Proceedings of the 1980 IEEE International Conference on Circuits and Computers

A NOVEL FORMULATION FOR STEADY-STATE SENSITIVITY ANALYSIS IN POWER NETWORKS

J.W. Bandler and M.A. El-Kady

Group on Simulation, Optimization and Control, Faculty of Engineering

McMaster University, Hamilton, Canada L8S 4L7

ABSTRACT

We present a novel approach for direct and compact derivation of sensitivity expressions in electrical power networks. Expressions for first-order change and total derivatives of a general real function w.r.t. control and design variables are effectively derived by exploiting a special complex notation. The approach employs only complex matrix manipulations.

INTRODUCTION

Expressions for first-order changes of system states and other functions of interest and their total derivatives w.r.t. practical control and design variables are required in various applications [6]. The first-order changes are valuable in estimating the effects of transmission system contingencies and ranking them, generation outages, device malfunctions and other defects expected in power systems operation which may result in subsequent service deterioration. On the other hand, the total derivatives (reduced gradients) are required in different planning problems when, for example, applying gradient-type optimization techniques.

Several approaches to sensitivity calculations in power systems have been described. The class of these approaches [2-4] which exploits elements of the Jacobian matrix, available from the load flow solution computed by the Newton-Raphson method, employs the power flow equations, originally complex, in a real form. Ease of derivation and compactness in formulation of the required sensitivity expressions can, however, be gained by preserving this basic complex form.

The conjugate notation [1] provides a useful tool for direct treatment of the complex power flow equations. In this notation sensitivity expressions can be derived and formulated in terms of formal [5] (or symbolic) partial derivatives of complex functions w.r.t. general complex variables.

In this paper we exploit the conjugate notation to derive, using only complex matrix manipulations, the required sensitivity expressions in the compact complex form. First, we introduce the notation. Then the problem formulation in terms of complex state and control variables and the method proposed for sensitivity calculations are presented successively in ensuing sections.

This work was supported by the Natural Sciences and Engineering Research Council of Canada under Grant A7239.

NOTATION

In the conjugate notation, a complex variable

$$\zeta_i = \zeta_{i1} + j\zeta_{i2} \tag{1}$$

and its complex conjugate ζ_i^* replace, as independent quantities, the real and imaginary parts of the variable. Hence, the first-order change of a continuous function f of a set of complex variables arranged in a column vector $\underset{\sim}{\zeta}$,

$$\underset{\sim}{\zeta} = \underset{\sim}{\zeta}_1 + j\underset{\sim}{\zeta}_2 \tag{2}$$

and their complex conjugate $\underset{\sim}{\zeta}^*$ can be expressed as

$$\delta f = \left(\frac{\partial f}{\partial \underset{\sim}{\zeta}}\right)^T \delta\underset{\sim}{\zeta} + \left(\frac{\partial f}{\partial \underset{\sim}{\zeta}^*}\right)^T \delta\underset{\sim}{\zeta}^*, \tag{3}$$

where δ denotes first order change, T denotes transposition and $\partial f/\partial\underset{\sim}{\zeta}$ and $\partial f/\partial\underset{\sim}{\zeta}^*$ are column vectors representing the formal partial derivatives of f w.r.t. $\underset{\sim}{\zeta}$ and $\underset{\sim}{\zeta}^*$, respectively.

The formal partial derivatives are mathematically defined as

$$\frac{\partial f}{\partial \underset{\sim}{\zeta}} \triangleq \left(\frac{\partial f}{\partial \underset{\sim}{\zeta}_1} - j\frac{\partial f}{\partial \underset{\sim}{\zeta}_2}\right)/2 \tag{4}$$

and

$$\frac{\partial f}{\partial \underset{\sim}{\zeta}^*} \triangleq \left(\frac{\partial f}{\partial \underset{\sim}{\zeta}_1} + j\frac{\partial f}{\partial \underset{\sim}{\zeta}_2}\right)/2. \tag{5}$$

However, we can utilize the possibility of obtaining them formally using the ordinary differentiation rules [5].

It can be shown that, for a real function f, we can write.

$$\frac{\partial f}{\partial \underset{\sim}{\zeta}^*} = \left(\frac{\partial f}{\partial \underset{\sim}{\zeta}}\right)^*. \tag{6}$$

COMPLEX FORMULATION OF POWER FLOW EQUATIONS

The power flow equations of an electrical power network are represented by a set of complex equality constraints in the form

$$S_M^* - E_M^* Y_T V_M = 0, \tag{7}$$

where S_M is a vector of the bus powers,

$$S_M = P_M + j\, Q_M, \tag{8}$$

V_M is a vector of bus voltages, Y_T is the bus admittance matrix of dimension $n \times n$, n denoting number of buses in the power network and E_M is a diagonal matrix of components of V_M in corresponding order.

The variables in (7) are classified, in practice, as complex state and control variables. The complex state variables include the load-type bus voltages arranged in the vector

$$\zeta_x^L \triangleq V_L = V_{L1} + j\, V_{L2}, \tag{9}$$

the generator bus state variables defined as

$$\zeta_x^G \triangleq Q_G + j\, \delta_G, \tag{10}$$

where δ_G and Q_G are vectors of phase angles of generator bus voltages and generator bus reactive powers, respectively, and the slack bus power

$$\zeta_x^n \triangleq S_n = P_n + j\, Q_n. \tag{11}$$

Also, the complex control variables include the load bus powers

$$\zeta_u^L \triangleq S_L = P_L + j\, Q_L, \tag{12}$$

the generator bus control variables defined as

$$\zeta_u^G \triangleq P_G + j\, |V_G|, \tag{13}$$

where $|V_G|$ is a vector of magnitudes of generator bus voltages and the slack bus control variable

$$\zeta_u^n \triangleq \delta_n + j\, |V_n|, \tag{14}$$

where $|V_n|$ and δ_n denote, respectively, the magnitude and phase angle of the slack bus voltage V_n. We may also consider the line admittances arranged in the vector ζ_u^t which are contained in the bus admittance matrix Y_T.

SENSITIVITY CALCULATIONS

When solving (7) in the conventional load flow problem by the Newton-Raphson method [7] using cartesian coordinates, the state variables ζ_x^G of (10) and ζ_x^n of (11) are preferably considered in formulating the equations, as

$$\tilde{\zeta}_x^G \triangleq V_G = V_{G1} + j\, V_{G2} \tag{15}$$

and

$$\tilde{\zeta}_x^n \triangleq V_n = V_{n1} + j\, V_{n2}. \tag{16}$$

Hence, we may write the power flow equations in the form

$$h(\zeta_x, \zeta_x^*, \zeta_u^t, \zeta_u^{t*}) = \zeta_u^{m*}, \tag{17}$$

where

$$\zeta_x = \zeta_{x1} + j\, \zeta_{x2} \tag{18}$$

is a vector of the state variables ζ_x^L, $\tilde{\zeta}_x^G$ and $\tilde{\zeta}_x^n$ of (9), (15) and (16), respectively, and

$$\zeta_u^m = \zeta_{u1}^m + j\, \zeta_{u2}^m \tag{19}$$

is a vector of the control variables ζ_u^L, ζ_u^G and ζ_u^n of (12), (13) and (14), respectively.

We write (17) in the perturbed form

$$K\, \delta\zeta_x + \bar{K}\, \delta\zeta_x^* = \delta\zeta_u^{m*} - H_{\zeta u}^t\, \delta\zeta_u^t - \bar{H}_{\zeta u}^t\, \delta\zeta_u^{t*}, \tag{20}$$

where K, $\bar{K}$, $H_{\zeta u}^t$ and $\bar{H}_{\zeta u}^t$ denote, respectively, the formal derivatives $(\partial h^T/\partial \zeta_x)^T$, $(\partial h^T/\partial \zeta_x^*)^T$, $(\partial h^T/\partial \zeta_u^t)^T$ and $(\partial h^T/\partial \zeta_u^{t*})^T$.

We remark that the elements of the complex matrices K and $\bar{K}$ constitute the well-known Jacobian matrix of the load flow problem in the rectangular form.

We now write (20) in the consistent form

$$\begin{bmatrix} K & \bar{K} \\ \bar{K}^* & K^* \end{bmatrix} \begin{bmatrix} \delta\zeta_x \\ \delta\zeta_x^* \end{bmatrix} = \begin{bmatrix} \delta\zeta_u^{m*} \\ \delta\zeta_u^m \end{bmatrix} - \begin{bmatrix} H_{\zeta u}^t & \bar{H}_{\zeta u}^t \\ \bar{H}_{\zeta u}^{t*} & H_{\zeta u}^{t*} \end{bmatrix} \begin{bmatrix} \delta\zeta_u^t \\ \delta\zeta_u^{t*} \end{bmatrix}. \tag{21}$$

For a real function f of ζ_x, ζ_x^*, ζ_u^m, ζ_u^{m*}, ζ_u^t and ζ_u^{t*}, we may, using (6), write

$$\delta f = [f_{\zeta x}^T \;\; f_{\zeta x}^{*T}] \begin{bmatrix} \delta\zeta_x \\ \delta\zeta_x^* \end{bmatrix} + [f_{\zeta u}^{mT} \;\; (f_{\zeta u}^{m*})^T] \begin{bmatrix} \delta\zeta_u^m \\ \delta\zeta_u^{m*} \end{bmatrix} + [f_{\zeta u}^{tT} \;\; (f_{\zeta u}^{t*})^T] \begin{bmatrix} \delta\zeta_u^t \\ \delta\zeta_u^{t*} \end{bmatrix}, \tag{22}$$

where $f_{\zeta x}$, $f_{\zeta u}^m$ and $f_{\zeta u}^t$ denote, respectively, the formal partial derivatives of f w.r.t. ζ_x, ζ_u^m and ζ_u^t.

Using (21) and (22), it can be shown that

$$\delta f = [\hat{V}^T \; \hat{V}^{*T}] \left\{ \begin{bmatrix} \delta\zeta_u^{m*} \\ \\ \delta\zeta_u^{m} \end{bmatrix} - \begin{bmatrix} H^t_{\zeta u} & \bar{H}^t_{\zeta u} \\ \\ \bar{H}^{t*}_{\zeta u} & H^{t*}_{\zeta u} \end{bmatrix} \begin{bmatrix} \delta\zeta_u^{t} \\ \\ \delta\zeta_u^{t*} \end{bmatrix} \right\}$$

$$+ [f^{mT}_{\zeta u} \; (f^{m*}_{\zeta u})^T] \begin{bmatrix} \delta\zeta_u^{m} \\ \\ \delta\zeta_u^{m*} \end{bmatrix} + [f^{tT}_{\zeta u} \; (f^{t*}_{\zeta u})^T] \begin{bmatrix} \delta\zeta_u^{t} \\ \\ \delta\zeta_u^{t*} \end{bmatrix} , \qquad (23)$$

where $\hat{V}$ are complex adjoint variables obtained from solving the adjoint equations

$$[K^T \quad \bar{K}^{*T}] \begin{bmatrix} \hat{V} \\ \hat{V}^* \end{bmatrix} = f_{\zeta x}. \qquad (24)$$

The required total formal derivatives of f w.r.t. ζ_u^m and ζ_u^t are obtained directly from (23) as follows

$$\frac{df}{d\zeta_u^m} = \hat{V}^* + f^m_{\zeta u} \qquad (25)$$

and

$$\frac{df}{d\zeta_u^{tT}} = f^{tT}_{\zeta u} - \hat{V}^T H^t_{\zeta u} - \hat{V}^{*T} \bar{H}^{t*}_{\zeta u} . \qquad (26)$$

In practice, gradients w.r.t. real and imaginary parts of the defined control variables are of direct interest. These gradients are simply obtained from

$$\frac{df}{d\zeta_1} = 2 \; \mathrm{Re}\{ \frac{df}{d\zeta} \} \qquad (27)$$

and

$$\frac{df}{d\zeta_2} = - 2 \; \mathrm{Im}\{ \frac{df}{d\zeta} \}, \qquad (28)$$

where ζ can be ζ_u^m or ζ_u^t.

For a given real function f, the adjoint equations (24) are formulated using the elements of the Jacobian of the load flow problem at the solution point. The solution of (24) is then substituted into (25)-(28) to obtain the required total derivatives of f w.r.t. control variables.

It should be noticed that when the polar formulation of power flow equations is used, the vectors $\delta\zeta_x$ and $\delta\zeta_x^*$ of (20) are defined accordingly. In this case, the complex matrices K and $\bar{K}$ constitute the Jacobian matrix of the load flow problem in the polar form and the approach described thereafter is directly applicable.

CONCLUSIONS

A compact complex notation has been utilized to describe a novel approach for compact derivation of sensitivity expressions required in power system studies. The approach has been described using only complex matrix manipulations. It employs a sparse set of linear adjoint equations formed by exploiting the Jacobian matrix of the original load flow problem. The approach has been illustrated using the cartesian coordinates. The approach is directly applicable to the polar form using the corresponding perturbed form of power flow equations.

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