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FAULT LOCATION OF ANALOG CIRCUITS

R.M. Biernacki and J.W. Bandler

Group on Simulation, Optimization and Control, Faculty of Engineering

McMaster University

Hamilton, Canada, L8S 4L7

ABSTRACT

This paper deals with fault detection for linear analog circuits. The methods described are based on measurements of voltage using current excitations and have been developed for the location of single as well as for multiple faults. They utilize certain algebraic invariants of faulty elements. Computationally, they depend on checking the consistency or inconsistency of suitable sets of linear equations. The equations themselves are formulated via adjoint circuit simulations.

INTRODUCTION

The main objective of testing is to check whether a circuit, already manufactured, meets the required specifications or not. If not, testing should detect the source which causes the circuit to be wrong, principally, to indicate the element(s) which is (are) at fault. By a fault we mean not only an unwanted short or open circuit but also, more generally, any large change in the value of an element w.r.t. its nominal value. We assume that the network design, i.e., the topology as well as the nominal values of the parameters are known.

Fault location can be done by the method which identifies all element values (e.g., [1]) and then comparing the nominal and actual values. However, we usually look for one, two or several faults and there is no need to identify everything as though we did not know anything about the network.

There are a few papers dealing with fault analysis without identifying all elements, mostly to locate single faults. This can be done by constructing a fault dictionary using computer simulation of mainly single catastrophic faults [2,3]. Another approach uses certain analytical or geometrical invariants of element value changes [4-8]. The latter approach is worth consideration since it enables us to deal also with other than catastrophic faults and the computational effort required is much smaller than in the case of fault dictionaries.

This paper presents a new approach to fault detection in the foregoing sense. Analog linear and lumped networks are considered. Methods for single as well as for multiple fault location are proposed. The methods are based on checking consistency or inconsistency of certain equations which are independent of faulty elements. The measurement tests are assumed to be performed at a single frequency.

SINGLE-FAULT DETECTION

Consider two different network functions f_1 and f_2 of the same element Y as

$$f_1 = \frac{A_1 + B_1 Y}{C_1 + D_1 Y}, \quad f_2 = \frac{A_2 + B_2 Y}{C_2 + D_2 Y}. \tag{1}$$

If the two functions essentially depend on Y then each of them can be solved for Y as

$$Y = \frac{A_1 - C_1 f_1}{-B_1 + D_1 f_1} = \frac{A_2 - C_2 f_2}{-B_2 + D_2 f_2}. \tag{2}$$

From (2) we find the relation

$$(C_1B_2 - D_1A_2)f_1 + (A_1D_2 - B_1C_2)f_2 = (A_1B_2 - B_1A_2) + (C_1D_2 - D_1C_2)f_1f_2. \tag{3}$$

If the two network functions are of the same type (3) becomes the linear relation

$$af_1 + bf_2 = c, \tag{4}$$

where $a \triangleq C_1B_2 - D_1A_2$, $b \triangleq A_1D_2 - B_1C_2$ and $c \triangleq A_1B_2 - B_1A_2$.

Equation (4) gives us the relationship between values of f_1 and f_2 when all network elements except Y are kept unchanged. Similar relationships between f_1 and f_2 can be derived for all other elements. This is done for nominal values of all elements. Therefore, we obtain p equations

$$a^i f_1 + b^i f_2 = c^i, \quad i = 1, 2, \ldots, p, \tag{5}$$

each of them corresponding to a certain element of the network.

Based on measurements, we find the actual values of f_1 and f_2. If there is a single fault within the network then the equation corresponding to the faulty element is satisfied since all other elements are at their nominal values. All other equations are likely to be unsatisfied. To be able to locate uniquely the faulty element it is required that

$$\det \begin{bmatrix} a^k & b^k \\ a^\ell & b^\ell \end{bmatrix} \neq 0, \tag{6}$$

for any k, ℓ, $k \neq \ell$.

Since the nominal values satisfy equations (5) we can use the changes Δf_1 and Δf_2 instead of

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f_1 and f_2 and we have homogeneous equations

$$a^i \Delta f_1 + b^i \Delta f_2 = 0, \quad i = 1, 2, \ldots, p. \quad (7)$$

The actual values of the network functions f_1 and f_2 are to be identified by measurements. Using, preferably, current excitation and voltage measurements the two network functions should be certain impedances or trans-impedances

$$f_j = V_j^m / I_{gj}, \quad j = 1, 2. \quad (8)$$

The two excitations I_{g1} and I_{g2} do not need to be applied to the same port, but if they are then the voltage measurements V_1^m and V_2^m can be taken simultaneously at the same measurement test. We now derive a simple method which supplies the coefficients of equation (7) for the latter case.

Consider the representation of the network shown in Fig. 1. Note that the 4-port network

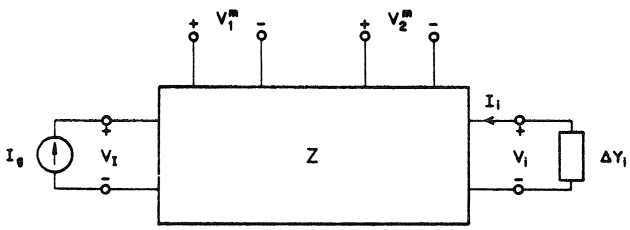

Fig. 1 Network with a single fault.

consists of elements which are at their nominal values. According to Fig. 1 we have

$$\underset{\sim}{V} \triangleq [V_1^m \ V_2^m \ V_I \ V_i]^T = \underset{\sim}{Z}[\ 0 \ 0 \ I_g \ -V_i \Delta Y_i]^T. \quad (9)$$

Since the left hand side of (9) can be expressed as $\underset{\sim}{V} = \underset{\sim}{V}^0 + \Delta \underset{\sim}{V}$, where $\underset{\sim}{V}^0$ is the nominal vector obtained for $\Delta Y_i = 0$, we find

$$\Delta \underset{\sim}{V} = \underset{\sim}{Z}[0 \ 0 \ 0 \ I_i]^T. \quad (10)$$

Thus

$$\begin{bmatrix} \Delta V_1^m \\ \Delta V_2^m \end{bmatrix} = I_i \begin{bmatrix} Z_{14} \\ Z_{24} \end{bmatrix} . \quad (11)$$

Eliminating I_i from (11) we obtain

$$Z_{24} \Delta V_1^m - Z_{14} \Delta V_2^m = 0. \quad (12)$$

Note that in order to be able to eliminate I_i at least one of Z_{14} and Z_{24} has to be different from zero.

The equation (12) is one of the equations (7). It corresponds to the ith element. In this way we can find all equations (7). But it would be inconvenient to consider as many different 4-port networks as the number of elements. We propose to use the adjoint network simulation for this purpose. The method is explained in Fig. 2.

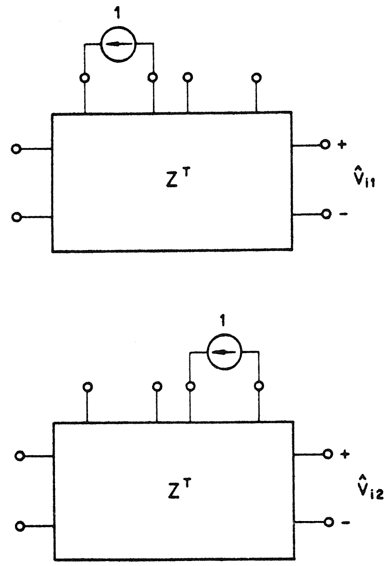

Fig. 2 Adjoint network simulation.

We find

$$\hat{V}_{i1} = Z_{14} \text{ and } \hat{V}_{i2} = Z_{24}. \quad (13)$$

Thus, the equation (12) can be rewritten as

$$\hat{V}_{i2} \Delta V_1^m - \hat{V}_{i1} \Delta V_2^m = 0. \quad (14)$$

Therefore, in order to obtain the coefficients of the equations (7) two simulations of the adjoint network are required. First, we apply a unit current to the first measurement port and calculate the voltages across all elements $\hat{V}_{11}$, $\hat{V}_{21}$, ..., $\hat{V}_{p1}$. Second, applying a unit current to the second measurement port we find $\hat{V}_{12}$, $\hat{V}_{22}$, ..., $\hat{V}_{p2}$. Finally, we formulate the equations (14) for $i = 1, 2, \ldots, p$. In fact, only one simulation is required since in both cases we have to solve exactly the same system of equations with different right hand sides. It can be calculated simultaneously, or alternatively, using the same LU factorization.

Finally, it is to be noted that the above method can be used to detect more general faults like shorts between nonincident nodes. We can simply consider nonexisting elements between such nodes as elements of nominal value Y = 0.

MULTIPLE-FAULT DETECTION

We now generalize the foregoing approach in order to be able to deal with several simultaneous faults within the network. These faults are represented as external loads of (n+k)-port network shown in Fig. 3. We consider n ports of measurement with the voltage vector $\underset{\sim}{V}^m$ and the current vector $\underset{\sim}{I}^m$. The ports of fault are described by the voltage vector $\underset{\sim}{V}^x$ and the current vector

$$\underset{\sim}{I}^x = -[V_1^x \Delta Y_1^x \ V_2^x \Delta Y_2^x \ldots V_k^x \Delta Y_k^x]^T, \quad (15)$$

where $k \leq n-1$.

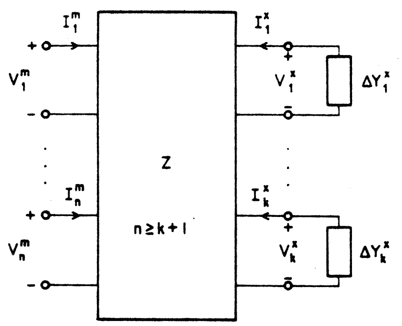

Fig. 3 Network with k simultaneous faults.

Assuming that the impedance matrix Z of the (n+k)-port network exists we have

$$\begin{bmatrix} V^m \\ V^x \end{bmatrix} = \begin{bmatrix} Z_{mm} & Z_{mx} \\ Z_{xm} & Z_{xx} \end{bmatrix} \begin{bmatrix} I^m \\ I^x \end{bmatrix}. \qquad (16)$$

If the ports of measurement are open-circuited or are excited by independent current sources we find that the voltage change vector can be expressed as

$$\begin{bmatrix} \Delta V^m \\ \Delta V^x \end{bmatrix} = Z \begin{bmatrix} 0 \\ I^x \end{bmatrix}, \qquad (17)$$

and, in particular,

$$\Delta V^m = Z_{mx} I^x. \qquad (18)$$

Z_{mx} is a rectangular matrix having more rows than columns. Assuming that Z_{mx} is a full column rank matrix we can find the solution of the equation (18) as

$$I^x = (Z_{mx}^T Z_{mx})^{-1} Z_{mx}^T \Delta V^m. \qquad (19)$$

Therefore, eliminating I^x from (18) and (19) we find the equation

$$[Z_{mx}(Z_{mx}^T Z_{mx})^{-1} Z_{mx}^T - 1]\Delta V^m = 0, \qquad (20)$$

which is a generalization of equation (12). Given a vector of voltage changes ΔV^m we check the equation (20). It is consistent regardless of the element changes $\Delta Y_1, \Delta Y_2, \ldots, \Delta Y_k$ if all other elements are kept at their nominal values.

In order to be able to detect k simultaneous faults we need to know equations similar to (20) for all possible combinations consisting of k elements.

As before, the matrix Z_{mx} can be found by means of the adjoint network. For the adjoint network we have

$$\begin{bmatrix} \hat{V}^m \\ \hat{V}^x \end{bmatrix} = \begin{bmatrix} Z_{mm}^T & Z_{xm}^T \\ Z_{mx}^T & Z_{xx}^T \end{bmatrix} \begin{bmatrix} \hat{I}^m \\ \hat{I}^x \end{bmatrix}. \qquad (21)$$

Let $\hat{I}^x = 0$. Then we obtain

$$\hat{V}^x = Z_{mx}^T \hat{I}^m, \qquad (22)$$

where $\hat{I}^m$ is the vector of an adjoint network excitation. Taking n linearly independent excitations, e.g.,

$$[\hat{I}^{m1} \ldots \hat{I}^{mn}] = 1, \qquad (23)$$

we obtain

$$Z_{mx}^T = [\hat{V}^{x1} \ldots \hat{V}^{xn}]. \qquad (24)$$

Thus, we need n simulations of the adjoint network (with the same LU factorization) in order to obtain the coefficients of the equations (20) for all possible combinations of k elements. We apply a unit source to the measurement ports and calculate voltages across all elements of the adjoint nominal network. Taking the values corresponding to a certain combination of elements we find the corresponding matrix Z_{mx}.

If there are k faults within the network we can detect them by checking the equations (20) for all possible combinations of k elements. The expression which corresponds to the elements at fault is equal to zero while the other expressions are likely to be different from zero. This enables us to indicate the suitable combination. However, the approach is limited. Some problems which may arise are discussed in the following section.

INTERPRETATION

We now discuss the assumptions and the capacity of the approach presented in this paper. In order to use it we have to formulate an appropriate set of p equations for single-faults, $\binom{p}{2}$ matrix equations for double-faults, $\binom{p}{3}$ matrix equations corresponding to three simultaneous faults, etc. This can be done by practically one simulation of the adjoint nominal network (with n different excitations). Given measured voltages we calculate the voltage changes w.r.t. nominal values and check the equations. We start with equations corresponding to single faults. If all equations except one are not satisfied we can suppose that there is a single fault in the element which corresponds to the satisfied equation. If all equations corresponding to single faults are not satisfied we have to go further and check the equations corresponding to double faults, etc.

To be able to uniquely detect the suitable fault combination the equations (20) should be block independent [9] of each other. It can be shown that two different equations (20) are block dependent if the columns of the corresponding matrices Z_{mx} generate exactly the same k-dimensional subspace in n-dimensional space. Checking this we can find out which equations of the form of (20) are dependent. In other words, we can determine the combinations, whose influence on the vector ΔV^m is similar, i.e., based on ΔV^m we cannot distinguish these combinations. Then, we should change measurement tests to be able to determine which combination actually occurs.

The approach presented in this section is based on the assumption of the existence of the impedance matrix. This assumption, however, is not essential since the impedance matrix exists for most practical networks. A more crucial assumption is the one which concerns the matrix Z_{mx} to be of full column rank, i.e., that there exist exactly k linearly independent rows of Z_{mx}. These rows correspond to those voltages which we can use to uniquely determine I^x as well as V^x. This is simply the problem of the identification of elements ΔY_1^x, ΔY_2^x, ..., ΔY_k^x which was discussed in [1]. Hence, there is an upper bound of k for which we are able to construct the equation (20) and, as a consequence, to detect k simultaneous faults. If we want to consider more simultaneous faults we can use the method of identification of all elements described in, for instance, [1].

Finally, it is to be noted that in order to implement the approach described in this paper a concept of "fuzzy consistency" should be developed.

EXAMPLE

Consider a simple resistive network shown in Fig. 4 with nominal values of elements $G_i = 1$, $i = 1, \ldots, 5$. For double-fault location we choose the port 11′ as a port of excitation with $I_g = 1A$

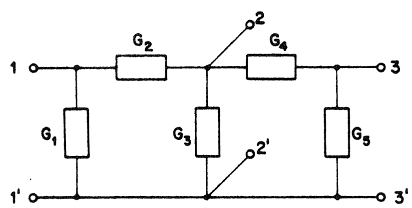

Fig. 4 A simple resistive network example.

and ports 11′, 22′ and 33′ as ports of measurement with nominal voltages $V_{11'} = 5/8$, $V_{22'} = 2/8$ and $V_{33'} = 1/8$. According to (24) we find

$$Z_{mx}^{12} = \frac{1}{8}\begin{bmatrix} 5 & 3 \\ 2 & -2 \\ 1 & -1 \end{bmatrix}, \qquad Z_{mx}^{13} = \frac{1}{8}\begin{bmatrix} 5 & 2 \\ 2 & 4 \\ 1 & 2 \end{bmatrix},$$

$$Z_{mx}^{14} = \frac{1}{8}\begin{bmatrix} 5 & 1 \\ 2 & 2 \\ 1 & -3 \end{bmatrix}, \qquad Z_{mx}^{15} = \frac{1}{8}\begin{bmatrix} 5 & 1 \\ 2 & 2 \\ 1 & 5 \end{bmatrix},$$

$$Z_{mx}^{23} = \frac{1}{8}\begin{bmatrix} 3 & 2 \\ -2 & 4 \\ -1 & 2 \end{bmatrix}, \qquad Z_{mx}^{24} = \frac{1}{8}\begin{bmatrix} 3 & 1 \\ -2 & 2 \\ -1 & -3 \end{bmatrix},$$

$$Z_{mx}^{25} = \frac{1}{8}\begin{bmatrix} 3 & 1 \\ -2 & 2 \\ -1 & 5 \end{bmatrix}, \qquad Z_{mx}^{34} = \frac{1}{8}\begin{bmatrix} 2 & 1 \\ 4 & 2 \\ 2 & -3 \end{bmatrix},$$

$$Z_{mx}^{35} = \frac{1}{8}\begin{bmatrix} 2 & 1 \\ 4 & 2 \\ 2 & 5 \end{bmatrix}, \qquad Z_{mx}^{45} = \frac{1}{8}\begin{bmatrix} 1 & 1 \\ 2 & 2 \\ -3 & 5 \end{bmatrix}.$$

Let the voltage measured be $V^m = [1/2\ 3/8\ 1/8]^T$. Then the equation (20) is satisfied for the matrix Z_{mx}^{24} and is not satisfied for every other combination. Thus, the elements G_2 and G_4 are indicated as faulty elements. It can be checked that $G_2 = 4$ and $G_4 = 0.5$ cause this situation.

CONCLUSIONS

Fault analysis, even though it can be carried out by methods of identification, should have its own special approaches especially in the case when only a few faults occur. Methods based on the bilinear dependence of network functions on a circuit parameter have been developed for single-fault detection. A particular approach utilizing a single current excitation and measurements of two voltages has been proposed. The adjoint network simulation has been found to be a convenient way for the necessary calculations. This approach has been successfully extended in order to deal with multiple-fault detection. However, there is a limit to the number of simultaneous faults which can be considered.

REFERENCES

[1] R.M. Biernacki and J.W. Bandler, "Postproduction parameter identification of analog circuits", IEEE Int. Symp. Circuits and Systems (Houston, TX, 1980).

[2] J.D. Bastian, "Fault isolation of analog circuits", Proc. 12th Asilomar Conf. Circuits, Systems and Computers, Western Periodicals (North Hollywood, Nov. 1978).

[3] R.E. Tucker and L.P. McNamee, "Computer-aided design application to fault detection and isolation techniques", Proc. IEEE Int. Symp. Circuits and Systems (Phoenix, AZ, 1977), pp. 684-687.

[4] G.O. Martens and J.D. Dyck, "Fault identification in electronic circuits with the aid of bilinear transformations", IEEE Trans. Reliability, vol. R-21, 1972, pp. 99-104.

[5] E.C. Neu, "A new n-port network theorem", Proc. 13th Midwest Symp. Circuit Theory (Minneapolis, 1970), pp. IV.5.1 - IV.5.10.

[6] E.C. Neu, "A general method for the identification of branch parameter changes", Proc. 14th Midwest Symp. Circuit Theory (Denver, 1971), pp. 17.2-1 to 17.2-8.

[7] S. Seshu and R. Waxman, "Fault isolation in conventional linear systems - a feasibility study", IEEE Trans. Reliability, vol. R-15, pp. 11-16, 1966.

[8] T.N. Trick and C.J. Aljajian, "Fault diagnosis of analog circuits", Proc. 20th Midwest Symp. Circuits and Systems (Lubbock, Texas, 1977), pp. 211-215.

[9] R.M. Biernacki and J.W. Bandler, "Fault location of analog circuits", Faculty of Engineering, McMaster University, Hamilton, Canada, Report SOC-236, 1979.