ANALYSIS AND SENSITIVITY EVALUATION OF 2p-PORT CASCADED NETWORKS

J.W. Bandler and M.R.M. Rizk
Group on Simulation, Optimization and Control
Faculty of Engineering
McMaster University, Hamilton, Ontario L8S 4L7

An exact analysis approach for efficiently evaluating the response and its sensitivities with respect to all design parameters for cascaded 2p-port networks is presented for any value of p. It is illustrated via a quasi-optical bandpass filter.

Introduction

A generalization of an analysis approach for 2-port cascaded networks [1] to handle 2p-port networks is presented. The generalized approach has the same advantages as those for 2-port networks. These advantages include efficient and fast analytical and numerical investigations of response, first-order sensitivities of the response w.r.t. variable parameters, and large change sensitivities. The need for this generalization evolved from the fact that many microwave networks are represented as a cascade of 2p-port elements.

Thevenin and Norton equivalents for these cascaded networks can be obtained systematically using this approach. These in turn are very useful for worst-case analysis [2]. As an example a quasi-optical bandpass filter has been analyzed using this approach and the exact sensitivities of the response w.r.t. a parameter appearing in two of the 2p-port elements, representing the filter elements, have been evaluated.

Theory

The analysis approach consists of two principal types. The first, which we call the forward analysis, consists of initializing a $\bar{\mathbf{U}}^T$ matrix as $\mathbf{E}_1^T$ or $\mathbf{E}_2^T$, which are defined as

$$\mathbf{E}_1 \triangleq \begin{bmatrix} \mathbf{1}_p \\ \mathbf{0}_p \end{bmatrix}, \qquad \mathbf{E}_2 \triangleq \begin{bmatrix} \mathbf{0}_p \\ \mathbf{1}_p \end{bmatrix}$$

where

$\mathbf{1}_p$ is the unit matrix of order p,

$\mathbf{0}_p$ is the null matrix of order p,

and successively premultiplying each constant chain matrix by the resulting matrix until an element of interest (which contains a variable parameter), a reference plane or a termination is reached. The second type of analysis is the reverse analysis which consists of initializing a $\mathbf{V}$ matrix as either $\mathbf{E}_1$ or $\mathbf{E}_2$ and successively postmultiplying each constant matrix by the resulting matrix until an element of interest, a reference plane or a termination is reached.

Consider the 2p-port element shown in Fig. 1, possessing p input ports and p output ports. Its transmission matrix is given by

$$\mathbf{A} \triangleq \begin{bmatrix} \mathbf{A}_{11} & \mathbf{A}_{12} \\ \mathbf{A}_{21} & \mathbf{A}_{22} \end{bmatrix},$$

where $\mathbf{A}_{11}$, $\mathbf{A}_{12}$, $\mathbf{A}_{21}$ and $\mathbf{A}_{22}$ are p x p matrices. The input quantities in this case are

$$\bar{\mathbf{y}} = [\bar{y}_1\ \bar{y}_2\ \ldots\ \bar{y}_p\ \bar{y}_{p+1}\ \bar{y}_{p+2}\ \ldots\ \bar{y}_{2p}]^T,$$

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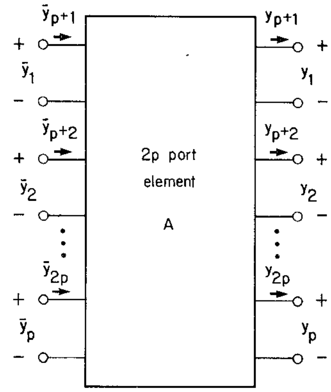

Fig. 1 A 2p-port element.

and the output quantities are

$$\mathbf{y} = [y_1\ y_2\ \ldots\ y_p\ y_{p+1}\ y_{p+2}\ \ldots\ y_{2p}]^T,$$

where the elements with subscripts 1 to p denote voltages and from p+1 to 2p denote currents.

For the forward and reverse analyses the matrices $\bar{\mathbf{U}}_1$, $\bar{\mathbf{U}}_2$, $\mathbf{V}_1$ and $\mathbf{V}_2$ are initialized such that

$$\mathbf{E}_1 \Longrightarrow \bar{\mathbf{U}}_1 \text{ or } \mathbf{V}_1, \quad \mathbf{E}_2 \Longrightarrow \bar{\mathbf{U}}_2 \text{ or } \mathbf{V}_2.$$

We can now derive in an analogous manner to the derivation of (9) of [1]

$$\mathbf{V}_S = (\bar{\mathbf{U}}_1^T + \mathbf{Z}_S\,\bar{\mathbf{U}}_2^T)\,\mathbf{A}\,(\mathbf{V}_1\,\mathbf{V}_L + \mathbf{V}_2\,(\mathbf{Y}_L\,\mathbf{V}_L - \mathbf{I}_L)), \quad (1)$$

where

$\bar{\mathbf{U}}_1$, $\bar{\mathbf{U}}_2$, $\mathbf{V}_1$ and $\mathbf{V}_2$ are the matrices obtained from forward and reverse analyses,

$\mathbf{V}_S$ is the vector containing the p source voltages,

$\mathbf{V}_L$ is the vector of load voltages,

$\mathbf{I}_L$ is the vector of current sources at the loads (if any),

$\mathbf{Z}_S$ and $\mathbf{Y}_L$ are diagonal matrices containing the source impedances and the load admittances, respectively.

To evaluate the unknowns $\mathbf{V}_L$, having obtained numerical values for (1), a system of p linear equations is solved.

To obtain the Thevenin voltages of the subnetwork on the l.h.s. of the element $\mathbf{A}$, we let $\mathbf{I}_L = \mathbf{0}$ and $\mathbf{Y}_L = \mathbf{0}$ in (1), which gives

$$\mathbf{V}_S = (\bar{\mathbf{U}}_1^T + \mathbf{Z}_S\,\bar{\mathbf{U}}_2^T)\,\mathbf{A}\,\mathbf{V}_1\,\mathbf{V}_L = (\mathbf{Q}_{11} + \mathbf{Z}_S\,\mathbf{Q}_{21})\,\mathbf{V}_L, \quad (2)$$

where

$$\mathbf{Q}_{11} = \bar{\mathbf{U}}_1^T\,\mathbf{A}\,\mathbf{V}_1, \quad \mathbf{Q}_{21} = \bar{\mathbf{U}}_2^T\,\mathbf{A}\,\mathbf{V}_1,$$

and from (2)

$$\mathbf{V}_{TH} = \mathbf{V}_L = (\mathbf{Q}_{11} + \mathbf{Z}_S\,\mathbf{Q}_{21})^{-1}\,\mathbf{V}_S. \quad (3)$$

The output impedance matrix or the Thevenin impedance is obtained one column at a time by letting $\mathbf{V}_S = \mathbf{0}$, $\mathbf{Y}_L = \mathbf{0}$ and $\mathbf{I}_L = \mathbf{0}$ except I_{Li} (which is the current source at the load end for the ith port) which leads to an equation from which the ith column of the p x p $\mathbf{Z}_{TH}$ matrix is obtained. Fig. 2 shows the $\mathbf{Z}_{TH}$ and $\mathbf{V}_{TH}$ of the subnetwork preceeding the element $\mathbf{A}$. Similar formulas can be derived (analogous to (13) and (14) of [1]) for the input admittance matrix and the Norton current equivalent matrix.

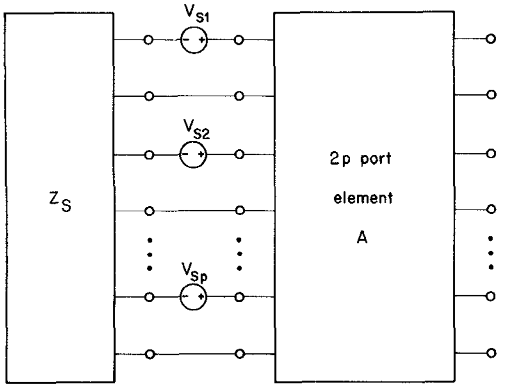

Fig. 2 Equivalent Thevenin voltages and impedance matrix for a subnetwork consisting of 2p-port elements.

As a special case when $\mathbf{Z}_S$ and $\mathbf{Y}_L$ are $\mathbf{0}$ or when they are considered as the first and last elements, respectively, (1) becomes

$$\mathbf{V}_S = \bar{\mathbf{U}}_1^T \mathbf{A} \mathbf{V}_1 \mathbf{V}_L = \mathbf{Q}_{11} \mathbf{V}_L, \tag{4}$$

so that the load voltages are given by

$$\mathbf{V}_L = \mathbf{Q}_{11}^{-1} \mathbf{V}_S. \tag{5}$$

When $\mathbf{A}$ is perturbed to $\mathbf{A} + \Delta\mathbf{A}$, the new values for the load voltages ($\mathbf{V}_L + \Delta\mathbf{V}_L$) can be obtained by 6 p^3 additional multiplications and the solution of a p-system of linear equations. Alternatively, the Sherman-Morrison formula [3] can be used to find $(\mathbf{Q}_{11}+\Delta\mathbf{Q}_{11})^{-1}$. Note that the reanalysis of the cascaded network is not performed. We use the results of only one analysis.

Differentiating (4) w.r.t. a parameter ϕ which appears in the matrix $\mathbf{A}$ only we get

$$\mathbf{0} = \bar{\mathbf{U}}_1^T (\partial\mathbf{A}/\partial\phi)\, \mathbf{V}_1 \mathbf{V}_L + \bar{\mathbf{U}}_1^T \mathbf{A} \mathbf{V}_1 (\partial\mathbf{V}_L/\partial\phi), \tag{6}$$

so that the sensitivity of the load voltages can be obtained from

$$\partial\mathbf{V}_L/\partial\phi = -\mathbf{Q}_{11}^{-1}(\partial\mathbf{Q}_{11}/\partial\phi)\mathbf{V}_L, \tag{7}$$

where

$$\partial\mathbf{Q}_{11}/\partial\phi = \bar{\mathbf{U}}_1^T(\partial\mathbf{A}/\partial\phi)\mathbf{V}_1. \tag{8}$$

Numerical Example

The analysis and sensitivity evaluation of the response of a quasi-optical bandpass filter have been performed using the analysis approach described. The filter consists of three metallic (copper) wire grids in space with separations of 12.5 mm ($5/4\lambda$). The equivalent circuit of the filter is shown in Fig. 3 [4]. The first and third gratings (in the x-y plane) have their wires parallel to the x axis, while the middle grating has the wires oriented at an angle ϕ with respect to the x axis. The circuits $R(\phi)$ and $R(-\phi)$ are used to connect the middle grating with the adjacent local coordinates (the equivalent circuit of the filter is based on the local coordinate concept [4]). The free space between the gratings is represented by the uncoupled transmission lines (with lengths equal to the separations between the gratings) as shown in Fig. 3. The parameters B_a, B_b, X_a and X_b can be found in [5] and R_p and X_p are from [4]. The dimensions of the gratings are given in Fig. 4. The

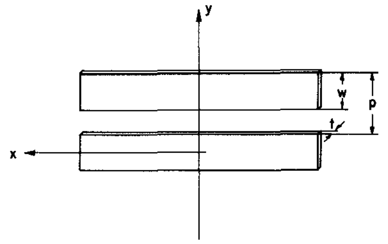

Fig. 4 Physical dimensions of the wire grid:
p = 0.2 mm, w = 0.12 mm, t = 0.01 mm.

dielectric sheets supporting the metal gratings were not considered in our analysis. The filter is excited by a source representing an incident wave linearly polarized in the direction perpendicular to the first grating (i.e., polarized in the y direction). The transmitted wave is represented by the output voltage at port 3.

The insertion loss of the filter (the center frequency f_0 is equal to 30 GHz) is shown in Fig. 5 for various angles ϕ. The exact sensitivity of the voltage at port 3 w.r.t. ϕ is plotted in Figs. 6, 7 and 8 for

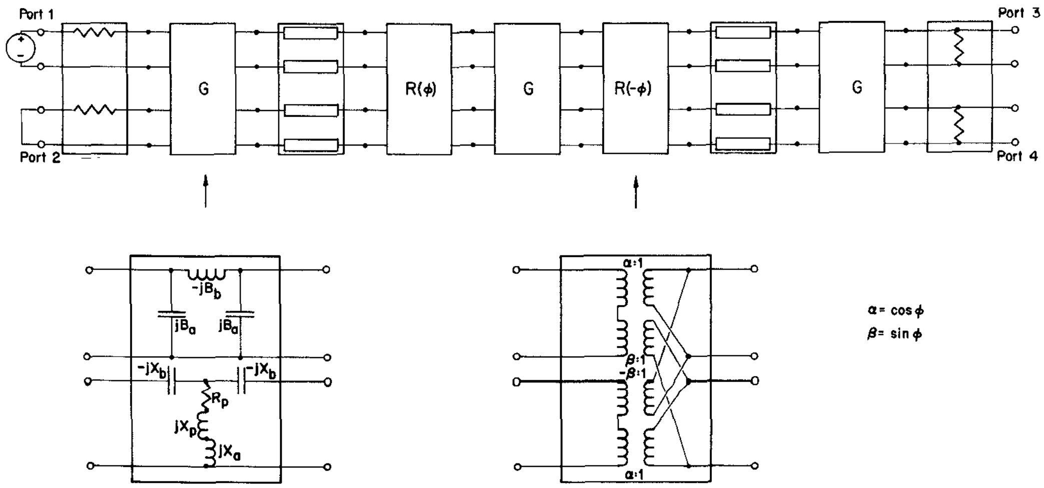

Fig. 3 Equivalent circuit of the quasi-optical filter [4].

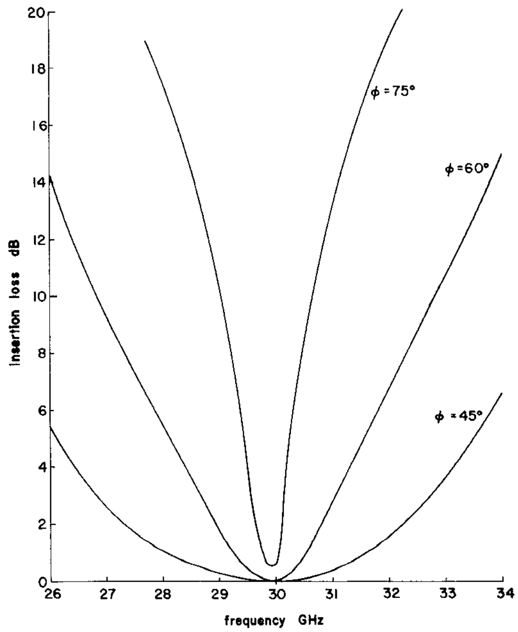

Fig. 5 Insertion loss of the filter for different values of ϕ.

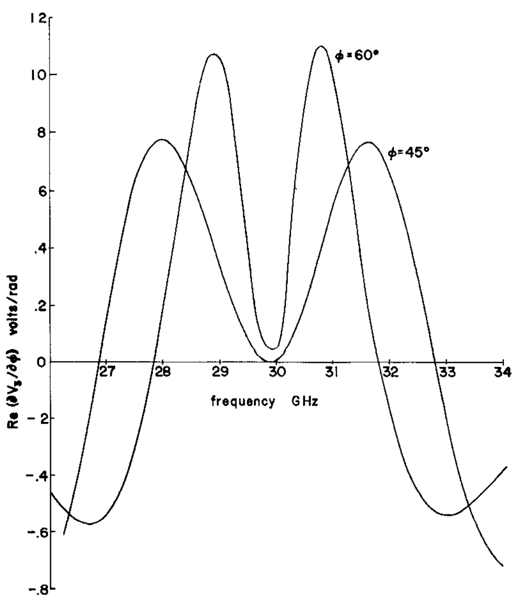

Fig. 6 Real part of $\partial V_3/\partial\phi$ at $\phi = 45^{\circ}$ and $\phi = 60^{\circ}$.

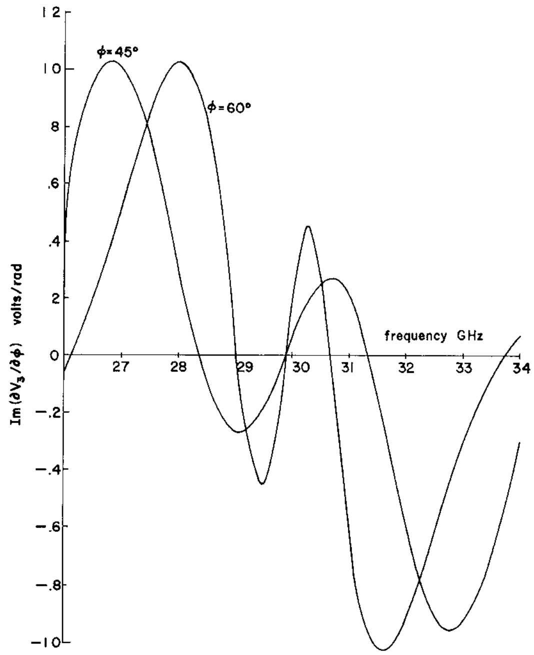

Fig. 7 Imaginary part of $\partial V_3/\partial\phi$ at $\phi = 45^{\circ}$ an $\phi = 60^{\circ}$.

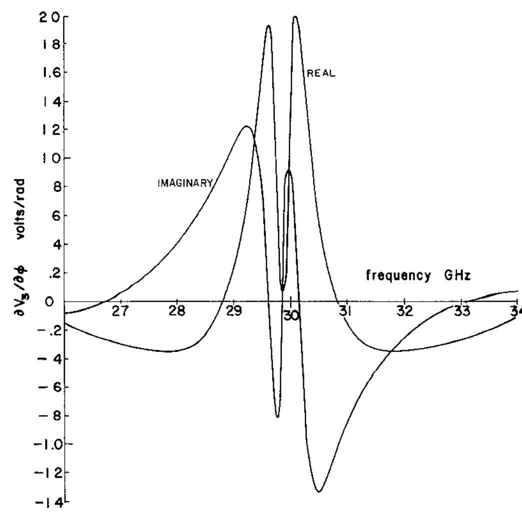

Fig. 8 Real and imaginary parts of $\partial V_3/\partial\phi$ at $\phi = 75^{\circ}$.

different values of ϕ. A slightly more complicated formula than (7) was used since ϕ appears in two elements of the cascade.

Conclusions

The use of this analysis approach avoids the need for reanalyzing the cascaded networks to evaluate large change sensitivities. It also facilitates the evaluation of first-order sensitivities of the response w.r.t. variable design parameters without defining and analyzing any additional network (adjoint network). These advantages lead to a considerable saving in computational time and effort.

References

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