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A HIERARCHICAL DECOMPOSITION APPROACH FOR NETWORK ANALYSIS

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ABSTRACT

A novel approach for analyzing large electrical networks is presented in which the network is decomposed into subnetworks in a hierarchical manner by removing some interconnections. These subnetworks are solved separately and then interconnected at a number of computing levels to obtain the solution of original network.

INTRODUCTION

The idea of decomposition or tearing was originated by Kron [1,2], in which a part of a given network is torn away so that the remaining subnetworks can be analyzed independently. The solutions of the separate subnetworks are then interconnected to take the part torn away into consideration and thus the solution of the original network is obtained at two levels [3]. Happ [4] has generalized the two-level computation into a multilevel computation process. However, the calculations at the levels except for the first can not be carried out in parallel and thus this method may not be suitable for analyzing large networks.

In this paper, a method is presented to solve a large network by decomposing it in a hierarchical manner. The network is decomposed into subnetworks and blocks by removing some interconnections and applying arbitrary current sources at the terminals created by removal of interconnections. As the decomposition imposes a hierarchical structure on the computations, the calculation at each level can be done in parallel.

NOTATION

N_k subnetwork of original network N_1.

N_k' subnetwork made up of equivalent multipoles of divisions of subnetwork N_k.

T_{ij} set of interconnection nodes common to subnetworks N_i and N_j.

$\underset{\sim}{Y}_k$ nodal admittance matrix of subnetwork N_k.

$\underset{\sim}{Z}_k$ $\underset{\sim}{Y}_k^{-1}$

$\underset{\sim}{V}_{k\ell}$ column vector of voltages on the external nodes of subnetwork N_k incident to subnetwork N_ℓ.

$\underset{\sim}{I}_{k\ell}$ column vector of arbitrary impressed currents applied at the external nodes of subnetwork N_k connected to $T_{k\ell}$.

$\underset{\sim}{V}_k^I$, $\underset{\sim}{I}_k^I$ column vectors of voltages and currents, respectively, on the internal nodes of subnetwrok N_k.

$\underset{\sim}{V}_k^O$, $\underset{\sim}{I}_k^O$ column vectors of voltages and currents, respectively, on external nodes of subnetwork N_k.

$\underset{\sim}{V}_k$, $\underset{\sim}{I}_k$ column vectors of voltages and currents, respectively, of subnetwork N_k.

$\underset{\sim}{V}_{km}^I$, $\underset{\sim}{I}_{km}^I$ column vectors of voltages and currents, respectively, on nodes external to subnetwork N_k' and internal to subnetwork N_m', where $N_k' \in Q^{L'}$ and $N_m' \in Q^{L-1'}$.

$\underset{\sim}{V}_{km}^O$, $\underset{\sim}{I}_{km}^O$ column vectors of voltages and currents, respectively, on nodes external to subnetworks N_k' and N_m', where $N_k' \in Q^{L'}$ and $N_m' \in Q^{L-1'}$.

$\underset{\sim}{E}_{k\ell} \equiv \underset{\sim}{E}^m$ $\underset{\sim}{V}_{\ell k} - \underset{\sim}{V}_{k\ell}$ if $T_{\ell k}$ exists and $N_k, N_\ell \subset N_m$.

Q^L set of subnetworks at decomposition level L.

$Q^{L'}$ set of subnetworks made up of multipoles at decomposition level L.

NETWORK DECOMPOSITION

Consider a large network N_1. Let us decompose N_1 into subnetworks $N_2, N_3, \ldots, N_i$ connected by a small number of interconnections. Each of them can be still too large for direct analysis so we decompose $N_2, \ldots, N_i$ into smaller subnetworks and continue this process until we reach sufficiently small subnetworks. The last ones, which are not further divided, we call blocks. This decomposition procedure gives us a hierarchical

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structure of subnetworks as illustrated in Fig. 1.

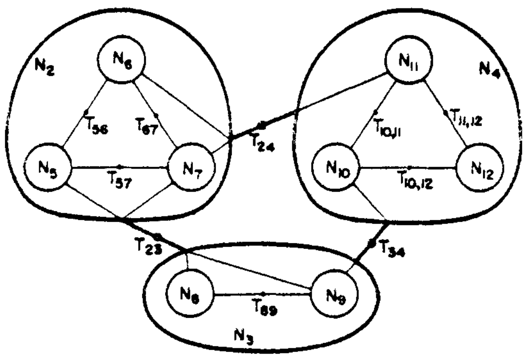

Fig. 1(a) Decomposed network with interconnections and blocks.

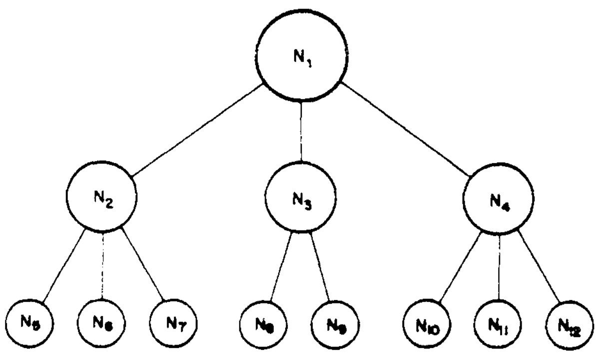

Fig. 1(b) Hierarchical structure of subnetworks obtained by decomposition. Level 1: $\{N_1\}$, Level 2: $\{N_2, N_3, N_4\}$, Level 3: $\{N_5, \ldots, N_{12}\}$

Subnetworks N_i and N_k are connected by T_{ik} interconnection nodes (see Fig. 1(a)). The network is decomposed, such that blocks are mutually uncoupled. For simplicity, it is assumed that each block contains a common ground node. When some blocks do not contain a common ground, the analysis can be performed in the same way after slight modification of these blocks [5].

ANALYSIS OF BLOCKS

Assume that the circuit N_1 is linear and every block is described by nodal equations. In order to decompose network N_1 into subnetworks, we apply the arbitrary current sources to all the interconnection nodes as shown in Fig. 2 and compute voltages on them. The network with added current sources is equivalent to the orignal one when voltages on these sources are zero. We obtain the conditions on node to datum voltages as

$$\underset{\sim}{V}_{jk} = \underset{\sim}{V}_{kj}, \quad \forall\, T_{jk}. \tag{1}$$

Every block is now separated from the rest of the network by the set of added current sources which can be treated as external excitations. We solve them separately and obtain

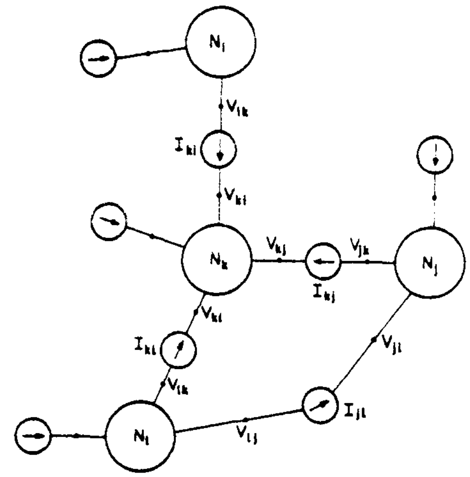

Fig. 2 Network blocks with added current sources.

$$\underset{\sim}{V}_k = [\underset{\sim}{Y}_k]^{-1}\, \underset{\sim}{I}_k \tag{2}$$

for all blocks. Equation (2) can be written as

$$\begin{bmatrix} \underset{\sim}{V}_k^I \\ \underset{\sim}{V}_k^O \end{bmatrix} = \begin{bmatrix} \underset{\sim}{Z}_k^{II} & \underset{\sim}{Z}_k^{IO} \\ \underset{\sim}{Z}_k^{OI} & \underset{\sim}{Z}_k^{OO} \end{bmatrix} \begin{bmatrix} \underset{\sim}{I}_k^I \\ \underset{\sim}{I}_k^O \end{bmatrix} . \tag{3}$$

If we assume current vector $\underset{\sim}{I}_k^O$ for all blocks of added sources arbitrarily and solve (3), then the outside voltages $\underset{\sim}{V}_k^O$ may not satisfy (1). Therefore, our aim is to correct $\underset{\sim}{I}_k^O$ by an amount $\Delta\underset{\sim}{I}_k^O$ to satisfy (1). From (3)

$$\begin{bmatrix} \underset{\sim}{V}_k^{I'} \\ \underset{\sim}{V}_k^{O'} \end{bmatrix} = \begin{bmatrix} \underset{\sim}{V}_k^I \\ \underset{\sim}{V}_k^O \end{bmatrix} + \begin{bmatrix} \underset{\sim}{Z}_k^{IO} \\ \underset{\sim}{Z}_k^{OO} \end{bmatrix} \Delta\underset{\sim}{I}_k^O, \tag{4}$$

where changes in outside voltages

$$\Delta\underset{\sim}{V}_k^O = \underset{\sim}{Z}_k^{OO}\, \Delta\underset{\sim}{I}_k^O \tag{5}$$

should satisfy the conditions

$$\Delta\underset{\sim}{V}_{ki} - \Delta\underset{\sim}{V}_{ik} = \underset{\sim}{V}_{ik} - \underset{\sim}{V}_{ki}, \quad k \neq i. \tag{6}$$

Let

$$\underset{\sim}{E}_{ki} = \underset{\sim}{V}_{ik} - \underset{\sim}{V}_{ki}, \quad k \neq i. \tag{7}$$

To find $\Delta\underset{\sim}{I}_k^O$ for all blocks we can put correction voltage sources (7) in place of added current sources $\underset{\sim}{I}_{ki}$ and calculate $\Delta\underset{\sim}{I}_k^O$ for blocks removing all other internal energy sources. The problem of network analysis has been reduced to determining $\Delta\underset{\sim}{I}_k^O$ flowing through interconnections. Blocks can now be represented by multipoles for which the matrix description is known (5).

ANALYSIS OF SUBNETWORKS

In this section we will discuss the hierarchical analysis of the network which is decomposed into subnetworks and blocks in a manner in which, at each level of decomposition, each subnetwork is decomposed into two smaller subnetworks only.

Now we will discuss the way of connecting two subnetworks described by equations of the form (5). Consider multipoles N_k' and N_ℓ', elements of $Q^{L'}$, being linked by E^m. The only difference between N_k, N_ℓ and N_k', N_ℓ' is that the latter do not contain independent sources inside them. Equations of $N_m \in Q^{L-1'}$ which consists of N_k' and N_ℓ' are obtained in the following way (see Fig. 3).

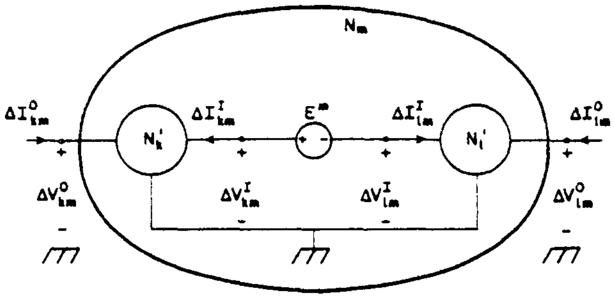

Fig. 3 Interconnection of two multipoles.

1. Present (5) for N_k' and N_ℓ' in the form

$$\begin{bmatrix} \Delta V^I_{tm} \\ \Delta V^O_{tm} \end{bmatrix} = \begin{bmatrix} Z^{II}_{tm} & Z^{IO}_{tm} \\ Z^{OI}_{tm} & Z^{OO}_{tm} \end{bmatrix} \begin{bmatrix} \Delta I^I_{tm} \\ \Delta I^O_{tm} \end{bmatrix}, \quad t = k, \ell \quad . \quad N_k, N_\ell \subset N_m \tag{8}$$

2. The nodal equations of N_m' can be written as

$$\begin{bmatrix} Z^{II}_{km} & Z^{IO}_{km} & 0 & 0 \\ Z^{OI}_{km} & Z^{OO}_{km} & 0 & 0 \\ 0 & 0 & Z^{II}_{\ell m} & Z^{IO}_{\ell m} \\ 0 & 0 & Z^{OI}_{\ell m} & Z^{OO}_{\ell m} \end{bmatrix} \begin{bmatrix} \Delta I^I_{km} \\ \Delta I^O_{km} \\ \Delta I^I_{\ell m} \\ \Delta I^O_{\ell m} \end{bmatrix} = \begin{bmatrix} \Delta V^I_{km} \\ \Delta V^O_{km} \\ \Delta V^I_{\ell m} \\ \Delta V^O_{\ell m} \end{bmatrix}, \tag{9}$$

where

$$\Delta I^I_{km} = -\Delta I^I_{\ell m}, \tag{10}$$

$$\Delta V^I_{km} - \Delta V^I_{\ell m} = E^m. \tag{11}$$

3. With the help of (10) and (11), (9) is reduced to

$$\begin{bmatrix} Z^{II}_{km} + Z^{II}_{\ell m} & Z^{IO}_{km} & -Z^{IO}_{\ell m} \\ Z^{OI}_{km} & Z^{OO}_{km} & 0 \\ -Z^{OI}_{\ell m} & 0 & Z^{OO}_{\ell m} \end{bmatrix} \begin{bmatrix} \Delta I^I_{km} \\ \Delta I^O_{km} \\ \Delta I^O_{\ell m} \end{bmatrix} = \begin{bmatrix} E^m \\ \Delta V^O_{km} \\ \Delta V^O_{\ell m} \end{bmatrix}. \tag{12}$$

4. Equation (12) can be written as

$$\begin{bmatrix} Z^{II'}_m & Z^{IO'}_m \\ Z^{OI'}_m & Z^{OO'}_m \end{bmatrix} \begin{bmatrix} \Delta I^I_m \\ \Delta I^O_m \end{bmatrix} = \begin{bmatrix} E^m \\ \Delta V^O_m \end{bmatrix}, \tag{13}$$

where

$$\Delta I^I_m = \Delta I^I_{km}, \quad \Delta I^O_m = \begin{bmatrix} \Delta I^O_{km} \\ \Delta I^O_{\ell m} \end{bmatrix}, \tag{14}$$

$$\Delta V^O_m = \begin{bmatrix} \Delta V^O_{km} \\ \Delta V^O_{\ell m} \end{bmatrix}. \tag{15}$$

From (13) we have

$$\Delta I^I_m = [Z^{II'}_m]^{-1} [E^m - Z^{IO'}_m \Delta I^O_m], \tag{16}$$

$$\Delta V^O_m = Z^{OI'}_m [Z^{II'}_m]^{-1} E^m + [Z^{OO'}_m - Z^{OI'}_m [Z^{II'}_m]^{-1} Z^{IO'}_m] \Delta I^O_m. \tag{17}$$

Now adjust V^O_m to

$$V^{O'}_m = V^O_m + Z^{OI'}_m [Z^{II'}_m]^{-1} E^m. \tag{18}$$

From (17) and (18) we have

$$\Delta V^{O'}_m = [Z^{OO'}_m - Z^{OI'}_m [Z^{II'}_m]^{-1} Z^{IO'}_m] \Delta I^O_m \tag{19}$$

or

$$\Delta V^{O'}_m = Z^{OO}_m \Delta I^O_m. \tag{20}$$

Again, to compute ΔI^O_m we replace subnetworks from level L by multipoles described by (20) and join them by correction voltage sources and put L = L-1. If L > 1, the form of the subnetwork is similar to N_m and equations (8)-(20) describing this subnetwork can be written and we can go for the next lower level. If L = 1, we obtain a subnetwork without outside current excitations and can determine ΔI^I_1 from (16), which is reduced to

$$\Delta I^I_1 = [Z^{II'}_1]^{-1} E^1, \tag{21}$$

where ΔI^I_1 describes the change in current excitations at the 2nd level. Using (5)-(16), we return to the highest level determining all the corrections in the arbitrary current sources and then the various node voltages of the original network are calculated from (4).

ALGORITHM FOR DECOMPOSED LINEAR NETWORK

Step 1 Assume I^O_k and solve the nodal equations for all blocks, i.e., (3).

Step 2 Set $L \leftarrow L-1$ and calculate E^m, $\forall N_m' \in Q^{L'}$ from (11).

Step 3 If L = 1, go to Step 7.

Step 4 Obtain $\underset{\sim}{Z}_m' \triangleq \begin{bmatrix} \underset{\sim}{Z}_m^{II'} & \underset{\sim}{Z}_m^{IO'} \\ \underset{\sim}{Z}_m^{OI'} & \underset{\sim}{Z}_m^{OO'} \end{bmatrix}$, $\forall N_m \in Q^{L'}$ with the help of (8) and (12).

Step 5 Adjust voltages from (18) as

$\underset{\sim}{V}_m^O \leftarrow \underset{\sim}{V}_m^O + \underset{\sim}{Z}_m^{OI'}[\underset{\sim}{Z}_m^{II'}]^{-1} \underset{\sim}{E}^m$, $\forall N_m' \in Q^{L'}$

Step 6 Calculate $\underset{\sim}{Z}_m^{OO} \triangleq [\underset{\sim}{Z}_m^{OO'} - \underset{\sim}{Z}_m^{OI'} [\underset{\sim}{Z}_m^{II'}]^{-1} \underset{\sim}{Z}_m^{IO'}]$, $\forall N_m' \in Q^{L'}$ and go to Step 2.

Step 7 Set $\underset{\sim}{Z}_1^{II} \leftarrow \underset{\sim}{Z}_2^{OO} + \underset{\sim}{Z}_3^{OO}$, calculate $\Delta\underset{\sim}{I}_1^I$ from (21). Using (14) and (10) we have $\Delta\underset{\sim}{I}_{21}^I = \Delta\underset{\sim}{I}_1^I$ and $\Delta\underset{\sim}{I}_{31}^I = -\Delta\underset{\sim}{I}_{21}^I$; using (5) and (8) we have $\Delta\underset{\sim}{I}_2^O = \Delta\underset{\sim}{I}_{21}^I$ and $\Delta\underset{\sim}{I}_3^O = \Delta\underset{\sim}{I}_{31}^I$.

Step 8 Set $L \leftarrow L+1$, calculate $\Delta\underset{\sim}{I}_m^I$ from (17), $\forall N_m' \in Q^{L'}$.

Step 9 Use (14) to determine $\Delta\underset{\sim}{I}_{km}^I$ and $\Delta\underset{\sim}{I}_{km}^O$, $\forall N_k' \in Q^{L+1'}$.

Step 10 Determine $\Delta\underset{\sim}{I}_k^O$ with the help of (5) and (8) $\forall N_k' \in Q^{L+1'}$.

Step 11 If L+1 is not the highest level then go to Step 8.

Step 12 Calculate $\underset{\sim}{I}_k^O \leftarrow \underset{\sim}{I}_k^O + \Delta\underset{\sim}{I}_k^O$ for all blocks and use (3) to find all nodal voltages.

An example of a linear network (Fig. 4) to illustrate the steps of the algorithm is described in detail in [5]. The final blocks are shown in Fig. 5.

CONCLUSIONS

A hierarchical decomposition approach for simulating a large network has been presented. First, analysis of blocks is performed and after this subnetworks are combined in a hierarchical manner joining two subnetworks at any time. Thus, combining the solution of the subnetworks can be performed in a series-parallel way. The analysis of very large networks is possible, therefore, in a short time. We have described the method for linear networks which can easily be extended to the case of nonlinear networks.

There is no efficient algorithm available for optimally decomposing large networks. In this approach, however, it is possible to use an efficient algorithm which gives suboptimal decompositon of large networks because only the number of external nodes of the subnetworks is important. Sparsity techniques at blocks or the subnetwork level can be used in implementing the algorithm.

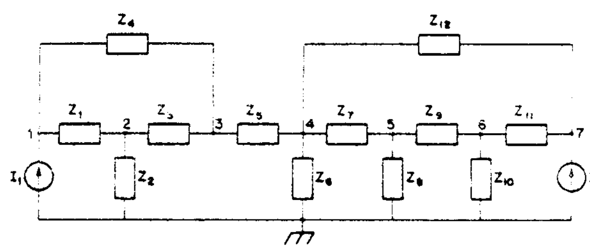

Fig. 4 Linear network example N_1.

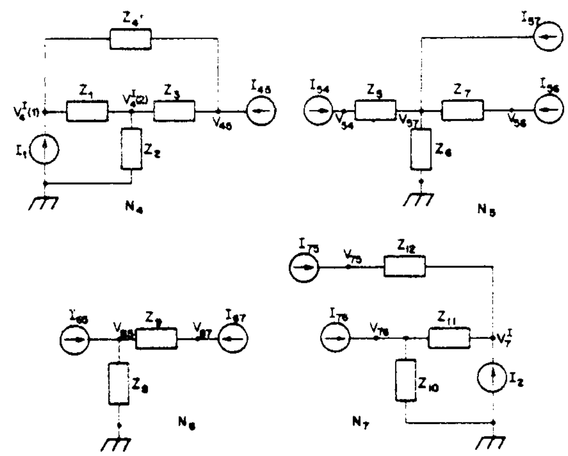

Fig. 5 Decomposed blocks $\{N_4, N_5\}$ of N_2 and $\{N_6, N_7\}$ of N_3.

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