of the amplifier due to the unit-impulse-voltage input can be obtained from (2) by means of convolution integral, and is given by

$$e_{np}^*(\omega_{0g}t) = (A_0\omega_{0g}/n) \sum_{k=1}^{n} \{2(n - k) + 1\} \cdot \int_0^{\omega_{0g}t} J_{2k-1}(\omega_{0g}t - z)J_{2(n-k)+1}(z/P)z^{-1}\, dz. \tag{4}$$

To find the unit-step-voltage response of the amplifier, a simple integration is performed wherein

$$e_{np}(\omega_{0g}t) = (A_0/n) \sum_{k=1}^{n} \{2(n - k) + 1\} \cdot \int_0^{\omega_{0g}t} \int_0^{w} J_{2k-1}(w - z)J_{2(n-k)+1}(z/P)z^{-1}\, dz\, dw, \tag{5}$$