PRACTICAL COMPLEX SOLUTION OF POWER FLOW EQUATIONS

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ABSTRACT

This paper applies and illustrates the compact, complex notation introduced by Bandler and El-Kady to the practical solution of the power flow equations. The solution of the complex linearized power flow equations, which is required by the Newton-Raphson method, is obtained by a direct method. The method, fully and exactly, incorporates generator buses as well as dummy load buses.

INTRODUCTION

Bandler and El-Kady [1-4] demonstrated the application of a compact, complex notation to power system simulation [2-4] through solution techniques for the ubiquitous power flow equations [5] and sensitivities of system states w.r.t. control or design variables. They retained the compact, complex form of the perturbed (or linearized) load flow equations and have derived a suitable elimination technique, which deals directly with the linear complex equations expressed in terms of a set of complex variables and their complex conjugates [3]. Departing from the conventional approach to the Newton-Raphson method, which employs the real mode, they invoked the formal interpretation in terms of first-order changes of problem complex variables [2,4].

Here, we explain the steps of the complex elimination scheme in simple, matrix form which exposes the sparsity structure. We introduce generator-type buses into the tableau from the beginning and handle dummy loads, which may be present in the system, in an explicit manner. We elaborate on the elimination of blocks of the conjugate tableau which exploits its sparsity. Finally, we illustrate the results of a computer program [6] written to implement Newton's method in the complex mode.

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COMPLEX FORMULATION

The power flow equations [2-3] are

$$\underset{\sim}{E}_M^* \underset{\sim}{Y}_T \underset{\sim}{V}_M = \underset{\sim}{S}_M^*, \tag{1}$$

where $\underset{\sim}{Y}_T$ is the complex bus admittance matrix of the network, $\underset{\sim}{V}_M$ is a column vector of the complex bus voltages, $\underset{\sim}{E}_M \triangleq \text{diag}\ \underset{\sim}{V}_M$, $\underset{\sim}{S}_M \triangleq \underset{\sim}{P}_M + j\underset{\sim}{Q}_M$ and * denotes the complex conjugate. Also, it has been shown that (1) can be rearranged for the generator buses to explicitly express the known (specified) quantities on the RHS. We write (1) as

$$\underset{\sim}{S}^* = \underset{\sim}{f}(\underset{\sim}{V}_M, \underset{\sim}{V}_M^*), \tag{2}$$

or in the perturbed form

$$\underset{\sim}{K}_{SM}\delta\underset{\sim}{V}_M + \overline{\underset{\sim}{K}}_{SM}\delta\underset{\sim}{V}_M^* = \underset{\sim}{b}_M, \tag{3}$$

where the elements of the vector $\underset{\sim}{b}_M$ are $b_\ell = \delta S_\ell^*$, $\ell = 1, \ldots, n_L$ for load buses, $b_d = \delta S_d^*$, $d = n_{L+1}, \ldots, n_L + n_D$ for dummy buses ($S_d = 0 + j0$) and $b_g = \delta S_g = \delta P_g + j\delta|V_g|$, $g = n_L + n_D + 1, \ldots, n-1$ for generator buses, P_g and $|V_g|$ denoting, respectively, the real power and magnitude of bus voltage associated with bus g. The matrices $\underset{\sim}{K}_{SM}$ and $\overline{\underset{\sim}{K}}_{SM}$ constitute the formal partial derivatives [1,2] of $\underset{\sim}{S}^*$ of (2) w.r.t. $\underset{\sim}{V}_M$ and $\underset{\sim}{V}_M^*$, respectively.

Newton-Raphson: jth Iteration

In the jth iteration of the complex mode Newton-Raphson method we solve the system (3) for $\delta\underset{\sim}{V}_M^j$ given

$$\underset{\sim}{b}_M^j = \underset{\sim}{S}^*_{(\text{scheduled})} - \underset{\sim}{f}(\underset{\sim}{V}_M^j, \underset{\sim}{V}_M^{j*}) \tag{4}$$

using (2). We let

$$\underset{\sim}{V}_M^{j+1} = \underset{\sim}{V}_M^j + \delta\underset{\sim}{V}_M^j \tag{5}$$

and continue in this manner until an appropriate criterion for $\delta\underset{\sim}{V}_M^j$ and $\underset{\sim}{b}_M^j$ has been satisfied.

COMPLEX TABLEAU

While properly accounting for the equation for the slack bus in the power flow equations (1), and deleting it from the rest of the perturbed equations (3) we can write

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$$K x + \bar{K} x^{*} = b, \qquad (6)$$

where

$$x_i \triangleq \delta V_i, \quad i \in \{1, 2, \ldots, n-1\} \qquad (7)$$

and where an equation of (3) has the form

$$k_i^T x + \bar{k}_i^T x^{*} = b_i. \qquad (8)$$

This is equivalent to

$$\bar{k}_i^{*T} x + k_i^{*T} x^{*} = b_i^{*}. \qquad (9)$$

We define coefficients of x to comprise the basic tableau and coefficients of x^* to comprise the conjugate tableau. The complex tableau as it stands at this stage in our presentation can be set out as follows, where the conjugate coefficients associated with the load buses have been normalized.

Initial Complex Tableau

$\leftarrow n_L \rightarrow$	$\leftarrow n_D \rightarrow$	$\leftarrow n_G \rightarrow$	$\leftarrow n_L \rightarrow$	$\leftarrow n_D \rightarrow$	$\leftarrow n_G \rightarrow$		
$K_{LL}^{(0)}$	$K_{LD}^{(0)}$	$K_{LG}^{(0)}$	1	0	0	$b_L^{(0)}$	
$K_{DL}^{(0)}$	$K_{DD}^{(0)}$	$K_{DG}^{(0)}$	0	0	0	$b_D^{(0)}$	(10)
$K_{GL}^{(0)}$	$K_{GD}^{(0)}$	$K_{GG}^{(0)}$	$\bar{K}_{GL}^{(0)}$	$\bar{K}_{GD}^{(0)}$	$\bar{K}_{GG}^{(0)}$	$b_G^{(0)}$	

In the tableau, 1 denotes the square unit matrix of appropriate dimensions and 0 is the null matrix. We remark here that, in practice, n_D is expected to be quite small in comparison with n_L. Obviously, retaining the complex form of these linearized equations results in immediate savings in computer storage. To prepare the tableau for subsequent conjugate elimination we diagonalize the conjugate tableau in the following way, noting that the appropriate storage locations are used for intermediate computations.

Transformation of $\bar{K}_{DD}$

Diagonalizing $K_{DD}^{(0)}$ and transferring it to the conjugate tableau by invoking the consistent form (9) we have

$\leftarrow n_L \rightarrow$	$\leftarrow n_D \rightarrow$	$\leftarrow n_G \rightarrow$	$\leftarrow n_L \rightarrow$	$\leftarrow n_D \rightarrow$	$\leftarrow n_G \rightarrow$		
$K_{LL}^{(0)}$	$K_{LD}^{(0)}$	$K_{LG}^{(0)}$	1	0	0	$b_L^{(0)}$	
0	0	0	$K_{DL}^{*(1)}$	1	$K_{DG}^{*(1)}$	$b_D^{*(1)}$	(11)
$K_{GL}^{(0)}$	$K_{GD}^{(0)}$	$K_{GG}^{(0)}$	$\bar{K}_{GL}^{(0)}$	$\bar{K}_{GD}^{(0)}$	$\bar{K}_{GG}^{(0)}$	$b_G^{(0)}$	

where superscripts 0 identify no changes in the tableau from the initial form, while $K_{DL}^{*(1)}$, $K_{DG}^{*(1)}$ and $b_D^{*(1)}$ represent appropriate changes in the consistent tableau associated with dummy loads and are stored in the K_{DL}, K_{DG} and b_D locations.

Reduction of $\bar{K}_{GD}$ and $\bar{K}_{GL}$, Diagonalization of $\bar{K}_{GG}$

Note that in the forward reduction process for K_{GD}, rows $n_L+1, \ldots, n_L+n_D$ of the basic tableau are zero, therefore the basic tableau for rows $n_L + n_D + 1, \ldots, n-1$ is unchanged. Reducing $\bar{K}_{GD}$ and $\bar{K}_{GL}$ in tableau (11) and diagonalizing $\bar{K}_{GG}$ we obtain

$\leftarrow n_L \rightarrow$	$\leftarrow n_D \rightarrow$	$\leftarrow n_G \rightarrow$	$\leftarrow n_L \rightarrow$	$\leftarrow n_D \rightarrow$	$\leftarrow n_G \rightarrow$		
$K_{LL}^{(0)}$	$K_{LD}^{(0)}$	$K_{LG}^{(0)}$	1	0	0	$b_L^{(0)}$	
0	0	0	$K_{DL}^{*(1)}$	1	$K_{DG}^{*(1)}$	$b_D^{*(1)}$	(12)
$K_{GL}^{(2)}$	$K_{GD}^{(2)}$	$K_{GG}^{(2)}$	0	0	1	$b_G^{(3)}$	

Full details of the manipulation of the tableau of (11) to reach tableau (12) are available [7].

Reduction of $\bar{K}_{DL}$ and $\bar{K}_{DG}$

Using the unit matrices $\bar{K}_{LL}$ and $\bar{K}_{GG}$ we carry out a simultaneous reduction process which affects only rows $n_L+1, \ldots, n_L+n_D$. This gives

$\leftarrow n_L \rightarrow$	$\leftarrow n_D \rightarrow$	$\leftarrow n_G \rightarrow$	$\leftarrow n_L \rightarrow$	$\leftarrow n_D \rightarrow$	$\leftarrow n_G \rightarrow$		
$K_{LL}^{(0)}$	$K_{LD}^{(0)}$	$K_{LG}^{(0)}$	1	0	0	$b_L^{(0)}$	
$K_{DL}^{(2)}$	$K_{DD}^{(1)}$	$K_{DG}^{(2)}$	0	1	0	$b_D^{*(2)}$	(13)
$K_{GL}^{(2)}$	$K_{GD}^{(2)}$	$K_{GG}^{(2)}$	0	0	1	$b_G^{(3)}$	

where we see the second change to the storage locations for K_{DL} and K_{DG} and the first to K_{DD}.

Final Complex Tableau

The final complex tableau, in which the conjugate tableau has been diagonalized and exhibiting explicitly the changes by computation necessary to achieve this is given by (13).

It is important to note that no changes have been made in the tableau associated with the load buses, which are usually in the majority, while relatively few rows, namely those associated with dummy loads and generators have undergone changes and, in general, fill-ins in the sparseness.

We summarize the tableau at the present stage as

$$[\ \underset{\sim}{K}^{(0)}\ \vdots\ \underset{\sim}{1}\ \vdots\ \underset{\sim}{b}^{(0)}\ ]. \tag{14}$$

Conjugate reduction combined with forward Gaussian elimination is employed in the manner presented by Bandler and El-Kady for a power system consisting only of load buses [3]. The ith step of the process is illustrated as follows. We write the consistent form (9) for the present situation as

$$[\ \underset{\sim}{u}_i^T\ \vdots\ \underset{\sim}{k}_i^{*T(i-1)}\ \vdots\ \underset{\sim}{b}_i^{*(i-1)}], \tag{15}$$

where $\underset{\sim}{u}_i$ is a unit vector with ith element of unity. Now the remaining rows i, i+1, ..., n-1 are used from the current original tableau to first eliminate the conjugate part of this consistent form resulting in the ith row

$$[\ \underset{\sim}{k}_i^{T(i)}\ \vdots\ \underset{\sim}{0}\ \vdots\ \underset{\sim}{b}_i^{(i)}\ ]. \tag{16}$$

The ith row is now used in a Gaussian forward reduction on the ith column, the result of which is

$$\begin{bmatrix} \underset{\sim}{k}_1^{T(1)} & \vdots & \underset{\sim}{0} & \vdots & b_1^{(1)} \\ \underset{\sim}{k}_2^{T(2)} & \vdots & \underset{\sim}{0} & \vdots & b_2^{(2)} \\ \cdot & & \cdot & & \cdot \\ \cdot & & \cdot & & \cdot \\ \cdot & & \cdot & & \cdot \\ \underset{\sim}{k}_i^{T(i)} & \vdots & \underset{\sim}{0} & \vdots & b_i^{(i)} \\ \hline \underset{\sim}{k}_{i+1}^{T(i)} & \vdots & \underset{\sim}{u}_{i+1}^T & \vdots & b_{i+1}^{(i)} \\ \cdot & & \cdot & & \cdot \\ \cdot & & \cdot & & \cdot \\ \cdot & & \cdot & & \cdot \\ \underset{\sim}{k}_{n-1}^{T(i)} & \vdots & \underset{\sim}{u}_{n-1}^T & \vdots & b_{n-1}^{(i)} \end{bmatrix}, \tag{17}$$

where we have created elements

$$k_{rs}^{(i)} = 0, \quad r > s, \quad s < i. \tag{18}$$

Backward substitution gives the desired solution.

Succinctly, the equations may be written as

$$\underset{\sim}{K}\,\underset{\sim}{x} + \underset{\sim}{x}^* = \underset{\sim}{b} \tag{19}$$

with accompanying consistent form

$$\underset{\sim}{x} + \underset{\sim}{K}^*\underset{\sim}{x}^* = \underset{\sim}{b}^* \tag{20}$$

yielding

$$(\underset{\sim}{1} - \underset{\sim}{K}^*\underset{\sim}{K})\underset{\sim}{x} = \underset{\sim}{b}^* - \underset{\sim}{K}^*\underset{\sim}{b}, \tag{21}$$

which can alternatively be solved by standard techniques.

APPLICATION TO TEST POWER SYSTEMS

We consider here two power systems: 23-bus [7,8] and 26-bus [7,9,10] (see Fig. 1) to illustrate general computational aspects of the algorithm presented. The detailed data and solution of these systems are available [7,11]. The algorithm is programmed using rectangular coordinates. For determining the solution of the load flow problems of the systems, flat voltage profiles have been used as starting points. The computations have been performed on a CYBER 170 computer.

Tables I and II show the solutions of the load flow problems for the 23-bus and the 26-bus power systems, respectively. They are obtained in 5 iterations [7].

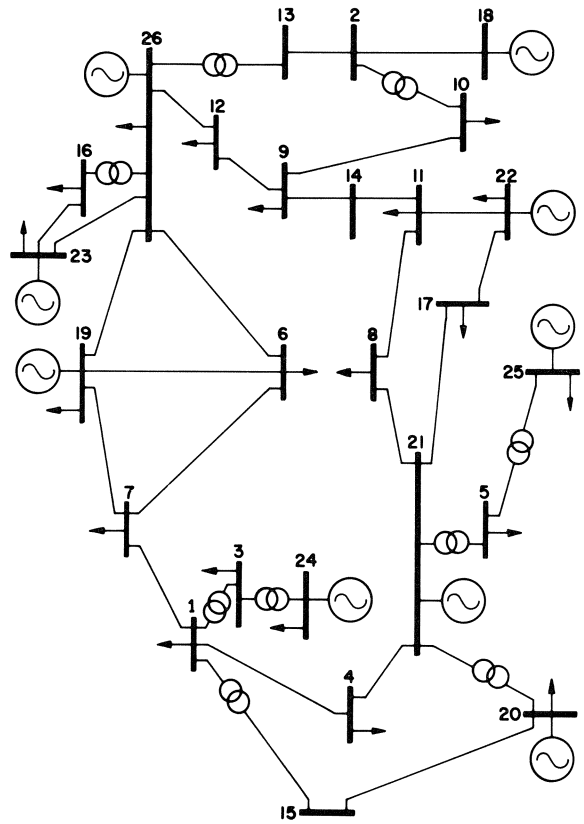

Fig. 1 26-bus power system.

CONCLUSIONS

The direct solution of the complex linearized power flow equations, required at the jth iteration of a complex Newton-Raphson method, has been described in this paper. By synthesis of complex variables consisting of the adjustable variables associated with voltage-controlled buses, we fully incorporate generators. The practical solution of large power systems has been emphasized. We have presented our algorithm in a tableau form which exposes the sparsity structure of the matrix of coefficients, and preserves the sparsity of the coefficients associated with the load buses until the conjugate tableau has been diagonalized. Subsequent elimination schemes to solve for the perturbed complex bus voltages are described. A computer program package called XLF1, which implements the work described here, is available [6].

TABLE I
LOAD FLOW SOLUTION OF THE 23-BUS POWER SYSTEM

Load Buses	
$V_1 = 1.0314 + j0.0179$	$V_{10} = 1.0123 + j0.2473$
$V_2 = 1.0059 + j0.0263$	$V_{11} = 0.9806 + j0.2486$
$V_3 = 1.0040 + j0.0864$	$V_{12} = 0.9430 + j0.3873$
$V_4 = 1.0015 + j0.0670$	$V_{13} = 0.9465 + j0.3528$
$V_5 = 0.9974 + j0.0546$	$V_{14} = 0.9529 + j0.3395$
$V_6 = 1.0061 + j0.1406$	$V_{15} = 0.9477 + j0.3418$
$V_7 = 0.9897 + j0.0807$	$V_{16} = 0.9408 + j0.4125$
$V_8 = 0.9931 + j0.0414$	$V_{17} = 0.9455 + j0.4007$
$V_9 = 1.0112 + j0.2137$	
Generator Buses	
$Q_{18} = 0.4204$	$V_{18} = 1.0282 + j0.0615$
$Q_{19} = 0.7228$	$V_{19} = 1.0475 + j0.0722$
$Q_{20} = 0.4510$	$V_{20} = 1.0233 + j0.2351$
$Q_{21} = 1.9016$	$V_{21} = 0.9301 + j0.4873$
$Q_{22} = 1.2589$	$V_{22} = 0.9340 + j0.4797$
Slack Bus	
$P_{23} = -0.6839$	$Q_{23} = 0.8913$

TABLE II
LOAD FLOW SOLUTION OF THE 26-BUS POWER SYSTEM

Load Buses	
$V_1 = 1.0328 + j0.0773$	$V_{10} = 1.0370 + j0.0692$
$V_2 = 1.0644 + j0.0943$	$V_{11} = 0.8982 - j0.0992$
$V_3 = 1.0424 + j0.0549$	$V_{12} = 0.9670 - j0.0741$
$V_4 = 0.9859 + j0.0979$	$V_{13} = 1.0463 + j0.0157$
$V_5 = 0.9741 + j0.2598$	$V_{14} = 0.9388 - j0.1071$
$V_6 = 1.0324 + j0.0554$	$V_{15} = 0.9273 + j0.0970$
$V_7 = 1.0132 + j0.0181$	$V_{16} = 1.0353 - j0.0471$
$V_8 = 0.9441 + j0.0403$	$V_{17} = 0.9318 + j0.0278$
$V_9 = 0.9614 - j0.1088$	
Generator Buses	
$Q_{18} = -0.4004$	$V_{18} = 1.0397 + j0.2528$
$Q_{19} = 0.1872$	$V_{19} = 1.0455 + j0.0966$
$Q_{20} = 0.7795$	$V_{20} = 0.9706 + j0.2408$
$Q_{21} = -0.0294$	$V_{21} = 0.9938 + j0.2295$
$Q_{22} = -0.1775$	$V_{22} = 0.8856 - j0.0885$
$Q_{23} = -0.1144$	$V_{23} = 0.9996 - j0.0265$
$Q_{24} = -0.1645$	$V_{24} = 0.9989 + j0.0458$
$Q_{25} = 0.1691$	$V_{25} = 0.9359 + j0.3522$
Slack Bus	
$P_{26} = 0.1334$	$Q_{26} = -0.0513$

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