Proc. ISCAS, Chicago, IL, April 1981

A LINEAR PROGRAMMING APPROACH TO FAULT LOCATION IN ANALOG CIRCUITS

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ABSTRACT

This paper deals with fault isolation of linear analog circuits taking the design tolerances on the nonfaulty elements into consideration. A two-step algorithm is proposed. First, an approximate method is used where a linear programming technique for isolating the elements most likely to be faulty is presented. Second, a verifying method for the faulty set is presented where we construct the algebraic invariants associated with the faulty set. Both methods can also be used individually as independent methods for fault isolation.

INTRODUCTION

This paper addresses itself to the problem of multi-fault location and identification in analog circuits. A fault, in this context, means that the change in the element value has exceeded its assigned tolerance and caused malfunction of the circuit under test.

Under the condition of a limited number of measurements, different methods have been used for locating and identifying the faulty set. Some of these methods are based on the use of an estimation criterion [1,2], and others are based on constructing analytical constraints which are invariant on the changes in the faulty set [3,4].

In this paper we present a two-step algorithm for fault isolation with a limited number of voltage measurements performed at a single frequency and taking the tolerances on the nonfaulty elements into consideration.

First, we consider an approximate method for fault isolation. The change in any measurement from its nominal value is given by a linear combination of special error parameters. A system of linear equations is constructed and a linear programming formulation is used to detect the most likely minimum number of faulty elements.

Second, we have extended an earlier formulation [3] for fault verification. We have considered the more practical case when the nonfaulty parameters are allowed to vary inside the tolerance region. Normally, the verification method is used to verify the results of the approximate method, but it can be used as an independent method for fault isolation as well. In both methods the evaluation of the faulty element values is carried out in a direct way after identifying the faulty set.

Finally, some examples illustrating the application of this two-step algorithm are included.

APPROXIMATE FAULT ISOLATION

In this section we consider an approximate method for fault isolation. The number of measurements is assumed to be less than the total number of elements in the faulty network.

Mainly we are looking for a solution which satisfies the given measurements and has the smallest number of faulty elements [2]. We define an error parameter for every network element. This is related to the deviation in the network element value from nominal. The change in any measurement from its nominal value is a linear function in these error parameters. The problem can be considered as solving the resulting underdetermined system of linear equations under the condition that the solution will have the minimum number of error parameters different from zero.

Normally, all elements in the faulty network deviate from nominal. A few are faulty and others are within their tolerance intervals. The change in any component value in linear active networks can be represented by a current or a voltage source as shown in Fig. 1. We apply a unit current source to the input port of the faulty network and m independent voltage measurements are performed simultaneously. The change in these measurements from their nominal values is given by

$$\Delta \underset{\sim}{V}^m = [\underset{\sim}{H}_{mx} \; \underset{\sim}{H}_{my}] \begin{bmatrix} \underset{\sim}{I}^x \\ \underset{\sim}{V}^y \end{bmatrix} = [\underset{\sim}{H}_{mx} \; \underset{\sim}{H}_{my}] \, \underset{\sim}{e} = \underset{\sim}{H}_m \, \underset{\sim}{e}, \qquad (1)$$

where $\underset{\sim}{I}^x$ and $\underset{\sim}{V}^y$ provide the equivalent current source vector and voltage source vector due to changes in the circuit elements from their nominal values. The vector $\underset{\sim}{e} \triangleq [e_1 \; e_2 \; \dots \; e_n]^T$ defines the error vector and n is the total number of network elements.

Equation (1) represents the underdetermined system of linear equations which should be

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This work was supported by the Natural Sciences and Engineering Research Council of Canada under Grant A7239.

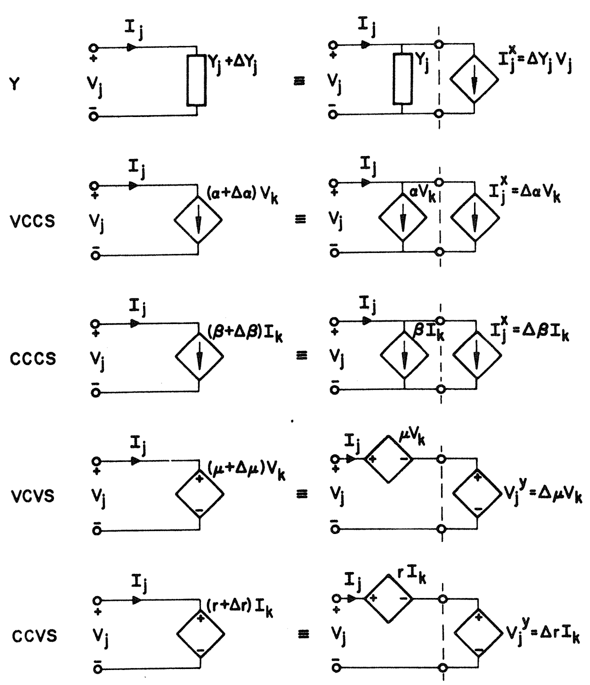

Fig.1 An equivalent representation of changes in element values.

satisfied by any solution. A linear programming formulation is constructed. The objective function is defined as the sum of the absolute values of the components of $\underset{\sim}{e}$. This least one objective function tends to satisfy the equality constraints, namely (1), with the minimum number of error parameters different from zero. This is consistent with the assumption that a few elements are faulty.

The optimization problem can be formally stated as follows.

$$\text{Minimize} \sum_{i=1}^{n} |e_i| \tag{2a}$$

subject to

$$\Delta \underset{\sim}{V}^m = \underset{\sim}{H}_m \underset{\sim}{e}. \tag{2b}$$

In general, the error parameters are complex and formulation (2) should be modified by redefining our error parameters. Let $r_i \overset{\Delta}{=} \mathrm{Re}(e_i)$, $g_i \overset{\Delta}{=} \mathrm{Im}(e_i)$, $i = 1, \ldots, n$. The optimization problem can then be restated as follows.

$$\text{Minimize} \sum_{i=1}^{n} (|r_i| + |g_i|) \tag{3a}$$

subject to

$$\mathrm{Re}[\Delta \underset{\sim}{V}^m] = \mathrm{Re}[\underset{\sim}{H}_m]\, \underset{\sim}{r} - \mathrm{Im}[\underset{\sim}{H}_m]\, \underset{\sim}{g}, \tag{3b}$$

$$\mathrm{Im}[\Delta \underset{\sim}{V}^m] = \mathrm{Im}[\underset{\sim}{H}_m]\, \underset{\sim}{r} + \mathrm{Re}[\underset{\sim}{H}_m]\, \underset{\sim}{g}. \tag{3c}$$

The optimization problems (2), (3) can be easily converted to the regular linear programming form by an appropriate transformation of the variables. The adjoint network has been used, see [5], in a very similar way to that proposed by Biernacki et al. [3] to evaluate the elements of the matrix $\underset{\sim}{H}_m$.

From the output of the linear program we obtain the vector $\underset{\sim}{e}$ which represents the deviations in the element values. We simulate the network with all components held at nominal values and the nonzero values of $\underset{\sim}{e}$ are connected across their corresponding elements. This simulation will provide the voltage and current for each network component. From Fig. 1, the output of the linear program and the output of the network simulation after testing, the change in every element parameter is calculated and checked against its assigned tolerance value. If the change exceeds the allowed tolerance we declare the element faulty, otherwise we consider it nonfaulty.

The presented approximate method can be used by itself as a separate way for isolating the faulty elements, or it can indicate which components should be tested first using the method of fault verification which is described in the next section.

Weighting factors [2] can be introduced in the objective function to reflect the previous experience, the reliability of the circuit elements and the sensitivities of the measurements employed relative to the circuit elements.

FAULT VERIFICATION IN THE PRESENCE OF TOLERANCES

Biernacki and Bandler [3] proposed a method for isolating k faulty elements based on k + 1 measurements. Here, we extend their formulation by including the effects of tolerances on the nonfaulty parameters.

We choose k elements in the circuit and consider them as candidates for the faulty set. The change in each element can be represented by either a current or a voltage source. We extract these sources from the network. The ports with voltage sources are called voltage ports and labelled by the superscript x and the ports with current sources are called current ports and labelled by the superscript y. Next we extract ℓ ports for measurement, where ℓ is greater than or equal to k+1, and these ports are labelled by the superscript m. Finally, we extract the excitation port and we label it by the superscript g. The excitation port should not coincide with any of the assumed ports of fault, but it can be any one of the measurement ports. The ports of measurement need not be different from the ports of fault, but we assume so for the sake of generality. This will lead to the $(\ell+k+1)$-port network shown in Fig. 2.

The port voltages and currents are related by the hybrid matrix which is assumed to exist. This is usually the case when the $(\ell+k+1)$-port network satisfies the conditions for a hybrid matrix representation [6]

$$\begin{bmatrix} \underset{\sim}{V}^m \\ V^g \\ \underset{\sim}{V}^x \\ \underset{\sim}{I}^y \end{bmatrix} = \begin{bmatrix} \underset{\sim}{H}_{mm} & \underset{\sim}{H}_{mg} & \underset{\sim}{H}_{mx} & \underset{\sim}{H}_{my} \\ \underset{\sim}{H}_{gm} & H_{gg} & \underset{\sim}{H}_{gx} & \underset{\sim}{H}_{gy} \\ \underset{\sim}{H}_{xm} & \underset{\sim}{H}_{xg} & \underset{\sim}{H}_{xx} & \underset{\sim}{H}_{xy} \\ \underset{\sim}{H}_{ym} & \underset{\sim}{H}_{yg} & \underset{\sim}{H}_{yx} & \underset{\sim}{H}_{yy} \end{bmatrix} \begin{bmatrix} \underset{\sim}{I}^m \\ I^g \\ \underset{\sim}{I}^x \\ \underset{\sim}{V}^y \end{bmatrix}, \tag{4}$$

where $\underset{\sim}{V}^m$, $\underset{\sim}{I}^m$ represent the ℓ measurement port variables, V^g, I^g represent the input port

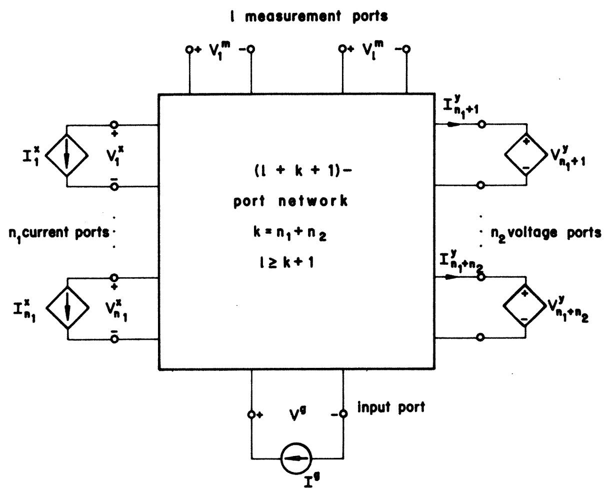

Fig. 2 $\ell + k + 1$ - port network.

variables, $\tilde{V}^x$, $\tilde{I}^x$ represent the n_1 current port variables, $\tilde{V}^y$, $\tilde{I}^y$ represent the n_2 voltage port variables, and $k = n_1 + n_2$.

Taking $\tilde{I}^m = \tilde{0}$ in (4), the measured voltages are given by

$$\tilde{V}^m = [\tilde{H}_{mg}\ \tilde{H}_{mx}\ \tilde{H}_{my}] \begin{bmatrix} I^g \\ \tilde{I}^x \\ \tilde{V}^y \end{bmatrix} = \tilde{H}_m \begin{bmatrix} I^g \\ \tilde{I}^x \\ \tilde{V}^y \end{bmatrix} . \quad (5)$$

From the available ℓ measurements we choose k+1 independent measurements and $\tilde{H}_m$ is reduced accordingly. Eliminating $\tilde{I}^x$ and $\tilde{V}^y$ variables from the resulting equations we get

$$I^g = \sum_{i=1}^{k+1} V_i^m \Delta_{i1}/\Delta = \sum_{i=1}^{k+1} V_i^m d_{i1}, \quad (6)$$

where Δ_{i1} is the cofactor of the element (i,1) of the reduced matrix and Δ is its determinant. Here we assume that $\Delta \neq 0$ which is a necessary condition for (6). Similar equations to (6) can be constructed but with a different set of k + 1 measurements. When the nonfaulty elements are restricted to their nominal values, the constructed system of equations should be satisfied in such a way that the k chosen elements could be considered as candidates for the exact faulty set.

Practically, the nonfaulty elements are not restricted to their nominal values, but are allowed to vary inside their tolerance region. Relative to the nominal point, the tolerance region is defined by

$$R_{\epsilon p} \triangleq \{\Delta \tilde{p} \mid -\epsilon_i \leq \Delta p_i \leq \epsilon_i,\ i \in I_p\}, \quad (7)$$

$$I_p \triangleq \{k + 1, \ldots, n\}, \quad (8)$$

where Δp_i is the change in the element value from nominal and ϵ_i is its associated tolerance. I_p defines the set of nonfaulty elements and n is the total number of the network elements.

Taking the tolerances on the nonfaulty elements into consideration, (6) becomes

$$I^g = \sum_{i=1}^{k+1} V_i^m (d_{i1} + \delta_{i1}), \quad (9)$$

where δ_{i1} is the change in the coefficient d_{i1} due to the variation of the nonfaulty elements from nominal values. The first-order approximation to δ_{i1} is given by

$$\delta_{i1} = \sum_{j \in I_p} \frac{\partial d_{i1}}{\partial p_j} \Delta p_j, \quad (10)$$

where the summation is over the set of the nonfaulty elements. Substituting (10) into (9) and rearranging we get

$$\sum_{j \in I_p} \left[\sum_{i \in I_m} \left(V_i^m \frac{\partial d_{i1}}{\partial p_j}\right)\right] \Delta p_j = \sum_{i \in I_m} (V_i^{m0} - V_i^m) d_{i1}, \quad (11)$$

where I_m identifies the set of k + 1 measurements used in constructing (6) and V_i^{m0} is the ith measurement value when the elements are at their nominal values. Let

$$C_j \triangleq \sum_{i \in I_m} \left(V_i^m \frac{\partial d_{i1}}{\partial p_j}\right), \quad E \triangleq \sum_{i \in I_m} (V_i^{m0} - V_i^m) d_{i1}. \quad (12)$$

Then (11) becomes

$$\sum_{j \in I_p} C_j \Delta p_j = E. \quad (13)$$

Equation (13) is linear in the changes of the nonfaulty parameters from their nominal values. To verify that the k elements are at fault, (13) should be satisfied by a vector $\Delta \tilde{p} \in R_{\epsilon p}$. Considering different combinations of k + 1 measurements we get a system of linear equations in the parameters Δp_j, j = k+1, ..., n and we may write (13) as

$$\sum_{j \in I_p} C_j^\lambda \Delta p_j = E^\lambda, \quad \lambda \in I_\Lambda \quad (14)$$

with

$$I_\Lambda \triangleq \{1, \ldots, \Lambda\}, \quad (15)$$

where Λ is the number of linear equations constructed from the available independent measurements.

In general, the coefficients C_j^λ, E^λ are complex and each equation can be separated into two equations with real coefficients

$$\sum_{j \in I_p} \mathrm{Re}(C_j^\lambda)\ \Delta p_j = \mathrm{Re}(E^\lambda), \quad \lambda \in I_\Lambda \quad (16a)$$

and

$$\sum_{j \in I_p} \mathrm{Im}(C_j^\lambda)\ \Delta p_j = \mathrm{Im}(E^\lambda), \quad \lambda \in I_\Lambda. \quad (16b)$$

Linear programming is used to find a feasible vector $\Delta \tilde{p}$ which satisfies (16) and (7) as the linear equality and inequality constraints, respectively. The objective function is chosen to enhance the physical meaning of the solution. The problem can be posed as follows.

$$\text{Minimize } F(\Delta\underset{\sim}{p}) \tag{17a}$$

subject to

$$\sum_{j\in I_p} \text{Re}(C_j^\lambda)\ \Delta p_j = \text{Re}(E^\lambda),\ \lambda \in I_\Lambda, \tag{17b}$$

$$\sum_{j\in I_p} \text{Im}(C_j^\lambda)\ \Delta p_j = \text{Im}(E^\lambda),\ \lambda \in I_\Lambda \tag{17c}$$

and

$$-\varepsilon_j \leq \Delta p_j \leq \varepsilon_j,\ j \in I_p, \tag{17d}$$

where $F(\Delta\underset{\sim}{p})$ is a linear function of the parameters $\Delta\underset{\sim}{p} \in R_{\varepsilon p}$.

Either the linear program is infeasible or a feasible solution which satisfies the chosen objective function is obtained. The sum of the absolute values of the parameters Δp_j, $j = k+1, \ldots, n$ has been taken as the objective function. This objective function will yield a feasible solution with the minimum number of nonfaulty elements different from nominal. Other objective functions can be chosen without affecting the feasibility of the solution.

A systematic procedure for calculating the coefficients of (14) is constructed [5]. For certain λ, C_j and E are functions of the coefficients d_{i1}, $i \in I_m$, and their sensitivities relative to the nonfaulty parameters. Using the adjoint network concept, (4) for the adjoint network is constructed and for $\hat{I}^g = 0$, $\hat{\underset{\sim}{I}}^x = \underset{\sim}{0}$ and $\hat{\underset{\sim}{V}}^y = \underset{\sim}{0}$ we get

$$\begin{bmatrix} \hat{V}^g \\ \hat{\underset{\sim}{V}}^x \\ -\hat{\underset{\sim}{I}}^y \end{bmatrix} = \begin{bmatrix} \underset{\sim}{H}_{mg}^T \\ \underset{\sim}{H}_{mx}^T \\ \underset{\sim}{H}_{my}^T \end{bmatrix} \hat{\underset{\sim}{I}}^m, \tag{18}$$

where $\hat{\underset{\sim}{I}}^m$ defines the adjoint currents of the k+1 measurement ports corresponding to the considered λ. Solving for the current $\hat{I}_i^m$, where $i \in I_m$, we get

$$\hat{I}_i^m = \hat{d}_{1i}\hat{V}^g + \sum_{j=2}^{n_1+1} \hat{d}_{ji}(\hat{V}_{j-1}^x) + \sum_{j=n_1+2}^{k+1} \hat{d}_{ji}(-\hat{I}_{j-1}^y). \tag{19}$$

Applying $m = k+1$ independent excitations to the measurement ports of the adjoint network we get

$$\begin{bmatrix} \hat{I}_i^{m1} \\ \cdot \\ \cdot \\ \cdot \\ \hat{I}_i^{mm} \end{bmatrix} = \begin{bmatrix} \hat{V}^{g1} & [\hat{\underset{\sim}{V}}^{x1}]^T & [-\hat{\underset{\sim}{I}}^{y1}]^T \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \hat{V}^{gm} & [\hat{\underset{\sim}{V}}^{xm}]^T & [-\hat{\underset{\sim}{I}}^{ym}]^T \end{bmatrix} \begin{bmatrix} \hat{d}_{1i} \\ \cdot \\ \cdot \\ \cdot \\ \hat{d}_{mi} \end{bmatrix} = \underset{\sim}{R}\ \hat{\underset{\sim}{d}}_i, \tag{20}$$

where the superscripts 1 to m stand for the m independent excitations applied to the adjoint network. Since $d_{i1} = \hat{d}_{1i}$, d_{i1} can be computed through solving (20). The sensitivities of d_{i1} relative to the nonfaulty circuit elements are computed through the computation of the sensitivities of the elements of the matrix $\underset{\sim}{R}$.

The computation of the faulty parameter values follows the same procedure that was described earlier in the approximate method with the exception that the nonfaulty elements are perturbed from their nominal value by the obtained feasible vector $\Delta\underset{\sim}{p}$ [5].

<u>Remarks</u>

Upper and lower bounds can be used to check whether (13) is satisfied for linear resistive networks. The upper and lower bounds are given, respectively, by

$$U = \sum_{j\in I_p} |C_j|\ \varepsilon_j - E \tag{21}$$

and

$$L = -\sum_{j\in I_p} |C_j|\ \varepsilon_j - E. \tag{22}$$

If $U \geq 0$ and $L \leq 0$ we can find a feasible vector $\Delta\underset{\sim}{p} \in R_{\varepsilon p}$ which satisfies (13). To extend the upper and lower bounds concept to general linear active networks, we are faced with the fact that (13) is usually a system of linear equations. The proposed linear programming formulation is a good way of finding the required feasible solution in that case.

The effect of inaccuracies in measurements has been handled in a similar way to that of the effect of tolerances on the nonfaulty elements, and the method proved to be successful [5].

In applying this method as a part of our algorithm, we first start by checking the faulty set detected by the approximate method. If we have not verified the faulty set, we use the verification method as a separate method for isolating the faulty set, and we follow the same procedure as outlined in [3].

EXAMPLES

<u>Example 1</u>

Consider the simple network shown in Fig. 3 with nominal values of elements $G_i = 1$ and tolerances $\varepsilon_i = \pm 0.05$, $i = 1, 2, \ldots, 5$. Assume that the network is excited at the port 11' and voltage measurements are taken at the ports 11', 22' and 33'.

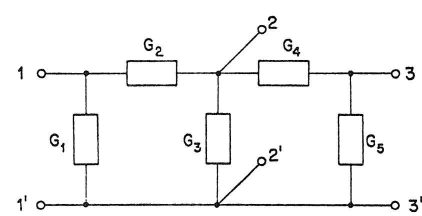

Fig. 3 A simple resistive network.

As an example we have taken the network elements to be $G_1 = 1.02$, $G_2 = 0.5$, $G_3 = 0.98$, $G_4 = 0.98$ and $G_5 = 0.95$.

<u>The approximate method</u> gave the following

values for the changes in the elements: $\Delta G_1 = 0.0$, $\Delta G_2 = -0.473$, $\Delta G_3 = 0.0331$, $\Delta G_4 = 0.03$ and $\Delta G_5 = 0.0$. The change in the second element exceeds its corresponding tolerance and we consider it faulty, which is a correct conclusion.

The verifying method was applied to check whether or not G_2 is faulty. The linear program gave a feasible solution. We have checked the single fault hypothesis for the other 4 elements and no feasible solution was found. This confirms that the second element as detected by the approximate method is really faulty. The computed faulty element value is $G_2 = 0.532$, which is very close to the actual value.

Example 2

Consider the active filter shown in Fig. 4 with the nominal element values given by $G_1 = G_2 = 1$, $C_1 = C_2 = 1$ and $K = 1$. Also, we assume that the amplifier has an output conductance $G_{out} = 1$. All elements are assumed to have design tolerances of ± 5 percent. The source resistance is assumed to be fault free, $R_g = 1$. We have considered ports 11', 22' and 33' as our ports of measurement.

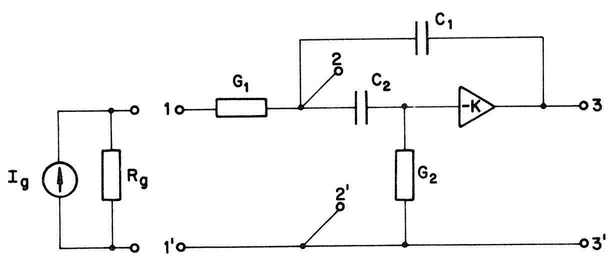

Fig. 4 RC active filter.

As an example we have taken $G_1 = 0.5$, $G_2 = 1.02$, $C_1 = 0.98$, $C_2 = 0.5$ and for the amplifier $K = 1.02$ and $G_{out} = 0.98$. The following values for the changes in the elements are given by the approximate method: $|\Delta G_1| = 0.5$, $|\Delta G_2| = 0.0185$, $|\Delta C_1| = 0.039$, $|\Delta C_2| = 0.487$, $|\Delta G_{out}| = 0.081$ and $|\Delta K| = 0.0$. From these results the changes in G_1 and C_2 well exceeded the design tolerances so we assume them faulty. Also, the change in G_{out} is slightly larger than the assigned tolerance so we are not quite sure whether or not it is faulty.

We have applied the verifying method to check whether or not G_1 and C_2 are faulty. We have considered our formulation for checking the double fault case. The linear program gives a feasible solution corresponding to the combination of G_1 and C_2. This proves that they are really at fault and we can exclude G_{out} from being faulty. We have checked the remaining 9 combinations and no feasible solution was detected. The calculated element values are $G_1 = 0.5$ and $C_2 = 0.488$, which are very close to the exact values.

CONCLUSIONS

In this paper we have presented a two-step algorithm for fault isolation taking into consideration the tolerances on the nonfaulty elements.

First, we introduced an approximate method for fault isolation. We used an estimation criterion for locating the faulty elements. This estimation criterion is based on physically realistic assumptions [2]. We formulated the problem using linear programming. The approximate method identifies the most likely faulty set according to our estimation criterion. It can be used as an independent method for fault isolation.

Second, we have extended an earlier formulation by Biernacki and Bandler [3]. We compensate for the uncertainty caused by the tolerances on the nonzero parameters by taking more measurements than the ones sufficient for the zero tolerance case. We then construct a system of linear equations which are invariant on the faulty elements. We check whether these equations are satisfied by searching for a feasible tolerance vector using the linear programming formulation.

By combining the approximate method and the subsequent verification method, we have, in general, reduced the computational effort needed substantially over that required by relying only on the verification approach.

ACKNOWLEDGEMENTS

The authors would like to express their great appreciation to Dr. M.R.M. Rizk of McMaster University for his help throughout the course of this work and to Dr. M.A. El-Kady for his helpful suggestions.

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