EXACT POWER NETWORK SENSITIVITIES VIA GENERALIZED COMPLEX BRANCH MODELLING

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ABSTRACT

This paper presents an application of the Tellegen's theorem approach to power network sensitivity calculations. Our theory employs an adjoint network concept based upon a novel, generalized complex branch modelling procedure allowing the exact steady-state component models of power networks to be considered without any approximation. Exact formulas for first-order change and reduced gradients are derived and tabulated.

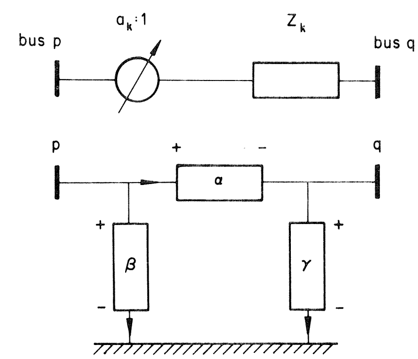
INTRODUCTION

A number of papers have applied the Lagrange multiplier approach [1,2] and the Tellegen's theorem [3] approach [4-7] to sensitivity calculations in electrical power networks.

Previous work based on Tellegen's theorem approximates the a.c. power model to permit direct application of the theorem. These approximations have been successively improved from the use of the d.c. load flow model [4] to the use of an improved a.c. approximate model [5]. In this paper, we employ a novel concept of generalized, perturbed complex branch modelling together with a pertinent adjoint technique of derivation to attain general sensitivity formulas based upon the exact a.c. power model without any approximation.

POWER NETWORK STEADY-STATE ELEMENT MODELS

We denote by n and n_B, respectively, the number of buses (nodes) and the number of branches in the network. We shall use $b = 1,2,\ldots, n_B$ to denote a branch index. In general, we denote by ζ_b a complex variable associated with branch b. ζ may represent voltage V, current I, power S, admittance Y, transformer tap ratio a, etc. The complex conjugate of ζ is written as ζ^* and δ will be used to denote the first-order change. Bus-type branches are denoted by $b = m = 1,2,\ldots,n$ and line-type branches by $b=t=n+1,\ldots,n_B$. Furthermore, $m = \ell = 1,2,\ldots,n_L$ identify load branches associated with P, Q-type buses for which the complex power $S_\ell = P_\ell + jQ_\ell$ is to be specified, $g = n_L + 1, \ldots, n_L + n_G$ identify generator branches associated with P, V-type buses for which the real (active) power P_g and the voltage magnitude $|V_g|$ are to be specified and $n = n_L + n_G + 1$ identifies the slack generator branch for which the bus voltage is to be specified. The line-type branches, on the other hand, may contain the ordinary passive elements of equivalent π-networks representing, for example, the transmission lines and transformers with real turns ratio as well as the elements of equivalent π-networks, derived [8] using the general branch modelling of Fig. 1, for the transformers with complex turns ratio (phase shifting transformers).



$$I_\alpha - a_k^* I_\alpha^* = V_\alpha/(Z_k a_k^*) - a_k^* V_\alpha^*/(Z_k^* a_k)$$

$$I_\beta - a_k^* I_\beta^* = (1-a_k)V_\beta/(Z_k a_k a_k^*) - (1-a_k^*)V_\beta^*/(Z_k^* a_k)$$

$$I_\gamma + a_k I_\gamma^* = (a_k-1)V_\gamma/(Z_k a_k) - a_k(1-a_k^*)V_\gamma^*/(Z_k^* a_k^*)$$

Fig. 1 Modelling of transformers with complex turns ratio.

In general, we deal with branch models as

$$h_b(I_b, I_b^*, V_b, V_b^*, U_b, U_b^*) = 0, \tag{1}$$

where U_b denotes an independent branch (control) variable. We write (1) in the perturbed form

$$h_{bi}\ \delta I_b + \bar{h}_{bi}\ \delta I_b^* = h_{bv}\ \delta V_b + \bar{h}_{bv}\ \delta V_b^* + W_b^S, \tag{2}$$

where the coefficients h_{bi}, $\bar{h}_{bi}$, h_{bv} and $\bar{h}_{bv}$ represent the formal [8] partial derivatives of h_b w.r.t. I_b, I_b^*, V_b and V_b^*, respectively. These formal derivatives may be evaluated using the ordinary differentiation rules. Moreover, it can be shown [8] that, for real h_b, we have $\bar{h}_{bi} = h_{bi}^*$ and

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$\bar{h}_{bv} = h^*_{bv}$. Observe that the perturbed branch models [9] are special cases of the general form (2). Note also that in the perturbed branch models of typical electronic circuits we may exclude the coefficients $\bar{h}_{bi}$ and $\bar{h}_{bv}$ of the conjugate variables (e.g., the constant voltage sources are modelled by $h_{bi} = \bar{h}_{bi} = \bar{h}_{bv} = 0$ and $h_{bv} = 1$).

THE AUGMENTED FORMS OF TELLEGEN'S THEOREM

Tellegen's theorem, based only upon Kirchhoff's laws and topology, states in perturbed form that

$$\sum_b \hat{I}_b \, \delta V_b = 0 \quad \text{and} \quad \sum_b \hat{V}_b \, \delta I_b = 0, \tag{3}$$

where the summation is taken over all branches, the ^ distinguishing the variables associated with the topologically similar adjoint network.

We may consider some [10] or all [11] of the exhaustive valid perturbed forms in terms of variables and their complex conjugate. Such terms are then added, subtracted or augmented via arbitrary complex coefficients [6] together or to other valid expressions [8] to formulate an augmented Tellegen sum. In terms of variations in the basic variables it is of the form

$$\sum_b \hat{\underset{\sim}{f}}{}_b^T \, \delta \underset{\sim}{w}_b = 0, \tag{4}$$

where

$$\underset{\sim}{w}_b = \begin{bmatrix} \underset{\sim}{w}_{bv} \\ \underset{\sim}{w}_{bi} \end{bmatrix}, \quad \underset{\sim}{w}_{bv} = \begin{bmatrix} V_b \\ V_b^* \end{bmatrix}, \quad \underset{\sim}{w}_{bi} = \begin{bmatrix} I_b \\ I_b^* \end{bmatrix}, \quad \hat{\underset{\sim}{f}}_b = \begin{bmatrix} \hat{\underset{\sim}{f}}_{bi} \\ \hat{\underset{\sim}{f}}_{bv} \end{bmatrix}, \tag{5}$$

T denotes transposition, $\hat{\underset{\sim}{f}}_b$ is a complex vector the elements of which are, in general, linear functions of the adjoint current and voltage variables and their complex conjugate, $\hat{\underset{\sim}{f}}_{bi}$ and $\hat{\underset{\sim}{f}}_{bv}$ being 2-component vectors.

STANDARD BRANCH JACOBIAN MATRICES

The element variables as distinct from the foregoing basic variables will be denoted by the vector $\underset{\sim}{z}_b$ of four components describing the practical state $\underset{\sim}{x}_b$ and control $\underset{\sim}{u}_b$ variables associated with branch b,

$$\underset{\sim}{z}_b = \begin{bmatrix} \underset{\sim}{x}_b \\ \underset{\sim}{u}_b \end{bmatrix}, \tag{6}$$

$\underset{\sim}{x}_b$ and $\underset{\sim}{u}_b$ being 2-component real and/or complex vectors. See, for example, Table I.

We relate the variations of the element variables $\underset{\sim}{z}_b$ to those of the basic variables $\underset{\sim}{w}_b$ by

$$\delta \underset{\sim}{z}_b = \underset{\sim}{J}_b \, \delta \underset{\sim}{w}_b, \tag{7}$$

where $\underset{\sim}{J}_b$ contains the conventional and/or formal derivatives of $\underset{\sim}{z}_b$ w.r.t. $\underset{\sim}{w}_b$. Of major interest is

$$(\underset{\sim}{J}_b^{-1})^T = \begin{bmatrix} \underset{\sim}{M}_{11}^b & \underset{\sim}{M}_{12}^b \\ \underset{\sim}{M}_{21}^b & \underset{\sim}{M}_{22}^b \end{bmatrix}, \tag{8}$$

whose submatrices are 2x2 Jacobian matrices which are standard for a branch type of a network. The branch Jacobian matrices for different branch types of power networks are shown in Table I.

TRANSFORMED ADJOINT VARIABLES AND NETWORK SENSITIVITIES

Let

$$\hat{\underset{\sim}{\eta}}_{bx} = \underset{\sim}{M}_{11}^b \hat{\underset{\sim}{f}}_{bi} + \underset{\sim}{M}_{12}^b \hat{\underset{\sim}{f}}_{bv} \tag{9}$$

$$\hat{\underset{\sim}{\eta}}_{bu} = \underset{\sim}{M}_{21}^b \hat{\underset{\sim}{f}}_{bi} + \underset{\sim}{M}_{22}^b \hat{\underset{\sim}{f}}_{bv} \tag{10}$$

be transformed adjoint variables associated with the bth branch, where $\hat{\underset{\sim}{\eta}}_{bx}$ and $\hat{\underset{\sim}{\eta}}_{bu}$ are 2-component vectors, the elements of which are linear functions of the adjoint current and voltage variables and their complex conjugate. Hence, using (5)-(8), the augmented Tellegen sum (4) is written in terms of variations in the element variables as

$$\sum_b (\hat{\underset{\sim}{\eta}}{}_{bx}^T \, \delta \underset{\sim}{x}_b + \hat{\underset{\sim}{\eta}}{}_{bu}^T \, \delta \underset{\sim}{u}_b) = 0. \tag{11}$$

Hence,

$$\delta f = \sum_b \left[\left(\frac{\partial f}{\partial \underset{\sim}{x}_b} \right)^T \delta \underset{\sim}{x}_b + \left(\frac{\partial f}{\partial \underset{\sim}{u}_b} \right)^T \delta \underset{\sim}{u}_b \right]. \tag{12}$$

Assuming a possible consistent modelling [12] of the adjoint system, we set

$$\hat{\underset{\sim}{\eta}}_{bx} = \frac{\partial f}{\partial \underset{\sim}{x}_b}, \tag{13}$$

hence, from (11) and (12)

$$\delta f = \sum_b \left[\left(\frac{\partial f}{\partial \underset{\sim}{u}_b} \right)^T - \hat{\underset{\sim}{\eta}}{}_{bu}^T \right] \delta \underset{\sim}{u}_b, \tag{14}$$

which expresses the first-order change of f (real or complex) solely in terms of variations in the control variables so that the total derivatives (the reduced gradients) of f are obtained as

$$\frac{df}{d\underset{\sim}{u}_b} = \frac{\partial f}{\partial \underset{\sim}{u}_b} - \hat{\underset{\sim}{\eta}}_{bu}. \tag{15}$$

CONSISTENT MODELLING AND SOLUTION OF ADJOINT SYSTEM

The adjoint network is defined, for a given function, by (13) which in general requires two complex relationships to be satisfied for each branch. Satisfying these two relationships simultaneously depends [12] on the form and the mode (i.e., real or complex) of the function f as well as on the form of the augmented Tellegen sum considered in the analysis. For a power network represented as in Table I, f is a general real function and the augmented Tellegen sum used is of the real form

$$\sum_b (\hat{I}_b V_b + \hat{I}_b^* V_b^* - \hat{V}_b I_b - \hat{V}_b^* I_b^*) = 0. \tag{16}$$

The application of Kirchhoff's laws results in

TABLE I
A REPRESENTATION OF A POWER SYSTEM VIA ELEMENT VARIABLES

Branch Type	Element Variables b	$\underset{\sim}{x}_b$	$\underset{\sim}{u}_b$	Branch Jacobian Matrices $\underset{\sim}{M}^b_{11}$	$\underset{\sim}{M}^b_{12}$	$\underset{\sim}{M}^b_{21}$	$\underset{\sim}{M}^b_{22}$
Load	ℓ	$\begin{bmatrix} \lvert V_\ell \rvert \\ \delta_\ell \end{bmatrix}$	$\begin{bmatrix} P_\ell \\ Q_\ell \end{bmatrix}$	$\begin{bmatrix} V_\ell/\lvert V_\ell \rvert & V^*_\ell/\lvert V_\ell \rvert \\ jV_\ell & -jV^*_\ell \end{bmatrix}$	$\begin{bmatrix} -I_\ell/\lvert V_\ell \rvert & -I^*_\ell/\lvert V_\ell \rvert \\ jI_\ell & -jI^*_\ell \end{bmatrix}$	$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$	$\begin{bmatrix} 1/V^*_\ell & 1/V_\ell \\ -j/V^*_\ell & j/V_\ell \end{bmatrix}$
Generator	g	$\begin{bmatrix} \delta_g \\ Q_g \end{bmatrix}$	$\begin{bmatrix} \lvert V_g \rvert \\ P_g \end{bmatrix}$	$\begin{bmatrix} jV_g & -jV^*_g \\ 0 & 0 \end{bmatrix}$	$\begin{bmatrix} jI_g & -jI^*_g \\ -j/V^*_g & j/V_g \end{bmatrix}$	$\begin{bmatrix} V_g/\lvert V_g \rvert & V^*_g/\lvert V_g \rvert \\ 0 & 0 \end{bmatrix}$	$\begin{bmatrix} -I_g/\lvert V_g \rvert & -I^*_g/\lvert V_g \rvert \\ 1/V^*_g & 1/V_g \end{bmatrix}$
Slack Generator	n	$\begin{bmatrix} P_n \\ Q_n \end{bmatrix}$	$\begin{bmatrix} \lvert V_n \rvert \\ \delta_n \end{bmatrix}$	$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$	$\begin{bmatrix} 1/V^*_n & 1/V_n \\ -j/V^*_n & j/V_n \end{bmatrix}$	$\begin{bmatrix} V_n/\lvert V_n \rvert & V^*_n/\lvert V_n \rvert \\ jV_n & -jV^*_n \end{bmatrix}$	$\begin{bmatrix} -I_n/\lvert V_n \rvert & -I^*_n/\lvert V_n \rvert \\ jI_n & -jI^*_n \end{bmatrix}$
Line	t	$\begin{bmatrix} I_{t1} \\ I_{t2} \end{bmatrix}$	$\begin{bmatrix} G_t \\ B_t \end{bmatrix}$	$\begin{bmatrix} 1/Y_t & 1/Y^*_t \\ j/Y_t & -j/Y^*_t \end{bmatrix}$	$\begin{bmatrix} 1 & 1 \\ j & -j \end{bmatrix}$	$\begin{bmatrix} -V_t/Y_t & -V^*_t/Y^*_t \\ -jV_t/Y_t & jV^*_t/Y^*_t \end{bmatrix}$	$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$S_m = P_m + jQ_m$ $\quad V_m = |V_m| \underline{/\ \delta_m}$; m can be ℓ, g or n $\quad I_t = I_{t1} + jI_{t2}$, $Y_t = G_t + jB_t$

a set of adjoint linear equations to be solved for the unknown adjoint current and voltage variables. A generalized form of the adjoint network equations which is common to all forms of augmented Tellegen sum and for general complex functions has been derived [8]. The solution of the adjoint system is then substituted into (14) and (15) to obtain the first-order change and the total derivatives of f.

AN IMPORTANT SPECIAL VERSION

For the augmented Tellegen sum (16)

$$\hat{\underset{\sim}{f}}_{bi} = [\hat{I}_b \ \hat{I}^*_b]^T \quad \text{and} \quad \hat{\underset{\sim}{f}}_{bv} = -[\hat{V}_b \ \hat{V}^*_b]^T \ . \tag{17}$$

Using the expressions of Table I, the adjoint branch modelling is obtained from (9) and (13). Observe that the adjoint branch models are also of the general form (1) in terms of the adjoint currents and voltages and their complex conjugate. The application of KCL and KVL to the adjoint network of the branch models obtained results in [7] the following real structure of the adjoint equations.

$$\begin{bmatrix} \hat{\underset{\sim}{G}} & \hat{\underset{\sim}{B}} \\ \hat{\underset{\sim}{B}}' & \hat{\underset{\sim}{G}}' \end{bmatrix} \begin{bmatrix} \hat{\underset{\sim}{V}}_1 \\ \hat{\underset{\sim}{V}}_2 \end{bmatrix} = \begin{bmatrix} \hat{\underset{\sim}{I}}_1 \\ \hat{\underset{\sim}{I}}_2 \end{bmatrix} , \tag{18}$$

where subscripts 1 and 2 denote, respectively, the real and imaginary parts of complex quantities, $\hat{\underset{\sim}{V}}$ is a vector of n-1 components representing the unknown adjoint load and generator bus voltages and $\hat{\underset{\sim}{I}}$ is a corresponding RHS vector,

$$\hat{\underset{\sim}{V}} = \begin{bmatrix} \hat{\underset{\sim}{V}}_L \\ \hat{\underset{\sim}{V}}_G \end{bmatrix} \quad \text{and} \quad \hat{\underset{\sim}{I}} = \begin{bmatrix} \hat{\underset{\sim}{I}}_L \\ \hat{\underset{\sim}{I}}_G \end{bmatrix} , \tag{19}$$

L and G denoting, respectively load and generator buses. The elements $\hat{I}_\ell$ and $\hat{I}_g$ of the vectors $\hat{\underset{\sim}{I}}_L$ and $\hat{\underset{\sim}{I}}_G$ are given by the formulas

$$\begin{aligned} \hat{I}_\ell \triangleq & - \frac{1}{2V_\ell} \left(|V_\ell| \frac{\partial f}{\partial |V_\ell|} - j \frac{\partial f}{\partial \delta_\ell} \right) \\ & + \frac{1}{2} y_{\ell n} V^*_n \left(\frac{\partial f}{\partial P_n} + j \frac{\partial f}{\partial Q_n} \right) \\ & - \frac{1}{2} \sum_t [\lambda_{\ell t} Y_t \left(\frac{\partial f}{\partial I_{t1}} - j \frac{\partial f}{\partial I_{t2}} \right)] \end{aligned} \tag{20a}$$

$$\begin{aligned} \hat{I}_g \triangleq & \frac{\partial f}{\partial \delta_g} - j |V_g|^2 \frac{\partial f}{\partial Q_g} \\ & - \mathrm{Im}\{V_g \sum_t [\lambda_{gt} Y_t \left(\frac{\partial f}{\partial I_{t1}} - j \frac{\partial f}{\partial I_{t2}} \right)]\} \\ & + \mathrm{Im}\{V_g y_{gn} V^*_n \left(\frac{\partial f}{\partial P_n} + j \frac{\partial f}{\partial Q_n} \right)\} , \end{aligned} \tag{20b}$$

where λ_{mt} denote elements of the bus incidence matrix of the network and y_{mm}, m = ℓ or g, are elements of the symmetric bus admittance matrix. The submatrices in (18) are given by

$$\hat{\underset{\sim}{G}} + j \hat{\underset{\sim}{B}} = \begin{bmatrix} (\underset{\sim}{Y}^*_{LL} + \underset{\sim}{\Psi}_L) & \underset{\sim}{Y}^*_{LG} \\ \bar{\underset{\sim}{Y}}^*_{GL} & (\bar{\underset{\sim}{Y}}^*_{GG} + \underset{\sim}{\Psi}_G) \end{bmatrix} , \tag{21a}$$

$$\hat{G}' + j\,\hat{B}' = \begin{bmatrix} (Y_{LL} - \Psi_L^*) & Y_{LG} \\ 0 & 2\,\mathrm{diag}\{V_g\} \end{bmatrix}, \qquad (21b)$$

where the bus admittance matrix Y, excluding the column and row corresponding to the slack bus, is

$$Y = G + j\,B = \begin{bmatrix} Y_{LL} & Y_{LG} \\ Y_{GL} & Y_{GG} \end{bmatrix}, \qquad (22)$$

$$[\bar{Y}_{GL}\ \bar{Y}_{GG}] \triangleq -j2\,\mathrm{diag}\{V_g\}[Y_{GL}\ Y_{GG}], \qquad (23)$$

$$\Psi_L \triangleq -\mathrm{diag}\{S_\ell/V_\ell^2\} \text{ and } \Psi_G \triangleq j2\,\mathrm{diag}\{S_g/V_g\}. \qquad (24)$$

DISCUSSION OF THE SPECIAL VERSION

In practice, the 2n-2 real adjoint equations (18) are to be solved for the adjoint bus voltages $\hat{V}$. Hence, the adjoint branch currents and voltages, which constitute the vectors of (17), are easily obtained. Using the standard expressions of Table I, the vector $\hat{\eta}_{bu}$ of (10) is evaluated and then substituted into (14) and (15). The adjoint matrix of coefficients of (18) is at least as sparse as the bus admittance matrix of the power network. It is simple, mostly constant, the majority of its elements are line conductances and susceptances representing basic data of the problem, available and already stored in the computer memory. Moreover, it is independent of the function f which is represented only on the RHS of the adjoint equations. Hence, several functions can be handled by repeat forward and backward substitutions using the LU factors of the adjoint matrix at a base-case point.

NUMERICAL RESULTS

We have numerical results for a 6-bus system [7]. Full details are available [9]. We also have results for a 26-bus system [13-15]. Details of data and results (not shown) are available [9]. Exact changes as calculated by new load flow solutions have been compared with those predicted by first-order estimates.

CONCLUSIONS

Instead of approximating the a.c. power flow model to cope with the conventional form and technique of analysis of Tellegen's theorem, we have employed a suitable augmented form of the theorem applicable to the generalized complex branch models of power networks. The proper adjoint network technique followed has led to simple derivation and elegant formulation of exact sensitivity formulas based on the a.c. power model without any approximation. Moreover, it offers the flexibility of working with any set of real and/or complex state and control variables of practical interest. The important special version described provides exact sensitivity formulas for general real functions while employing a simple and efficient adjoint analysis.

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