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POSTPRODUCTION PARAMETER IDENTIFICATION OF ANALOG CIRCUITS

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ABSTRACT

This paper deals with postproduction identification of network parameters for linear analog circuits. Methods for selected as well as for the identification of all parameters are discussed. The methods are based on measurements of voltage using current excitations. Tests are assumed to be performed at a single frequency. The well known nodal approach is used to formulate the appropriate systems of equations for identification of all parameters for passive as well as active circuits. A ladder network example is studied in some detail.

INTRODUCTION

Computer-aided circuit design, which has become one of the most powerful tools in the design of analog electrical devices [1,2], enables us to deal with manufacturing tolerance and tuning problems. So-called deterministic tuning requires not only knowledge as to which elements have to be altered, but also the actual values of network parameters in order to be able to calculate the amount of tuning to be carried out. This is the subject of the actual parameter identification.

The solvability of the all parameter identification problem was first considered by Berkowitz [3]. His approach was mostly based on current measurements. Later, several other authors [4-9] investigated the problem. Trick et al. [6,7] considered the identification using nodal voltage measurements only. They proved the very important result that, for linear networks, the problem can be solved by means of linear equations. Their approach, however, seems to be unnecessarily complicated, because many simulations of the adjoint have to be performed in order to formulate the equations.

Most papers on parameter identification assume tests to be performed at a single frequency. This provides the values of passive admittances and control coefficients of controlled sources. Repeating the identification at different frequencies enables us to identify the component values provided that there is a unique dependence of element values on the frequency response (as for canonical structures).

We also assume that there are no direct parallel connections of elements or, alternatively, we have to be satisfied with the knowledge of the admittance of the whole connection [3].

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IDENTIFICATION OF SELECTED PARAMETERS

In this section we discuss two extreme situations that (1) voltages across unknown elements are available to be measured, and (2) ports of measurement are different from ports of identification.

The first situation can be represented as an active n-port being terminated by unknown elements $Y_1, Y_2, \ldots, Y_n$ (Fig. 1(a)). Assume that there

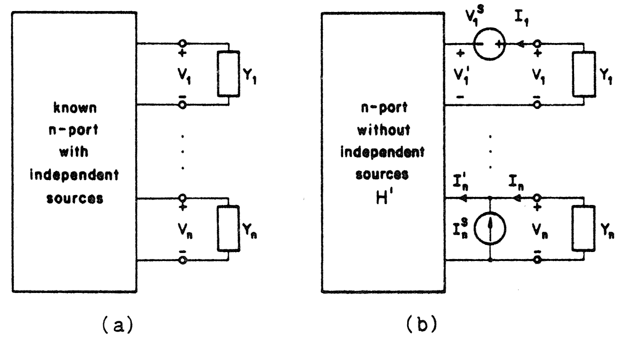

Fig. 1 Active n-port and its hybrid equivalent.

exists a hybrid equivalent of the active n-port shown in Fig. 1(b). The equivalent is described by the vector of port voltage sources $\mathbf{V}_a^S \triangleq [V_1^S\ V_2^S\ \ldots\ V_k^S]^T$, the vector of port current sources $\mathbf{I}_b^S \triangleq [I_{k+1}^S\ I_{k+1}^S\ \ldots\ I_n^S]^T$, and the hybrid matrix of the n-port without independent sources defined by

$$\begin{bmatrix} \mathbf{V}_a' \\ \mathbf{I}_b' \end{bmatrix} = \begin{bmatrix} \mathbf{H}_{aa} & \mathbf{H}_{ab} \\ \mathbf{H}_{ba} & \mathbf{H}_{bb} \end{bmatrix} \begin{bmatrix} \mathbf{I}_a \\ \mathbf{V}_b \end{bmatrix} . \quad (1)$$

From Fig. 1(b) we obtain

$$-\mathbf{I} = \begin{bmatrix} \mathbf{H}_{aa}^{-1} & \mathbf{0} \\ \mathbf{H}_{ba}\mathbf{H}_{aa}^{-1} & \mathbf{1} \end{bmatrix} \begin{bmatrix} \mathbf{V}_a^S \\ \mathbf{I}_b^S \end{bmatrix} + \begin{bmatrix} \mathbf{H}_{aa}^{-1} & -\mathbf{H}_{aa}^{-1}\mathbf{H}_{ab} \\ \mathbf{H}_{ba}\mathbf{H}_{aa}^{-1} & \mathbf{H}_{bb}-\mathbf{H}_{ba}\mathbf{H}_{aa}^{-1}\mathbf{H}_{ab} \end{bmatrix} \begin{bmatrix} \mathbf{V}_a \\ \mathbf{V}_b \end{bmatrix}, \quad (2)$$

where $\mathbf{1}$ is the identity matrix of appropriate order and $\mathbf{I}$ is the vector of currents through the unknown elements. Knowing $\mathbf{I}$ we obtain

$$Y_i = -I_i/V_i \quad \text{or} \quad Z_i = -V_i/I_i \quad (3)$$

for i = 1, 2, ..., n.

From (2) it is seen that the existence of the inverse of $\mathbf{H}_{aa}$ is necessary to obtain the solution. This is equivalent to the existence of the admittance matrix of the n-port. On the other hand we have considered another assumption, i.e., that the hybrid equivalent exists. It can be shown that the existence of a hybrid matrix is sufficient for the existence of the corresponding hybrid equivalent. Therefore, if we assume that the admittance matrix exists we can consider the Norton equivalent and, according to the above discussion, we can find the solution. This leads to the following theorem.

Theorem 1

Identification of n elements based on voltages across these elements is possible if and only if there exists the admittance matrix of the corresponding n-port (after shorting independent voltage sources and open-circuiting independent current sources).

Now, consider that the measurement ports are different from the ports of the elements which are to be identified. Using a similar hybrid equivalent approach it can be proved [10] that the existence of the transmission matrix linking ports of identification as the input with ports of measurement as the output is necessary for the identification. On the other hand, assuming that there exists a mixed "transmission-hybrid" representation of the network described by (see Fig. 2)

$$[\underset{\sim}{V}^x \ \underset{\sim}{I}^x \ \underset{\sim}{I}_V \ \underset{\sim}{V}_I]^T = \underset{\sim}{A}[\underset{\sim}{V}^m \ \underset{\sim}{I}^m \ \underset{\sim}{V}_V \ \underset{\sim}{I}_I]^T, \qquad (4)$$

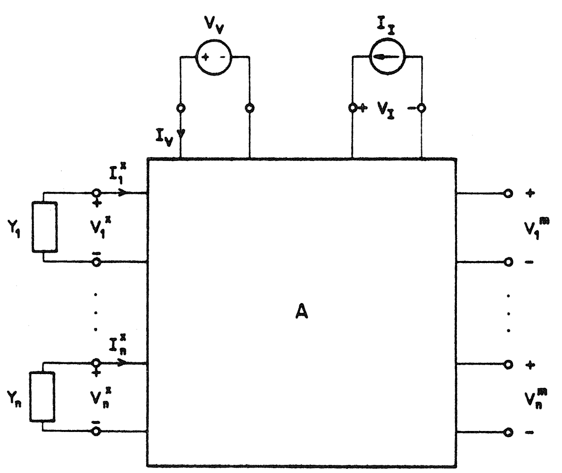

Fig. 2 (2n+m)-port with m external excitations.

we find, for $\underset{\sim}{I}^m = \underset{\sim}{0}$,

$$\begin{bmatrix} \underset{\sim}{V}^x \\ \underset{\sim}{I}^x \end{bmatrix} = \begin{bmatrix} \underset{\sim}{A}_{11} \\ \underset{\sim}{A}_{21} \end{bmatrix} \underset{\sim}{V}^m + \begin{bmatrix} \underset{\sim}{A}_{13} & \underset{\sim}{A}_{14} \\ \underset{\sim}{A}_{23} & \underset{\sim}{A}_{24} \end{bmatrix} \begin{bmatrix} \underset{\sim}{V}_V \\ \underset{\sim}{I}_I \end{bmatrix}. \qquad (5)$$

This discussion gives us the following theorem.

Theorem 2

Existence of the transmission-type matrix defined by (4) is necessary and sufficient for identification of n unknown elements based upon n voltage measurements if ports of measurement are different from ports of identification.

The requirements of Theorem 1 can easily be verified. The admittance matrix exists if and only if no port can be shorted by shorting all the remaining ports. In contrast, verifying the conditions of Theorem 2 is more difficult. This is simply because the elements of a general transmission matrix of a 2n-port network (unlike a 2-port) cannot be defined as ratios of single input and single output in the presence of shorts and opens of other ports. Nevertheless, we observe that in both cases there exists a limit to the number of elements which can be identified. Usually, Theorems 1 and 2 are satisfied as far as the identification of one or two elements is concerned. The more elements we want to consider the more unlikely it is to satisfy the corresponding theorem. The number of elements which can still be identified strongly depends upon topology and elements chosen.

IDENTIFICATION OF ALL PARAMETERS

We now consider the situation when all network elements are unknown. We assume that voltages across all elements are available. Since Kirchhoff's voltage law is satisfied (i.e., we assume that measurements are accurate enough) we can consider nodal voltages only. Using the current excitations we have a generalized branch shown in Fig. 3. As is well known, a network with

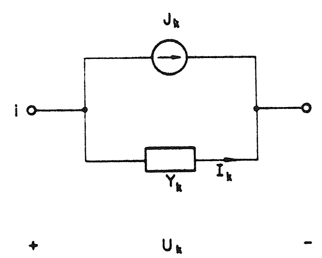

Fig. 3 Generalized branch.

p branches and r nodes can be described by the branch-node incidence matrix $\underset{\sim}{\Lambda} = [\lambda_{ik}]$, i = 1, ..., r-1 and k = 1, ..., p.

Following the typical nodal approach we write Kirchhoff's current law in the form

$$\underset{\sim}{\Lambda} \ \underset{\sim}{I} = - \underset{\sim}{I}^S, \qquad (6)$$

where $\underset{\sim}{I} \triangleq [I_1 \ I_2 \ \dots \ I_p]^T$ is the vector of branch currents and $\underset{\sim}{I}^S \triangleq \underset{\sim}{\Lambda}[J_1 \ J_2 \ \dots \ J_p]^T$.

Using the notation $\underset{\sim}{Y} = [Y_1 \ Y_2 \ \dots \ Y_p]^T$ for the vector of branch admittances, and $\underset{\sim}{U} = \mathrm{diag}(U_1 \ U_2 \ \dots \ U_p)$ for the matrix of branch voltages, we can write Ohm's law for all branches of the network as

$$\underset{\sim}{I} = -\underset{\sim}{U} \ \underset{\sim}{Y}. \qquad (7)$$

Since Kirchhoff's voltage law is satisfied automatically we note that equation (6) together with (7) are all the available equations for the network. The current vector I is of no interest, so eliminating it from (6) and (7) we find

$$(\tilde{\Lambda}\ \tilde{U})\tilde{Y} = \tilde{I}^S. \qquad (8)$$

This is simply the system of equations which we have sought. It contains r-1 equations with p unknown values of all admittances. Matrix $\tilde{\Lambda}$ consists of r-1 linearly independent rows, so if branch voltages are different from zero then the matrix $(\tilde{\Lambda}\tilde{U})$ also consists of r-1 linearly independent rows. Note that p can be equal to r-1 only if the network graph is a tree. In other cases we always have p > r-1 and we are not able to identify all elements. If some of these elements are known we can solve (8) for the remaining parameters provided that the resulting system contains an appropriate number of linearly independent equations. This is another approach to the problems considered in the foregoing section.

Now, we are interested in the identification of all parameters of the network. Since the number of equations in (8) is less than the number of unknowns we have to find additional equations based on other test(s). According to (8) one test gives us at most r-1 independent equations. This means that we need at least m tests, where

$$m = \text{int}\left(\frac{p}{r-1} \right) \qquad (9)$$

and int(x) denotes the smallest integer x_o such that $x \le x_o$. Because the number of branches p is between r-1 (for a tree-network) and r(r-1)/2 (for a complete-graph network), we find that

$$1 \le m \le \text{int}\left(\frac{r}{2} \right). \qquad (10)$$

For typical networks m is expected to equal 2 or 3. Every set of measurements $\tilde{U}^i$ provides the appropriate system of equations (8). All of those systems give us the final matrix equation

$$\begin{bmatrix} \tilde{\Lambda}^1\tilde{U}^1 \\ \tilde{\Lambda}^2\tilde{U}^2 \\ \cdot \\ \cdot \\ \tilde{\Lambda}^M\tilde{U}^M \end{bmatrix} \tilde{Y} = \begin{bmatrix} \tilde{I}^{S1} \\ \tilde{I}^{S2} \\ \cdot \\ \cdot \\ \tilde{I}^{SM} \end{bmatrix} . \qquad (11)$$

where $M \ge m$.

The system (11) is required to contain exactly p independent equations. How to arrange for the least number of independent measurements, however, is not known so far. Nevertheless, several directions can be proposed. It would seem to be optimal if the subsequent measurements provided equations which formed an independent system along with all previously obtained equations and, furthermore, if the final system was not ill-conditioned. To this end we would propose to use different locations for the excitations for the different tests. These excitations should be as remote from one another as possible.

METHODS FOR LADDER NETWORKS

Consider the ladder network shown in Fig. 4.

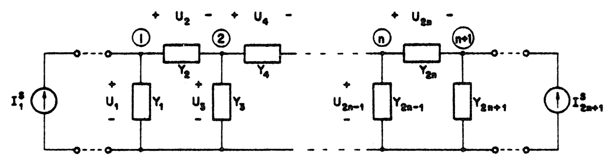

Fig. 4 Ladder network.

The branch-node incidence matrix Λ consists of r-1 = n+1 rows and p = 2n+1 columns. Using only the input source for the first test

$$\tilde{I}^{S1} = [I_1^{S1} \quad 0 \quad 0 \quad \dots \quad 0]^T, \qquad (12)$$

we obtain the first subsystem of (11).

According to (9) we find m = 2, so we have to arrange for another test. We will discuss three difference tests such that each of them can be chosen as the second test.

1. We use the same excitations (i.e., $\tilde{I}^{S2} = \tilde{I}^{S1}$) and the output port is shorted.

2. We use only the output source for the second test, i.e.,

$$\tilde{I}^{S2} = [0 \quad 0 \quad \dots \quad 0 \ I_{2n+1}^{S2}]^T, \qquad (13)$$

and the input port is shorted.

3. We do not make any shorts and we apply the output source only.

Note that regardless of the method chosen, for any row of $\tilde{\Lambda}^2$ we can find an <u>identical</u> row within the matrix $\tilde{\Lambda}^1$. Hence the linear independence or linear dependence of the final system (11) depends on the particular values of voltages from the first test in comparison with those from the second test. Because of this the first method is likely to be ill-conditioned. It can be caused by relatively insensitive behaviour of voltages across the elements located close to the input w.r.t. a change of the output load. From this point of view it is obvious that we are looking for quite a different excitation for the second test. The second and the third methods satisfy this requirement. We will discuss the third method but most of the following results will be applicable to the second method. The resulting system of equations (11), after reordering, can be expressed in the form

$$\tilde{A}\ \tilde{Y} = \tilde{B}, \qquad (14)$$

where

$$B = \begin{bmatrix} I^{S1} \\ \\ I^{S2} \end{bmatrix} = [I_1^{S1}\ 0\ 0\ \ldots\ 0\ I_{2n+1}^{S2}]^T \quad (15)$$

and

$$A = \begin{bmatrix} U_1^1 & U_2^1 & & & & \\ U_1^2 & U_2^2 & & & & \\ & -U_2^1 & U_3^1 & U_4^1 & & \\ & -U_2^2 & U_3^2 & U_4^2 & & \\ & & \ldots & & & \\ & & & & -U_{2n}^1 & U_{2n+1}^1 \\ & & & & -U_{2n}^2 & U_{2n+1}^2 \end{bmatrix} . \quad (16)$$

The particular sparse form of the matrix A enables us to find explicit formulae for the solution of (14). It is more convenient to use nodal voltages $V_1^i, \ldots, V_{n+1}^i$, i = 1, 2, such that

$$U_{2k-1}^i = V_k^i \quad \text{and} \quad U_{2k}^i = V_k^i - V_{k+1}^i \quad (17)$$

for i = 1, 2 and k = 1, 2, ... From the last two equations we obtain Y_{2n} and

$$Y_{2n+1} = \frac{I_{2n+1}^{S2}\ (V_n^1 - V_{n+1}^1)}{\Delta_n}. \quad (18)$$

Substituting Y_{2n} into the preceding two equations we can determine Y_{2n-1} and Y_{2n-2}. In this way we find the recurrent formulas

$$Y_{2k+1} = \frac{\Delta'_k}{\Delta_k} Y_{2k+2}, \; k = 1, 2, \ldots, n-1, \quad (19)$$

$$Y_{2k} = \frac{I_{2n+1}^{S2}\ V_{n+1}^1}{\Delta_k}, \; k = 1, 2, \ldots, n, \quad (20)$$

where

$$\Delta_k = \det \begin{bmatrix} V_k^1 & V_{k+1}^1 \\ V_k^2 & V_{k+1}^2 \end{bmatrix} \quad (21)$$

and

$$\Delta'_k = \det \begin{bmatrix} V_k^1 - V_{k+1}^1 & V_{k+1}^1 - V_{k+2}^1 \\ V_{k+1}^2 - V_k^2 & V_{k+2}^2 - V_{k+1}^2 \end{bmatrix} . \quad (22)$$

Finally, from the first equation of the system (14) we have

$$Y_1 = \frac{I_1^{S1} - (V_1^1 - V_2^1)Y_2}{V_1^1} . \quad (23)$$

Alternatively, since the system is overdetermined, we obtain from the second equation

$$Y_1 = \frac{V_2^2 - V_1^2}{V_1^2} Y_2 \quad (24)$$

and both solutions should be identical. Of course, this second equation does not appear in the second method (because of shorting the input port). The above solution may be described by the term <u>backward solution</u>.

Similarly, starting from the first two equations we can derive the <u>forward solution</u> [10]. For the second method, only the backward solution exists and Y_1 is expressed by (23). For the third method we can use the backward as well as the forward solution and the two solutions should be identical. They can be different from each other if the measurements are inaccurate. Then the question arises of how to take advantage of the fact that the system (14) is overdetermined.

The solvability of the problem depends on the determinants (21). Detailed discussion of this can be found in [10].

ACTIVE NETWORKS

We now consider a network which consists of passive as well as active lumped elements. Control sources are taken into account as models of active elements. We will consider only voltage controlled current sources (VCCS) which are typical for the nodal approach. It is sufficiently general for many practical cases.

The general formulation discussed earlier can easily be extended to identify unknown control coefficients besides all other passive admittances.

Consider a network with passive branches and s voltage controlled current sources. The VCCS elements are described by the equation

$$J_k^c = Y_k^c\ U_k^c, \; k = 1, 2, \ldots, s. \quad (25)$$

For our purposes we have to treat the controlled branches as different from those which contain passive elements and/or independent sources even if they are parallel. Now, Kirchhoff's current low can be written in the form

$$\Lambda_p\ I + \Lambda_a\ J^c = -I^S, \quad (26)$$

where $J^c \triangleq [J_1^c\ J_2^c\ \ldots\ J_s^c]^T$, Λ_p is the (r-1) x p incidence matrix concerning passive branches and Λ_a is an (r-1) x s matrix constructed for all controlled branches in the same way as Λ_p.

Using the notation $U = \text{diag}(U_1\ U_2\ \ldots\ U_p\ U_1^c\ U_2^c\ \ldots\ U_s^c)$ we finally find the equation

$$(\Lambda\ U)Y = I^S, \quad (27)$$

where $\Lambda = [\Lambda_p\ -\Lambda_a]$ and Y is the vector of unknown parameters $Y = [Y_1\ Y_2\ \ldots\ Y_p\ Y_1^c\ Y_2^c\ \ldots\ Y_s^c]^T$. The system (27) contains r-1 equations with p+s unknowns. As before, in order to obtain an appropriate number of independent equations we have to arrange for other tests. The number of tests which we need is at least

$$m = \text{int}\left(\frac{p+s}{r-1} \right). \quad (28)$$

The same approaches are valid as for the choice of independent measurements.

EXAMPLE

As an example consider the identification of unknown parameters G_1, G_2, G_3, G_4, G_5 and G^c of a resistive active circuit shown in Fig. 5. We have

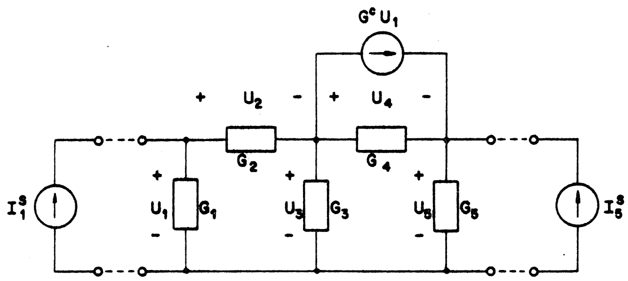

Fig. 5 An active network example.

$$\underset{\sim}{A} = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & -1 & 1 & -1 \end{bmatrix}$$

and $\underset{\sim}{U} = \text{diag}(U_1 \;\; U_2 \;\; U_3 \;\; U_4 \;\; U_5 \;\; U_1)$. The number of tests required is 2. First, we let $\underset{\sim}{I}^{S1} = [2\ 0\ 0]^T$ and measure voltages $\underset{\sim}{U}^1 = \text{diag}(1\ \ 1\ \ 0\ \ -1\ \ 1\ \ 1)$. Second, for $\underset{\sim}{I}^{S2} = [0\ \ 0\ \ 8]^T$, we measure $\underset{\sim}{U}^2 = \text{diag}(1\ -1\ 2\ -4\ 6\ 1)$. The two tests give the final equation

$$\begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & -1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & -1 \\ 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 2 & -4 & 0 & 1 \\ 0 & 0 & 0 & 4 & 6 & -1 \end{bmatrix} \begin{bmatrix} G_1 \\ G_2 \\ G_3 \\ G_4 \\ G_5 \\ G^c \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ 0 \\ 0 \\ 0 \\ 8 \end{bmatrix},$$

whose solution is $\underset{\sim}{G} = [1\ \ 1\ \ 0.5\ \ 1\ \ 1\ \ 2]^T$.

CONCLUSIONS

A very basic approach to the problem of postproduction parameter identification has been discussed. Methods presented here are oriented to linear analog electrical networks. They are based mainly on voltage measurements of the network, which is excited by current source(s). The limitations for the selected element identification have been derived and formulated in Theorems 1 and 2.

For identification of all elements, a simple approach based on nodal analysis has been proposed. This approach provides the maximum number of independent equations which can be formulated based on a single test. As a very important example we present a method for ladder networks. The method is much simpler than that of Trick and Sakla [6] and, because of a particular sparse form of the equations, we obtain explicit recurrent formulas for the solution. For arbitrary network topologies, however, there are still many open questions and unsolved problems.

It is to be noted that the presented nodal approach to the identification of all elements is also valid under limited measurements. In such a case we simply do not have all of equations in (11). The equations containing explicitly the voltages which are not available have to be dropped from (11). If necessary, we should perform more tests. Then, the identification can be done if the network is element-value-solvable.

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