Proc. 21st Midwest Symposium on Circuits and Systems, Ames, Iowa, August 1978.

ALGORITHMS FOR DESIGN CENTERING INVOLVING YIELD AND ITS SENSITIVITIES

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Invited Paper

Abstract

This paper reviews the approach to design centering and optimal assignment of component tolerances adopted by the authors. In particular, this paper surveys suitable algorithms for carrying out yield optimization involving explicit formulas for evaluating yield and its sensitivities with respect to the designable parameters. Examples and available programs are referenced. A comparison with the simplicial approximation is attempted.

1. INTRODUCTION

A review is made of the approach to design centering and optimal assignment of component tolerances adopted by the authors [1-11]. It permits the effective use of generally available simulation programs [9] such as SPICE2 [12]. The authors' approach to centering via nonlinear programming in which the production yield is allowed to drop below 100% [5-9] is explained. Arbitrary statistical distributions are handled through explicit formulas for yield and its sensitivities.

This paper surveys suitable algorithms for carrying out the yield optimization. Involved are steps to evaluate yield and its sensitivities concentrating for computational efficiency on constraints which are candidates for being violated, avoiding as far as possible errors due to the overlapping of nonfeasible regions described by different constraints within the tolerance region [11]. Exact description of the boundary of the constraint region via a generalized function of the least pth type leads to new results applicable to postproduction tuning [5,10]. Here, a tolerance problem equivalent to the tolerance and tuning problem of Bandler, Liu and Tromp [1] is presented. Based on this equivalence a mathematical definition of postproduction yield has been developed and interpreted [5,10]. Results, for example, have been obtained for optimal worst-case tolerance assignment and design centering on an active filter in which a postproduction tuning variable was taken into account [10].

Central to our computational approach is the quadratic modeling of the constraint functions [2-5]. These models have to be determined and updated efficiently. To this end inherent symmetry [5] in the functions w.r.t. the parameters is exploited and sparsity is forced in solving for the quadratic models [7,8].

2. THEORETICAL REVIEW

2.1 QUADRATIC MODELING AND SOME IMPLICATIONS

A nonlinear programming approach based on work by Bandler [13] and Bandler, Liu and Tromp [1], but employing approximations to the design constraints has been described in detail by the present authors [3,11]. An interpolation region centered at the initial guess to the nominal design is chosen. The simulation program is used to provide the value of the response functions (constraints) at a certain set of base points (see Fig. 1). The base points are points within the interpolation region and defined in terms of values of the designable parameters. Based upon the corresponding values of the resulting responses, multidimensional quadratic polynomials are constructed. These quadratic polynomials have the general form

$$P(\underset{\sim}{\phi}) = a_0 + \underset{\sim}{a}^T(\underset{\sim}{\phi} - \bar{\underset{\sim}{\phi}}) + \frac{1}{2}(\underset{\sim}{\phi} - \bar{\underset{\sim}{\phi}})^T \underset{\sim}{H}(\underset{\sim}{\phi} - \bar{\underset{\sim}{\phi}}) , \quad (1)$$

where a_0 and $\underset{\sim}{a}$ are, respectively, a constant scalar and a constant vector, $\underset{\sim}{H}$ is a constant

This work was supported by the National Research Council of Canada under Grant A7239 and by a Postdoctoral Fellowship to H.L. Abdel-Malek.

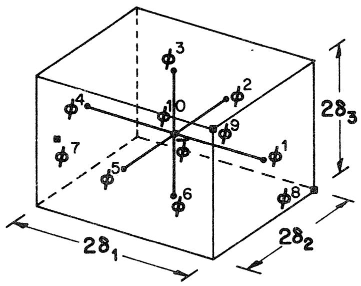

Fig. 1 Arrangement of the base points w.r.t. the center of an interpolation region in 3 dimensions. To exploit sparsity ϕ^7, ϕ^8 and ϕ^9 are, respectively, placed in the planes containing $\{\bar{\phi}, \phi^1, \phi^2\}$, $\{\bar{\phi}, \phi^1, \phi^3\}$ and $\{\bar{\phi}, \phi^2, \phi^3\}$[7].

symmetric Hessian matrix of the quadratic and $\bar{\phi}$ is the center of the chosen interpolation region.

The base points are simply those points where the approximated response function and the quadratic polynomial coincide. Since the quadratic polynomial is a linear function in the coefficients (a_0, elements of a and elements of H), a system of simultaneous linear equations has to be solved to obtain the polynomial. The number of base points (exactly equal to the number of simulations required) is the minimum necessary to fully describe the responses by the multidimensional quadratic polynomials. This minimum number is given by

$$N = (k+1)(k+2)/2 \ , \qquad (2)$$

where k is the number of designable parameters, i.e., elements of variable vector ϕ. The number N is, of course, also the number of the unknown coefficients (a_0, a and H).

The authors have suggested ways of reducing the computational effort in solving the resulting system of N simultaneous linear equations ($N^3/3 + N^2 - N/3$ multiplications or divisions for Gauss elimination). Sparsity was forced in the system matrix [7] by a special choice of base points.

The base points are chosen according to the equation

$$[\phi^1 \ \phi^2 \ \ldots \ \phi^N] = \begin{bmatrix} \delta_1 & & & \\ & \delta_2 & & \\ & & \ddots & \\ & & & \delta_k \end{bmatrix} [1_k \ \ -1_k \ \ B \ \ 0_k] + [\bar{\phi} \ \bar{\phi} \ \ldots \ \bar{\phi}] \ , \qquad (3)$$

where ϕ^i, i = 1, 2, ..., N, are the base points, δ_i, i = 1, 2, ..., k, are parameters defining the size of the interpolation region (see Fig. 1), 1_k is the identity matrix of order k, 0_k is the zero vector of order k and B is a k × k(k-1)/2 matrix having only two entries in each row and considering all possible combinations. In other words, the base points consist of (a) the center of interpolation $\bar{\phi}$, (b) points with only one parameter at a time set to its positive or negative extreme value within the interpolation region, while other parameters are fixed at their values at $\bar{\phi}$ and (c) points where only two parameters are different from their values at $\bar{\phi}$. According to this scheme, the number of operations (multiplications or divisions) is reduced to only $5k^2 - 2k$.

The following useful theorem dealing with the preservation of parameter symmetry has been proved [5].

<u>Theorem</u> If $f(\phi)$ is symmetric w.r.t. a matrix S, i.e.,

$$f(S\,\phi) = f(\phi) \ , \qquad (4)$$

where S is a k×k permutation matrix obtained by interchanging suitable rows of a unit matrix, then the approximating polynomial $P(\phi)$ will be symmetric w.r.t. S if, for each base point ϕ^i, $S\phi^i$ is also a base point.

Symmetry will appear, for example, in certain cascaded structures. Its preservation is effected by appropriate choice of base points. Computational effort is reduced by not repeating unnecessary analyses.

Another useful theorem, which follows, deals with the preservation of one-dimensional convexity [5,11].

<u>Theorem</u> If there exist three distinct base points ϕ^1, ϕ^2 and ϕ^3 in the ith direction, i.e.,

$$\phi^j = \phi^1 + c_j\, e_i \ , \ j = 2, 3, \qquad (5)$$

where c_j, j = 2, 3, are scalars and e_i is the unit vector in the ith direction, then the approximating polynomial $P(\phi)$ is one-dimensionally convex/concave in the ith variable if the approximated function $f(\phi)$ is so.

Being the property which makes vertices of the tolerance region (Fig. 2) worst cases [13], one-dimensional convexity maybe an important property to preserve. We should note here that the choice of base points in (3) satisfies the requirement of this theorem. See also Fig. 1.

The quadratic approximations are updated as the optimization process or accuracy may require [11]. If the optimization indicates an optimum far away from the interpolation region the approximations are updated. Also, if higher accuracy is required the size of the interpolation region is reduced and hence the approximations are updated. Since

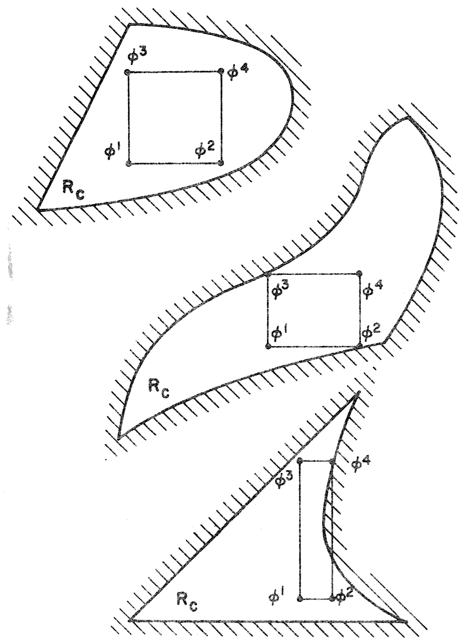

Fig. 2 Convex, one-dimensionally convex and nonconvex regions with representative tolerance regions. Note that, in the nonconvex case, the vertices are not worst cases [13].

vertices of the tolerance region are considered to be critical, approximations should be updated to cover potentially active vertices. In order to save effort as many vertices as possible may be collected within an interpolation region as shown in Fig. 3.

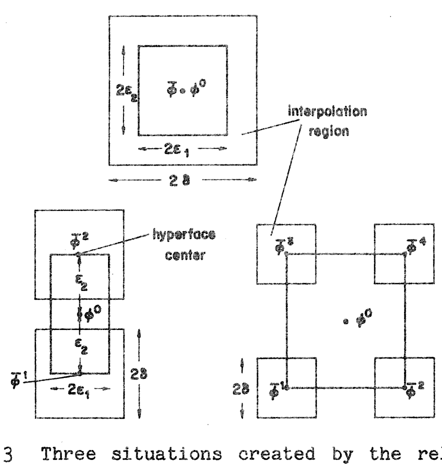

Fig. 3 Three situations created by the relative sizes of the tolerance and interpolation regions [11].

An algorithm employing the following simple formulas based on Taylor expansions has been developed [11].

$$P(\underset{\sim}{\phi}^b) = P(\underset{\sim}{\phi}^a) + 2\varepsilon_i \frac{\partial P(\underset{\sim}{\phi}^a)}{\partial \phi_i} + 2\varepsilon_i^2 H_{ii} , \qquad (6)$$

$$\underset{\sim}{\nabla} P(\underset{\sim}{\phi}^b) = \underset{\sim}{\nabla} P(\underset{\sim}{\phi}^a) + 2\varepsilon_i \underset{\sim}{H}_i , \qquad (7)$$

where

$$\underset{\sim}{\phi}^b = \underset{\sim}{\phi}^a + 2\varepsilon_i \underset{\sim}{e}_i , \qquad (8)$$

and where $\underset{\sim}{\phi}^a$ and $\underset{\sim}{\phi}^b$ are two neighboring vertices of the tolerance region, H_{ii} is the ith diagonal element of $\underset{\sim}{H}$ and $\underset{\sim}{H}_i$ is the ith column of $\underset{\sim}{H}$. The algorithm is used to compute the values and gradients of the quadratic polynomial approximations required by the optimization program in an efficient manner.

Since the approximations embody information about the sensitivities of the approximated constraints, w.r.t. the designable parameters, they can be utilized for investigating the effects of slightly perturbing some constraints without requiring any additional simulations. This is simply effected by altering the scalar a_0 in the approximation of a perturbed constraint.

2.2 YIELD DETERMINATION VIA LINEAR CUTS

It is inexpensive to conduct a Monte Carlo analysis in conjunction with the approximation, however, the resulting yield will not be a continuous function of the design parameters due to the finite number of Monte Carlo analyses. Also, yield sensitivities are not available from the Monte Carlo analysis. Both continuity of yield and the availability of its sensitivites are of particular relevance if optimization (of yield or cost) is used. The authors have, therefore, directed their efforts to developing a method incorporating these features. This method can also be used by itself for yield analysis only [6].

The basic idea is to use weighted hypervolumes for evaluating the yield [6]. Evaluating hypervolumes, in general, is expensive because it involves a multidimensional integration. For the special case of cutting an orthotope (generalization of a rectangle in a multidimensional space) representing the tolerance region by a linear constraint, a simple formula can be found [5,6]. The volume removed by the linear cut in Fig. 4, for example, is given by

$$V = \frac{1}{6}\alpha_1\alpha_2\alpha_3 \left[1 - \left(1-\frac{2\varepsilon_1}{\alpha_1}\right)^3 - \left(1-\frac{2\varepsilon_2}{\alpha_2}\right)^3 - \left(1-\frac{2\varepsilon_3}{\alpha_3}\right)^3 + \left(1-\frac{2\varepsilon_1}{\alpha_1}-\frac{2\varepsilon_2}{\alpha_2}\right)^3\right] . \qquad (9)$$

A general formula has been derived [6] and,

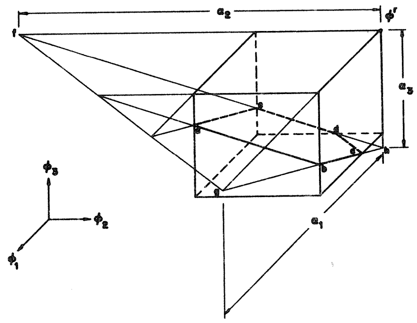

Fig. 4 Three dimensional illustration of the intersection of a linear cut with the tolerance region. $\underset{\sim}{\phi}^r$ is a reference vertex [6].

accordingly, the yield in the case of a uniform distribution of outcomes within the tolerance region can be expressed as

$$Y = 1 - \sum_{\ell=1}^{m} V^{\ell} , \qquad (10)$$

where V^{ℓ} is the ratio of the nonfeasible hypervolume defined by the ℓth linear cut to the hypervolume of the tolerance region ($2^k \prod_{i=1}^{k} \varepsilon_i$), m the number of these cuts which are supposed to approximate the boundary of the constraint region. The assumptions made are

(1) It is possible to locally approximate the boundary of the constraint region by linear cuts with reasonable accuracy.

(2) The nonfeasible hypervolumes defined by the different linear cuts do not overlap within the tolerance region in order to be able to sum them as in (10).

The method does not assume that these linear cuts are fixed in the parameter space. It is possible [7,11] that these linear cuts be continuously updated to follow the generally nonlinear constraints. This facilitates a good approximation to the boundary of the constraint region as the tolerance region is allowed to move in the parameter space during, for example, an optimization process. Methods for continuously updating the linear cuts have been given [7,11].

Arbitrary statistical distributions can be handled through regionalization [14]. The tolerance region is partitioned to orthocells (see Fig. 5). Equation (10) can also be used to provide the yield but V^{ℓ} is now the weighted nonfeasible hypervolume within the tolerance region defined by the ℓth linear cut. The weight $W(i_1, i_2, \ldots, i_k)$ assigned to the orthocell $(i_1, i_2, \ldots, i_k)$ is the probability per unit volume of having an outcome (a set of parameter values) within this cell.

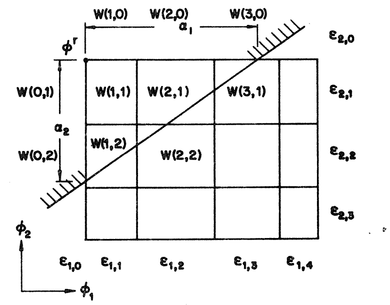

Fig. 5 Two-dimensional illustration of the partitioning of the tolerance region into cells indicating dimensions and weighting of those cells relevant to the calculation of the weighted nonfeasible hypervolume [6,7].

The freedom in assigning the dimensions of the orthocells allows the use of previous information, such as a worst-case design or a histogram representing the relative frequencies of the outcomes.

Having an analytical formula for the yield facilities inexpensive yield and yield sensitivity evaluations. The procedure is suitable for optimization, in particular due to the relatively large number of yield analyses required.

3. EXAMPLES AND COMPUTER PROGRAMS

This work was first introduced in conjunction with 2-section, lossless transmission-line transformers [2]. These examples verified the accuracy attainable by quadratic approximations, particularly if they are updated. Worst-case tolerance optimization of inhomogeneous waveguide transformers [11] justified the use of available simulation programs that do not provide sensitivity information. These examples also confirm the accuracy of running Monte Carlo analyses in conjunction with the quadratic models.

Karafin's bandpass filter [15,16] provided a vehicle for testing the yield evaluation via linear cuts derived at an optimal worst-case toleranced solution. Three types of distributions were tested: the uniform distribution, the bimodal distribution, and the normal distribution [6].

Optimal centering and optimization of the tolerances of the components of a tunable active filter provided further verification of the use of this work with general purpose simulators, in this case SPICE2 [12]. In fact, by carefully preparing the data for evaluation of responses at the base points, only two runs of SPICE2 sufficed to produce an excellent optimal result [9].

A current switch emitter follower was optimized in a number of ways [8]. A worst-case optimization of the nominals and tolerances of 4 circuit parameters was carried out. The quadratic models were updated once hence, using (2), 30 simulations (complete integrations) were required, as shown in Table 1. Table 1 also indicates the effort required to maximize yield assuming a total of 8 parameters subject to statistical fluctuations, 4 of which were the previously mentioned circuit parameters and 4 were correlated transistor model parameters. The number of further simulations required was 45 (substitute k = 8 into (2)). the yield was increased from 39% to 89%.

TABLE 1
SUMMARY OF EFFORT FOR THE CSEF

Optimization problem	N.O.S.	N.O.Y.E.	CDC time M	CDC time O
worst-case	30	0	48 s	55 s
yield	75	49	122 s	96 s
perturbed constraints and specifications	0	~50	0	~45 s

N.O.S. = number of simulations
N.O.Y.E. = number of yield evaluations
M = modeling
O = optimization

The integrations in the paper by Abdel-Malek and Bandler [8] were executed through a specially written program called CSEF [17] for solving the state equations associated with this nonlinear circuit. The analysis has, however, been verified both by SPICE2 [9] and by another specially written program employing the companion network [18]. A comparison for the various integration schemes has been presented [9].

Selecting 3 appropriate different values for a_0 in (1) to investigate 3 sets of specifications and defining one different parameter constraint and reoptimizing did not involve further quadratic modeling, as indicated in Table 1. As seen from the table less than one second was required to carry out a complete yield and yield sensitivity evaluation, while more than 1.5 seconds are generally required for only one numerical integration using Gear's method [19,20].

A documented program package is available [21] for evaluating the coefficients of the quadratic approximation using the sparse approach [7]. The program is called MODEL4. The evaluation of the approximating polynomials and their gradients at vertices of the tolerance region is carried out by a subroutine called QPE [21]. The optimization methods used by the authors are based upon the exact minimax approach developed by Bandler and Charalambous [22] in conjunction with Fletcher's quasi-Newton program for unconstrained functions [23]. The least pth approach with extrapolation is employed [24]. The optimization package is called FLOPT4 [25]. The required gradient information is either already available or is obtained from the above mentioned quadratic models.

We should also mention that quadratic modeling has been successfully applied to a microwave filter example consisting of lossless transmission lines and stubs [26]. The filter is symmetrical around a central plane. Eight variables were considered: seven characteristic impedances and frequency. 45 base points are altogether involved, however, due to symmetry only 24 distinct points required actual analysis. The approximation w.r.t. frequency permitted an evaluation of group delay.

4. COMPARISON WITH SIMPLICIAL APPROXIMATION

In this section we commit to print our temerity in comparing our approach based upon quadratic approximations and linear cuts with the continual evolution and refinement of the simplicial approach originally devised by Director and Hachtel [27-32]. In this connection we refer to Table 2, and add the following amplification within three relatively well-defined contexts: Monte Carlo analysis, design centering and yield maximization.

TABLE 2
COMPARISON OF TWO PRINCIPAL APPROACHES

Assumptions, features	Quadratics, linear cuts	Simplicial approximation
sensitivities	none	none
linearization	yes	yes
convexity	no	yes
implementation	difficult	easy
design centering	yes	yes
design centering (some nominals fixed)	reduced effort	same effort
optimal worst-case tolerances	yes	no
M.C. analysis	inexpensive	inexpensive
M.C. analysis (different distributions)	inexpensive	inexpensive
optimal yield	yes	no
perturbed specifications	inexpensive	restart

Both approaches make an attempt to model the constraint boundary and, hence, can be used for Monte Carlo analysis. Presently, simplicial approximation is restricted to convex regions, whereas the quadratic model, of course, is not. Neither approach yields approximations accurate everywhere. Simplicial approximation develops a relatively large number of linear constraints or point bases while we develop quadratic constraints generally of the order of the number of actual constraints.

Both approaches provide design centers but our approach permits optimal worst case tolerances w.r.t. any proposed objective function, whereas the objective of the simplicial method seems to us nebulous. In our approach the dimensionality of the space of designable parameters is independent of the space of statistically fluctuating parameters (important in dealing with parasitic effects and model uncertainties, e.g., see references [8,33]). The simplicial method assumes a variable nominal point for each parameter subject to uncertainty.

Our dynamically updated linear cuts along with explicit formulas for weighted hypervolumes address the yield optimization problem explicitly. Furthermore, our approach can utilize histograms involving arbitrary distributions. The latest developments in the simplicial method appear to handle (albeit indirectly) only up to second order moments of the distribution and hence assume symmetrical distributions.

Finally, we note again that we can inexpensively consider perturbed specifications, while the simplicial method appears to require restarting.

5. CONCLUSIONS

This paper has reviewed the theoretical work of the authors on optimal design centering, the optimal assignment of component tolerances, tuning, yield determination and optimization. Attention has been directed at realistic engineering examples, algorithms for implementing the theory and available documented computer programs. Finally, we have attempted some comparison between our approach and that of Director, Hachtel and their coworkers. We concede that the comparison may be premature: we are convinced that neither approach has yet reached its limit.

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