We have the largest bandwidth when dealing with non unity aspect ratios and low dielectric constant. In contrast, to obtain good wave guiding ability and low radiation from discontinuities a higher dielectric constant is required. If only one of these conditions is critical we can adjust the parameters of the guide to obtain a bandwidth at Cfl = 2.5.

We have used for the upper limit the first higher order mode cutoff frequency. In practice it seems that it is possible to maintain a single mode operation slightly above this frequency since the first higher order mode has extensive external fields at its cutoff frequency and is unlikely to be propagated in any circuit until a frequency is sufficiently high enough to reduce the evanescent fields down to a manageable size.

The purpose of this paper has been to give some guidance about the many parameters involved in the bandwidth of image guide in order to help in the design of broad-band circuits.

REFERENCES

Tolerance Analysis of Cascaded Structures

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Abstract—This paper presents an analysis scheme to obtain the response of a cascaded network and its first-order sensitivities with respect to design variables at the vertices of the tolerance region in an efficient and systematic way. This information is needed in worst-case search algorithms to identify the worst vertex or in a general tolerance assignment. A substantial saving in computational effort is achieved by using the new approach over the basic approach of reanalyzing the circuit at every vertex.

I. INTRODUCTION

A recent approach for the analysis of cascaded networks (using the chain matrix) has been used efficiently to perform response analysis as well as simultaneous and arbitrary large-change sensitivity evaluation [1]. This paper presents an analysis and sensitivity evaluation scheme using the recent approach in a form suitable for tolerance analysis of such networks.

In tolerance assignment each parameter $\phi$ has a tolerance associated with it so that it can have a value lying between $\phi^0 + \epsilon$ and $\phi^0 - \epsilon$, where $\phi^0$ is the nominal value and $\epsilon$ is the tolerance. The number of vertices of the tolerance region is $2^k$, where $k$ is the number of the tolereanced parameters, which includes all different combinations of parameter values.

The tolerances on elements as well as the nominal parameter values are optimized, consequently the response and its first-order sensitivity at the vertices of the tolerance region are needed by the optimization algorithms. This information is particularly useful if a worst-case search algorithm [3] has to identify the worst vertex [4], [5].

A specific algorithm designed for evaluating the response and its sensitivities at the vertices of a tolerance region is presented. An example is given along with a comparison between the new approach and the conventional way (the reanalysis) for evaluating the response and its sensitivities at the vertices.

II. NECESSARY BACKGROUND

The analysis approach is based on two types of analyses. The first is the forward analysis which consists of initializing a $\bar{u}^T$ row vector as either $e_1^T, e_2^T$, where

$$e_1 = [1 \ 0] \quad e_2 = [0 \ 1]$$

or a suitable linear combination and successively premultiplying each constant chain matrix by the resulting row vector until an element of interest or a termination is reached. The second is the reverse analysis, which is similar to conventional analysis of cascaded networks, proceeds by initializing a $v \times 1$ column vector as either $e_1$, $e_2$ or a suitable linear combination and successively postmultiplying each constant matrix by the resulting column vector, again until either an element of interest or a termination is reached.

Consider the network in Fig. 1. Applying forward and reverse analyses up to $A$, we obtain the expression

$$Q \overset{\triangle}{=} \bar{u}^T A v$$

(1)

where

$A$ transmission or chain matrix of the element of interest,

$\bar{u}$ vector obtained from forward analysis (initiated at the source and ending at the input port of $A$),

$v$ vector obtained from reverse analysis (initiated at the load and ending at the output port of $A$).
With minimal additional effort we can obtain the partial derivative

\[ Q' = \vec{u}^T \frac{\partial A}{\partial \phi} \frac{\partial e}{\partial \phi} \]  

and the change

\[ \Delta Q = \vec{u}^T \Delta A \frac{\partial e}{\partial \phi} \]  

where the parameter \( \phi \) is contained in \( A \). Note that \( Q \) relates output and input variables, namely, voltage and/or currents depending on how \( \vec{u}^T \) and \( e \) are initialized (see Table I).

Usually more than one element in the cascade is considered. The network is divided into subnetworks by reference planes which are chosen so that no more than one element is to be explicitly considered between any pair of reference planes (see Fig. 2(a) where the superscripts denote the subnetworks). Forward and reverse analyses are initiated at these reference planes.

Equivalent Thevenin sources of the subnetworks can be determined by performing a sequence of forward analyses (equivalent Norton sources can be determined by reverse analyses) [1]. The Thevenin voltage for the \( i \)th subnetwork (see Fig. 2) is given by [1]

\[ V'_i = \frac{V_i}{(\vec{u}_1 + Z_i \vec{u}_2)^T A' v_1} = \frac{V_i}{Q_{1i} + Z_{2i} Q_{2i}} \]  

where

\[ V_i \]  
source voltage for subnetwork \( i \),
\[ Z_i \]  
source impedance of subnetwork \( i \),
\[ V'_i \]  
open circuit voltage at the load end of subnetwork \( i \),
\[ \vec{u}^T \]  
equivalent Thevenin voltage for subnetwork \( i \) which is initialized as \( e \).

The terms \( \Delta Q_i \) and \( \Delta Q_{2i} \) are defined as

\[ \Delta Q_i = \vec{u}_1^T A' v_2 \]
\[ \Delta Q_{2i} = \vec{u}_2^T A' v_1 \]

A special case of (4) applicable to Fig. 1 (\( Z_S = Y_L = I_L = 0 \), where \( Y_L \) is the load admittance and \( I_L \) is the current source at the load end) is

\[ V_L = \frac{V_S}{\vec{u}_1^T A' v_1} = \frac{V_S}{Q_{11}} \]  

III. THE EVALUATION OF \( V_L \) AND ITS SENSITIVITIES WRT DESIGN PARAMETERS AT ALL VERTICES OF THE TOLERANCE REGION

Assume that we have partitioned the network by reference planes into subnetworks such that each subnetwork contains one chain matrix containing a variable parameter. Each reference plane is chosen to fall immediately after a variable element. The Thevenin voltage/impedance of the \( i \)th subnetwork is considered as the source voltage/impedance of the \((i+1)\)th subnetwork, given by (4) and (5), respectively, where \( j = i + 1 \).

We have to note here that the terms \( Q_{i1} \) and \( Q_{i2} \) are as defined previously with \( v_1 \) and \( v_2 \) set to \( e_1 \) and \( e_2 \), respectively, since the appropriate reference plane immediately follows the element \( A' \). The number of pairs of terms \( V'_j + Z'_j \) and \( Z'_{j+1} \) to be evaluated is 2, since each subnetwork contains one variable element with two extreme values (assuming that each \( A' \) contains only one variable parameter).

Differentiating (4) wrt \( \phi_h \), where \( \phi_h \) does not belong to \( A' \), but \( V'_i \) and \( Z'_i \) are functions of \( \phi_h \) (i.e., \( \phi_h \) is in a subnetwork \( h \) before the \( i \)th subnetwork) we get

\[ \frac{\partial V'_{j+1}}{\partial \phi_h} = \frac{(Q_{i1} + Z_{2i} Q_{2i}) \frac{\partial v'_1}{\partial \phi_h} - V'_i Z_{2i} Q_{2i}}{(Q_{1i} + Z_{2i} Q_{2i})^2} \]  

and differentiating (5) wrt \( \phi_h \), we get

\[ \frac{\partial Z'_{j+1}}{\partial \phi_h} = \frac{\partial Z'_i}{\partial \phi_h} \frac{(Q_{i1} Q_{2i} - Q_{i2} Q_{2i})}{(Q_{1i} + Z_{2i} Q_{2i})^2} \]  

On the other hand, the derivatives wrt \( \phi_h \) which is contained in \( A' \) (\( Z'_i \) and \( V'_i \) are not functions of \( \phi_h \)) are

\[ \frac{\partial V'_{j+1}}{\partial \phi_h} = \frac{3 Q_{1i} Z_{2i} Q_{2i}}{(Q_{1i} + Z_{2i} Q_{2i})^2} \]  

\[ \frac{\partial Z'_{j+1}}{\partial \phi_h} = \frac{3 Z'_i Q_{1i} Q_{2i} - Z'_i Q_{i2} Q_{1i}}{(Q_{1i} + Z_{2i} Q_{2i})^2} \]
Fig. 3. Illustration of the principal stages of the algorithm. Here we are considering a cascade of six elements, where the three variables $\phi_1$, $\phi_2$, and $\phi_3$ are in elements number 2, 4, and 5, respectively.

and

$$\frac{\partial Z_{i+1}}{\partial \phi_i} = \frac{(Q_{11}^i + Z_5^i Q_{15}^i) \left( \frac{\partial Q_{12}^i}{\partial \phi_i} + Z_5^i \frac{\partial Q_{13}^i}{\partial \phi_i} \right) - (Q_{12}^i + Z_3^i Q_{13}^i) \left( \frac{\partial Q_{11}^i}{\partial \phi_i} + Z_5^i \frac{\partial Q_{21}^i}{\partial \phi_i} \right)}{(Q_{11}^i + Z_5^i Q_{15}^i)^2}$$

where $\frac{\partial Q_{11}^i}{\partial \phi_i}$, $\frac{\partial Q_{12}^i}{\partial \phi_i}$, $\frac{\partial Q_{13}^i}{\partial \phi_i}$, and $\frac{\partial Q_{21}^i}{\partial \phi_i}$ correspond to (2) and Table I. This sensitivity information is carried out through the analysis for each subnetwork. The number of variables for which sensitivities of $V_{S,i+1}^i$ and $Z_{S,i+1}^i$ exist at the $(i+1)$th subnetwork is $i$ so that $2^i$ sensitivity calculations are performed. The expression relating $V_{S,i}$ and the last sets of $V_S$ and $Z_S$, is given by (4), so that $2^k$ values for $V_S$ and its sensitivities can be obtained from appropriate values of $V_{S,i}$, $Z_{S,i}$, and $A$.

Fig. 3 shows an example of the stages involved in the following algorithm to obtain the responses and its sensitivities at the vertices (3 variables+8 vertices) of the tolerance region.

Algorithm

Step 1: Initialize $u_1$, $u_2$ and $v_i$. (See Table I).
Set $i\leftarrow 1$, $m\leftarrow 1$, $j\leftarrow n$.
Comment: $n$ is the total number of elements in the cascade, and $m$ is a counter for the variable elements. In the example of Fig. 3 $n=6$.

Step 2: If $i=i_m$ go to Step 6.
Comment: $i_m$ is an element of $L$, an index set containing superscripts of the $k$ matrices containing the $k$ variable parameters and ordered consecutively. It is assumed that each matrix contains only one variable. In the example of Fig. 3 $k=3$ and $L=\{2,4,5\}$.

Step 3:

- $u_{1T}^T \rightarrow u_{2T}^T A^T$.
- $u_{2T}^T \rightarrow u_{3T}^T A^T$.
Set $i\leftarrow i+1$.

Step 4: If $i=i_m$ go to Step 5.
Go to Step 3.
Comment: Step 2 and Step 4 check whether or not we have reached a chain matrix containing a variable parameter. If we reached a variable we calculate the Thevenin voltages and impedances and their sensitivities, otherwise we proceed with the forward analysis performed in Step 3.

Step 5: If $m=k$ go to Step 7.
Comment: This step checks if we have reached the last chain matrix containing a variable or not.

Step 6: Calculate $V_S$, $Z_S$

$$\frac{\partial V_S}{\partial \phi_1}, \ldots, \frac{\partial V_S}{\partial \phi_m}, \frac{\partial Z_S}{\partial \phi_1}, \ldots, \frac{\partial Z_S}{\partial \phi_m}$$

$2^k$ sets all together.

IV. EXAMPLE

The cascaded seven-section bandpass filter shown in Fig. 4[6] was considered. All sections are quarter-wave at 2.175 GHz. The optimal minimax characteristic impedances [7] are taken as nominal values. They are

- $Z_1 = Z_7 = 0.606595$
- $Z_2 = Z_6 = 0.303547$
- $Z_3 = Z_5 = 0.722287$
- $Z_4 = 0.235183.$

A tolerance of $\pm 0.03$ on $Z_1$, $Z_4$, and $Z_5$ was chosen. The algorithm was used to evaluate $V_L$, $\partial V_L/\partial Z_1$, $\partial V_L/\partial Z_4$, and $\partial V_L/\partial Z_5$ at the eight vertices of the tolerance region ($2^k$ vertices where $k$ is the number of tolerated variables). The results are
TABLE II

<table>
<thead>
<tr>
<th>Vertex</th>
<th>$v_L$</th>
<th>$\frac{\delta v_L}{\delta z_1}$</th>
<th>$\frac{\delta v_L}{\delta z_2}$</th>
<th>$\frac{\delta v_L}{\delta z_5}$</th>
<th>Sign of Tolerance Extreme</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.4935+0.02351</td>
<td>-0.02450+0.05953</td>
<td>0.25004+0.11934</td>
<td>0.02549+0.32944</td>
<td>- - -</td>
</tr>
<tr>
<td>2</td>
<td>0.48819+0.02571</td>
<td>-0.02776+0.01588</td>
<td>0.28336+0.05326</td>
<td>0.00954+0.34878</td>
<td>+ - -</td>
</tr>
<tr>
<td>3</td>
<td>0.49673+0.04662</td>
<td>0.03751+0.15916</td>
<td>-0.06831+0.94430</td>
<td>0.04573+0.29165</td>
<td>- + -</td>
</tr>
<tr>
<td>4</td>
<td>0.49671+0.04046</td>
<td>-0.03388+0.11197</td>
<td>-0.00426+0.87724</td>
<td>0.03578+0.31848</td>
<td>+ + -</td>
</tr>
<tr>
<td>5</td>
<td>0.49204+0.04391</td>
<td>-0.04367+0.08072</td>
<td>0.29407+0.11930</td>
<td>-0.00103+0.33324</td>
<td>- + +</td>
</tr>
<tr>
<td>6</td>
<td>0.48786+0.04670</td>
<td>-0.09378+0.03123</td>
<td>0.32067+0.07952</td>
<td>-0.02042+0.35007</td>
<td>+ - -</td>
</tr>
<tr>
<td>7</td>
<td>0.49889+0.03101</td>
<td>0.02600+0.18868</td>
<td>-0.05742+0.07346</td>
<td>0.02462+0.29849</td>
<td>+ + +</td>
</tr>
<tr>
<td>8</td>
<td>0.49818+0.02127</td>
<td>-0.04526+0.13735</td>
<td>0.01130+0.09019</td>
<td>0.01130+0.32057</td>
<td>+ + +</td>
</tr>
</tbody>
</table>

The response $v_L$ and its sensitivities at the vertices of the tolerance region at normalized frequency 0.7.

V. CONCLUSIONS

The algorithm for evaluating the response and its sensitivities at the vertices of the tolerance region proved to be very efficient. The seven-section filter example was run with tolerances on the characteristic impedances of the stubs and transmission lines (all seven). It took 0.269 s CPU time, on a CDC 6400 computer, to evaluate the response (only) at the $128(2^7)$ vertices. Using the conventional method of reanalyzing the circuit for different component values would take $0.269 \times 128 = 9.472$ s CPU, where one analysis is performed in approximately 0.074 s. For the case of evaluating the response and its sensitivities at vertices discussed in Section IV, it took 0.118 s CPU time compared with $8 \times 0.074 = 0.592$ s for eight analyses. The savings in computational effort is substantial.

REFERENCES