NEW RESULTS IN NETWORK SIMULATION, SENSITIVITY AND TOLERANCE ANALYSIS FOR CASCADED STRUCTURES

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ABSTRACT

An attractive, exact and efficient approach to network analysis for cascaded structures is presented. It is useful for sensitivity and tolerance analyses, in particular, for a multiple of simultaneous large changes in design parameter values.

Introduction

This paper presents a new and comprehensive treatment of computer-oriented cascaded network analysis. The approach permits efficient exact analysis, exact evaluation of first-order response sensitivities, exact evaluation of the effects of simultaneous large changes in any elements (as well as growing elements), exploitation of network structure: branches, symmetry, reciprocity, etc. All calculations are applied directly to the given network: no adjoint network is defined. All calculations involve at most the premultiplication of two by two matrices by row vectors or postmultiplications by column vectors. Response functions, sensitivities or large-change effects are represented analytically in terms of the parameters to be investigated.

Theoretical Foundation

Consider the two-port element depicted in Fig. 1(a). The basic iteration, also summarized by Table 1, is $\bar{y} = A\, y$, where A is the transmission or chain matrix, y contains the output voltage and current and $\bar{y}$ the corresponding input quantities. Forward analysis (Fig. 1(b), Table 1) consists of initializing a u^T row vector as either [1 0], [0 1] or a suitable linear combination and successively premultiplying each constant chain matrix by the resulting row vector until an element of interest, a reference plane or a termination is reached. Reverse analysis, similar to conventional analysis of cascaded networks, proceeds by initializing a v column vector as u above but uses postmultiplication.

In summary, assuming a cascade of n two-ports

$$\bar{y}^1 = y^0 = A^1 A^2 \ldots A^i \ldots A^n y^n \qquad (1)$$

and, applying forward and reverse analysis up to A^i, this reduces to an expression of the form

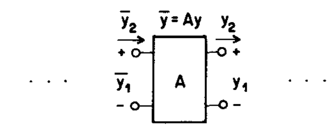
$$d = \bar{u}^{1T}\, \bar{y}^1 = c\, \bar{u}^{iT} A^i\, y^i , \qquad (2)$$

where

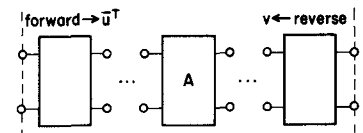
$$y^n = c\, y^n \qquad (3)$$

and c and d relate selected output and input variables of interest explicitly with A^i. The typical formula will, therefore, contain factors of the forms shown in Table 2. The (*) denotes either A, δA, $\partial A/\partial \phi$, (where ϕ is a variable parameter contained in A) or ΔA and the (†) denotes Q, δQ, Q' or ΔQ for function evaluation, first-order sensitivity, partial derivative or large-change sensitivity, respectively.

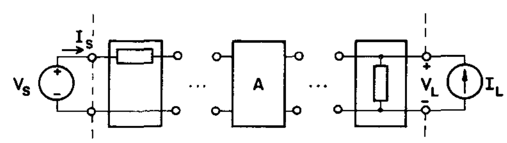
A full reverse analysis taking $[v_1^n \; v_2^n] = [e_1 \; e_2]$ yields



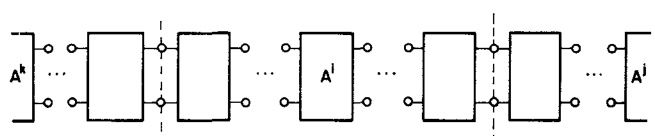
(a) Reference directions



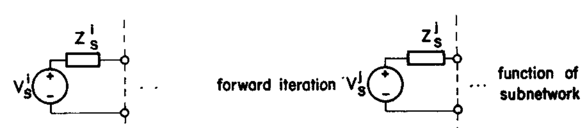
(b) Forward and reverse analyses initiated from arbitrary reference planes to an element



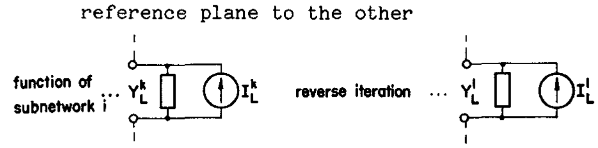
(c) Typical analysis of a cascaded network with source and load impedances assumed constant



(d) Subnetwork i cascaded with subnetworks k (at source end) and j (at load end)



(e) Forward iteration for (d), transferring an equivalent source accounting for design variables from subnetwork k from one reference plane to the other



(f) Reverse iteration for (d) transferring an equivalent source accounting for design variables from subnetwork j from one reference plane to the other

Figure 1 Notation and illustration of problems

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Table 1
Principal Concepts Involved in the Analyses

Concept	Definition	Implication
Basic iteration	$\bar{\mathbf{y}} = \mathbf{A}\,\mathbf{y}$	$\mathbf{y} \Longrightarrow \bar{\mathbf{y}}$
Forward operation	$\bar{\mathbf{u}}^T\mathbf{A} = \mathbf{u}^T$	$\bar{\mathbf{u}}^T\bar{\mathbf{y}} = \bar{\mathbf{u}}^T\mathbf{A}\mathbf{y} = \mathbf{u}^T\mathbf{y}$
Reverse operation	$\bar{\mathbf{v}} = \mathbf{A}\mathbf{v}$	$\mathbf{y} = c\mathbf{v} \Longrightarrow \bar{\mathbf{y}} = c\bar{\mathbf{v}}$
Voltage selector	$\mathbf{e}_1^T \triangleq [1 \quad 0]$	$\mathbf{e}_1 \Longrightarrow \mathbf{u}_1$ or $\mathbf{v}_1$
Current selector	$\mathbf{e}_2^T \triangleq [0 \quad 1]$	$\mathbf{e}_2 \Longrightarrow \mathbf{u}_2$ or $\mathbf{v}_2$
Equivalent source	$\mathbf{y} = \begin{bmatrix} V_S - Z_S I_S \\ I_S \end{bmatrix}$	$\mathbf{e}_1^T\mathbf{y} = V_S - Z_S I_S$, $\mathbf{e}_2^T\mathbf{y} = I_S$
Equivalent load	$\mathbf{y} = \begin{bmatrix} V_L \\ Y_L V_L - I_L \end{bmatrix}$	$\mathbf{y} = V_L\mathbf{e}_1 + (Y_L V_L - I_L)\mathbf{e}_2$

Table 2
Notation and Implied Initial Conditions

Factor	Identification	Initial Conditions Forward	Initial Conditions Reverse
$\bar{\mathbf{u}}_1^T$ (*) $\mathbf{v}_1$	$(\dagger)_{11}$	voltage	voltage
$\bar{\mathbf{u}}_1^T$ (*) $\mathbf{v}_2$	$(\dagger)_{12}$	voltage	current
$\bar{\mathbf{u}}_2^T$ (*) $\mathbf{v}_1$	$(\dagger)_{21}$	current	voltage
$\bar{\mathbf{u}}_2^T$ (*) $\mathbf{v}_2$	$(\dagger)_{22}$	current	current

* † See text for explanation: * is associated with $\mathbf{A}$ and † with Q

$$[\mathbf{v}_1^i \quad \mathbf{v}_2^i] = \mathbf{A}^{i+1}\,\mathbf{A}^{i+2} \cdots \mathbf{A}^n\,[\mathbf{e}_1 \quad \mathbf{e}_2]$$

and a corresponding full forward analysis taking

$$[\bar{\mathbf{u}}_1^1 \quad \bar{\mathbf{u}}_2^1]^T = [\mathbf{u}_1^0 \quad \mathbf{u}_2^0]^T = [\mathbf{e}_1 \quad \mathbf{e}_2]$$

yields

$$[\mathbf{e}_1 \quad \mathbf{e}_2]\;\mathbf{A}^1\,\mathbf{A}^2 \cdots \mathbf{A}^{i-1} = [\bar{\mathbf{u}}_1^i \quad \bar{\mathbf{u}}_2^i]^T .$$

Full analyses are, however, generally not required.

Reference planes to initialize the analyses are chosen so that no more than <u>one</u> element is considered between any pair of reference planes, as shown by Fig. 1. In Fig. 1(c) the element $\mathbf{A}$ is the only element whose effect is to be considered. In Fig. 1(d) the element $\mathbf{A}^i$ is considered in the ith subnetwork of the cascade. A forward iteration of the structure of Fig. 1(d) is illustrated in Fig. 1(e), where equivalent (Thevenin) sources are iteratively determined. Reverse iteration is shown in Fig. 1(f), where equivalent (Norton) sources are determined. The objectives are to explicitly highlight only specific elements of interest.

General expressions for Fig. 1 include

$$V_S^i = (\bar{\mathbf{u}}_1 + Z_S^i\,\bar{\mathbf{u}}_2)^T\,\mathbf{A}^i(V_L^i\,\mathbf{v}_1 + (Y_L^i V_L^i - I_L^i)\mathbf{v}_2) = V_L^k + Z_S^i I_S^i, \quad (4)$$

$$I_S^i = \bar{\mathbf{u}}_2^T\,\mathbf{A}^i(V_L^i\mathbf{v}_1 + (Y_L^i V_L^i - I_L^i)\mathbf{v}_2) = V_L^k Y_L^k - I_L^k . \quad (5)$$

From (4), letting $I_L^i = 0$ and $Y_L^i = 0$, we have $I_S^j = 0$ and the Thevenin voltage

$$V_S^j = V_L^i = \frac{V_S^i}{(\bar{\mathbf{u}}_1 + Z_S^i\bar{\mathbf{u}}_2)^T\,\mathbf{A}^i\mathbf{v}_1} = \frac{V_S^i}{Q_{11}^i + Z_S^i Q_{21}^i} . \quad (6)$$

Letting $V_S^i = 0$ and $Y_L^i = 0$, we have $I_S^j = -I_L^i$ and the output impedance

$$Z_S^j = \frac{V_L^i}{I_L^i} = \frac{(\bar{\mathbf{u}}_1 + Z_S^i\bar{\mathbf{u}}_2)^T\,\mathbf{A}^i\mathbf{v}_2}{(\bar{\mathbf{u}}_1 + Z_S^i\bar{\mathbf{u}}_2)^T\,\mathbf{A}^i\mathbf{v}_1} = \frac{Q_{12}^i + Z_S^i Q_{22}^i}{Q_{11}^i + Z_S^i Q_{21}^i} . \quad (7)$$

These expressions for V_S^j and Z_S^j permit equivalent Thevenin sources to be moved in a forward iteration.

From (4) and (5), letting $I_L^i = 0$ and $Z_S^i = 0$ we have $I_L^k = 0$ and the input admittance

$$Y_L^k = \frac{I_S^i}{V_S^i} = \frac{\bar{\mathbf{u}}_2^T\,\mathbf{A}^i(\mathbf{v}_1 + Y_L^i\mathbf{v}_2)}{\bar{\mathbf{u}}_1^T\,\mathbf{A}^i(\mathbf{v}_1 + Y_L^i\mathbf{v}_2)} = \frac{Q_{21}^i + Y_L^i Q_{22}^i}{Q_{11}^i + Y_L^i Q_{12}^i} . \quad (8)$$

Letting $V_S^i = 0$ and $Z_S^i = 0$, we have $V_L^k = 0$ and the Norton current

$$I_L^k = -I_S^i = -I_L^i(Y_L^k\bar{\mathbf{u}}_1 - \bar{\mathbf{u}}_2)^T\,\mathbf{A}^i\mathbf{v}_2 = -I_L^i(Y_L^k Q_{12}^i - Q_{22}^i) . \quad (9)$$

These expressions for I_L^k and Y_L^k permit equivalent Norton sources to be moved in a reverse iteration.

The input current I_S^i for $I_L^i = 0$ is obtained via (8) as

$$I_S^i = \frac{V_S^i(Q_{21}^i + Y_L^i Q_{22}^i)}{Q_{11}^i + Y_L^i Q_{12}^i + Z_S^i Q_{21}^i + Z_S^i Y_L^i Q_{22}^i} . \quad (10)$$

Useful special cases of these formulas for I_S and V_L in Fig. 1(c) are, from (10) and (6), respectively,

$$I_S = V_S\frac{\bar{\mathbf{u}}_2^T\mathbf{A}\mathbf{v}_1}{\bar{\mathbf{u}}_1^T\mathbf{A}\mathbf{v}_1} = V_S\frac{Q_{21}}{Q_{11}} , \quad (11)$$

$$V_L = \frac{V_S}{\bar{\mathbf{u}}_1^T\mathbf{A}\mathbf{v}_1} = \frac{V_S}{Q_{11}} . \quad (12)$$

Table 3 gives useful formulas obtained for variations in a particular element $\mathbf{A}$. We note, for example, that, since $\mathbf{A}$ is arbitrary and at most only one full analysis yields all Q_{11}, δQ_{11}, Q_{11}' and ΔQ_{11}, the corresponding V_L, δV_L, $\partial V_L/\partial\phi$ and ΔV_L for all possible parameters anywhere in the cascade can be evaluated exactly for one network analysis. This special case is equivalent to the results of previous researchers[1,2].

For a symmetrical network det $\mathbf{A}$ = 1 and

Table 3
Functions of Input Current I_S and Output Voltage V_L for Changes in $\underset{\sim}{A}$ Only

Variable	Input	Output
$\underset{\sim}{A}$	$I_S = V_S \frac{Q_{21}}{Q_{11}}$	$V_L = \frac{V_S}{Q_{11}}$
$\delta\underset{\sim}{A}$	$\delta I_S = \frac{V_S\delta Q_{21} - I_S\delta Q_{11}}{Q_{11}}$	$\delta V_L = -\frac{V_L^2}{V_S}\delta Q_{11}$
$\frac{\partial\underset{\sim}{A}}{\partial\phi}$	$\frac{\partial I_S}{\partial\phi} = \frac{V_S Q'_{21} - I_S Q'_{11}}{Q_{11}}$	$\frac{\partial V_L}{\partial\phi} = -\frac{V_L^2}{V_S} Q'_{11}$
$\Delta\underset{\sim}{A}$	$\Delta I_S = \frac{V_S\Delta Q_{21} - I_S\Delta Q_{11}}{Q_{11}+\Delta Q_{11}}$	$\Delta V_L = -\frac{V_L^2}{V_L + V_S/\Delta Q_{11}}$

$$[\underset{\sim}{e}_1 \; -\underset{\sim}{e}_2] \; \underset{\sim}{A} \; [\underset{\sim}{e}_1 \; -\underset{\sim}{e}_2] = \underset{\sim}{A}^{-1}$$

and it may be shown that, for such networks,

$$[\underset{\sim}{v}_1^i \quad \underset{\sim}{v}_2^i] = [\bar{\underset{\sim}{u}}_1^{n-i+1} \quad \bar{\underset{\sim}{u}}_2^{n-i+1}]^T$$

can be used to reduce computational effort.

Numerical Example

The cascaded seven-section bandpass filter[3,4] shown in Fig. 2, was analyzed using the suggested

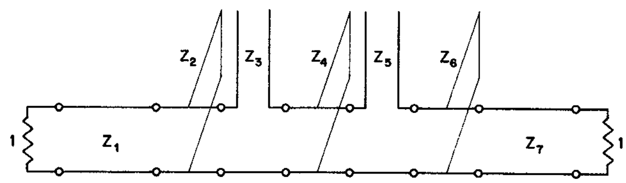

Figure 2 Seven-section filter containing unit elements and stubs. All sections are quarter-wave at 2.175 GHz

approach. All sections are quarter-wave at 2.175 GHz. The optimal characteristic impedances are

$$Z_1^0 = Z_7^0 = 0.606463 \qquad Z_2^0 = Z_6^0 = 0.303051$$

$$Z_3^0 = Z_5^0 = 0.722061 \qquad Z_4^0 = 0.235593$$

The output voltage at a normalized frequency of 0.7 $V_L = 0.49740790 - j3.9011594x10^{-3}$ was verified twice using (6): once associating $\underset{\sim}{A}^1$ with Z_3 and once with Z_4. Furthermore, one analysis yielded

$$V_L(Z_4^0+0.03) = 0.49838950 - j\ 0.034901610$$

$$V_L(Z_4^0-0.03) = 0.49062912 + j\ 0.034959186$$

The open-circuit voltage at the load end was calculated using (6) as

$$V_{OC} = 0.98624507 + j\ 0.092266904$$

and the Thevenin impedance using (7) is

$$Z_{TH} = 0.98119253 + j\ 0.20103391$$

which further verified V_L.

One analysis yielded for $\varepsilon_2 = 0.021$, $\varepsilon_5 = 0.024$

$$V_L(Z_2^0-\varepsilon_2, Z_5^0-\varepsilon_5) = 0.49719716 + j\ 2.2191360x10^{-3}$$

$$V_L(Z_2^0+\varepsilon_2, Z_5^0-\varepsilon_5) = 0.49583538 - j\ 2.3636314x10^{-2}$$

$$V_L(Z_2^0-\varepsilon_2, Z_5^0+\varepsilon_5) = 0.49732462 + j\ 1.7909912x10^{-2}$$

$$V_L(Z_2^0+\varepsilon_2, Z_5^0+\varepsilon_5) = 0.49751427 - j\ 8.3726470x10^{-3}$$

A multidimensional quadratic approximation[5] was carried out for V_L. The variables for the approximation were the characteristic impedances as well as the normalized frequency. The center base point had the characteristic impedances as given before and a normalized frequency of 0.7. 45 base points[5] with characteristic impedances perturbed by ±0.03 and normalized frequency by ±0.01 were needed. The symmetry of the structure was taken into consideration in choosing the base points. The following characteristic impedances were chosen:

$$Z_1 = Z_7 = 0.606595 \qquad Z_2 = Z_6 = 0.303547$$

$$Z_3 = Z_5 = 0.722287 \qquad Z_4 = 0.235183$$

The group delay using the derivative of V_L w.r.t. ω obtained from the quadratic approximation is 0.893 ns while the exact group delay[6] is 0.895 ns.

Conclusion

An important claim we make in this paper is that (4) - (10) generate, following differencing or differentiating (as appropriate), any desired exact formulas for multiple network analyses, sensitivity and tolerance analysis with simultaneous large changes. All calculations are carried forward simultaneously and redundant calculations are obviated.

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